Recap

Response to AC current

- Resistors $V = I_0 R \cos(\omega t)$
- Capacitors $V = \frac{I_0}{\omega C} \cos\left(\omega t - \frac{\pi}{2}\right)$ Voltage lags current
- Inductors $V = I_0 \omega L \cos\left(\omega t + \frac{\pi}{2}\right)$ Voltage leads current

Complex impedance (Z)

- Describes amplitude and phase of *V* response to AC I
- $\mathrm{Z} = |Z|e^{i\phi} = R + iX$
- Generalised Ohm's law: $\tilde{V} = \tilde{I}Z$ (or often $V = IZ$)
- Combining in series $Z_T = \sum Z_n$ and parallel $\frac{1}{Z_T} = \sum \frac{1}{Z_n}$

$$Z_R = R \qquad Z_C = \frac{1}{i\omega C} \qquad Z_L = i\omega L$$

Today: some examples

- Series and parallel LCR circuits

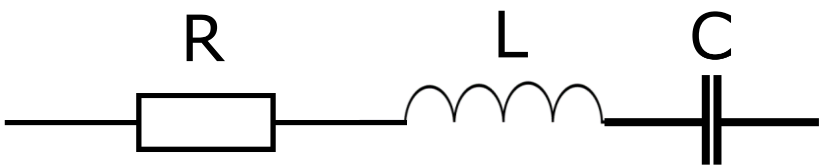
- Filters
  - RC low pass filter
  - RL high pass filter
  - Band pass filter

Bode plots

Limits of Z_c and Z_L

- **Capacitor**
  - At low frequencies (like DC) → **Open circuit**
    - Not a surprise, it's got a gap!
  - At high frequencies ("fast") → **Short circuit!**
- **Inductor**
  - At low frequencies → **Short circuit**
    - Not a surprise, it's just a wire really
  - At high frequencies → **Open circuit!**
- **Sometimes this can help you with your intuition on the circuit's behaviour.**

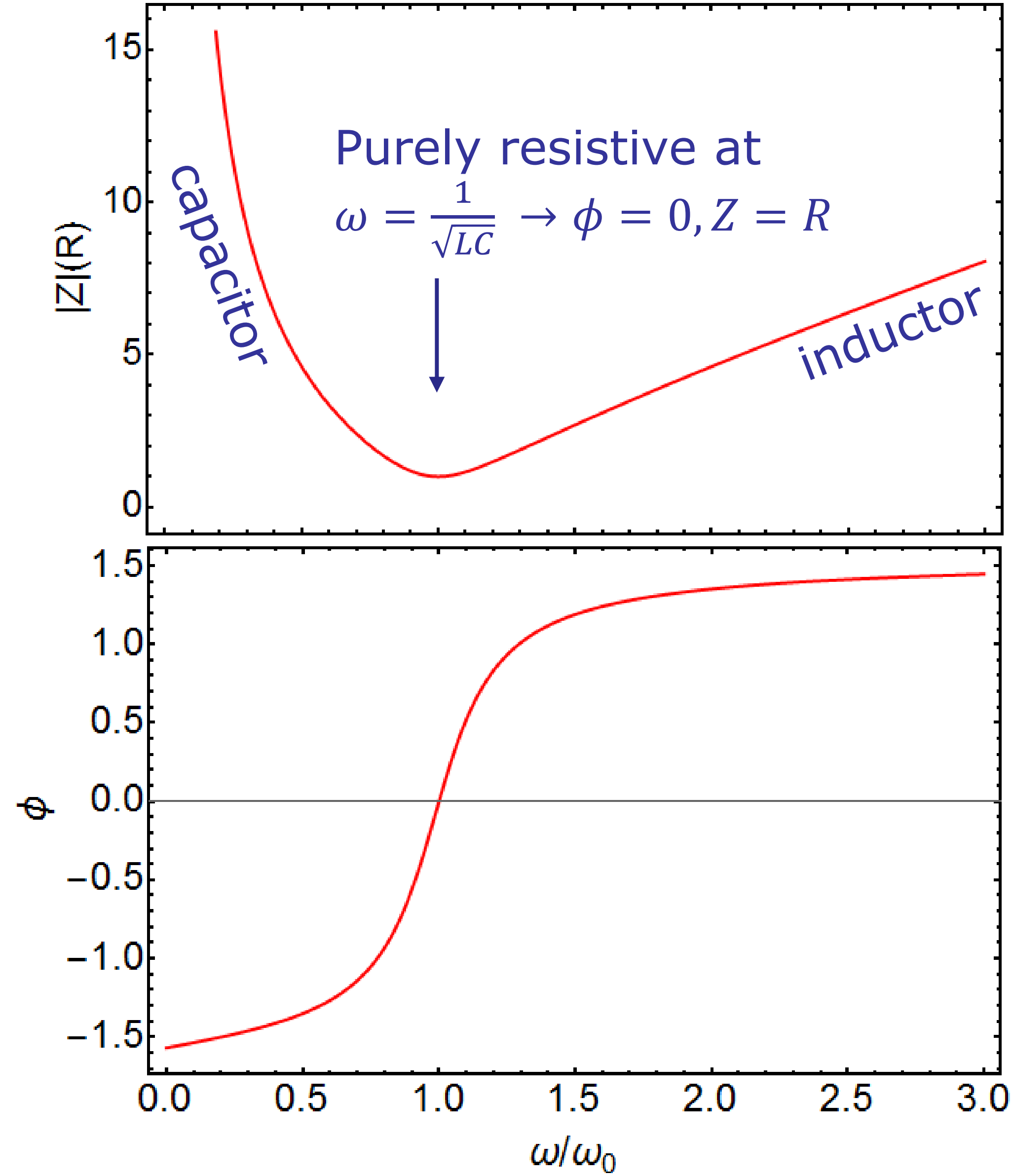
LCR series circuit



$$Z = R + i\left(\omega L - \frac{1}{\omega C}\right)$$

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

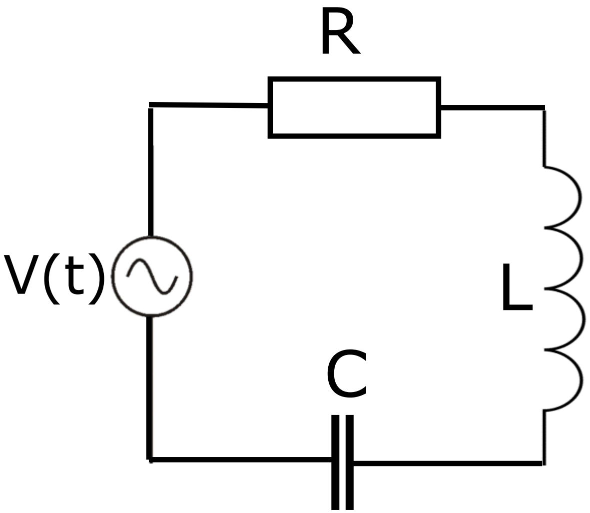
$$\phi = \arctan\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$



LCR series circuit – voltage driven

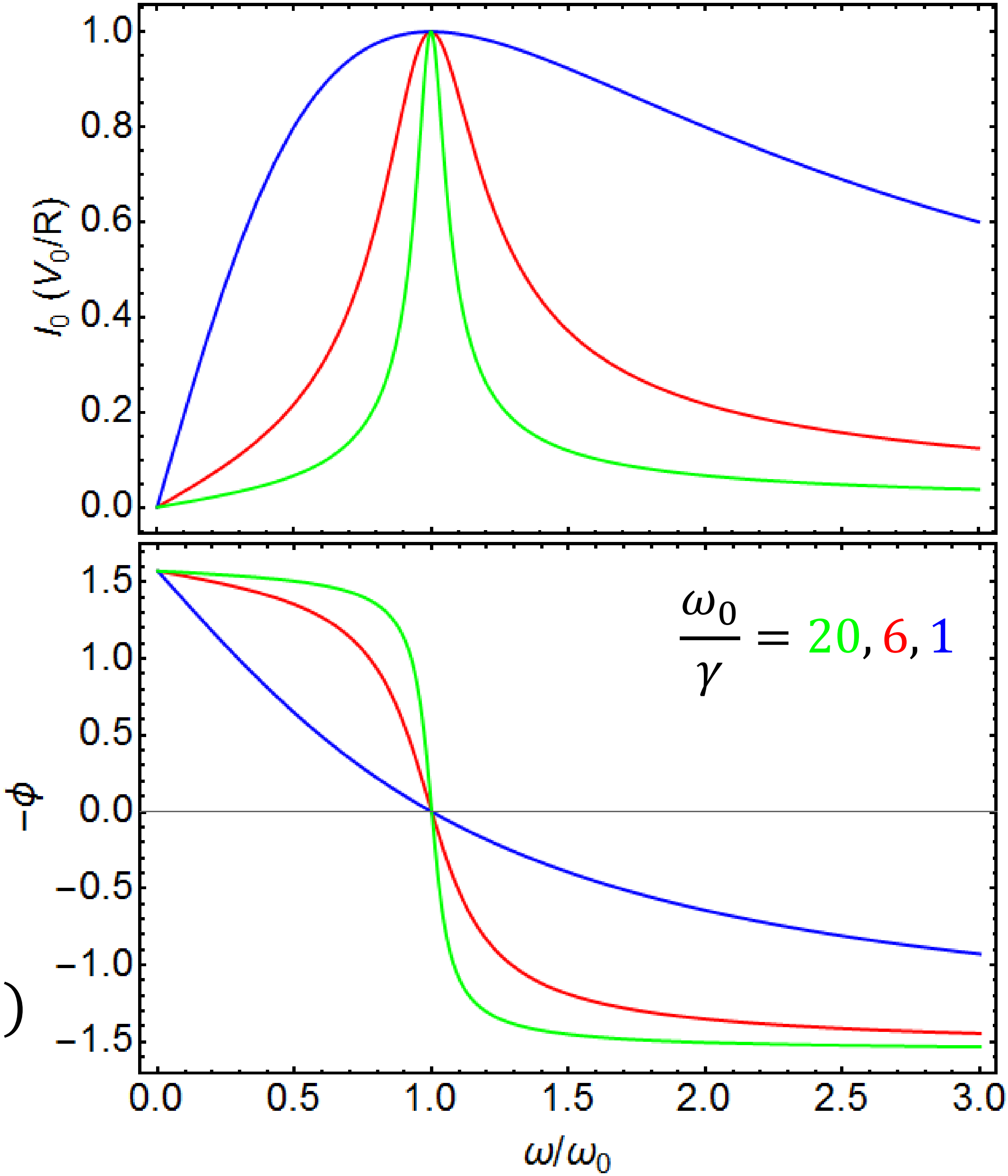
$$V = V_0 \cos(\omega t)$$

$$\tilde{V} = V_0 e^{i\omega t}$$

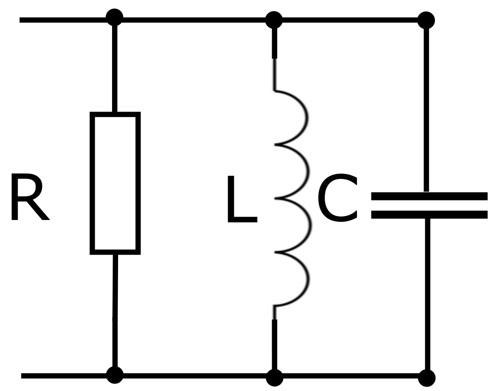


$$\tilde{I} = \frac{\tilde{V}}{Z} = \frac{\tilde{V}}{|Z|} e^{-i\phi}$$

$$I = I_0 \cos(\omega t + (-\phi))$$



LCR parallel circuit

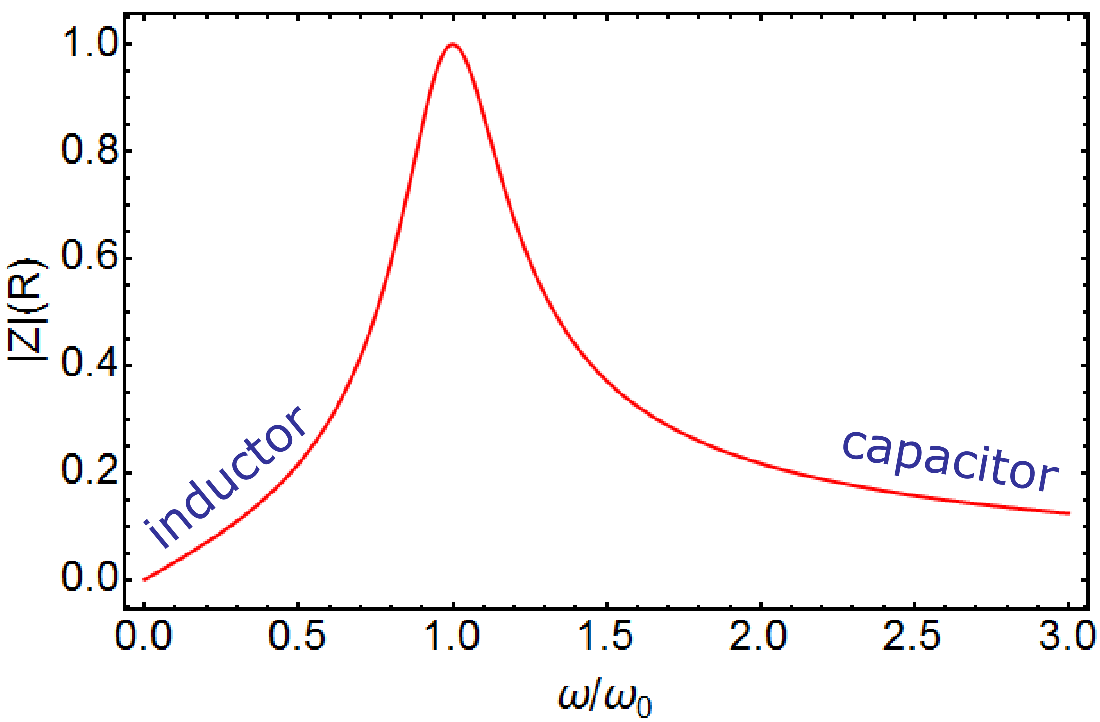


$$\frac{1}{Z} = \frac{1}{R} + \frac{1}{i\omega L} + i\omega C$$

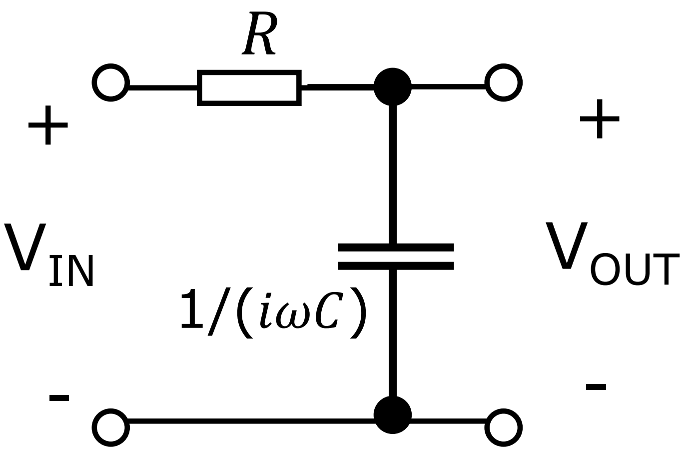
$$\frac{1}{Z} = \frac{1}{R} + i\left(\omega C - \frac{1}{\omega L}\right)$$

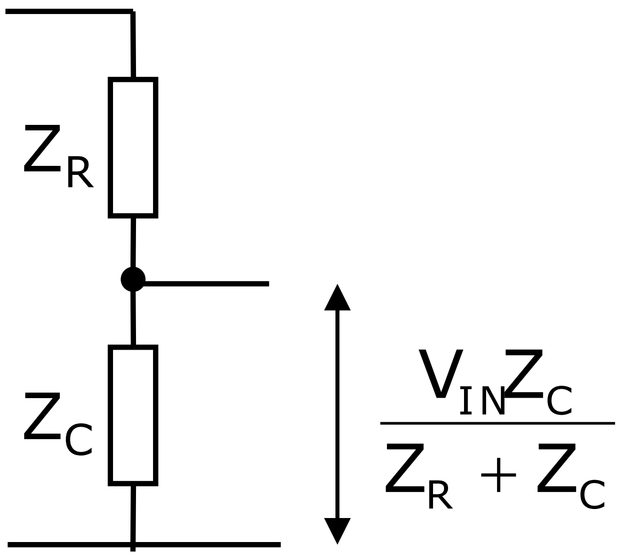
Purely resistive at

$\omega = \frac{1}{\sqrt{LC}} \rightarrow \phi = 0, Z = R$



RC low pass filter





$$\frac{V_{out}}{V_{in}} = \frac{Z_c}{Z_R + Z_C}$$

$$\frac{V_{out}}{V_{in}} = \frac{\frac{1}{i\omega C}}{R + \frac{1}{i\omega C}}$$

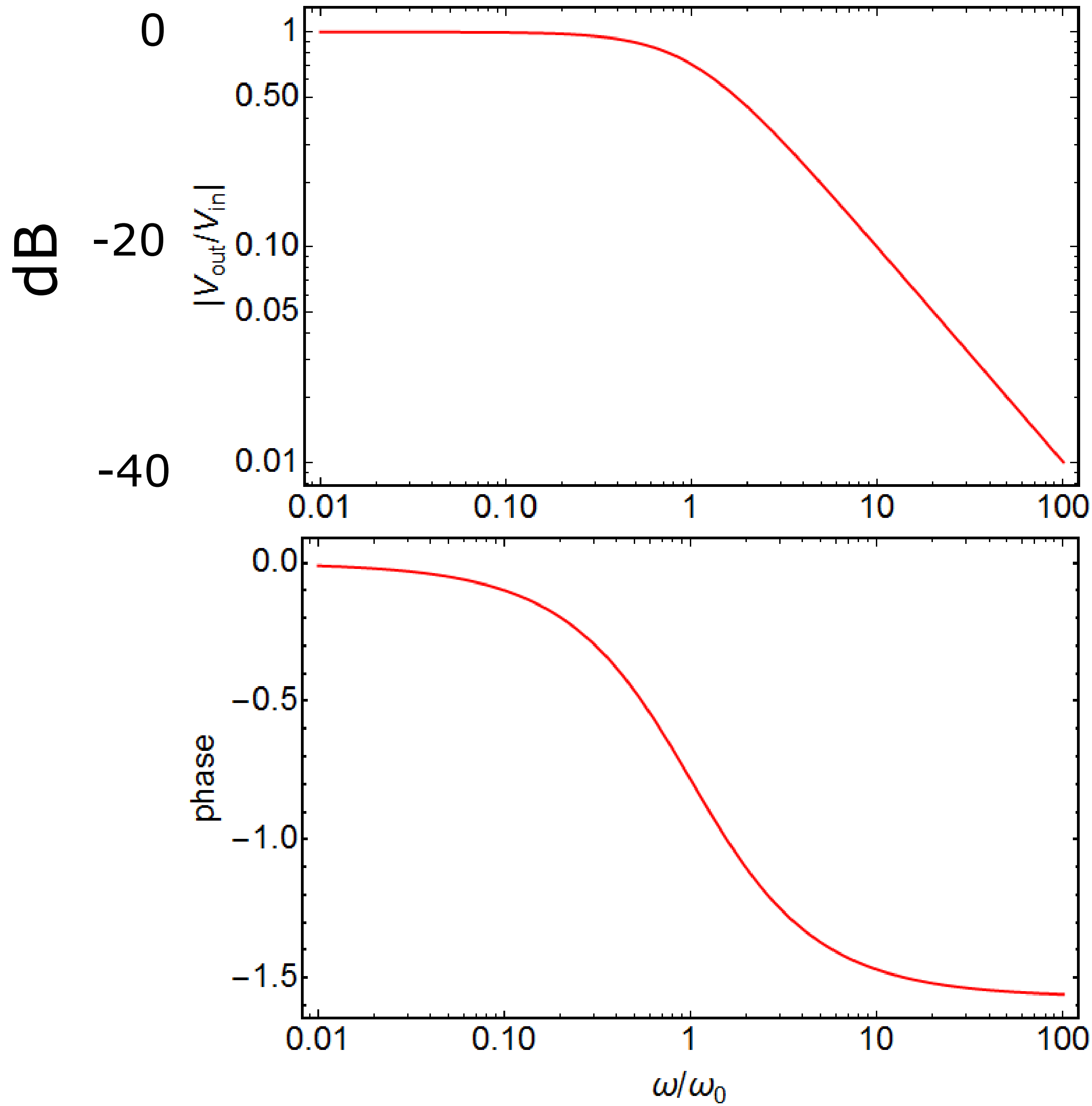
Complex result

$$\frac{V_{out}}{V_{in}} = \frac{1}{1 + i\omega RC}$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{\sqrt{1 + (\omega RC)^2}} e^{i\phi}$$

$$\phi = -\arctan(\omega RC)$$

RC low pass filter – Bode plots



$$\omega_0 = \frac{1}{RC}$$

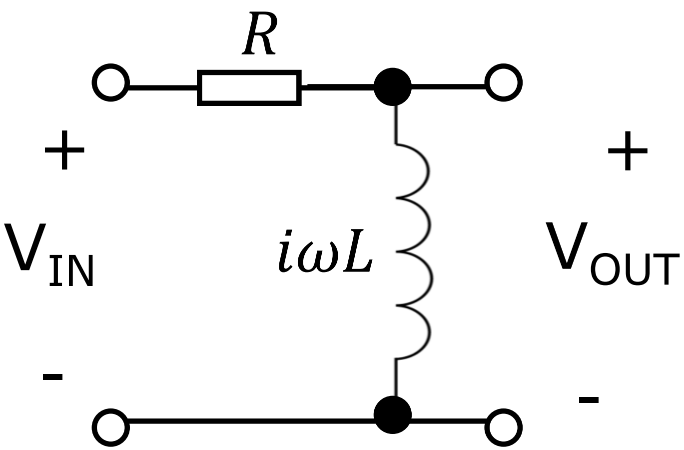
Decibels

Logarithm of power ratio

$$\text{dB} = 10\log_{10}\left|\frac{V_{out}^2}{V_{in}^2}\right| = 20\log_{10}\left|\frac{V_{out}}{V_{in}}\right|$$

V_{OUT}/V_{IN}	dB
10	20
1	0
0.1	−20
0.01	−40
0.001	−60

RL high pass filter

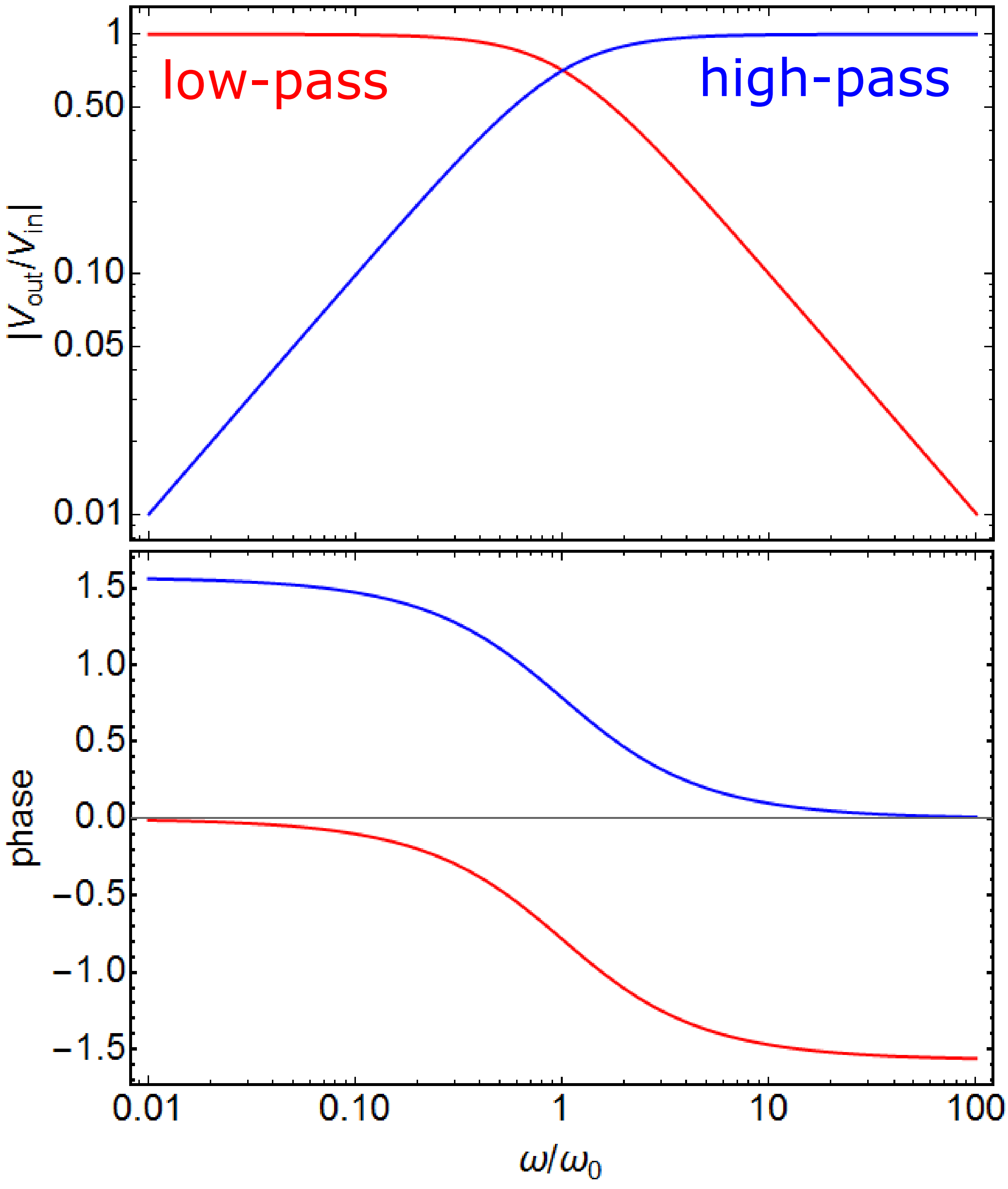


$$\frac{V_{out}}{V_{in}} = \frac{i\omega L}{R + i\omega L}$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{\sqrt{1 + \left(\frac{R}{\omega L}\right)^2}} e^{i\phi}$$

$$\phi = \arctan\left(\frac{R}{\omega L}\right)$$

Bode plots (amplitude + phase)



$$\omega_0 = \frac{R}{L}$$

$$\omega_0 = \frac{1}{RC}$$

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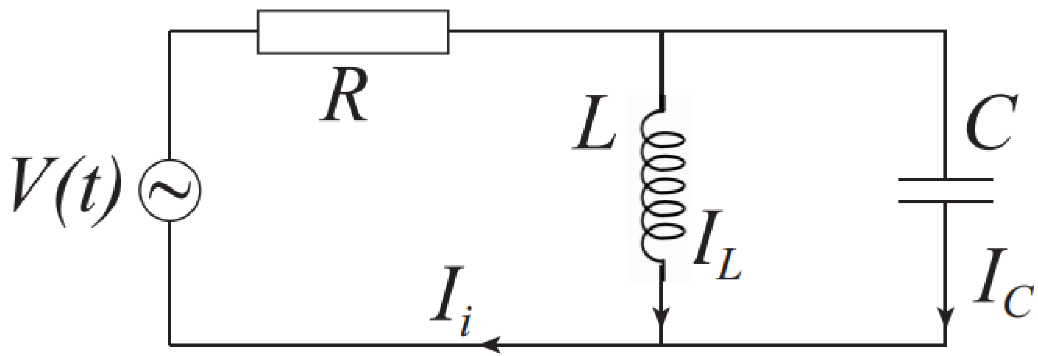
7. The diagram below shows a parallel LC resonance circuit in series with a resistance R, driven by a voltage source $V(t) = V_i \sin \omega t$.



(i) By replacing the voltage source and resistor combination with a Norton equivalent, or otherwise, show that the complex amplitude of the current in the inductor can be written as

$$I_L = \frac{1}{1 + j\omega\frac{L}{R} - \omega^2 LC} \times \frac{V_i}{R},$$

and find a similar expression for the complex amplitude of the current in the capacitor.

[8]