C2: Quantum information basics

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Lecture 3

5.2 Rabi oscillations

Driving the state $|\psi(t=0)\rangle = |g\rangle$ with radiation δ detuned from ω_0 with strength V results in the excited state population $|c_\mathrm{e}|^2$ undergoing *Rabi oscillations* such that

$$|c_\mathrm{e}|^2 = \frac{V^2}{\Omega^2} \sin^2(\Omega t/2) \tag{24}$$

where $\Omega = \sqrt{V^2 + \delta^2}$.

5.3 Gate implementation: approximations

In deriving the effect of radiation on a atom/ion qubit we made several assumptions:

1. Two level system.
2. The upper level has no spontaneous decay.
3. Semi-classical approximation: treat the the radiation as a classical wave.
4. Dipole approximation: the electric field is constant across the atom (basically $\lambda \gg a_0$ where a_0 is the Bohr radius).
5. Rotating wave approximation (RWA) - ignore counter-rotating wave in $\cos(\omega t)$.

Approximations (3) and (4) are usually well met and we will not discuss further. Approximations (1) and (5) both rely on the effect of driving far-detuned transitions and are discussed in subsection 5.3.1 whereas the validity of approximation (2) depends on the relative strength of driving versus decay which is discussed in subsection 5.3.2.

5.3.1 Off-resonant excitations

In the limit that $\delta \gg V$, in Eq.(24) we have $\Omega \approx \delta$ such that

$$|c_\mathrm{e}|^2 \approx \frac{V^2}{\delta^2} \sin^2(\delta t/2) = \frac{V^2 t^2}{4} \operatorname{sinc}^2(\delta t/2). \tag{25}$$

Note that this equation can also be derived from 1st-order perturbation theory. The relative strength of the transition (for different δ is thus given by $\operatorname{sinc}^2(\delta t/2)$ which means driving fields with $\delta \ll 2\pi/t$ are insensitive to detuning while driving fields with $\delta \gg 2\pi/t$ are massively suppressed. This dependence on pulse time t reflects the fact that for a short pulse the drive frequency is not well defined (from Fourier arguments).

Therefore, other levels can be ignored (for the moment assuming they have similar V) if their detuning $|\delta_\mathrm{other}|$ is both $\gg |\delta_\mathrm{eg}|$ and $\gg 2\pi/t$. The same argument applies to the RWA for which we can take δ_other as being $\omega_0 + \omega$ such that we require $\delta \ll \omega_0, \omega$ and $t \gg 2\pi/\omega$. It is usually not too difficult to ensure these conditions are met.

5.3.2 Strong and weak driving

So far have assumed that $|e\rangle$ has an infinite lifetime but this is not necessarily the case; what happens if $|e\rangle$ has a finite lifetime $\tau(=1/\Gamma)$? Given that the operation time t is typically $t \sim \pi/V$ the relevant parameter is $\tau/t \sim V\tau$ leading to two regimes:

- $V\tau \gg 1$; *strongly driven transitions.* In this regime we can ignore spontaneous emission, everything we have done so far holds, and we can perform the *coherent control* necessary.
- $V\tau \ll 1$; *weakly driven transitions.* The system can be described by rate equations and Fermi's golden rule applies.

In between, things are more complicated but we can, in fact, still describe the behavior using a Bloch sphere picture using the *optical Bloch equations.*

For an atomic electric dipole transition (such as we have considered so far) the strong driving criteria is difficult to meet as

$$\hbar\Gamma \sim \frac{\omega_0^3}{\epsilon_0 c^3} e^2 \left|\langle g|z|e\rangle\right|^2 , \tag{26}$$

which for a typical transition (where $\langle g|z|e\rangle \sim a_0$) gives $\tau \sim 10\,\mathrm{ns}$. This problem can be overcome by using magnetic-dipole transitions or Raman transitions.

5.4 Magnetic dipole transitions

An obvious way to reduce Γ (increase τ) is to reduce ω_0 as $\Gamma \propto \omega_0^3$. However, transitions from the ground state to such lower-lying excited states are generally electric-dipole forbidden. Fortunately *magnetic dipole transitions* (which have different selection rules) are often allowed. These states typically have $\omega_0/2\pi$ on the MHz or GHz scale (instead of the 10^{15} Hz of an optical transition) which massively reduces Γ; in addition the Γ of magnetic dipole allowed states is further suppressed by a factor of α^2 (where $\alpha \approx 1/137$ is the fine structure constant). This leads to several advantages:

- effectively infinite lifetime - can easily reach strongly driven regime,
- RF fields/radiation easier to generate and manipulate,

and disadvantages

- Can't focus RF radiation to a single qubit,
- Readout harder.

5.5 Raman transitions

Raman transitions are two-photon transitions which use coupling to a third level $|a\rangle$ to allow optical transitions between between $|g\rangle$ and $|e\rangle$ even though direct dipole transitions are not allowed. To do this a state $|a\rangle$ is chosen which has allowed transitions between both $|g\rangle$ and $|e\rangle$; both these allowed transitions are driven (with strengths V_1 and V_2) with a large (but not too large) detuning Δ ($\Delta \gg \Gamma_a$, $\Delta \gg V_{1,2}$ but $\Delta \ll \omega_{ga}, \omega_{ea}$). The remarkable result is that this couples states $|g\rangle$ and $|e\rangle$ with an effective coupling strength (Rabi-frequency) of

$$V_{\mathrm{eff}} = \frac{V_1 V_2}{2\Delta} \tag{27}$$

even though the population of the state $|a\rangle$ can be made vanishingly small; its average is $\frac{1}{2}(V/\Delta)^2$ for $V_1 = V_2 = V$. This allows us to be in the strong driving regime with the added benefits (e.g. spatial resolution) of optical laser radiation.