C2: Quantum information basics

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Lecture 4

5.6 Ramsey fringes and spin echoes

Below we discuss two specific well known gate (operation) sequences.

5.6.1 Ramsey sequence

If we wanted to measure the energy gap ω_0 in a two level system how could we do it? This is an important question for many topics ranging from atomic clocks to quantum computing. One approach would be to use a single pulse of radiation and look for the peak response of the system when driving with a pulse weight $Vt \le \pi$. This method (often called the Rabi method) gives a peak at $\delta = 0$ of width $\approx V$. An alternative method developed by Ramsey is to instead apply two pulses separated by a time T during which the system undergoes free evolution. The simplest implementation is to use $\pi/2$ rotations around the x–axis:

- starting in the $|0\rangle$ state,
- the first $\pi/2-$pulse brings the Bloch vector to the equator,
- during free evolution (for time T) the Bloch vector precesses around the z–axis at ω_0,
- the second $\pi/2-$pulse effectively maps y onto z,
- the populations, which oscillate with T at ω_0, are measured.

This method not only has increased accuracy but also has several advantages in terms of systematic effects — for example, the applied field might actually change the level-spacing which would corrupt the Rabi measurement but not the Ramsey one as the system undergoes free evolution during the interrogation time T.

The effect of a Ramsey sequence can also be worked out more formally using the propagators (see Eq.(9)) for a $\pi/2-$pulse and for free evolution over a time T:

$$U_{\pi/2} = \frac{1}{\sqrt{2}}(\sigma_0 - i\sigma_x)R, \qquad U_{\text{evolve}}(T) = \cos(\omega_0 T/2)\sigma_0 + i\,\sin(\omega_0 T/2)\sigma_z. \tag{28}$$

where brings us back out of the rotating frame. The entire evolution is given by $U_{\pi/2}\, U_{\text{evolve}}(T)\, U_{\pi/2}$ which acting on $|0\rangle$ gives (up to a global phase):

$$|\psi\rangle = \begin{pmatrix} \sin(\omega_0(T+\tau)/2) \\ -\cos(\omega_0(T+\tau)/2) \end{pmatrix}. \tag{29}$$

Hence the amplitudes oscillate at $\omega_0/2$ and so the populations oscillate at ω_0.

Note that here we have assumed that the $\pi/2$ pulse has an identical phase to the first one. If the driving field continues to oscillate between pulses and the phase is not reset then the entire operation can be treated in the rotating frame (the frame rotating with the field) which instead gives (up to a global phase):

$$|\psi\rangle = \begin{pmatrix} \sin(\delta T/2) \\ -\cos(\delta T/2) \end{pmatrix}. \tag{30}$$

5.6.2 Spin echoes

A spin-echo is a technique which makes it possible to cancel the evolution under the background Hamiltonian. This is useful if you want to maintain a given state while waiting for some time T or to prevent an ensemble of qubits each with slightly different ω_0 from dephasing. The basic idea is to apply π rotation around the x–axis (i.e. a X or NOT gate) halfway through the evolution (i.e. at $T/2$). This can be visualized using the Bloch sphere and can also be formally shown using propagators:

$$U_{\text{total}} = U_{\text{evolve}}(T/2)\ U_{\text{X}}\ U_{\text{evolve}}(T/2) = U_{\text{X}} \tag{31}$$

where we have used U_{evolve} from Eq.(28) and $U_{\text{X}} = \sigma_x$. Applying a further U_x at the end would return the qubit(s) to their initial state.

5.7 Initialisation and measurement

These will be discussed in more detail in later lectures but for the moment we briefly outline how they can be performed in the case of atoms or ions.

- Initialisation is generally carried out using *optical pumping*. A laser is used to excite atoms that are in any state other than the target state to a higher excited state. This, in combination with random relaxation processes, leads to preferential population of the target state.
- Readout is generally done by applying a laser which is on resonance with a transition from one of the two qubit states (say $|0\rangle$) to a higher excited state resulting in fluorescence (due to spontaneous decay of the higher excited state). The other qubit state ($|1\rangle$ in this case) is unaffected by the laser and so remains dark. By carefully measuring the fluorescence the relative populations of states $|0\rangle$ and $|1\rangle$ can be extracted.

5.8 Photons

In the case of photons the qubit can be encoded using either the polarisation or a spatial degree of freedom (in both cases these can define two orthogonal states even though they have the *same* energy). Below we consider using horizontal and vertical polarisation as the basis states (lets say $|0\rangle$ and $|1\rangle$ respectively) .

- Initialisation: while the initial polarisation state of a photon can readily be prepared (e.g. using a polariser) reliably obtaining single photons is more difficult (but can for example be reasonably achieved using a pulsed laser).
- Single qubit gates can also be readily implemented using standard optical elements such as waveplates; a waveplate with a retardation along one axis of ϕ, placed at an angle θ to $|0\rangle$, corresponds to the propagator

  $$U(\theta,\phi) = \begin{pmatrix} \cos^2(\theta) + \sin^2(\theta)\mathrm{e}^{i\phi} & \cos(\theta)\sin(\theta)(1-\mathrm{e}^{i\phi}) \\ \cos(\theta)\sin(\theta)(1-\mathrm{e}^{i\phi}) & \cos^2(\theta)\mathrm{e}^{i\phi} + \sin^2(\theta) \end{pmatrix}. \tag{32}$$

  Using half ($\phi = \pi$) and quarter ($\phi = \pi/2$) waveplates we can implement many gates, for example $U(\pi/4,\pi) = \sigma_x = \mathrm{X}$ and $U(0,\pi) = \sigma_z = Z$ and $U(0,\pi/2) = \mathrm{S}$.
- Readout is done using a polarising beam splitter in combination with single-photon detectors.

While this ready manipulation, and relatively simple technology are big advantages, disadvantages of photons include that (i) they are hard to store and (ii) they interact very weakly making the implementation of 2-qubit gates challenging.