CP2 Circuit Theory

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https://www2.physics.ox.ac.uk/contacts/people/robertsmith ('Teaching' tab):

- Problem set
- Synopsis and reading list
- Lecture summaries
- Slides

Thanks to
Todd Huffman

But do make your own notes because: (i) it is helpful for you to learn, (ii) I will say extra things, (iii) I will do some stuff on the blackboard.

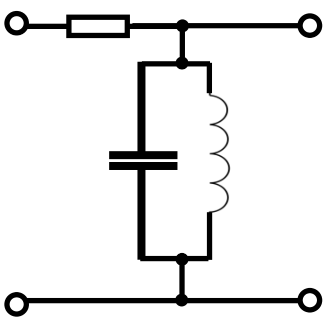
Why study circuit theory?

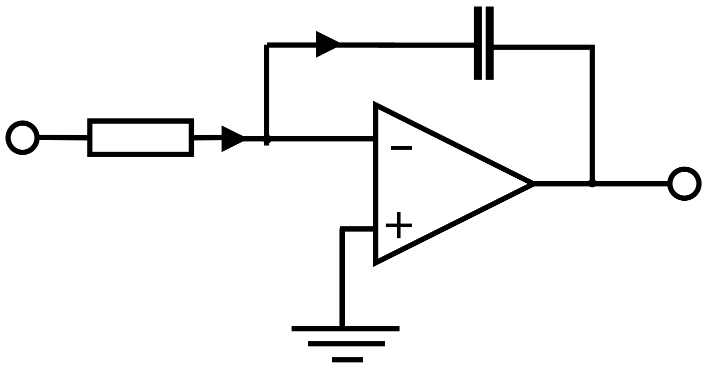
- Foundations of electronics: analogue circuits, digital circuits, computing, communications…
- Scientific instruments: readout, measurement, data acquisition…
- Physics of electrical circuits, electromagnetism, transmission lines, particle accelerators, thunderstorms…
- Not just electrical systems, also thermal, pneumatic, hydraulic circuits, vacuum, control theory

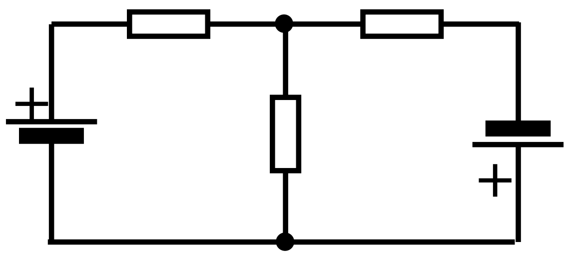
Aims of this course:

Understand basic circuit components (resistors, capacitors, inductors, voltage and current sources, op-amps)

Analyse and design simple linear circuits (considering both DC, AC and transient response)







Circuit Theory: Synopsis

- Basics: voltage, current, Ohm's law, ideal voltage and current sources…
- Kirchoff's laws and tricks for solving: mesh currents, node voltage, Thevenin and Norton's theorem, superposition…
- Capacitors:
- Inductors:

  } Stored energy, RC, RL and LCR transient circuits.
- AC theory: complex notation, phasor diagrams, RC, RL, LCR circuits, resonance, bridges…
- Op amps: ideal operational amplifier circuits…

Mathematics required

- Differential equations $$\frac{d^2I}{dt^2} + \frac{R}{L}\frac{dI}{dt} + \frac{1}{LC}I = 0$$
- Complex numbers $$V(t) = V_0 e^{j\omega t}$$

$$I = \frac{V}{Z} \qquad Z = R + jX$$

- Linear equations

$$V_0 - I_1R_1 - (I_1 - I_2)R_3 = 0$$

$$(I_1 - I_2)R_3 - I_2R_2 + 2 = 0$$

Covered by Complex Nos & ODEs / Vectors & Matrices lectures

Charge, voltage, current, power

Charge: determines strength of electromagnetic force
quantised: $e = 1.602 \times 10^{-19} C$ [coulombs]

Potential difference: $V = V_B - V_A$ [volts]

Energy to move unit charge from A to B in electric field

$$V = -\int_A^B \mathbf{E} \cdot \mathbf{ds} \qquad \mathbf{E} = -\nabla V$$

$$W = -\int_A^B Q\mathbf{E} \cdot \mathbf{ds}$$

Current: rate of flow of charge

$$I = \frac{dQ}{dt} = nAve$$

- Charge Q=e
- No. electrons/unit vol
- Cross-section area of conductor
- Drift velocity

[amps]

Power: work done per unit time

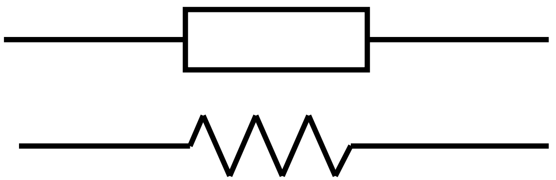
$$P = \frac{dW}{dt} = \frac{d(QV)}{dt} = IV$$

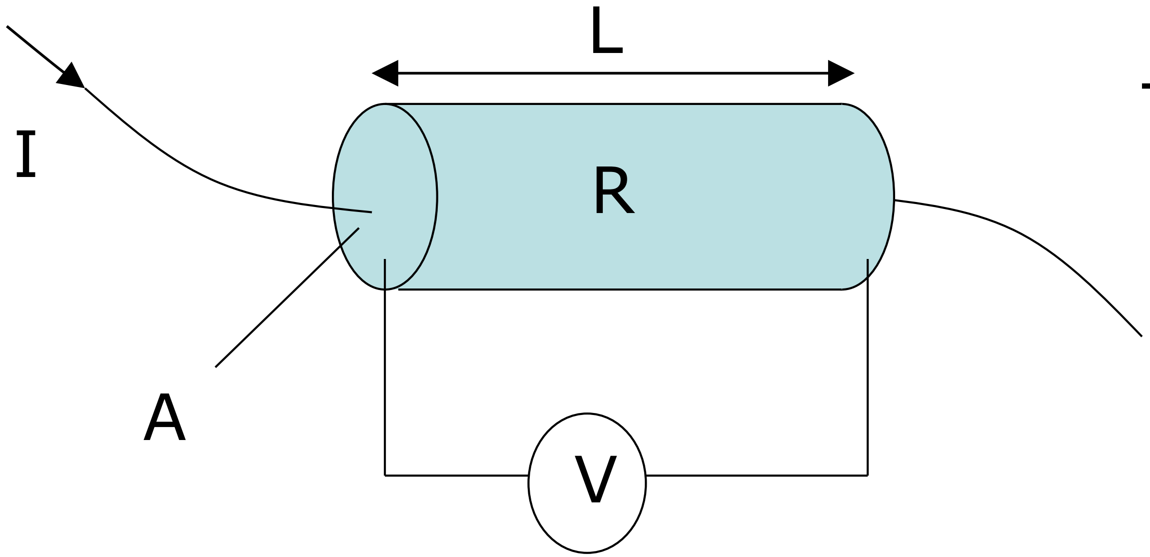
[watts = J/s]

Ohm's law

Voltage difference ∝ current

Resistor symbols:





$$V = IR$$

R=Resistance Ω[ohms]

$$R = \frac{\rho L}{A}$$

ρ=Resistivity Ωm

Resistivities

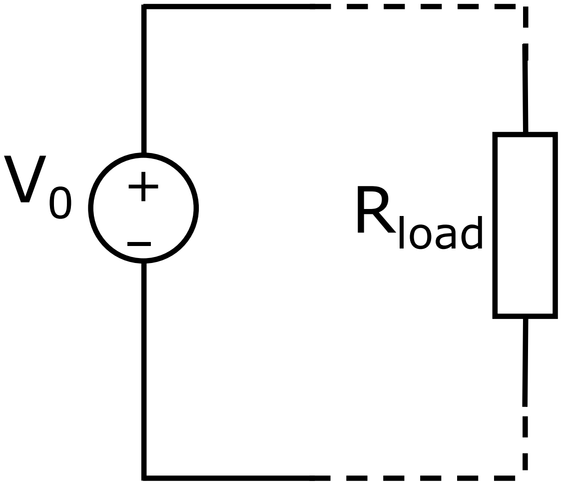
Silver	$1.6\times10^{-8}\ \Omega m$
Copper	$1.7\times10^{-8}\ \Omega m$
Manganese	$144\times10^{-8}\ \Omega m$
Distilled water	$5.0\times10^{3}\ \Omega m$
PTFE (Teflon)	$\sim10^{19}\ \Omega m$

Conductance [seimens] $$g = \frac{1}{R}$$

conductivity [seimens/m] $$\sigma = \frac{1}{\rho}$$

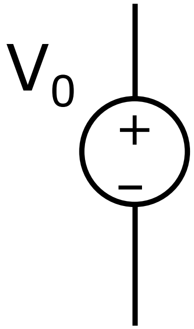
Power dissipation by resistor: $$P = IV = I^2R = \frac{V^2}{R}$$

Voltage source



Ideal voltage source: supplies V_0 independent of current

Symbol:



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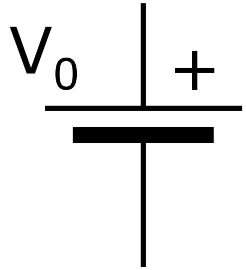
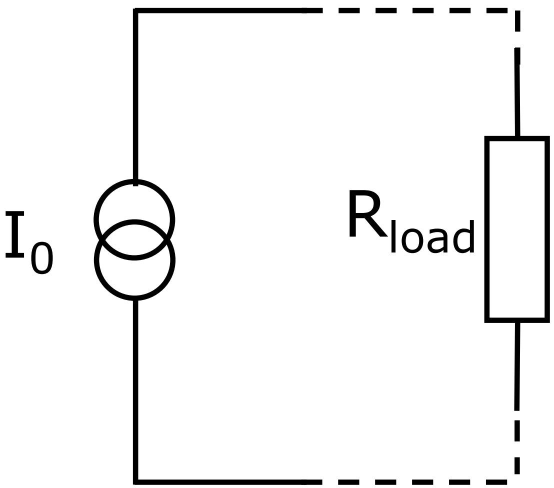
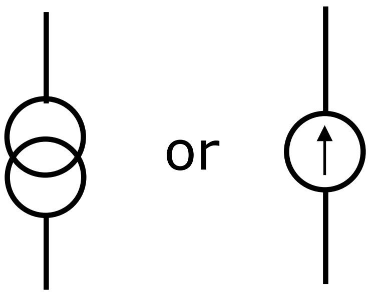


Constant current source



Ideal current source: supplies I_0 amps independent of voltage

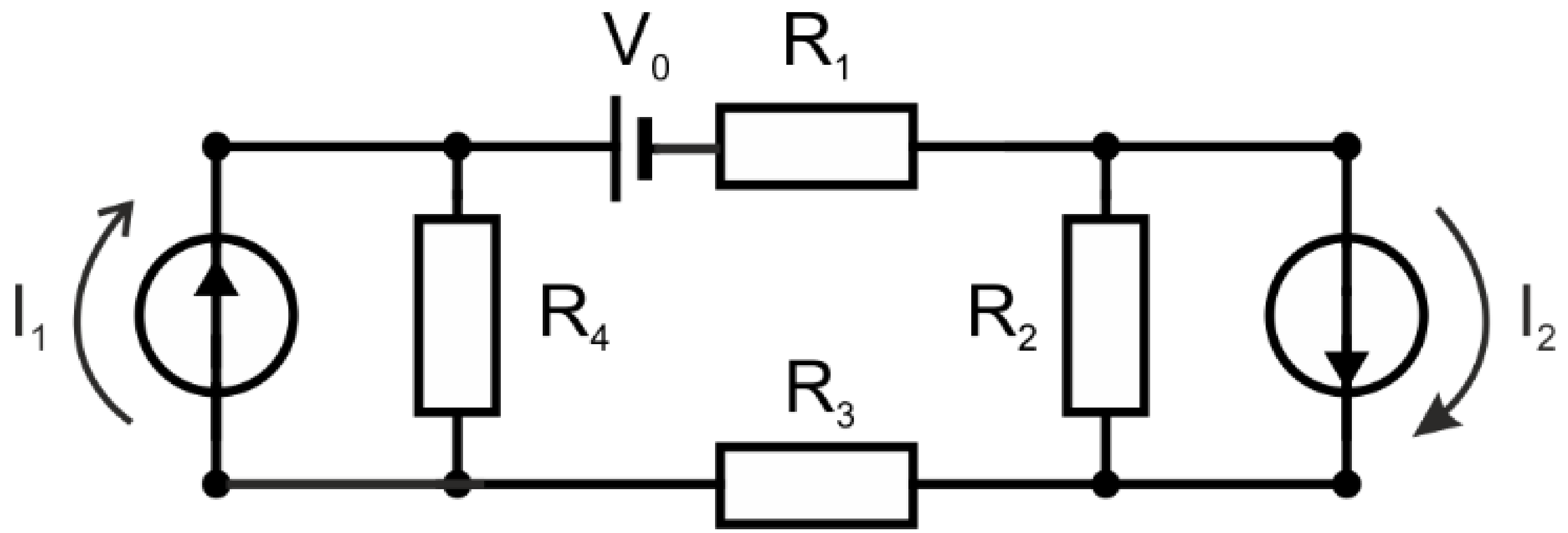
Symbol:



or

Circuits

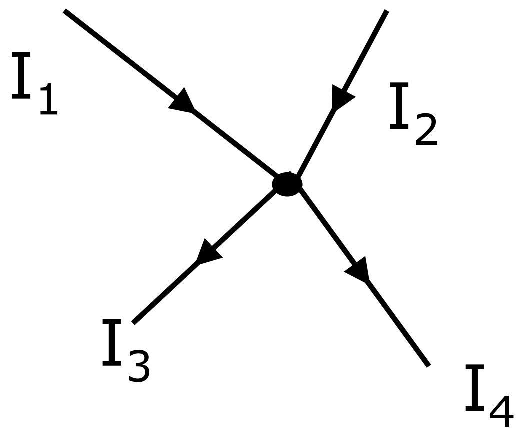
Out of these components we can make (arbitrarily complicated) circuits:



But how do we work out what they do…

Kirchoff’s Laws

I Kirchoff’s current law: Sum of all currents at a node is zero



$$I_1+I_2-I_3-I_4=0$$

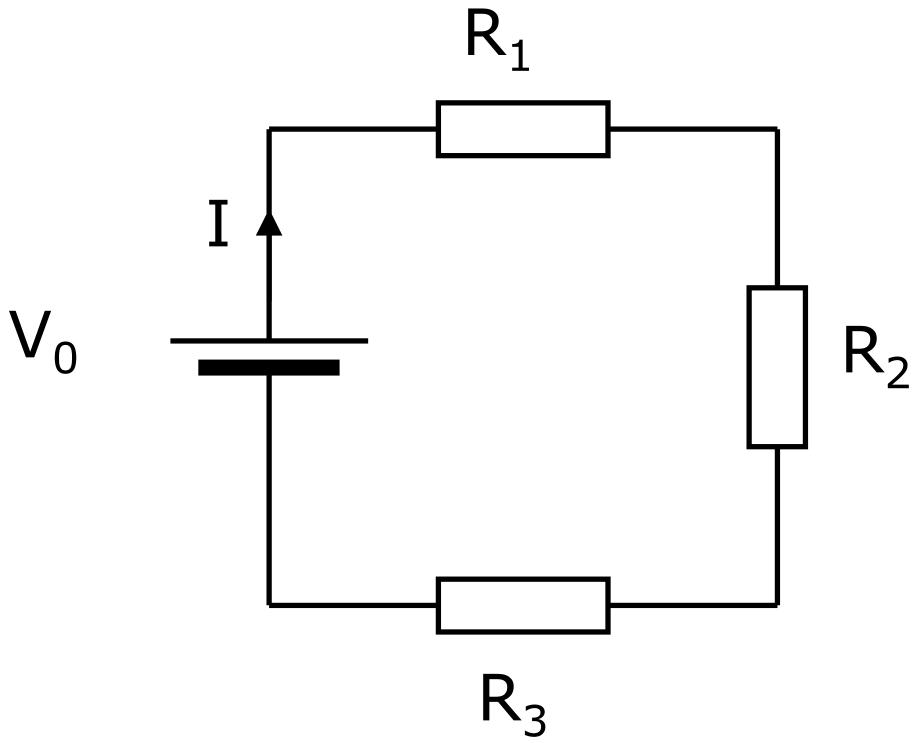
$$\sum I_n = 0$$ KCL

(conservation of charge)

It does not matter whether you pick “entering” or “leaving” currents as positive.

BUT keep the same convention for all currents on one node!

II Kirchoff's voltage law: Around a closed loop the net change of potential is zero (Conservation of Energy)

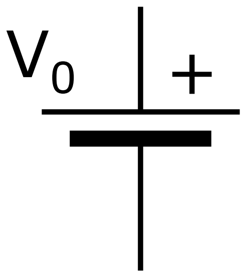


$$\sum V_n = 0$$

KVL

But what about the signs of V_n?

Passive Sign Convention



Sources have a + sign on the terminal the current normally leaves

Where do we put the + sign on a resistor (or other passive component)?

$$V = IR$$

Procedure

- Choose direction of current you are defining as positive.
- For **any passive component** make a + sign on the side of that component that the current is ***entering***.
- When applying KVL, as you go round a loop a – to + component has a negative voltage and a + to – component has a positive voltage.

Learn it; **Live it**; **Love it**!

Passive Sign Convention

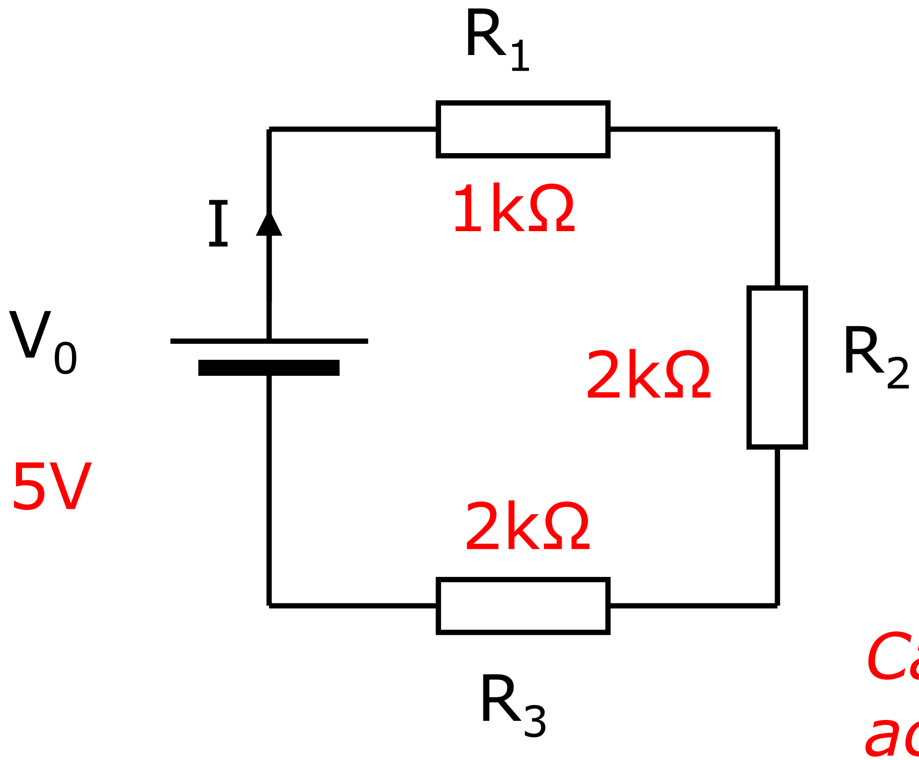
The ‘convention’ is related to how power input/output from a circuit is defined:

- Power flowing out of a circuit into an electrical component is defined as positive.

- Power flowing into a circuit from an electrical component is defined as negative.

Power conservation $$\sum I_n V_n = 0$$

II Kirchoff's voltage law: Around a closed loop the net change of potential is zero (Conservation of Energy)

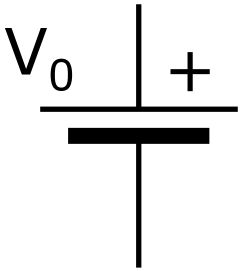


$$\sum V_n = 0$$

Calculate the voltage across R_2

Show on blackboard

Passive Sign Convention



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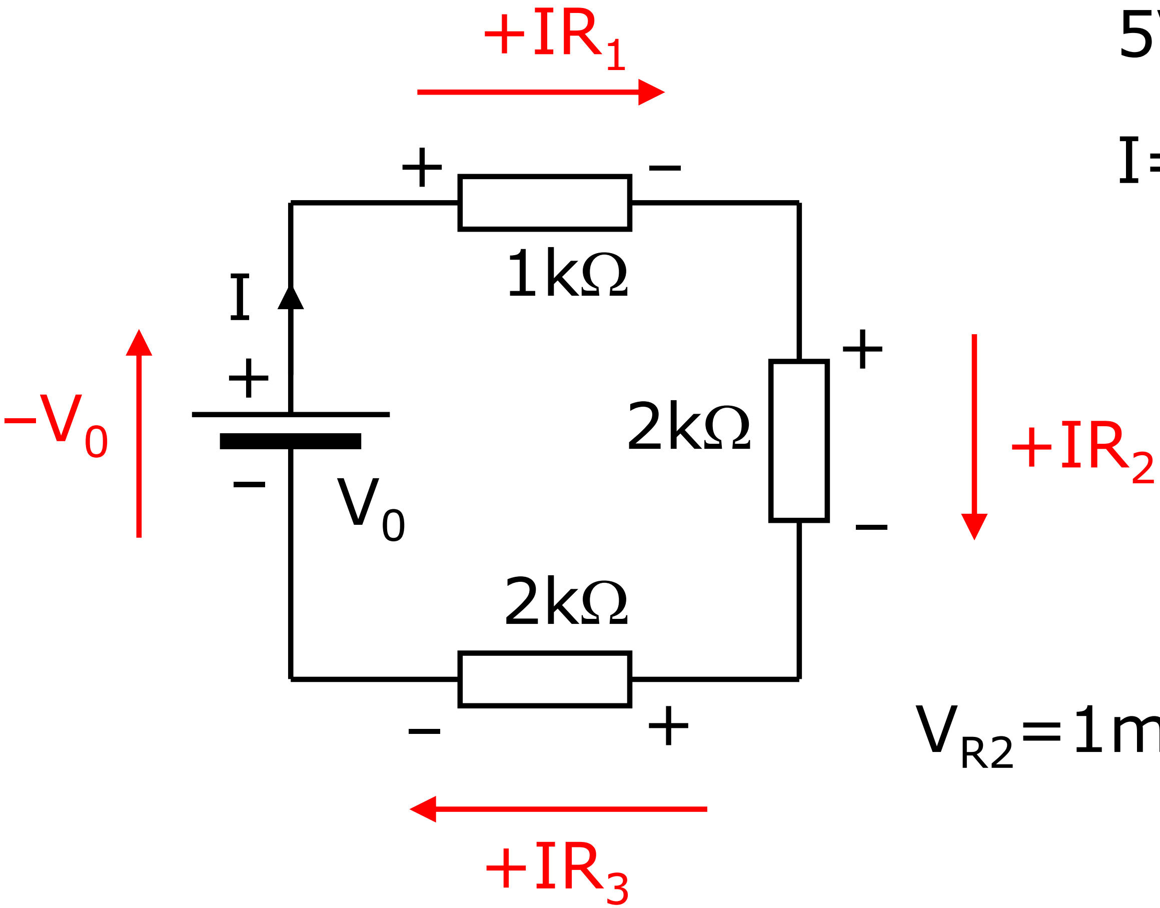
Learn it; **Live it**; **Love it**!

Kirchoff's voltage law: $\sum V_n = 0$

$-V_0 + IR_1 + IR_2 + IR_3 = 0$



$5V = I(1+2+2)k\Omega$

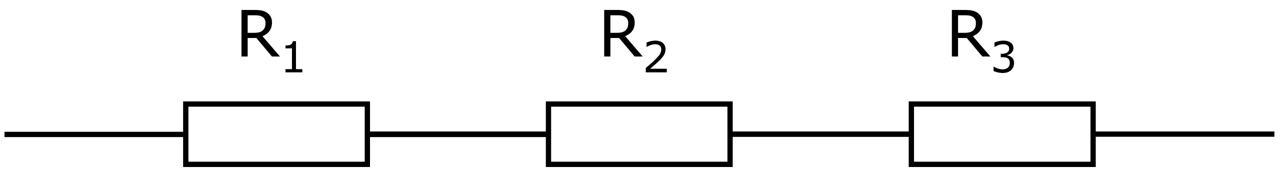
$I = 1mA$

$V_{R2} = 1mA \times 2k\Omega = 2V$

Series / parallel circuits

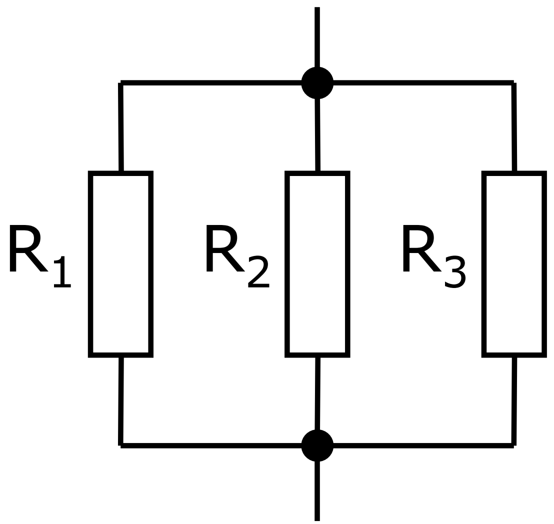
Show on blackboard

Series / parallel circuits



$$R_T = \sum_n R_n$$

Resistors in series: $R_{Total} = R_1 + R_2 + R_3 \ldots$

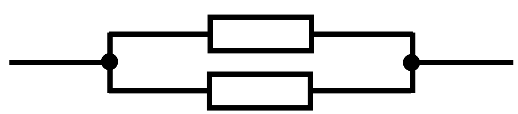


Resistors in parallel

$$\frac{1}{R_T} = \sum_n \frac{1}{R_n}$$

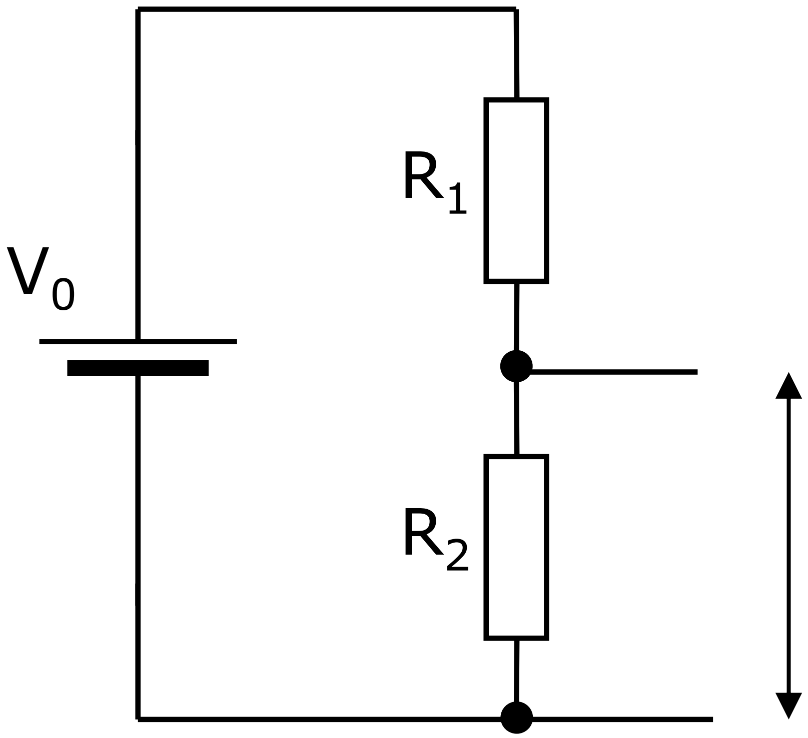
$$= \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \ldots$$

Two parallel resistors:



$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

Potential divider

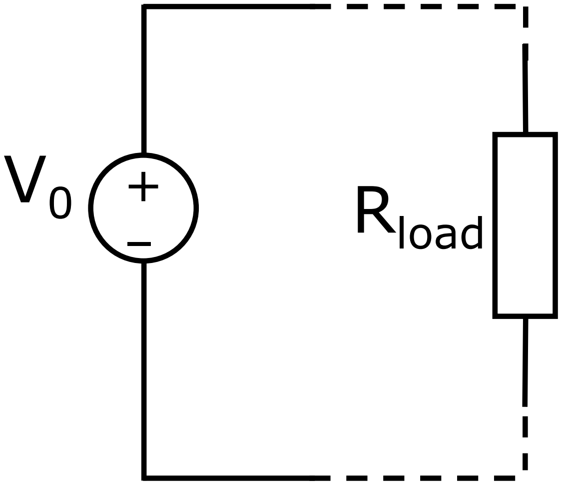


$$\frac{V_0 R_2}{R_1 + R_2}$$

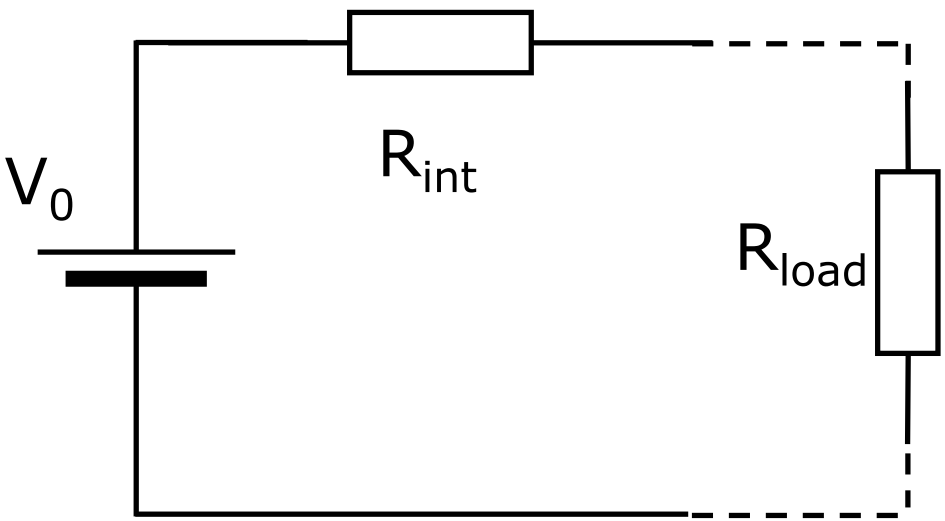
USE PASSIVE SIGN CONVENTION!!!

Show on blackboard

Voltage source



Ideal voltage source: supplies V_0 independent of current

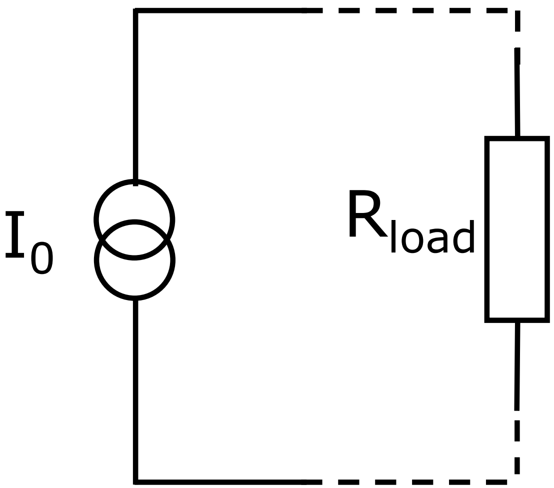


Real voltage source:

$$V_{\text{load}} = V_0 - IR_{\text{int}}$$

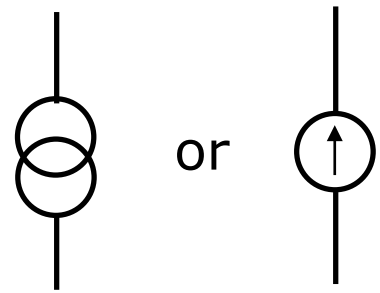
$$V_{\text{load}} = V_0 \times \frac{R_{load}}{R_{\text{load}} + R_{\text{int}}}$$

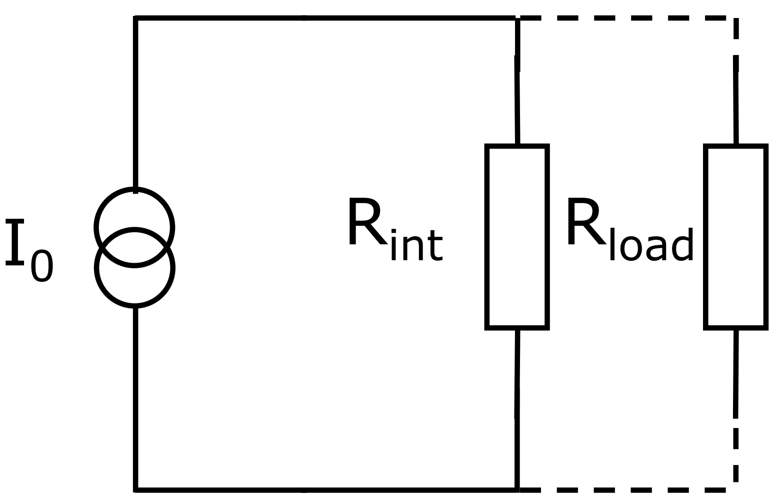
Constant current source



Ideal current source: supplies I_0 amps independent of voltage

Symbol:





Real current source:

$$I_{\mathrm{load}} = I_0 - \frac{V}{R_{\mathrm{int}}}$$

$$I_{\mathrm{load}} = I_0 \times \frac{R_{\mathrm{int}}}{R_{\mathrm{load}} + R_{\mathrm{int}}}$$

End of Lecture 1