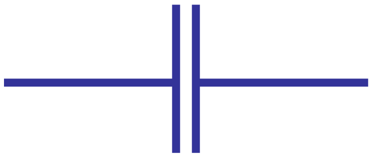
Recap

- Resistors (V=IR), voltage and current sources
- Kirchoff's laws KCL $\sum_{node} I_n = 0$ KVL $\sum_{loop} V_n = 0$
- Techniques
  - Mesh currents
  - Node voltages
  - Superposition
  - Thevenin theorem
  - Norton theorem
- Impedance matching

Today

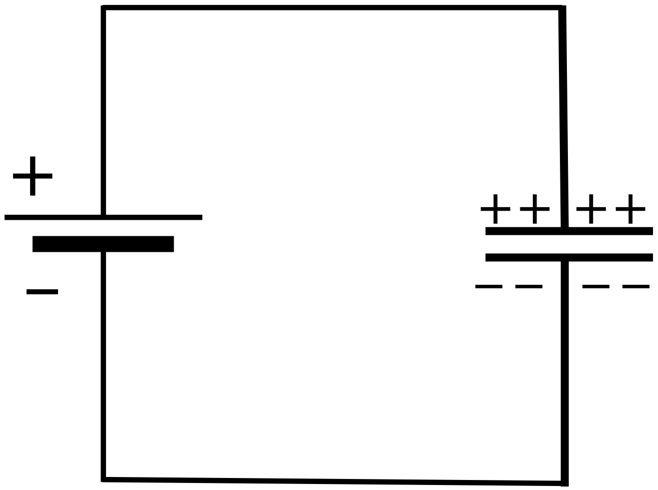
- Capacitors and inductors
- Transient circuits

Capacitors



Capacitors store charge

Definition: $C = \frac{Q}{V}$

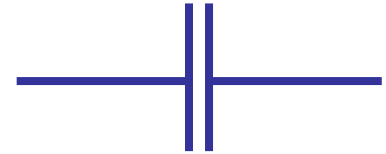


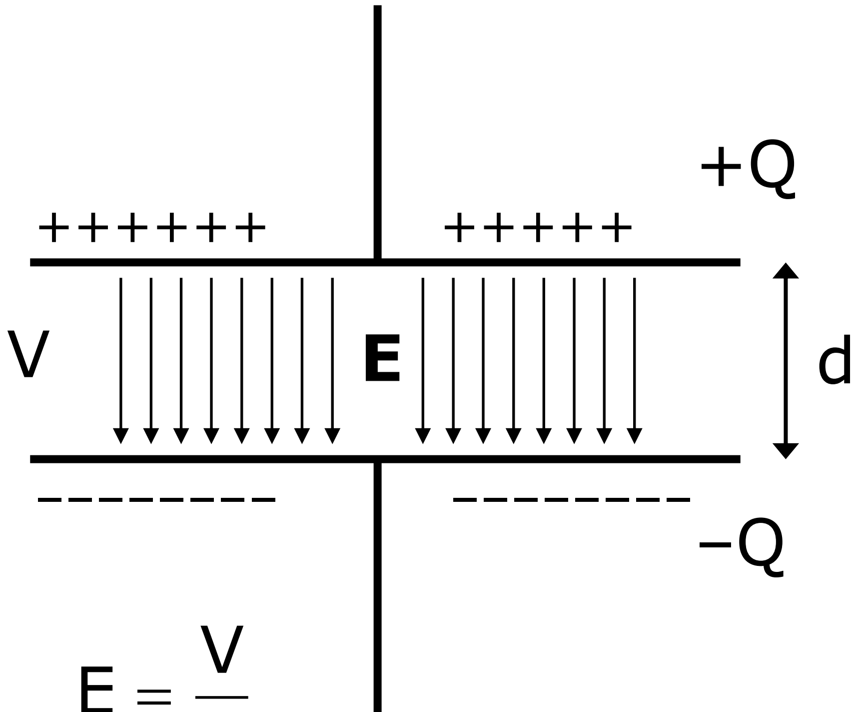
Units: farad [F]=[CV^{-1}]=[AsV^{-1}]

more usually μF, nF, pF

$$Q = CV \qquad I = \frac{dQ}{dt} \longrightarrow I = C\frac{dV}{dt}$$

Parallel plate capacitor





$$E = \frac{V}{d}$$

plate area

$$C = \frac{\varepsilon A}{d}$$

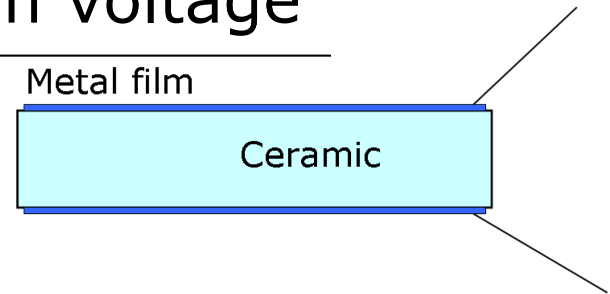
Dielectric permittivity

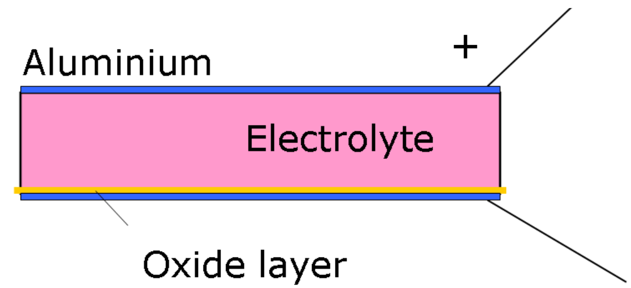
$\varepsilon = \varepsilon_r \varepsilon_0$

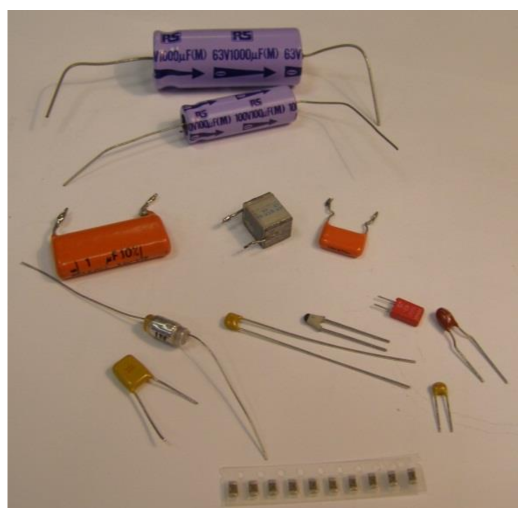
$$\varepsilon_0 = 8.854 \times 10^{-12} CV^{-1}m^{-1}$$

Capacitor types

	Range	Maximum voltage	
Ceramic	1pF–1µF	30kV	
Mica	1pF–10nF	500V	(very stable)
Plastic	100pF–10µF	1kV	
Electrolytic	0.1µF–0.1F	500V	Polarised
Parasitic	~pF		



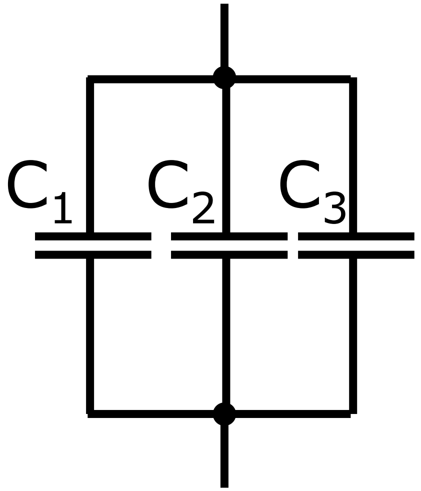




Polarised

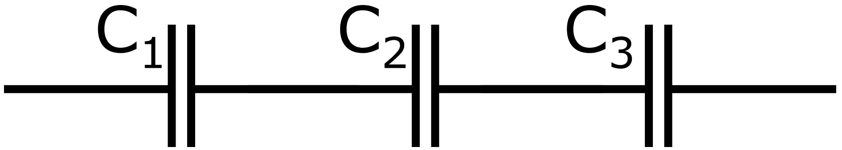
Capacitors

Capacitors in parallel:



$$C_{Total} = \sum_n C_n = C_1 + C_2 + C_3 \ldots$$

Capacitors in series:



$$\frac{1}{C_T} = \sum_n \frac{1}{C_n} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \ldots$$

Capacitors – energy stored

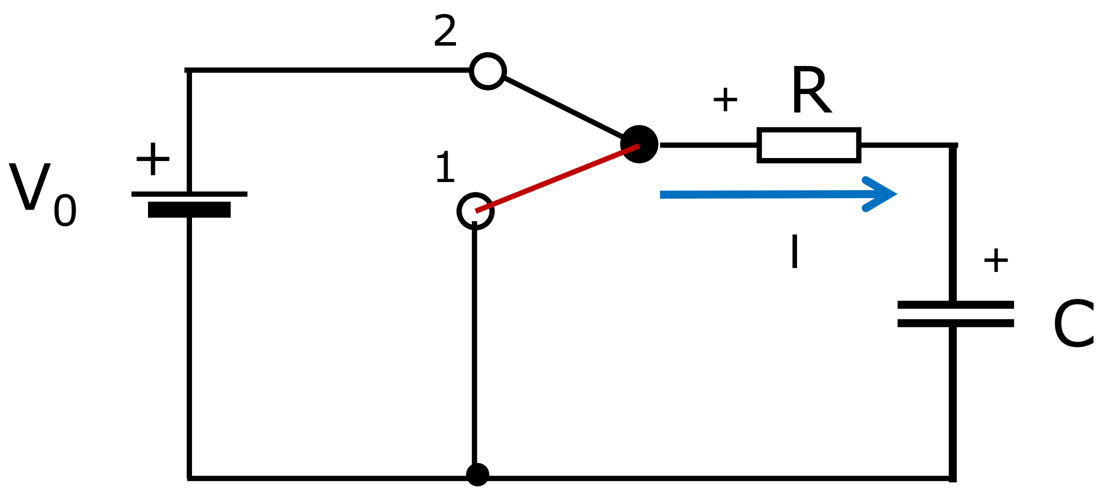
$$W = \int_0^t P(t')dt' = \int_0^t I(t')V(t')dt' = \int_0^t C\frac{dV(t')}{dt'}V(t')dt'$$

$$= C\int_0^V V'\,dV' = \tfrac{1}{2}CV^2$$

$$W = \tfrac{1}{2}CV^2 = \tfrac{1}{2}QV = \frac{Q^2}{2C}$$

Energy stored as electric field

RC circuits



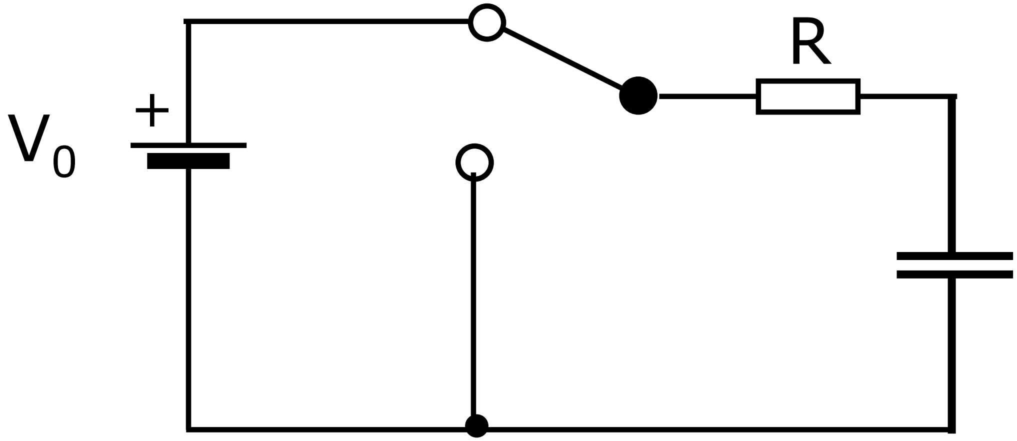
- Switch in Position "1" for a long time.
- Then instantly flips at time t = 0.
- How do we analyse this?

and Apply KVL

ALWAYS start by asking these three questions:

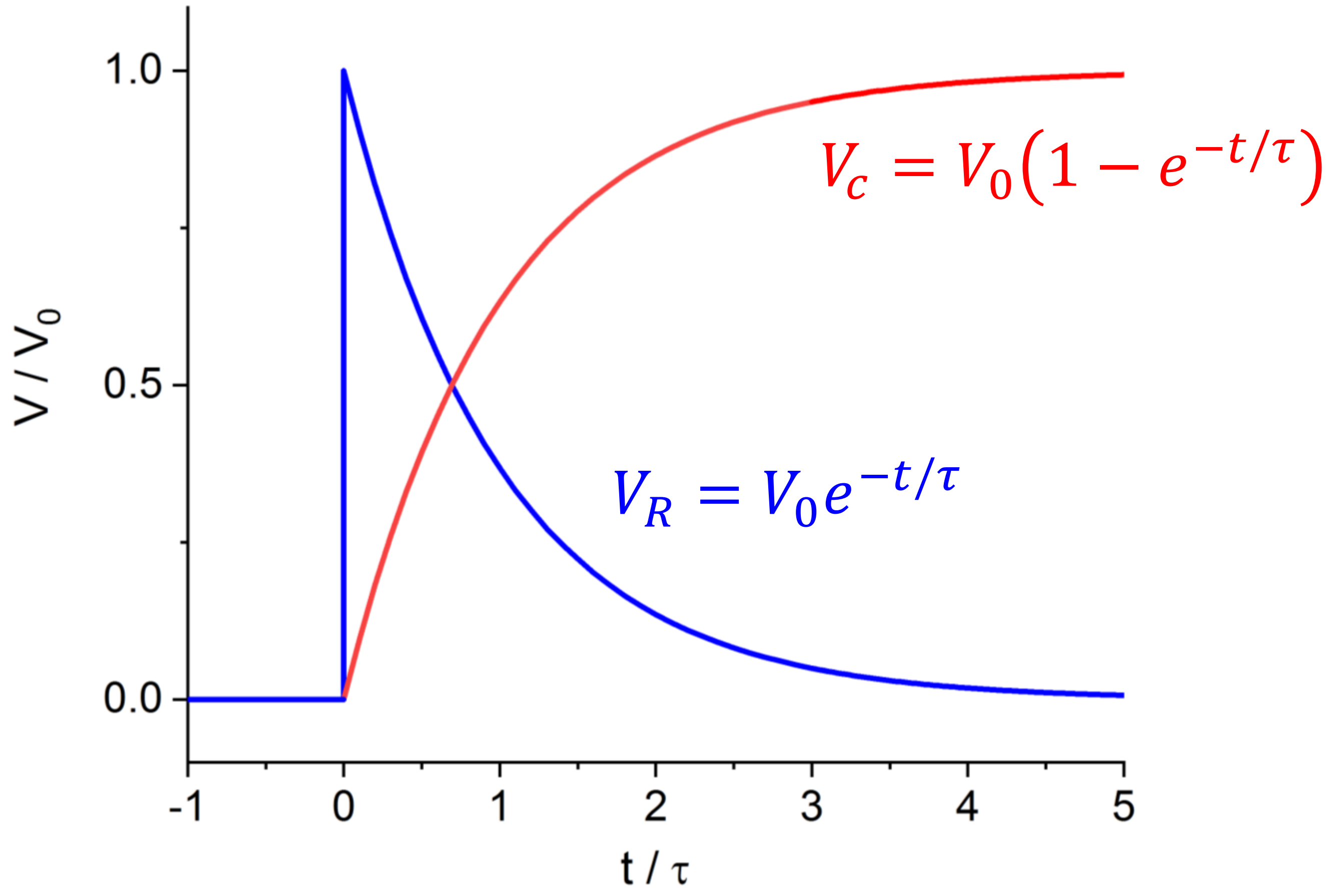
1. What does the Circuit do up until the switch flips? (Switch has been at pos. 1 for a VERY long time. easy)
2. What does the circuit do a VERY long time AFTER the switch flips? (easy)
3. What can we say about the INSTANT after switch flips? (easy if you know the rule)

It is impossible to change the voltage on a capacitor instantly!



$$I(t) = \frac{V_0}{R} e^{-t/\tau}$$

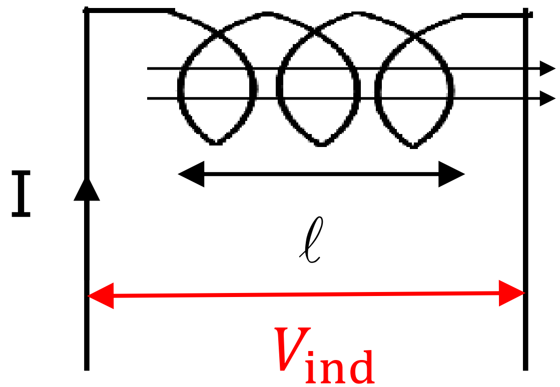
$$\tau = RC$$



Inductors

Oppose a change in current

Solenoid
N turns



$$B = \frac{\mu_0 N I}{l}$$

$$\Phi = \int B.dA = BAN = \frac{\mu_0 N^2 A}{l} I$$

Try to change I:

Electromagnetic induction
- Faraday’s law;
- Lenz’s law - V_{ind} will oppose change

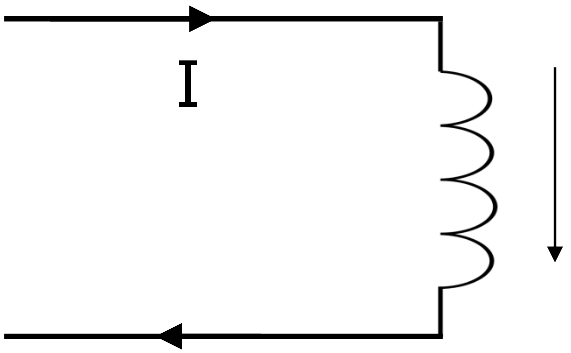
$$V_{\mathrm{ind}} = \frac{d\Phi}{dt} = L\frac{dI}{dt}$$

inductance $$L = \frac{\mu_0 N^2 A}{l}$$

Units: henrys [H] = [VsA^{-1}]

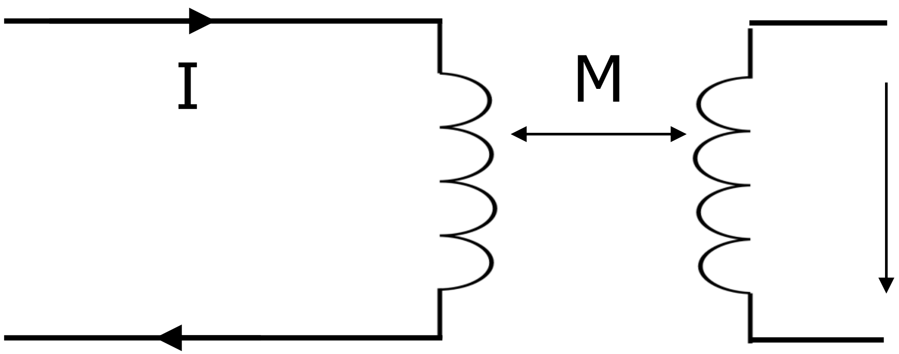
Mutual inductance

Self Inductance



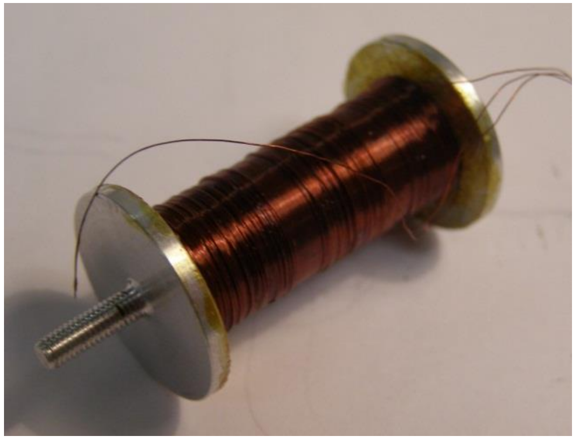
$$V = L\frac{dI}{dt}$$

Mutual Inductance



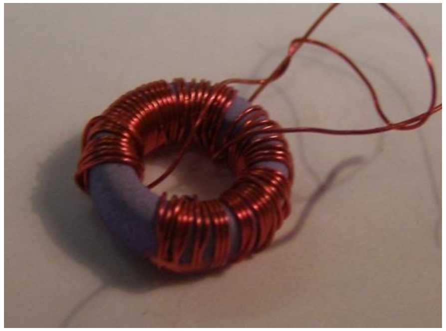
$$V = M\frac{dI}{dt}$$

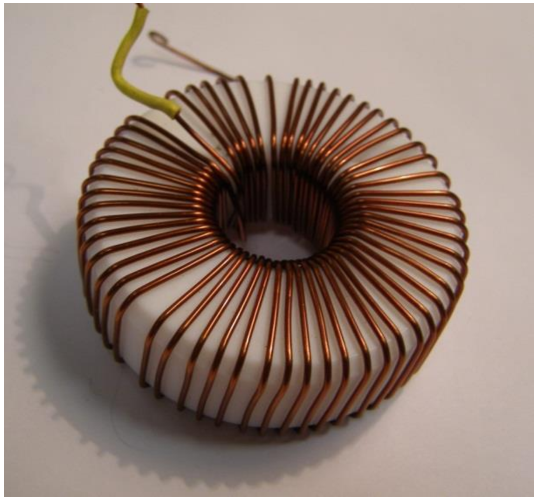
Inductors

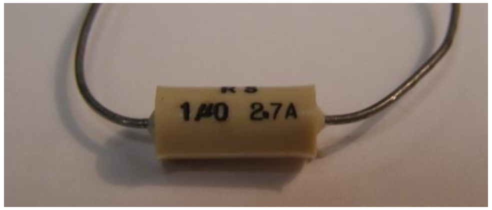


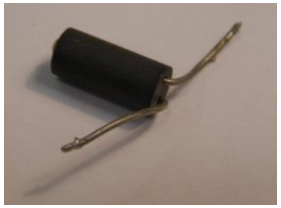
Wire wound coils - air core

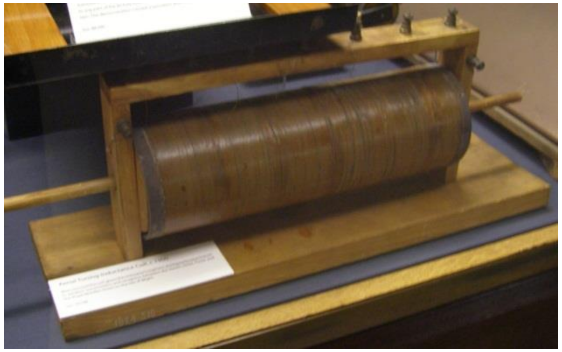
- ferrite core





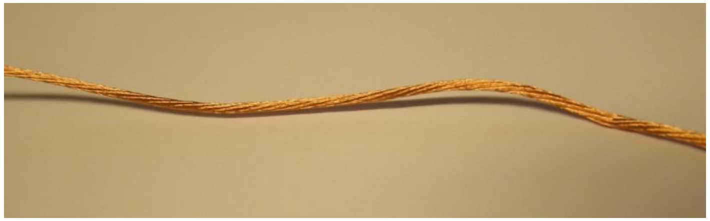






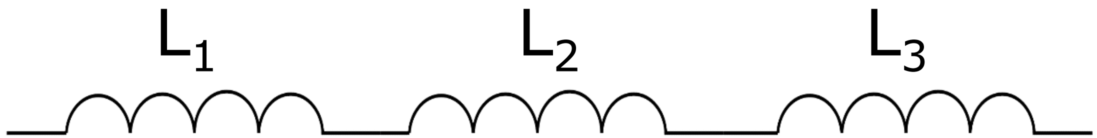
Wire loops

Straight wire



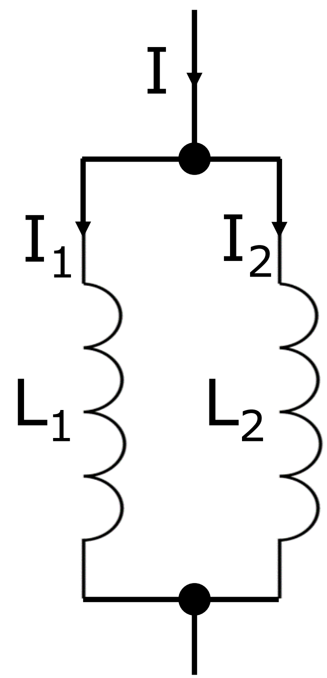
Inductors

Inductors in series:



$$L_{Total} = L_1 + L_2 + L_3 \ldots \qquad L_T = \sum_n L_n$$

Inductors in parallel



$$\frac{1}{L_T} = \sum_n \frac{1}{L_n}$$

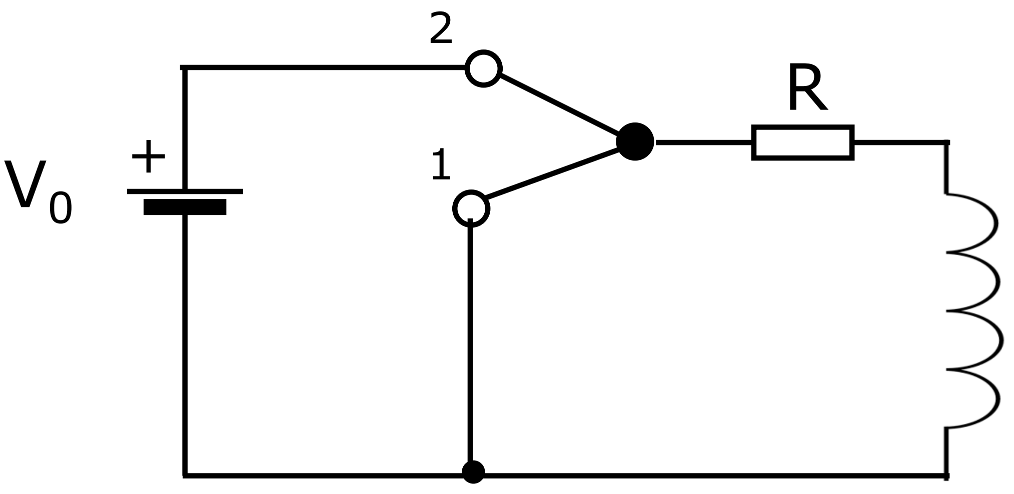
$$= \frac{1}{L_1} + \frac{1}{L_2} + \ldots$$

Inductors – energy stored

$$
\begin{aligned}
W &= \int_0^t P(t')\,dt' \\
&= \int_0^t I(t')V(t')\,dt' \\
&= \int_0^t I(t')L\frac{dI(t')}{dt}\,dt' \\
&= L\int_0^I I\,dI' \\
&= \tfrac{1}{2}LI^2
\end{aligned}
$$

Energy stored as magnetic field

RL circuits



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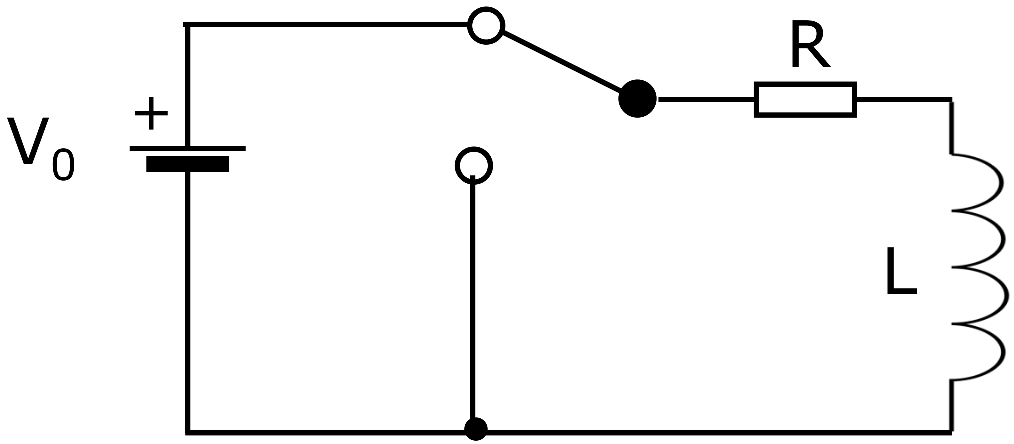
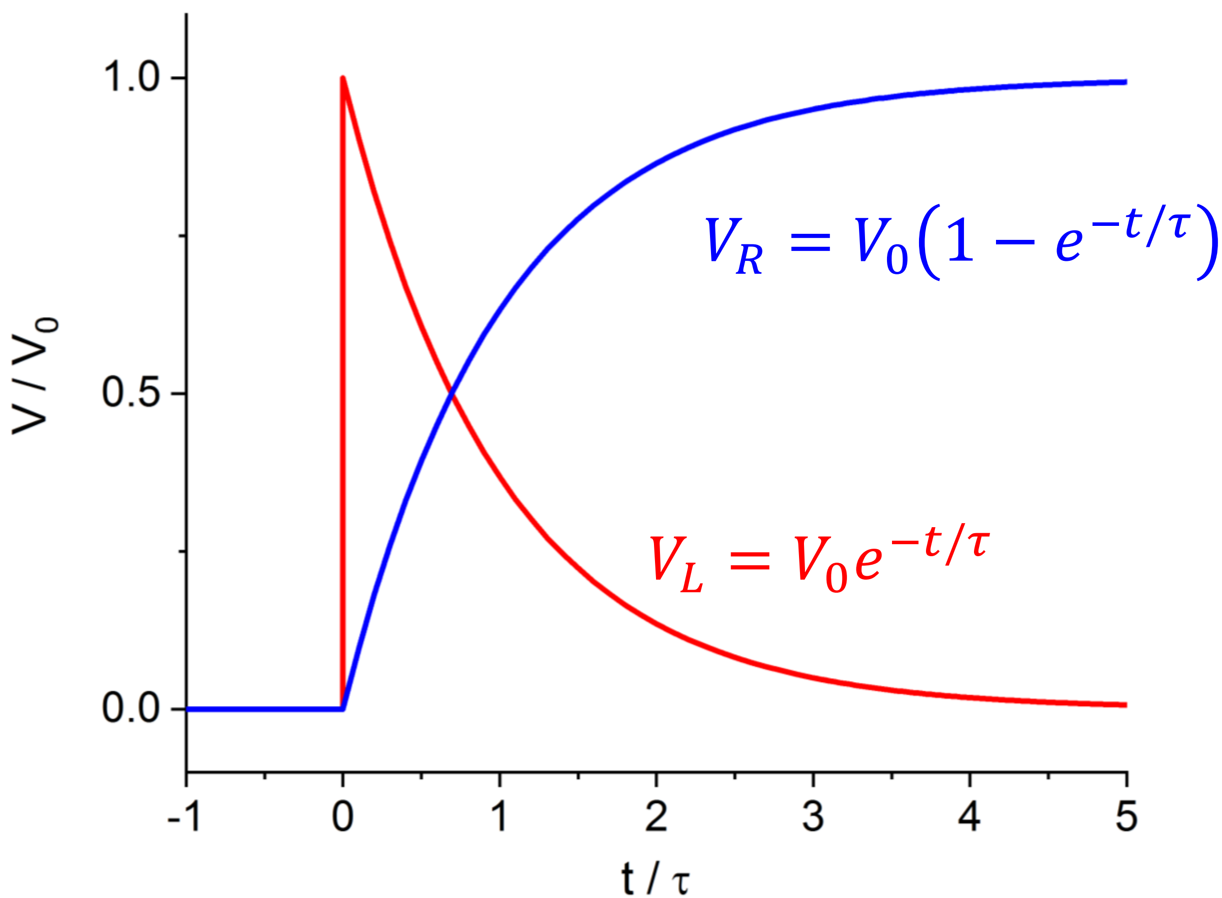
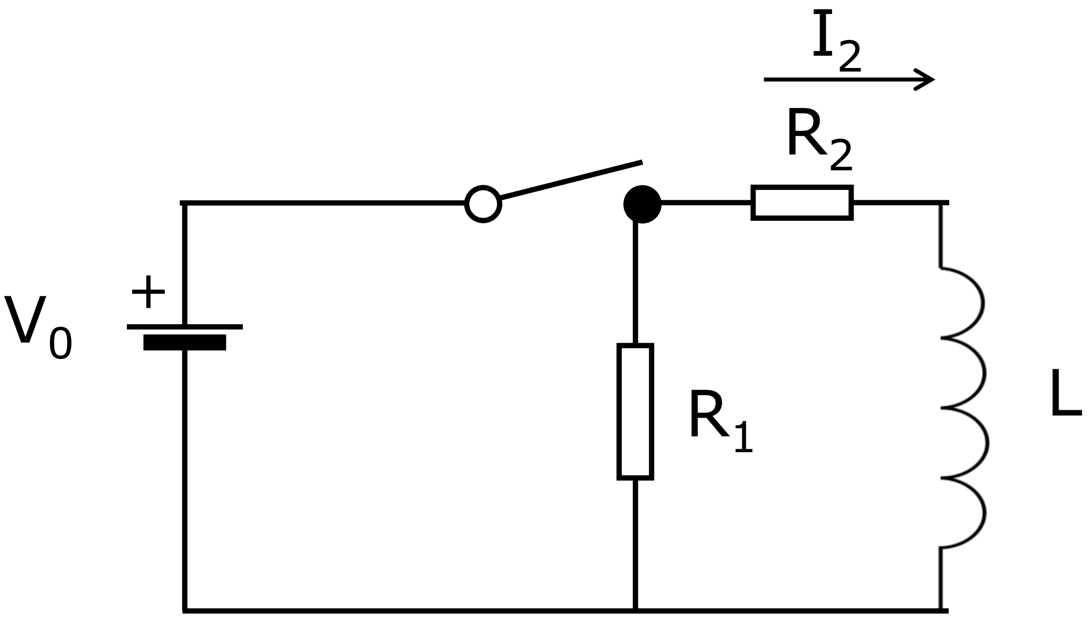
It is impossible to change the current on a inductor instantly!



$$I(t) = \frac{V_0}{R}\left(1 - \exp\left(-\frac{t}{\tau}\right)\right)$$

$$\tau = \frac{L}{R}$$





t<0 switch closed I_2=?
t=0 switch opened
t>0 $I_2(t)$=?

voltage across R_1=?
Let R_1 get very large:
What happens to V_L?

Do through if time otherwise try at home