

Homework IV Part II

MMathPhys Renormalisation Group

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1 Sine-Gordon RG

In the lecture we obtained the RG equations

$$\frac{dK}{d\tau} = 2\pi^3 g^2 + O(g^4), \quad \frac{dg}{d\tau} = \left(2 - \frac{\pi}{K}\right)g + O(g^3) \quad (1)$$

for the Sine-Gordon model

$$\mathcal{H} = \int d^2r \left(\frac{K}{2} (\nabla h)^2 + g \cos 2\pi h \right). \quad (2)$$

a) (adapted from Wen): Consider a more general potential which we write as¹

$$\mathcal{H} = \int d^2r \left(\frac{K}{2} (\nabla h)^2 + V(h) \right), \quad V(h) = \int d\ell \tilde{V}(\ell) e^{2\pi i \ell h}. \quad (3)$$

By generalising from the case we discussed, write the RG equation for $\tilde{V}(\ell)$ to linear order in \tilde{V} . Reexpress this as a partial differential equation for $V(h)$, in terms of the parameters τ and h . What would the initial periodic potential shown in Fig. 1 look like when $\tau \gg 1$? (But while V is still small, so the linear approximation is valid. Assume also that the average is zero, $\int dh V(h) = 0$, since we can always subtract a constant from \mathcal{H} .)

***** All the remaining questions on this set are optional! *****

b) Consider the initial non-periodic potential $V(h) = Ae^{-h^2/2\Delta}$. This is

¹In many physical applications $V(h)$ would be periodic, so ℓ would be restricted to integers. Here we allow a non-periodic V

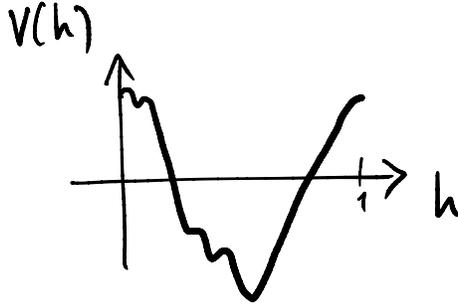


Figure 1: Periodic potential

a barrier disfavouring $h \sim 0$. What happens to $V(h)$ as the RG time increases (still within the linear approximation)?

c) Let us say we have a system with the potential in (b) that is defined on a spatial disc of radius L . Let us enforce the condition $h = 0$ on the boundary of the disc. Do you expect this system to exhibit spontaneous symmetry breaking in the limit of large L ?

2 Spin-glass

Consider an Ising model in d dimensions

$$\mathcal{H} = - \sum_{\langle ij \rangle} J_{ij} S_i S_j \quad (4)$$

J_{ij} is a random coupling associated with the bond $\langle ij \rangle$. It is equal to $+J$ with probability $1/2$ and to $-J$ with probability $1/2$. The J_{ij} s are independent, quenched random variables, representing non-fluctuating disorder in our system. Note that we must distinguish the *thermal* average, which we do in the background of a fixed random choice of J_{ij} ,

$$\langle \dots \rangle_{\{J_{ij}\}} = \frac{\sum_{\{S\}} e^{-\mathcal{H}[\{S\}; \{J_{ij}\}]} (\dots)}{Z[\{J_{ij}\}]} \quad (5)$$

from the disorder average,

$$\overline{(\dots)} = \sum_{\{J_{ij}\}} \frac{1}{2^{\text{no. bonds}}} (\dots). \quad (6)$$

Fully averaged expectation values

$$\overline{\langle S_i \dots \rangle_J} \tag{7}$$

are hard to compute because the random variables appear both in the numerator and denominator of (5).

a) Draw the phase diagram of the model in one dimension as a function of J . Exhibit an explicit equivalence to a model we already know in 1D.

b) In the one-dimensional **ferromagnet**, where $J_{ij} = J$, the (unstable) zero-temperature fixed point can be distinguished from the disordered phase by the long distance behaviour of correlation functions: $\langle S_i S_j \rangle = 1$ at zero temperature, while the correlator decays exponentially with distance when the temperature is nonzero. Can we use the long distance behaviour of the analogous correlator,

$$\overline{\langle S_i S_j \rangle_J} \tag{8}$$

in the spin-glass model (4) to distinguish the zero-temperature fixed point from the $T > 0$ phase? If not, why not?

c) Can you find an alternative correlation function $G(i, j)$ which decays to zero at long distance for $T > 0$, but is nonzero at long distances at $T = 0$? We are still in 1D here. Note: while G should be a disorder-averaged quantity, it is useful to note that we do not always have to perform the disorder average and the thermal average at the same time.

d) Assuming that in high enough dimensions $G(i, j)$ can be nonvanishing at long distances even at nonzero temperature, draw a phase diagram with two phases. Think about the properties of the new phase.