Quark-Meson-Coupling Model for Finite Nuclei
Nuclear Matter and beyond

J. R. Stone
Tennessee/Oxford/RIKEN/Adelaide

jirina.stone@physics.ox.ac.uk
The Fundamental Interactions

**Strong force**
- Gluons
- Atomic nuclei
- Fission and fusion
- Quark structure of proton and neutron

**Electromagnetic force**
- Photon
- Atoms and molecules
- Chemical reactions
- Light waves
- Electronics

**Weak force**
- W and Z bosons
- Neutron decay
- Beta radioactivity
- Muon and tau decay
- Neutrino interactions

**Gravitation**
- Graviton?
- Solar system
- Galaxies
- Curved spacetime
- Black holes

Macroscopic: Sand castle
Microscopic: Sand stones

$10^{-7}$ cm: Crystal

$10^{-9}$ cm: Atom

$10^{-12}$ cm: Atomic nucleus

$<10^{-16}$ cm: Quark
Nuclear force is not fundamental!!

Two nucleons:
nucleon-nucleon scattering
tractable with many parameters
no unique model as yet

Many nucleons: force depends on density and momentum –
strong, electro-weak interactions play role as well as sub-nucleon structure
Hadrons are objects made of quarks.

Baryons are Nucleons, and strange Hyperons, $\Lambda$, $\Sigma$, $\Xi$.

Net baryonic density normalized to nuclear density $d_0 = 0.16$ fm$^{-3}$.
NUCLEON-NUCLEON INTERACTION IN NUCLEAR MEDIUM:

NUCLEAR MANY-BODY PROBLEM (NMBP)

Forces and potentials derived from free nucleon systems do not work directly in finite nuclei
No exact direct solution of the NMBP possible
– approximation have to be sought

Do we have enough constraints to uniquely determine parameters of existing theories?

Do we believe this is a relevant question?

Astrophysics
Neutron stars
Supernovae
Nuclear synthesis
Star evolution
Early Universe
Gravity waves

Nuclear matter:
Cold and hot:
Symmetric
Asymmetric
Pure neutron

Nuclei and particles
Nuclear structure
Nuclear reactions
Fusion and fission
Heavy ion collisions:
Range of energies
Outline of the talk:

Two approaches to the problem:

1 Construction of models with enough variable parameters to reproduce experiment with less regard to physical content of the model

   Examples: density dependent energy functional, realistic potentials, ab-initio models, one-boson-exchange relativistic models

2 Construction of models with limited number of parameters strongly constrained by theory and seek agreement with experiment by physical content

Example: Quark meson coupling model
Astrophysics and nuclear matter

Infinite dense matter:

System of an infinite number of interacting particles in an infinite volume with a finite ratio of a number of particles per unit volume.

No Coulomb force present – no surface effects –
- translational invariance

interior of neutron stars, core-collapse supernovae,
possibly large heavy nuclei

Testing theories under simplified conditions
Phases of nuclear matter:

**Symmetric** (equal number of protons and neutrons)
**Asymmetric** (unequal)
**Pure neutron**

Symmetric matter has at density $\rho_0 = 0.16 \text{ fm}^{-3}$:

- Bound state $(E/A) = -16 \text{ MeV}$
- Symmetry energy $E/A_{\text{sym}} - E/A_{\text{pn}} = 30 - 32 \text{ MeV}$
- Slope of the symmetry energy $L = 40 - 100 \text{ MeV}$
- Incompressibility:
  - traditional $K = 240+/-30 \text{ MeV}$
  - NEW VALUE $250 - 315 \text{ MeV}$

Stone at al., *PRC 89, 044316* (2014)
The Equation of State (EoS):

**Ideal gas:**

**Average pressure:**

\[ p = \frac{1}{3} \frac{N}{V} m v^2 \]  
N # of molecules of mass m in volume V

**Average molecular kinetic energy:**

\[ \left\langle \frac{1}{2} m v^2 \right\rangle = \frac{3}{2} kT \]  
k Boltzmann constant, T temperature

**Equation of State**

\[ p = \frac{N k T}{V} = \varepsilon(\rho, T) \]

\[ \varepsilon \text{ total energy density of gas with number density } \rho = \frac{N}{V} \]
Nuclear matter:

\[ P = \varepsilon(\rho, T) \quad \varepsilon(\rho, T) = \sum_f \left( \frac{E}{A}(\rho, T) \rho \right)_f \quad \mu_B = (P + \varepsilon) / \rho \]

Two key points:

I. The EoS is dependent on composition
   CONSTITUENTS + INTERACTIONS

II. E/A and ITS DENSITY DEPENDENCE
    must be determined by nuclear and/or particle models.
### Realistic bare nucleon

- 20-60 adjustable parameters
- Several thousands of data points:
  - Free nucleon-nucleon scattering and properties of deuteron

Used in nuclear matter calculations, shell model, ab-initio theories

### Phenomenological density dependence

- 10-15 adjustable parameters
- Several tens of data points:
  - Symmetric nuclear matter at saturation,
  - Ground state properties of finite nuclei

Used in mean-field models non-relativistic Hartree-Fock

- **NO DENSITY DEPENDENT FURTHER TREATMENT NEEDED TO USE IN NUCLEAR ENVIRONMENT**

- **DENSITY DEPENDENCE INCLUDED IN AN EMPIRICAL WAY THROUGH PARAMETERS**

Ab-initio techniques
approach, we use the formalism developed in Refs. The nucleon-nucleon in-medium correlations weighted by the BHF defect function over the third nucleon in the medium, the average being an effective, density-dependent, two-body force by averaging potential, where possible.

The model are constrained to be compatible with the two-nucleon and nucleon-antinucleon). The meson parameters in this intermediate excitation of nucleon resonances (Delta, Roper, binding energy at saturation has been shown to introduce errors well below 1 MeV for the and single-particle energies in the Bethe-Goldstone equation.

Historically, there is the observation that the position of corrections are smaller, at least for Brueckner-type approaches. at larger density, more than twice saturation density in the most recent CD-Bonn, N3LO, and IS, yield strong overbinding between the results with the Paris, V18, and Bonn C potentials curves) one obtains in general too strong binding, varying our set of complete details, the reader is referred to Refs.

The resulting effective two-nucleon potential has the operator $V_{ij}$

$$V_{ij} = \tau \sum_k (g_{ik} \cdot r_{kj} + g_{jk} \cdot r_{ki})$$

Concerning the inclusion of three-body forces in the BHF approach amounts to including the TBF corresponding to nucleon-antinucleon excitation by 2

For the use in BHF calculations, this TBF is reduced to

Figure 6 shows the saturation points of symmetric matter. This is illustrated for the case of the V18

We begin in Fig.

Li et al., PRC74, 047304 (2006)

“Realistic” models obtained from free nucleon scattering and renormalized to nuclear medium.

NOT CALIBRATED TO SYMMETRIC NUCLEAR MATTER PARAMETERS.
EoS of pure neutron matter as calculated in different models

- Effect of 3BF

- Maris et al., PRC 87, 054318 (2013)
- Akmal et al., PRC58, 1084 (1998)
- Gezerlis et al., PRL 111,032501 (2013)
Neutron stars:

Masses are measured well – 2 solar mass
Radii are difficult and uncertain
Moments of inertia not available as yet
Gravitational waves signals not fully constraining

Models yield only mass as function of radius
Composition of the star uncertain but crucial for determination of the EoS

EOS CRITICALLY DEPENDENT ON NUCLEAR/PARTICLE MODELS

Empirical approach – polytropic EoS: \( P(n_B) = K n_B^{\Gamma} \)

Combination of models and observation data

Assumptions: There is only one EoS of high density matter

Latest models of the EoS calibrated on data from the gravitational wave observation of the GW170817 NS merger (Zhao and Lattimer, Phys. Rev. D98, 063020 (2018))

Binary tidal deformability $\Lambda$. 
Finite nuclei
Quantum Monte Carlo
Coupled Cluster Method
No Core Shell Model
Green's Function Approaches
Chiral effective field Theory

Shell model
Locally adjusted

Density functional models
Global or locally adjusted
Empirical mass models

Figure adapted from SCIDAC report
Example:

Skyrme non-relativistic energy functional (total energy of the system includes density dependence)

**Designed to model:**

Ground state properties of finite nuclei: binding energies, radii, shapes, electromagnetic moments, low amplitude collective excitations (giant resonances), single-particle states in nuclear ground state

Derived properties: neutron skin, particle separation energies, particle driplines

**Properties of nuclear matter**
PHENOMENOLOGICAL DENSITY DEPENDENT POTENTIAL: THE SKYRME INTERACTION

\[ \mathcal{E}_{\text{Sk}} = \mathcal{E}_{\text{Sk, even}} + \mathcal{E}_{\text{Sk, odd}}, \quad (6a) \]

\[ \mathcal{E}_{\text{Sk, even}} = C_0^\rho \rho_0^2 + C_0^{\rho,\alpha} \rho_0^{2+\alpha} + C_0^{\Delta \rho} \rho_0 \Delta \rho_0 + C_0^{\nabla J} \rho_0 \nabla \cdot J_0 + C_0^\tau \rho_0 \tau_0 + C_0^J J_0^2 \]

\[ + C_0^\sigma \sigma_0^2 + C_0^{\sigma,\alpha} \sigma_0^2 \rho_0^\alpha + C_0^{\Delta \sigma} \sigma_0 \Delta \sigma_0 + C_0^{\nabla J} \sigma_0 \cdot \nabla \times J_0 - C_0^\tau J_0^2 - \frac{1}{2} C_0^J \sigma_0 \cdot \tau_0 \]

\[ + C_1^\rho \rho_1^2 + C_1^{\rho,\alpha} \rho_1^2 \rho_0^\alpha + C_1^{\Delta \rho} \rho_1 \Delta \rho_1 + C_1^{\nabla J} \rho_1 \nabla \cdot J_1 + C_1^\tau \rho_1 \tau_1 + C_1^J J_1^2 \]

\[ + C_1^\sigma \sigma_1^2 + C_1^{\sigma,\alpha} \sigma_1^2 \rho_1^\alpha + C_1^{\Delta \sigma} \sigma_1 \Delta \sigma_1 + C_1^{\nabla J} \sigma_1 \cdot \nabla \times J_1 - C_1^\tau J_1^2 - \frac{1}{2} C_1^J \sigma_1 \cdot \tau_1 \]

Parameters adjustable to experiment:
t0, t1, t2, t3, t4, t5, x0, x1, x2, x3, x4, x5, α, β, γ

\[ C_0^\rho = \frac{3}{8} t_0 + \frac{3}{48} t_3 \rho^\alpha, \]
\[ C_1^\rho = -\frac{1}{4} t_0 \left( \frac{1}{2} + x_0 \right) - \frac{1}{24} t_3 (1 + x_3) \rho^\alpha, \]
\[ C_0^s = -\frac{1}{4} t_0 \left( \frac{1}{2} - x_0 \right) - \frac{1}{24} t_3 \left( \frac{1}{2} - x_3 \right) \rho^\alpha, \]
\[ C_1^s = -\frac{1}{8} t_0 - \frac{1}{48} t_3 \rho^\alpha, \]
\[ C_0^\tau = \frac{3}{16} t_1 + \frac{1}{4} t_2 \left( \frac{5}{4} + x_2 \right) + \frac{3}{16} t_4 \rho^\beta + \frac{1}{4} t_5 \left( \frac{5}{4} + x_5 \right) \rho^\gamma, \]
\[ C_1^\tau = -\frac{1}{8} t_1 \left( \frac{1}{2} + x_1 \right) + \frac{1}{8} t_2 \left( \frac{1}{2} + x_2 \right) - \frac{1}{8} t_4 \rho^\beta \left( \frac{1}{2} + x_4 \right) + \frac{1}{8} t_5 \rho^\gamma \left( \frac{1}{2} + x_5 \right), \]
\[ C_0^T = -\frac{1}{8} \left[ t_1 \left( \frac{1}{2} - x_1 \right) - t_2 \left( \frac{1}{2} + x_2 \right) + t_4 \rho^\beta \left( \frac{1}{2} - x_4 \right) - t_5 \rho^\gamma \left( \frac{1}{2} + x_5 \right) \right], \]

Infinite number of combinations of the parameters that give good agreement with experiment
NUCLEAR MATTER PROPERTIES FROM MEAN FIELD MODELS WITH DENSITY DEPENDENT SKYRME EFFECTIVE INTERACTION:

1. 240 non-relativistic models based on the Skyrme interaction - density dependent effective nucleon-nucleon force dependent on up to 15 adjustable parameters were recently tested against the most up-to-date constraints on properties of nuclear matter:

5 satisfied all the constraints M. Dutra et al., PRC 85, 035201 (2012)
BUT ONLY 2 OF THEM (SQMC(700) and KDEv1) ALSO WORK SATISFACTORILY) IN FINITE NUCLEI!!!
<table>
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<th>Skyrme</th>
<th>$t_0$</th>
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</table>
Energy per particle

\[ E(\rho, \delta) = E_0(\rho, \delta = 0) + S(\rho)\delta^2, \quad \delta = (\rho_n - \rho_p)/\rho \]

Symmetric nuclear matter:

\[ S(\rho) = S(\rho_0) - \frac{L(\rho_0 - \rho)}{3\rho_0} \]

\[ S(\rho_0) \approx a_{sym} \approx 30 \text{ MeV} \]

**AB-INIO: (eg. MR-iM-SRG, Coupled clusters, No-core shell model, Green’s function models, Quantum Monte Carlo etc.**

Techniques based on Effective field theory – include only nucleon-pion interaction.

**Advantage:** Provide ground-state and excited states properties (include correlations in the mean field) calculate interaction + operators, estimate errors

**Disadvantages:** Dependence on cut-off parameters, uncertainty in 3-body forces, computationally expensive to include higher orders in theories and to apply to heavier nuclei.

**ALREADY MULTIPLE MODELS EXIST**

R. Machleidt and D. R. Entem, Phys. Rept. 503, (2011) 1
* but current chiral NN+3N forces have deficiencies, are largest source of uncertainty

Courtesy of Heiko Hergert, NS 2016, Knoxville TN
Heavy Ion Collisions
Heavy Ion collisions:

GSI, MSU, Texas A&M, RHIC, LHC  existing
FAIR (GSI), NICA (Dubna, Russia)  planned

Measurement:  Beam energy  35 A MeV – 5.5 A TeV
Collisions (Au,Au), (Sn,Sn) , (Cu,Cu)
but also (p,p) for a comparison
Transverse and Elliptical particle flow

Calculation:  Transport models -- empirical mean field potentials
Fit to data $\rightarrow$ energy density $\rightarrow$ $P(\varepsilon)$ $\rightarrow$ the EoS
(extrapolation to equilibrium, zero temperature,

Quantum Molecular Dynamics
(e.g. Yingxun Zhang, Zhuxia Li, Akira Ono)
at much higher density values, and providing different dependencies on density. GeV per nucleon (29.6 to 197 GeV total mean field at high density is needed to reproduce this curve with the data shows that a repulsive 2 GeV per nucleon. The short dashed curve are most strongly influenced by pressures increases with incident energy; the flow data extraneously. The various lines are the transport theoretical predictions for the transverse flow discussed in the text.

The single-particle energies (that is, baryons) as well as pions (§) are the in-plane and out-of-plane components of the momentum perpendicular to the beam axis. Negative values for y are perpendicular to the compressed region expands while the spectator matter while it is present. We analyzed the flow of matter exist momentarily before expanding. We analyzed the flow of matter to the free nucleon cross-sections following collisions and the density and momentum potential energy contributions. By carefully avoiding an overcounting of participating matter predicted by the EOSs with densities up to about "saturation" density, matter EOSs discussed in the text. Several aspects of these innovations, and detailed analyses (5).

The particle density is of about 2 to 5 300 MeV generates about 60% more pressure than the EOS models with parameters fitted to data on elliptical and transverse flow occur in energetic nucleus-nucleus collisions. Collision dynamics play an important role in studies of the EOS. Two-body collisions (that is, baryons) as well as pions (4) may be present in the core collapse of type II supernovae (2). It is the azimuthal angle of K, <(2), R

Equation of State

RTICLES

William G. Lynch, 1,2 Ray Lacey, William G. Lynch,*

Danielewicz, 1,2 Boguta, 3,4,5 Akmal, 6

Transport models with parameters fitted to data on elliptical and transverse flow

ICM


17

Danielewicz, 1,2 Ray Lacey, William G. Lynch,*
Matter in HIC and compact objects have different EoS:

### Central A-A collision:
- Strongly beam energy dependent
- Beam energy < 1 GeV/ A:
  - Temperature: < 50 MeV
  - Energy density: ~ 1 - 2 GeV/fm$^3$
  - Baryon density < $\rho_0$
  - Time scale to cool-down: $10^{-22-24}$ s
  - No neutrinos

**Strong Interaction: (S, B and L conserved)**
- Time scale $10^{-24}$ s

**NEARLY SYMMETRIC MATTER**
- Inelastic NN scatterings, $N,N^*, \Delta's$
- LOTS of PIONS
- strangeness
- less important (kaons)

? (Local)EQUILIBRIUM?

### Proto-neutron star:
- (progenitor mass dependent)
- ~ 8 – 20 solar mass

**Temperature:** < 50 MeV
**Energy density:** ~ 1 GeV/fm$^3$
**Baryon density** ~ 2-3 $\rho_s$
**Time scale to cool-down:** 1 - 10 s
**Neutrino rich matter**

**Strong + Weak Interaction:** (B and L con)
- Time scale $10^{-10}$ s ($\rho$ and $T$ dependent)

### HIGHLY ASYMMETRIC MATTER
- Higher T: strangeness produced in weak processes
- Lower T: freeze-out
- N, strange baryons and mesons,
  NO PIONS, leptons

**EQUILIBRIUM**
Transport model results for maximum proton and neutron density and is isospin asymmetry $\delta$ in a collision of Sn nuclei as a function of the lab beam energy up to 800 MeV/nucl

Three observations: (i) The maximum density does exceed $1.4\ \rho_0$
(ii) The isospin asymmetry has not increased with beam energy - the system retains memory of the original asymmetry
(iii) The observables are only weakly dependent on $S(\rho)$

JRS, P. Danielewicz and Y. Iwata, PRC 96, 014612(2017)
New Physics (still on a phenomenological level)?

Quark-Meson-Coupling instead Nucleon-Meson Coupling?
FUNDAMENTAL QUESTIONS:

1. Is the nucleon immutable?

2. When immersed to a nuclear medium with applied scalar field with strength of order of half of its mass is it really unchangeable?

3. Is this effect relevant to nuclear structure?

Replace interaction between nucleons

By interaction between valence quarks in individual non-overlapping nucleons

Look for the modification of the quark dynamics in a nucleon due to presence of other nucleons

ACCOUNT FOR THE MEDIUM EFFECT
Quark-Meson-Coupling Model  
(many-body effective Hamiltonian)

History:
Original: Pierre Guichon (Saclay), PLB 200, 235 (1988)
Several variants developed in Japan, Europe, Brazil, Korea, China
Latest: JRS, Guichon, Reinhard and Thomas, PRL 116, 092501 (2016)

Exchange $\sigma, \omega, \rho$

Lattice QCD simulations of the structure of a nucleon
Bissey, PRD76, 114512 (2007)

Schematic illustration of a multi-baryon system: baryons can appear close enough to exchange mesons between quarks through disturbance in the QCD vacuum (Guichon)
Typical structure of gluon-field configurations, averaged over time, describing the vacuum properties of QCD (action density – left), charge density (right).

QCD vacuum fluctuations are expelled from the interior region of a baryon like the proton is animated at left and a quark-antiquark pair at the right.
Nucleon structure in QCD

Vacuum excitation (meson)
Non perturbative QCD vacuum

Bag model

confining boundary condition

Guichon, JRS, Thomas Prog.Part.Nucl.Phys. 2018
OUTLINE OF THE QMC MODEL

Guichon, Stone,Thomas: Progress in Particle and Nuclear Physics 100 (2018) 262–297

1. Take a baryon in medium as an MIT bag (with one qluon exchange)
   immersed in a mean density dependent scalar field \( \bar{\sigma} \)

2. Approximate the solution of the bag equations by a dynamical nucleon effective mass

\[
M_N^* (\sigma) = M_N - g_{\sigma N} \bar{\sigma} + \frac{d}{2} (g_{\sigma N} \bar{\sigma})^2
\]

The last term represents the response of a nucleon to the scalar field with \( d \) being the scalar polarizability – the origin of MANY-BODY forces in QMC
(no additional parameters – \( d \) is calculated)

\[ d = 0.0044 + 0.211 \times R_B - 0.0357 R_B^2 \]

3. The nucleon meson coupling constants relate to the quark meson couplings as

\[
g_{\sigma N} = 3 g^q \int \frac{d \vec{r}}{Bag} \bar{q}(\vec{r}), \quad g_{\omega N} = 3 g^q, \quad g_{\rho N} = g^q
\]

where \( q \) is the valence quark wave function for a free bag.
4. Solve self-consistently for the meson fields using the condition

\[ \frac{\partial E}{\partial \bar{\sigma}} = 0 \quad \frac{\partial E}{\partial \bar{\omega}} = 0 \quad \frac{\partial E}{\partial \bar{\rho}} = 0 \]

5. Construct a quantized Hamiltonian/Lagrangian for a given system (non-rel) for finite nuclei and relativistic for nuclear matter.

These depend only on nucleon dynamics and are solved by standard Hartree-Fock methods to determine observables of interest.

- Density dependence of the energy density functional is microscopically calculated
- Multi-body forces are automatically included
- Exchange terms are always included
- Spin-orbit term appears naturally in both NR and R models
- Proton and neutron s.p. potentials are calculated – no need for fitting.
Parameters *(very little maneuvering space)*:

I. 3 nucleon-meson coupling constants in vacuum $g_{\sigma N}, g_{\omega N}, g_{\rho N}$

We define (for convenience)

$$G_{\sigma N} = \frac{g_{\sigma N}^2}{m_{\sigma}^2} \quad G_{\omega N} = \frac{g_{\omega N}^2}{m_{\omega}^2} \quad G_{\rho N} = \frac{g_{\rho N}^2}{m_{\rho}^2}$$

II. Meson masses: $\omega, \rho, \pi$ keep their physical values

$650 \text{ MeV} < M_{\sigma} < 700 \text{ MeV}$

III. Bag radius (free nucleon radius):

$1 \text{ fm} \text{ (limited sensitivity within change +/- 20%)}$

All other parameters either calculated within the model or fixed by symmetry
I. Nuclear matter parameters used in the fit:

-17 MeV < $E_0$ < -15 MeV  
0.15 fm < $\rho_0$ < 0.17 fm$^3$  
28 MeV < $J$ < 34 MeV  
$L > 22$ MeV  
250 MeV < $K_0$ < 350 MeV

II. HF+BCS model including constrained calculation at fixed quadrupole moment $Q_{20}$  
(two more parameters – proton and neutron pairing strengths)

Calculated: ground state binding energies, axial and octupole deformations,  
two-neutron separation energies, electromagnetic properties,  
single-particle energies, pairing gaps, neutron skin, spin-orbit splitting

The parameter set is UNIQUE for each model Hamiltonian

Details in JRS, Guichon, Reinhard and Thomas, PRL 116, 092501 (2016)
Quadrupole deformation of even-even Gd isotopes across the isotopic chain

\[ R(\theta, \phi, t) = R_0 \left( 1 + \sum_{\lambda=0}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda\mu}^*(t) Y_{\lambda\mu}(\theta, \phi) \right) \]

Deformation parameters

- **\( \beta_2 \) quadrupole**
- **\( \beta_3 \) octupole**
- **\( \beta_4 \) hexadecapole**

**SV-min** Klupfel et al. (11 parameters)

**FRDM** Moller et al., ADNDT, 59, 185 (1995)
expect different values for nuclei with large isospin asymmetry. However, as can be seen from Table 4, the differences are rather small. This is primarily because the spin–orbit splitting depends on the product of the spin–orbit form factor and the corresponding single-particle wave functions. Thus, if the wave functions are not strongly localised in the surface region, where $W_{\tau}(r)$ is effective, the influence of the isospin dependence of $W_{\tau}(r)$ upon the splitting need not be so significant.

In Figs. 1, 2 we show the proton and neutron densities calculated with the QMC model and with the Sly4 Skyrme force [19]. In the proton case we also show the experimental values [23].

QMC proton density distribution compared with experiment and Skyrme SLy4
Spectrum of single-particle energies in the ground state of $^{78}$Ni

Data taken from Grawe et al., Rep.Prog.Phys. 70, 1525 (2007)
Ground state binding energy for selected SHE nuclei

**Figure 1.** (Color online). Ground state binding energies of selected 'benchmark' even-even superheavy nuclei. The experimental data were taken from [27, 28].

**Theory:**
- **Skyrme SV-min**
  - P. Klupfel et al.
- **Finite Range Droplet Model (FRDM)**
  - P. Moller et al., ADNDT, 59, 185 (1995)
- **Micro-macro model (MM)**

**Experiment:**
- H.J. Huang et al., Chinese Phys. C41, 030002 and 030003 (2017)

*Physics of even-even superheavy nuclei with 96 < Z < 110 in the Quark-Meson-Coupling Model*

J.R. Stone, K. Morita, P. A. M. Guichon, A. W. Thomas

Correlation of neutron skin, point proton and neutron radii in $^{48}$Ca

From Hagen et al., Nat.Phys. 12, 186 (2016)

Red circles: $\text{NNLO}_{\text{sat}}$ PRC 91, 051301(R) (2015)

Blue squares: Ch-Int. PRC 83, 031301 (2011)

Grey diamonds: DFT PRC 85, 041302 (2012)

Yellow symbols: QMC($\pi$)
Mass-radius of a neutron star prediction by the QMC model (relativistic version)

2 $M_{\text{solar}}$ mass neutron star with full hyperon octet predicted 3 years before its observation – no hyperon puzzle
Composition of matter in a neutron star core as calculated in the QMC model (nucleon-hyperon interaction calculated in a mean field approximation)

Existence of Λ - hypernuclei

Existence of cascade-hypernucleus

Non-Existence of bound Σ hypernuclei

The first evidence of a deeply bound state of Ξ--14N system"K.Nakazawa et al., Prog. Theor. Exp. Phys. (2015), 033D02

Stone, Guichon, Matevosyan, Thomas, NPA 792, 341 (2007)
Concluding remarks

1. Current low energy nuclear physics faces a number of challenges

1. We introduce the Quark-Meson-Meson Coupling model based on structure of the nucleon. The model has a small number of well constrained parameters – UNIQUE SET PER HAMILTONIAN

3. The formalism works for any baryons without increased number of parameters

4. The model is consistent when applied to nuclear matter, finite nuclei, hyperonic physics and compact objects

5. Is there an independent experimental confirmation of a medium effect a dynamical structure of a baryon? EMC?

6. What is the way forward?
Lattice QCD

increasingly successful numerical experiment for free systems
Quantum Chromodynamics (QCD): Complete description of the strong interaction

Parameters: Coupling constant $g$ and quark masses

Frank Wilczek, Physics Today, August 2000
The Lattice

**Ken Wilson (1974)**

- Quark fields reside on sites $\psi(x)$
- Gauge fields on the links $U_\mu(x) = e^{-iagA_\mu(x)}$
- Approximate the full QCD path integral by Monte Carlo methods

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}A \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{O}[A, \bar{\psi}, \psi] e^{-S[A, \bar{\psi}, \psi]}$$

With field configurations $U_i$ distributed according to $e^{-S[U]}$

Put it on a supercomputer

**TWO PROBLEMS:** THE EXPONENTIAL WEIGHTING FACTOR OSCILLATES AT FINITE CHEMICAL POTENTIALS

OVERLAP PROBLEM (PLEASE GOOGLE FOR DETAILS)
Thanks to:

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