1st-year Circuits
Lectures 1 and 2

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1 Basic definitions

- **Charge:** determines the strength of the electromagnetic force. Measured in Coulombs (C); quantised in units of the electron charge $e = 1.602 \times 10^{-19}$ C.
- **Potential difference:** energy to move unit charge from A to B; $V_{\text{AB}} = V_{\text{B}} - V_{\text{A}} = W/Q = -\int_{\text{A}}^{\text{B}} \boldsymbol{E.ds}$. Measured in volts(V), 1 V = 1 J/C.
- **Current:** rate of flow of charge $I = \frac{dQ}{dt}$. Measured in Amperes (A), 1 A = 1 C/s.
- **Power:** work done per unit time; $P = \frac{V dQ}{dt} = IV$. Measured in Watts, 1 W = 1 J/s.
- **Ohm's Law:** For a component obeying Ohm's law the voltage difference across it is proportional to the current flowing through it; $V = IR$ where R is the resistance measured in Ohms (Ω).
- An **ideal voltage source** maintains a fixed voltage independent of the current is supplying. The voltage is fixed independent of what is connected to it. A real voltage source can be thought of as an ideal voltage source *in series* with an internal resistance (the smaller the internal resistance the more ideal the voltage source).
- An **ideal current source** supplies a fixed current independent of the voltage required to do so. The current is fixed independent of what is connected to it. A real current source can be thought of as an ideal current source *in parallel* with an internal resistance (the larger the internal resistance the more ideal the current source).

2 Kirchoff's laws

1. **Kirchoff's current law (KCL).** The sum of all currents at a node is zero: $\sum_{\text{node}} I_n = 0$. This is consequence of conservation of charge.
2. **Kirchoff's voltage law (KVL).** Around a closed loop the net change of potential is zero: $\sum_{\text{loop}} V_n = 0$. This results from conservation of energy (in addition to conservation of charge).

3 Passive sign convention

When applying Kirchoff's laws it is easy to get in a muddle with signs, using passive sign convention helps you avoid this. The principle of the sign convention is as follows:

- Voltage and current sources have a '+' sign on the terminal that the current (normally) leaves.
- For all parts of the circuit, choose the direction of current that you are defining as positive.
- For any passive component (e.g. Resistor, Capacitor, Inductor) make a '+' sign on the side of the component that the current is *entering*.
- Now apply Kirchoff's laws. For KVL as you go round the loop a '–' to '+' component has a minus sign and a '+' to '–' component has a plus sign.

4 Methods for solving circuits

In terms of solving simple circuits involving just voltage and current sources and resistors we already have everything we need. However, there are some tricks which can sometimes help you get to the answer more quickly (or with less algebra).

- **Mesh currents:** (i) Label loop currents in all internal loops. (ii) Apply KVL around each loop (taking into account that the current in components that are shared between loops will be the sum or difference of the relevant loop currents). (iii) Solve for the loop currents and use these to obtain the required currents/voltages.
- **Node voltages:** (i) Choose a ground node and label all other nodes. (ii) Apply KCL to each node using the fact that the currents can usually be simply written down as a voltage difference divided by a resistance. (iii) Solve for the node voltages and use these to obtain the required currents/voltages.
- **Superposition.** A linear circuit with more than one voltage/current source can be analysed by considering one source at a time; the total response (i.e. currents or node voltages) of the circuit will be the sum of the individual responses. When analysing just one source, all other voltage sources should be replaced with a wire (short circuit) and all other current sources replaced with a break (open circuit).
- **Thevenin's theorem.** Any linear network of voltage/current sources and resistors can be written as an equivalent circuit of a perfect voltage source (of voltage V_{eq}) in series with a resistor (R_{eq}). V_{eq} is given by the open-circuit output voltage and $R_{\mathrm{eq}} = V_{\mathrm{eq}}/I_{\mathrm{eq}}$ where I_{eq} is the short-circuit output current.
- **Norton's theorem.** Any linear network of voltage/current sources and resistors can be written as an equivalent circuit of a perfect current source (of current I_{eq}) in parallel with a resistor (R_{eq}). I_{eq} is given by the short circuit output current and $R_{\mathrm{eq}} = V_{\mathrm{eq}}/I_{\mathrm{eq}}$ where V_{eq} is the open circuit output voltage. Note the simple relationship between the equivalent Thevenin and Norton circuits.

5 Choosing your impedance: measurements and power extraction

- When measuring a voltage the impedance (resistance) of the voltmeter should be $\gg R_{\mathrm{eq}}$ where R_{eq} is the equivalent resistance of the circuit (in a Thevenin sense).
- When measuring a current the impedance (resistance) of the ammeter should be $\ll R_{\mathrm{eq}}$ where R_{eq} is the equivalent resistance of the circuit (in a Norton sense).
- The maximum power transfer from a power source (with internal resistance R_{in}) to a load (with resistance R_{load}) will be when the $R_{\mathrm{load}} = R_{\mathrm{in}}$.