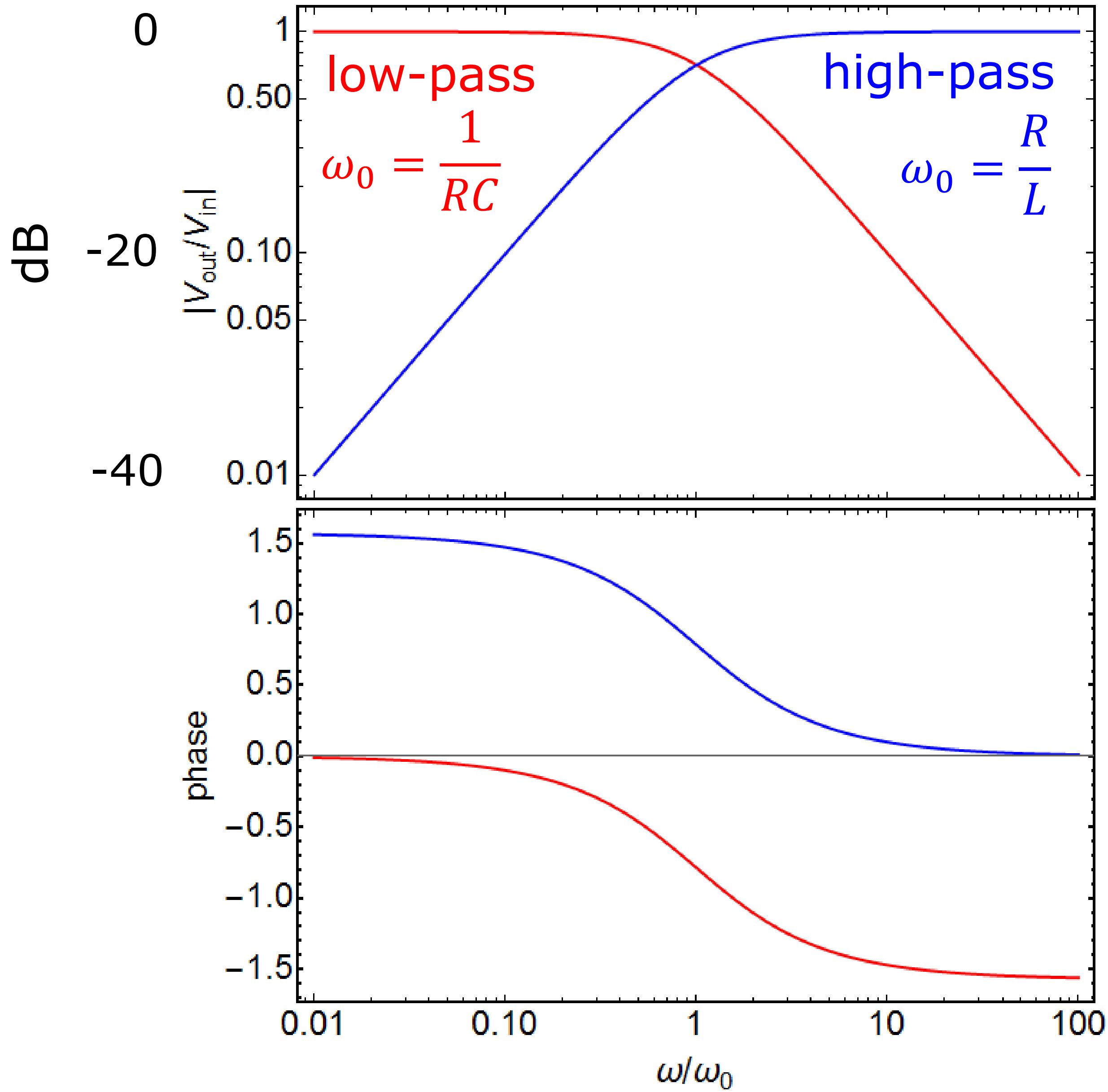
Recap

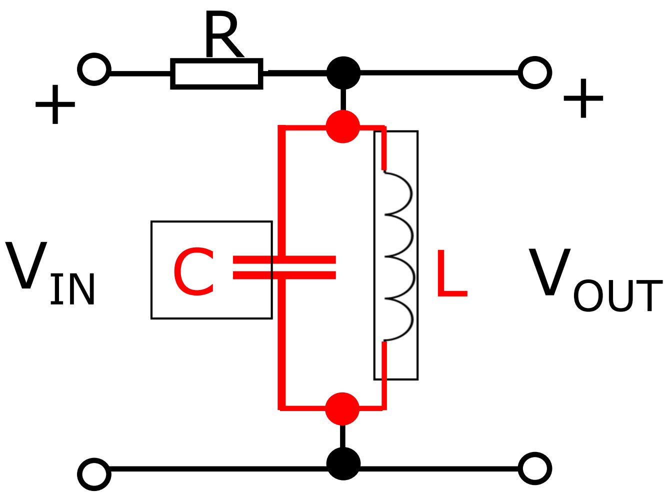
$$\mathrm{dB} = 20\log_{10}\left|\frac{V_{out}}{V_{in}}\right|$$



Today: AC circuit analysis

- Band pass filter
- Bridge circuits
- Power dissipation in AC circuits

Bandpass filter



$$\frac{V_{out}}{V_{in}} = \frac{Z_{LC}}{Z_{LC} + R} = \frac{1}{1 + \frac{R}{Z_{LC}}}$$

And we have $\frac{1}{Z_{LC}} = i\omega C + \frac{1}{i\omega L}$

(Note that at $\omega = \omega_0 = \frac{1}{\sqrt{LC}}$ we have $Z_{LC} \rightarrow \infty$)

So $\frac{V_{out}}{V_{in}} = \frac{1}{1+iR\left(\omega C - \frac{1}{\omega L}\right)}$

$$\frac{V_{out}}{V_{in}} = \frac{1}{1 + \frac{iR}{L\omega_0}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$$

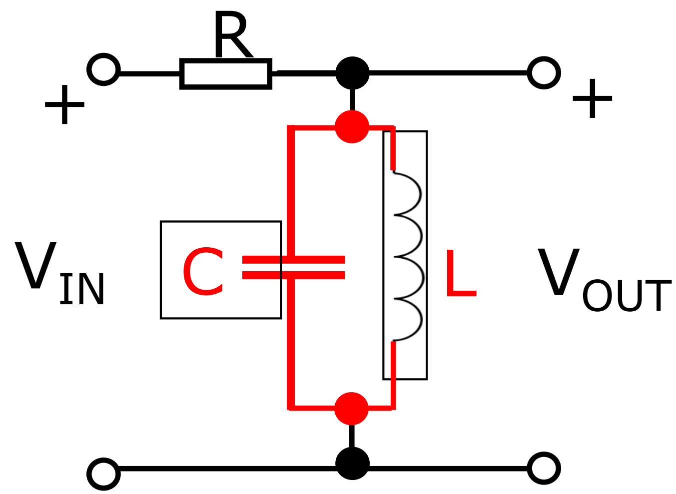
$\left|\frac{V_{out}}{V_{in}}\right| \rightarrow 1$ for $\omega = \omega_0$

$\left|\frac{V_{out}}{V_{in}}\right| \rightarrow 0$ for $\omega \rightarrow 0$

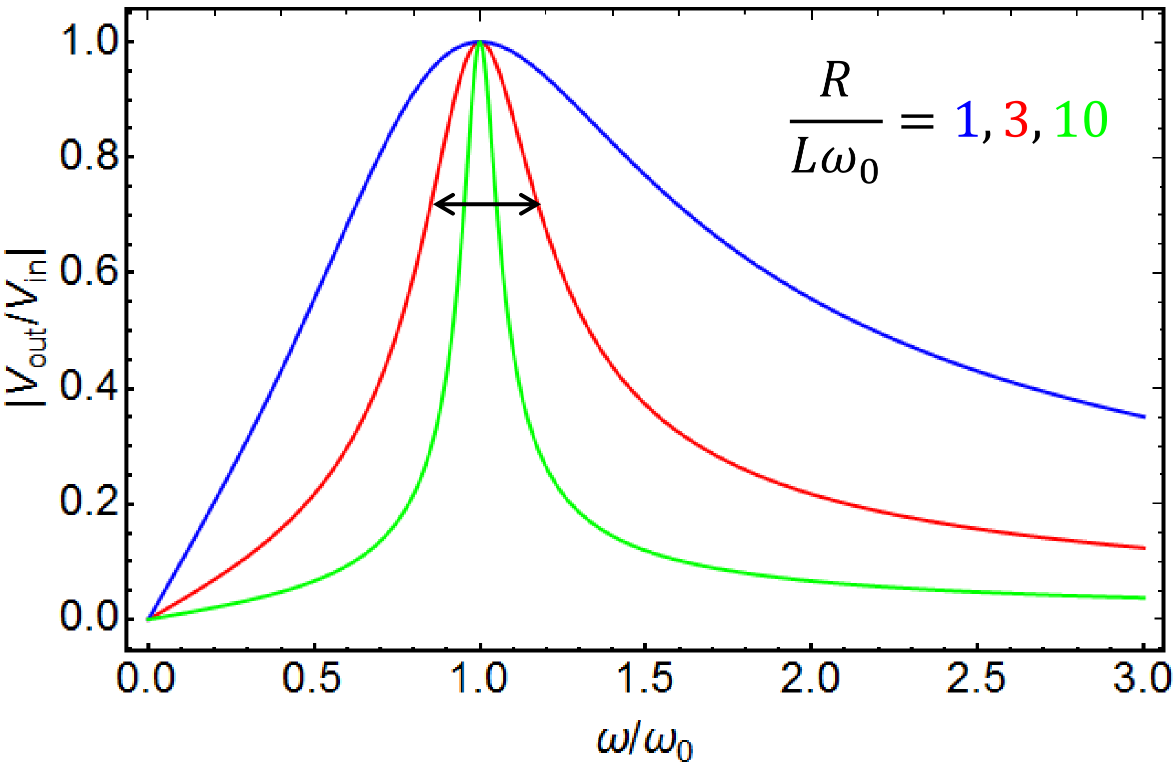
$\left|\frac{V_{out}}{V_{in}}\right| \rightarrow 0$ for $\omega \rightarrow \infty$

Bandpass filter

$$\frac{V_{out}}{V_{in}} = \frac{1}{1 + \frac{iR}{L\omega_0}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$$



$Z_{LC} \rightarrow 0$ $Z_{LC} \rightarrow \infty$ $Z_{LC} \rightarrow 0$



$\frac{R}{L\omega_0} = 1, 3, 10$

Here larger R gives sharper feature!!

$$\Delta\omega_{FWHM} = \frac{L\omega_0^2}{R}$$

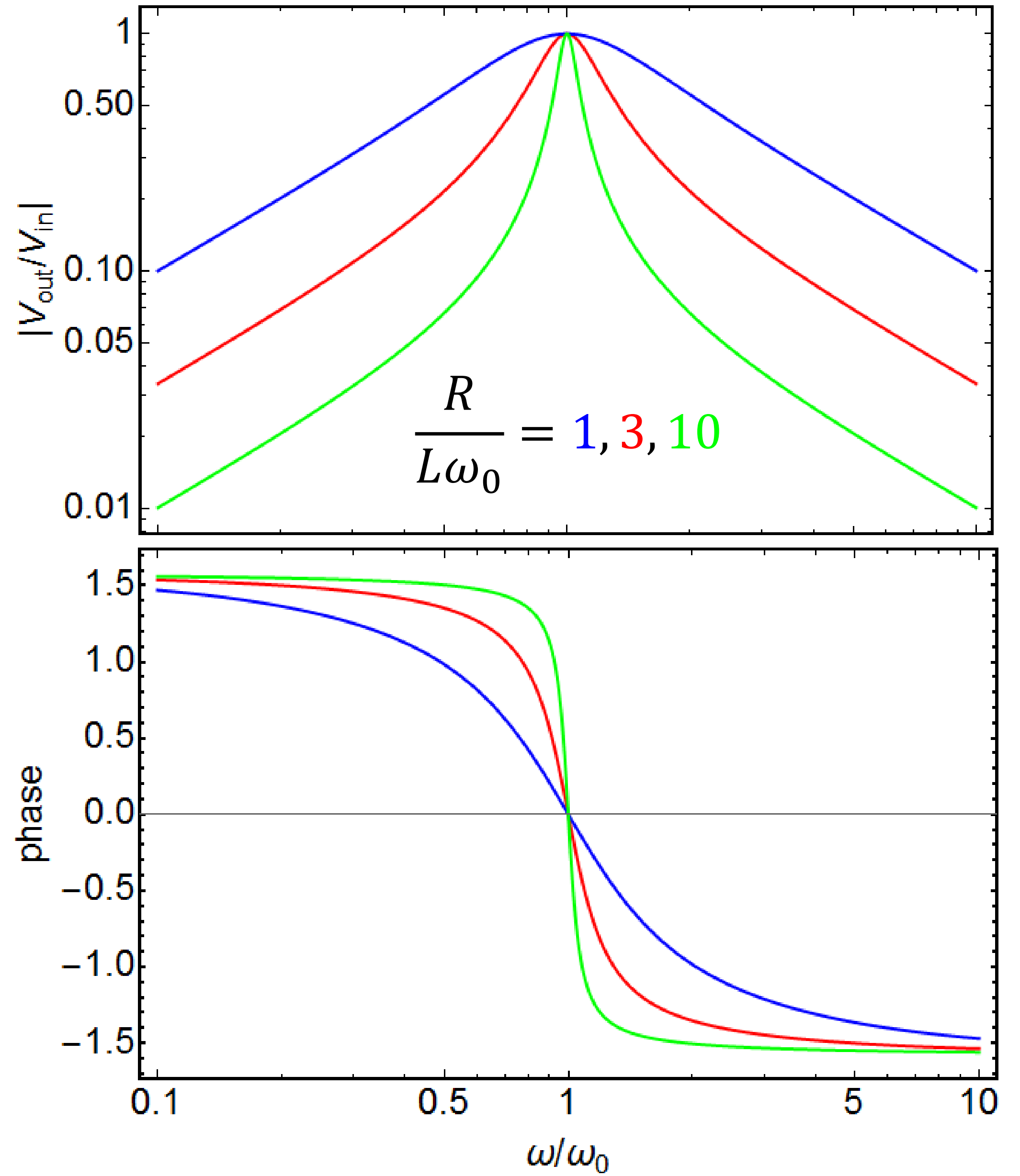
Quality factor

$$Q = \frac{\omega_0}{\Delta\omega_{FWHM}} = \frac{R}{L\omega_0} = R\sqrt{\frac{C}{L}}$$

Bandpass filter – Bode plots

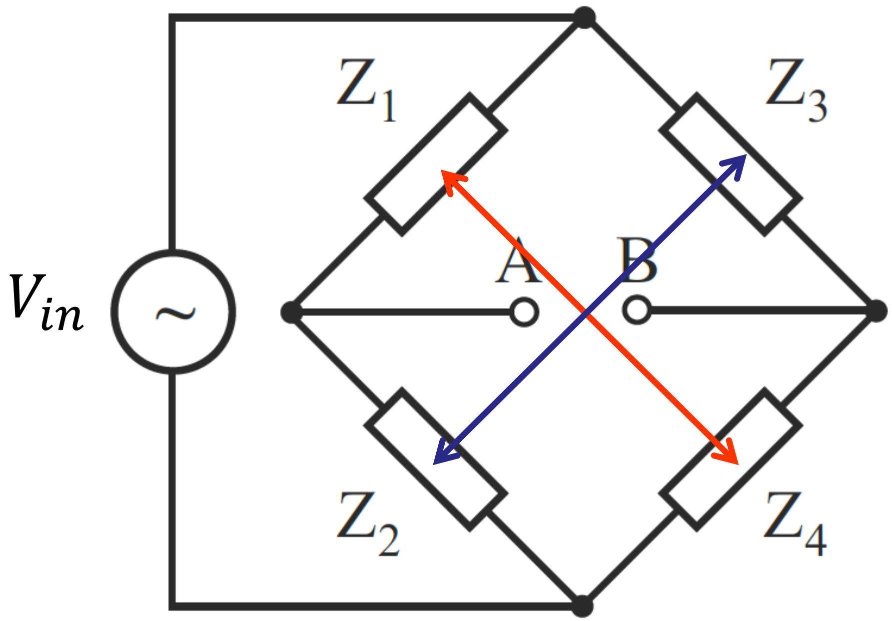
$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{1 + \left(\frac{R}{L\omega_0}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right)^2}}$$

$$\phi = -\arctan\left(\frac{R}{L\omega_0}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right)$$



Bridge circuits

Used to determine an unknown impedance



$$V_A = \frac{Z_2}{Z_1 + Z_2} V_{in}$$

$$V_B = \frac{Z_4}{Z_3 + Z_4} V_{in}$$

Bridge balanced when $V_{AB} = 0 \rightarrow \frac{Z_2}{Z_1 + Z_2} = \frac{Z_4}{Z_3 + Z_4}$

$$\rightarrow Z_1 Z_4 = Z_2 Z_3$$

(gives 2 equations – real and imaginary parts)

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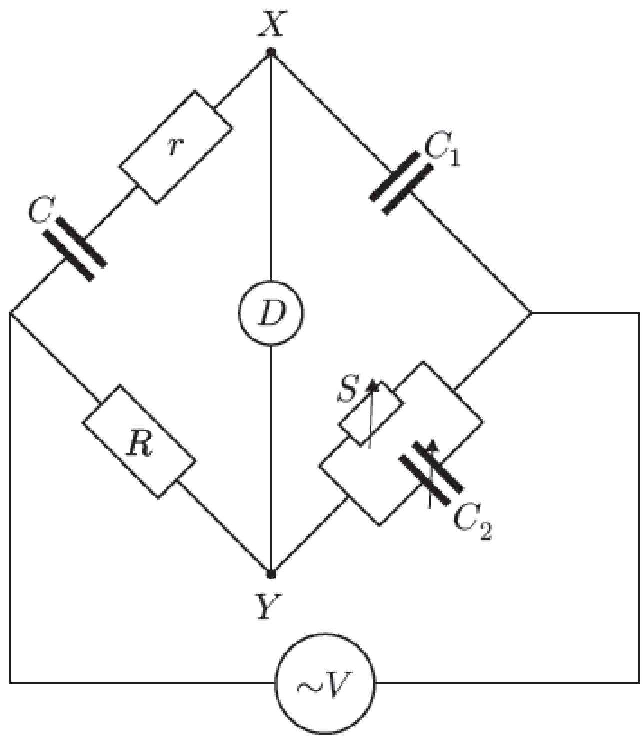
9. (i) A Schering bridge is used to determine the capacitance C and resistance r of a lossy capacitor. In the diagram below the instrument D detects when X and Y are at the same potential, i.e. the bridge is balanced.



Show that the conditions for balancing the bridge are

$$C = C_1 S/R \quad \text{and} \quad r = C_2 R/C_1 \,.$$ [10]

Power in AC circuits

Power into component: $P = VI$ (real values)

So remember $P = \mathrm{Re}(\tilde{I}) \times \mathrm{Re}(\tilde{V})$

$$P = I_0 \cos(\omega t) \times I_0\,|Z| \cos(\omega t + \phi)$$

Time average $\rightarrow \langle P \rangle = \frac{1}{2} I_0^2 |Z| \cos(\phi) = \frac{1}{2} I_0^2 \mathrm{Re}(Z)$

Power factor

Or $\rightarrow \langle P \rangle = \frac{1}{2} V_0 I_0 \cos(\phi) = V_{\mathrm{rms}} I_{\mathrm{rms}} \cos(\phi)$

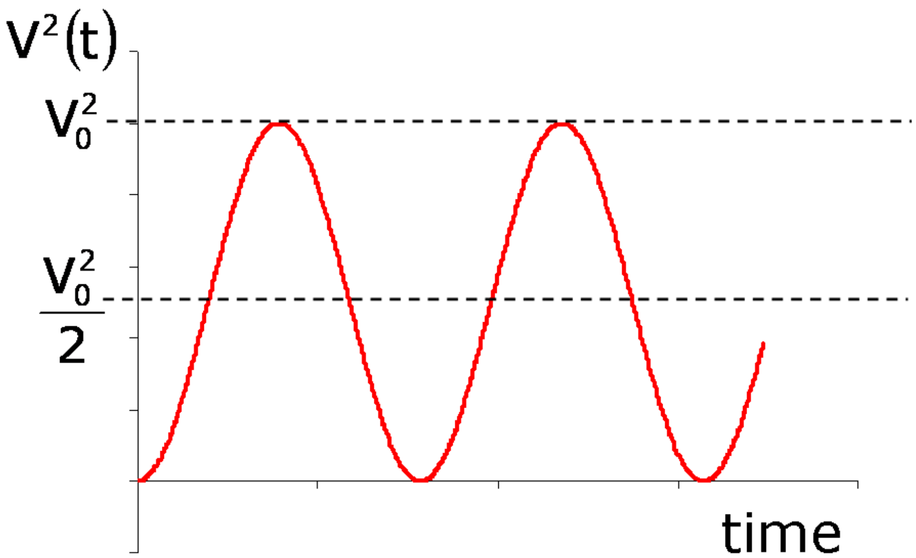
Resistive load $Z = R, \cos(\phi) = 1$

Reactive load $Z = iX, \cos(\phi) = 0$

RMS values

For resistive load $\langle P \rangle = \langle IV \rangle = \left\langle \frac{V^2}{R} \right\rangle = \frac{V_{\mathrm{rms}}^2}{R}$

$$V_{\mathrm{rms}} = \sqrt{\frac{1}{T}\int_0^T V^2(t)\, dt}$$



For sinusoid $V_{\mathrm{rms}} = \frac{V_0}{\sqrt{2}}$

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(ii) A source of alternating voltage $V_0 \exp(j\omega t)$ with internal complex impedance $Z_1 = R_1 + jX_1$ is connected to a load of complex impedance $Z_2 = R_2 + jX_2$.

Show that the current through the load is given by

$$I = I_0 \exp[j(\omega t - \phi)],$$

where

$$I_0 = V_0 \left[(R_1 + R_2)^2 + (X_1 + X_2)^2\right]^{-1/2},$$

and find an expression for ϕ. [4]

Hence show that the mean power $\langle P \rangle$ consumed by the load over one cycle is

$$\langle P \rangle = \tfrac{1}{2} I_0^2 R_2.$$ [6]