Recap

- Capacitors store charge; they resist a change of voltage

$$V = \frac{Q}{C} \qquad W = \frac{1}{2}CV^2$$

- Inductors oppose a change in current

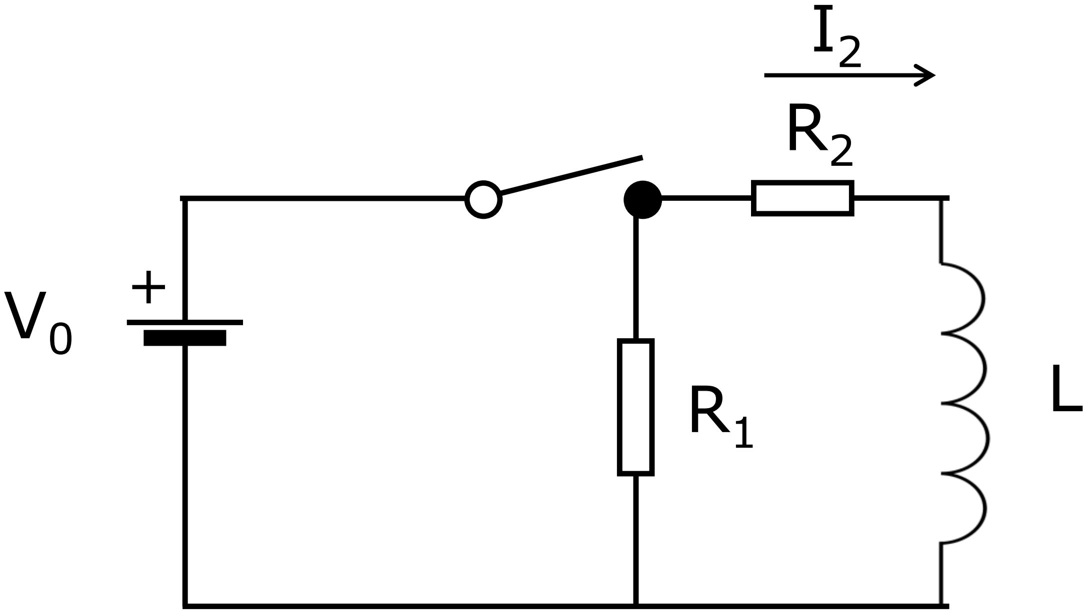
$$V = L\frac{dI}{dt} \qquad W = \frac{1}{2}LI^2$$

- Transient circuits

$$\tau = RC \qquad \tau = \frac{L}{R}$$

Capacitor and resistor in series

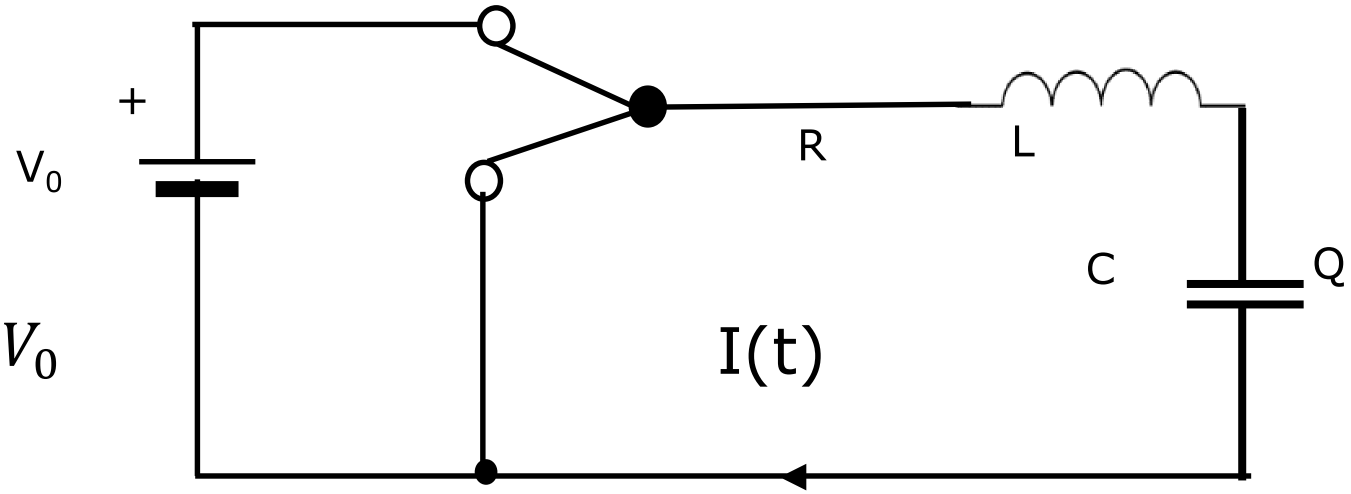
Inductor and resistor in series



t<0 switch closed I_2=?
t=0 switch opened
t>0 $I_2(t)$=?

voltage across R_1=?
Let R_1 get very large:
What happens to V_L?

LC circuit



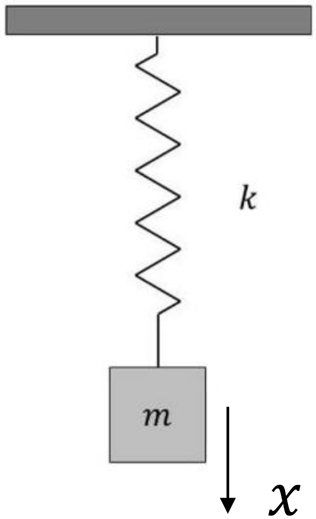
At $t = 0$

$Q = Q_0 = CV_0$

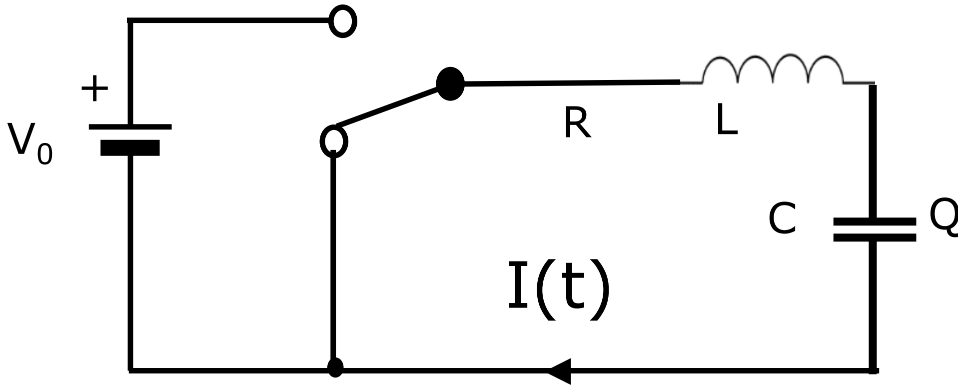
$$L\frac{dI}{dt} + \frac{Q}{C} = 0 \quad \rightarrow \quad L\frac{d^2Q}{dt^2} + \frac{I}{C} = 0 \quad \rightarrow \frac{d^2Q}{dt^2} + \omega_0^2 Q = 0$$

$$\rightarrow \quad Q = Q_0 \cos \omega_0 t \qquad\qquad \omega_0 = \sqrt{\frac{1}{LC}}$$

Harmonic oscillator

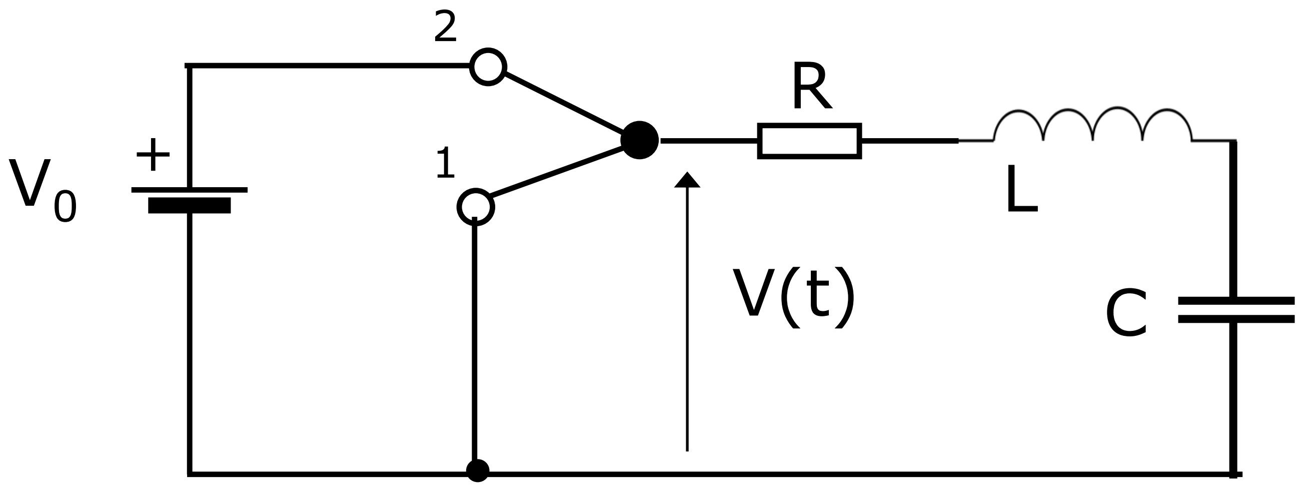


equivalent



$$x \rightarrow Q$$
$$\dot{x} \rightarrow I$$
$$m \rightarrow L$$
$$k \rightarrow \frac{1}{C}$$

LCR circuit



$$V(t) = \begin{cases} 0 & \text{for } t < 0 \\ V_0 & \text{for } t \geq 0 \end{cases}$$

$I(t)=0$ for $t<0$

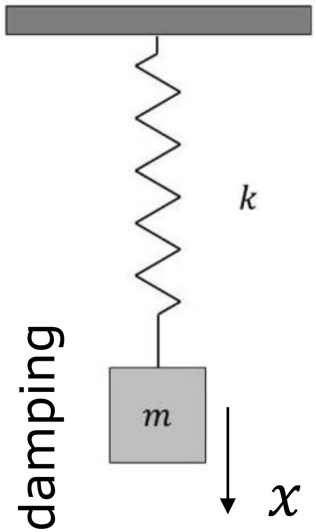
$t \geq 0$

KVL $\quad V_R + V_L + V_C = V_0$

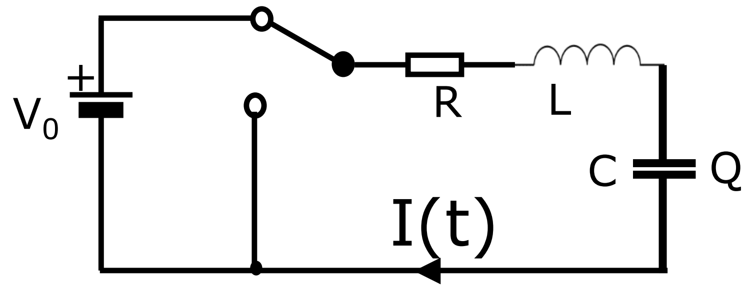
Component Laws $\quad IR + L\frac{dI}{dt} + \frac{Q}{C} = V_0$

Get Rid of "Q" $\quad \frac{d^2I}{dt^2} + \frac{R}{L}\frac{dI}{dt} + \frac{1}{LC}I = 0$

Damped harmonic oscillator



equivalent



$$x \to Q$$

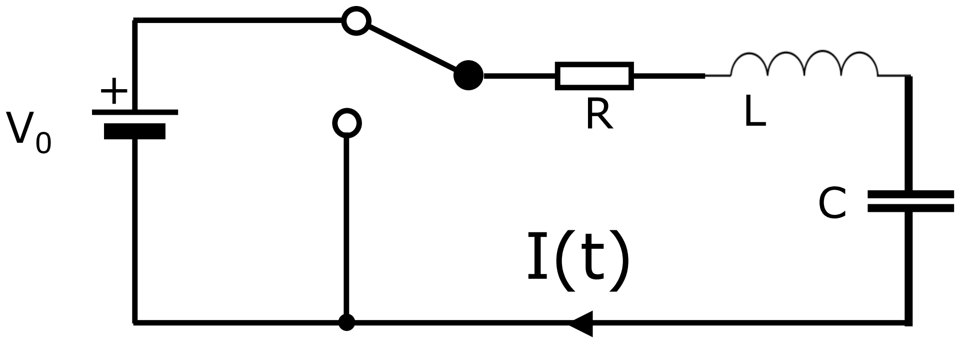
$$\dot{x} \to I$$

$$m \to L$$

$$k \to \frac{1}{C}$$

Damping $\to R$

LCR circuit – initial conditions



It is impossible to change the current on a inductor instantly!

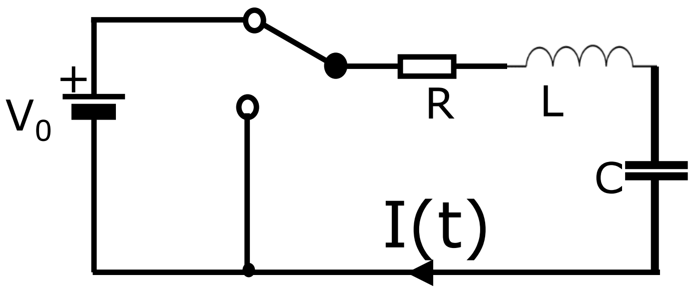
$$I(t=0)=0$$

It is impossible to change the voltage on a capacitor instantly!

$$L\frac{dI}{dt}\Big|_{t=0}=V_0$$

Also using I=0

LCR circuit



Equation for I

$$\frac{d^2I}{dt^2} + \frac{R}{L}\frac{dI}{dt} + \frac{1}{LC}I = 0$$

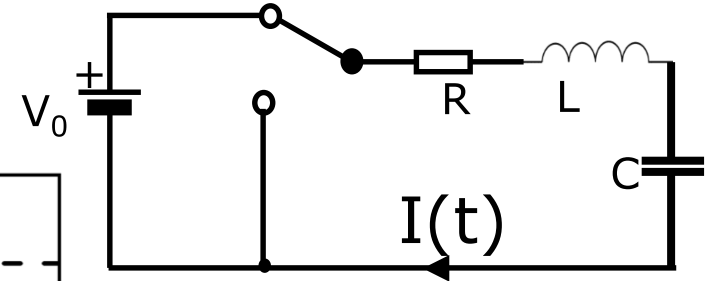
$$\frac{d^2I}{dt^2} + 2\gamma\frac{dI}{dt} + \omega_0^2 I = 0 \qquad \gamma = \frac{R}{2L} \qquad \omega_0^2 = \frac{1}{LC}$$

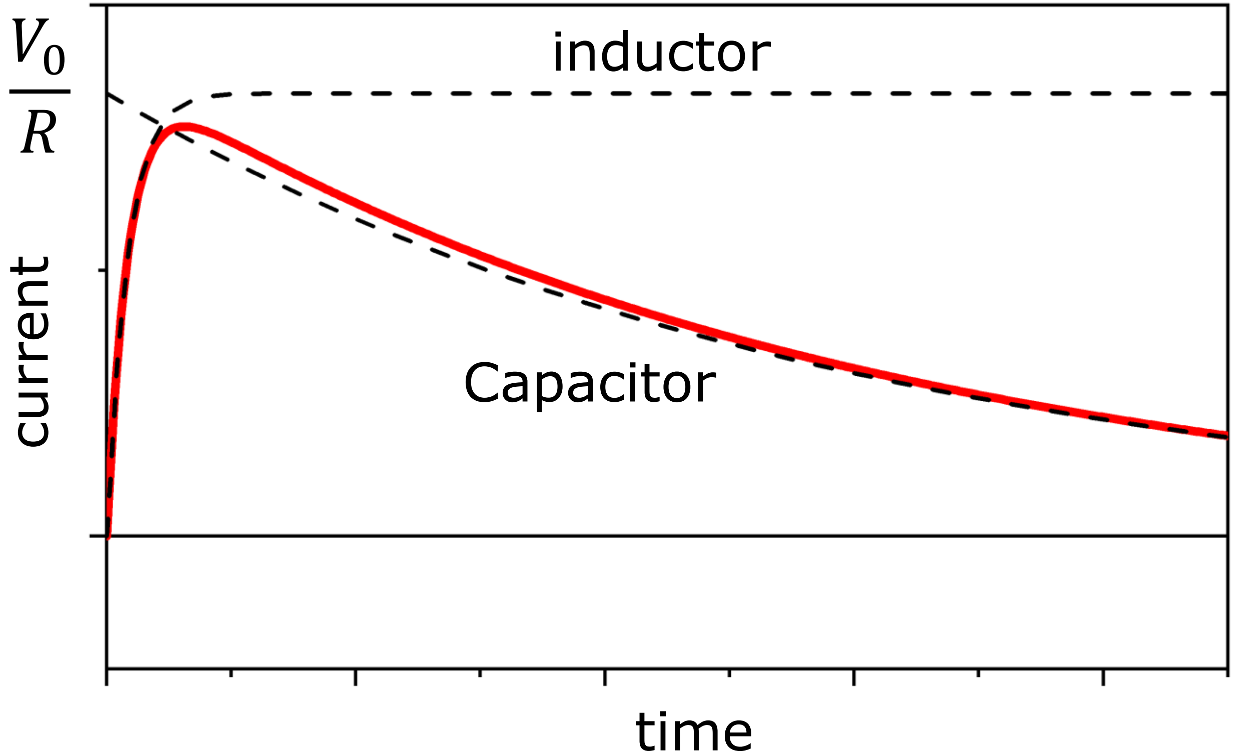
general solution

$$I = Ae^{\lambda_1 t} + Be^{\lambda_2 t} \qquad \lambda_{1,2} = -\gamma \pm \sqrt{\gamma^2 - \omega_0^2}$$

Form of solution depends on $\frac{\gamma}{\omega_0}$

Over-damped $(\gamma > \omega_0)$



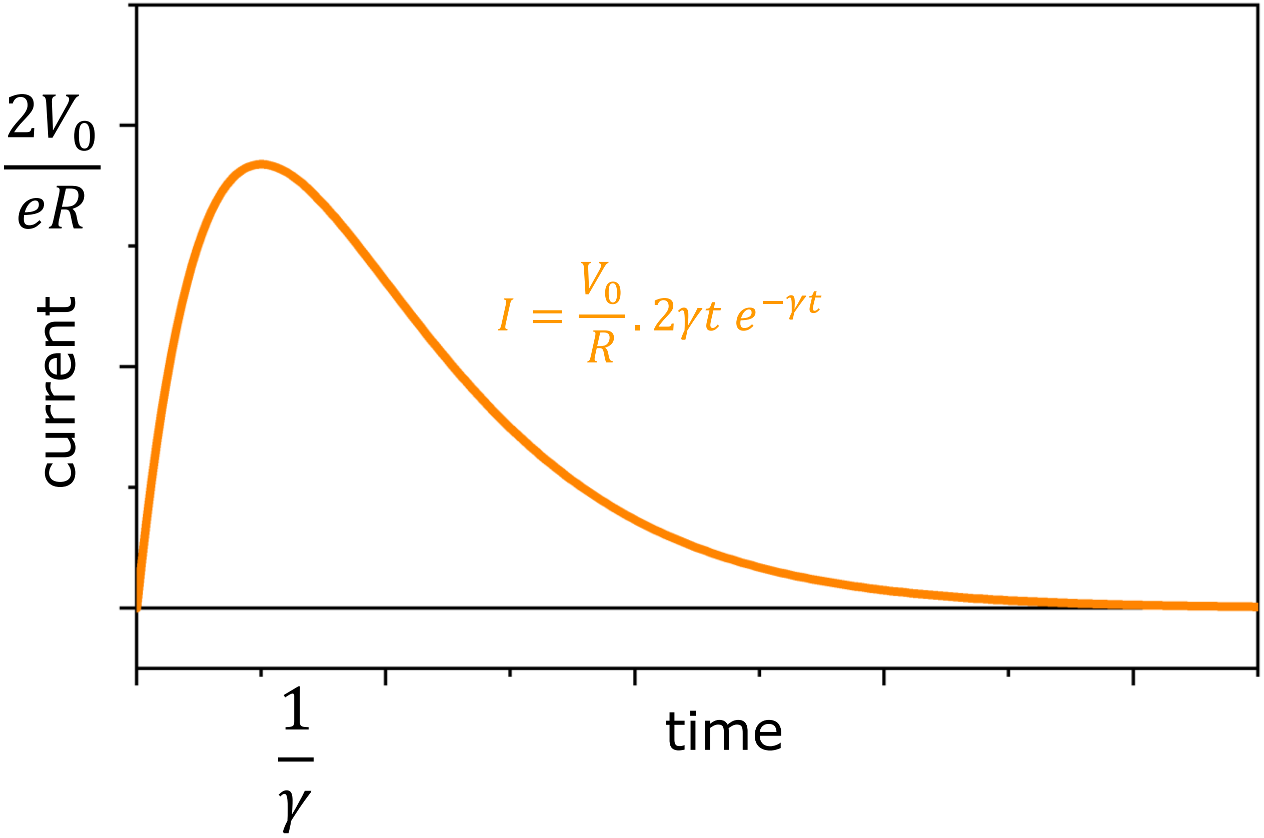


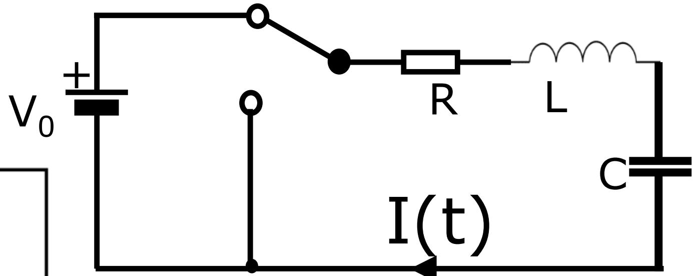
$$\lambda_{1,2} = -\gamma \pm \sqrt{\gamma^2 - \omega_0^2}$$

$$I = \frac{V_0}{R\sqrt{1 - \omega_0^2/\gamma^2}}\left[\exp\left(-\left(1 - \sqrt{1 - \frac{\omega_0^2}{\gamma^2}}\right)\gamma t\right) - \exp\left(-\left(1 + \sqrt{1 - \frac{\omega_0^2}{\gamma^2}}\right)\gamma t\right)\right]$$

$$I \approx \frac{V_0}{R}\left[\exp\left(-\frac{t}{\tau_C}\right) - \exp\left(-\frac{t}{\tau_L}\right)\right] \quad (\gamma \gg \omega_0) \quad \tau_C = RC \quad \tau_L = \frac{L}{R}$$

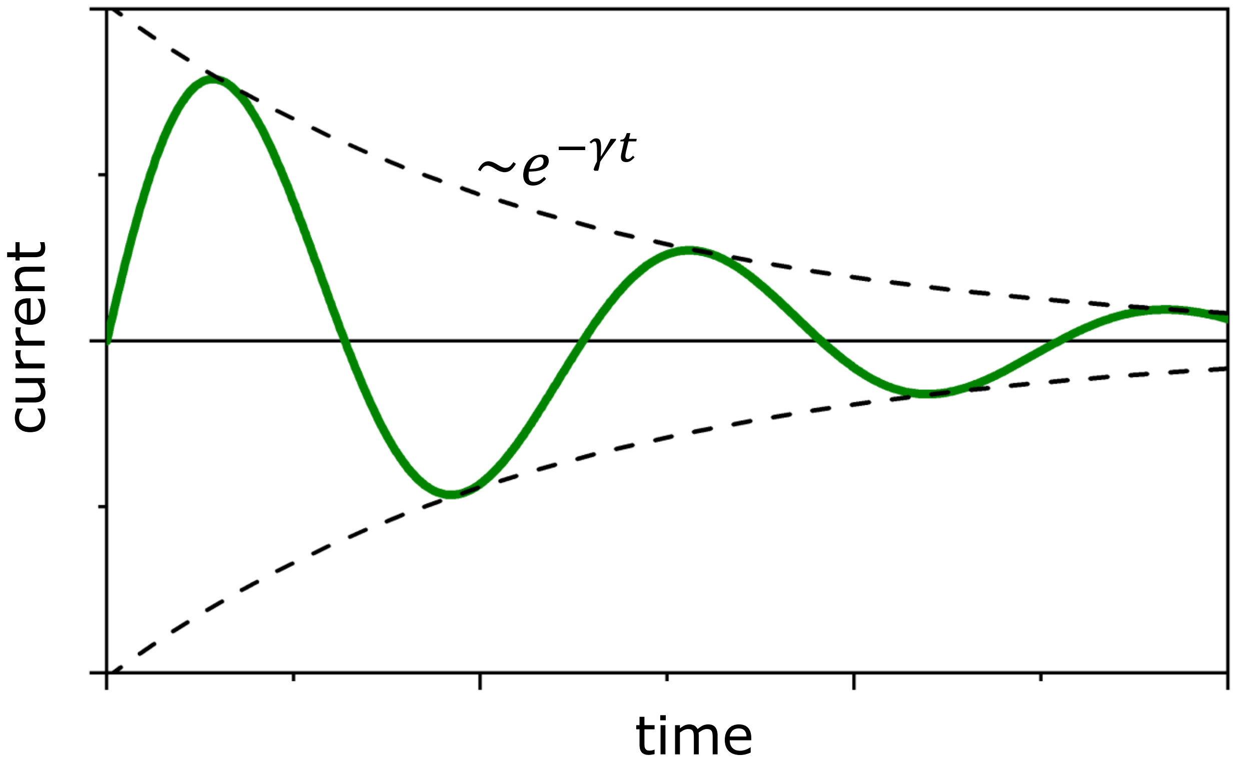
Critically-damped $(\gamma = \omega_0)$

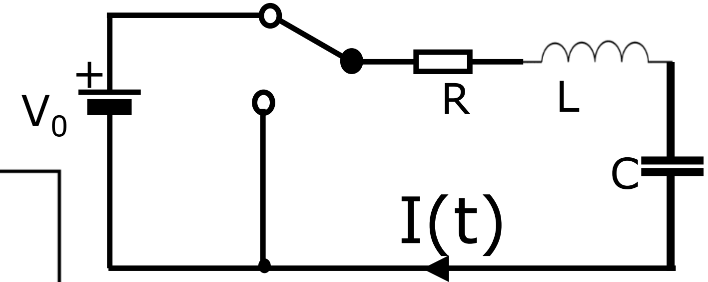




$$\lambda_1 = \lambda_2 = -\gamma$$

Under-damped $(\gamma < \omega_0)$





$$\lambda_{1,2} = -\gamma \pm i\sqrt{\omega_0^2 - \gamma^2}$$

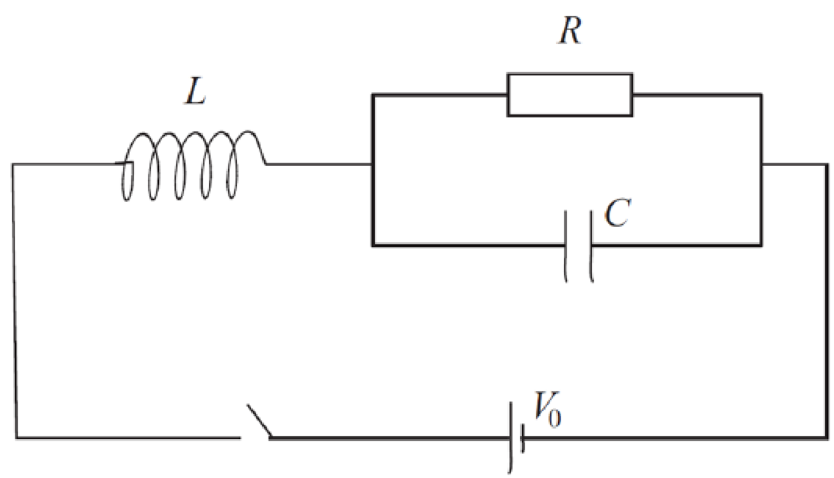
$$I = \frac{V_0}{R}\frac{2}{\sqrt{\omega_0^2/\gamma^2 - 1}} e^{-\gamma t} \sin\left(\sqrt{\omega_0^2 - \gamma^2}\ t\right)$$

Close to ω_0 but not exactly

$$I = V_0\sqrt{\frac{C}{L}}\frac{1}{\sqrt{1 - \gamma^2/\omega_0^2}} e^{-\gamma t} \sin\left(\sqrt{\omega_0^2 - \gamma^2}\ t\right)$$

Exam problem

b) The same network is connected to a battery, voltage V_0, as shown below. At time $t = 0$ the capacitor is uncharged and the switch is closed. By solving an appropriate differential equation, show that the current through the resistor is oscillatory provided $L < 4CR^2$. By considering the boundary conditions at $t = 0$ and as $t \to \infty$, sketch the form of this current as a function of time.



R, L, C circuits

- What if we have more than just one of each?
- What if we don't have switches but have some other input instead?
  - Power from the wall socket comes as a sinusoid, wouldn't it be good to solve such problems?
- **Yes! There is a much better way!**
  - **Stay Tuned for the Unexpected!**