Problem Set 3

MMathPhys: The Standard Model

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Higgs and electroweak theory

Problems marked with * are more advanced. Try to solve at least one of the two starred problems. To be completed and handed in by **Tuesday Week 8**.

1. **Higgs decay width into fermions**

   Calculate the decay width of the Higgs boson to quarks.

2. **Electron-neutrino scattering**

   The process $e\nu_e \to f_m f_n$ (f neither e nor ν) proceeds via a single diagram. Draw that diagram and show that, taking all external states to be massless, it yields a cross section of

$$\frac{\mathrm{d}\sigma}{\mathrm{d}u\mathrm{d}t} = -\frac{G_F^2}{\pi} N_c |U_{nm}|^2 \frac{u^2}{s^2} \left(\frac{M_W^2}{s - M_W^2} \right)^2 \delta(s + t + u). \tag{1}$$

   Use crossing symmetry to find $d\sigma/\mathrm{d}u\mathrm{d}t$ for $e\nu_\mu \to \mu\nu_e$ from this result.

3*. **Decay of the top quark**

   Consider the top quark, with a mass of $m_t \simeq 174$ GeV.

   (a) Identify the only interaction term in the Lagrangian which is linear in the top quark. Can a single insertion of this interaction term cause the top quark to decay? What are the decay products?

   (b) Write an expression for the matrix element for the dominant top-quark decay process.

   (c) Find a compact expression for the square of the matrix element, summing over final-state spin or helicity states and averaging over the initial top-quark helicity state.

   (d) Compute the width of the top quark. Neglect the masses of any other fermions in comparison to the top-quark mass, but treat the masses of W and Z bosons as comparable to the top-quark mass. You should be able to find an analytic expression for the decay rate. Then, substitute in physical values and express the answer in GeV.

4*. **Two Higgs doublet models**

Suppose the Higgs doublet of the standard model is supplemented by a second complex doublet, ψ, transforming as $(1, 2, 1)$ under $SU_c(3) \times SU_L(2) \times U_Y(1)$.

(a) If ψ is written $\psi = \begin{pmatrix} \chi \\ \xi \end{pmatrix}$, what are the electric charges of the component fields χ and ξ?

(b) Write out the covariant derivative $D_\mu \psi$ explicitly in terms of the gauge fields G^α_μ, W^a_μ and B_μ.

(c) Assuming the potential must be a function of the invariants $a = \varphi^\dagger \varphi$, $b = \psi^\dagger \psi$, and $c = \varphi^T \epsilon \varphi$, where φ is the usual Higgs doublet, what is the most general renormalizable form? How many independent real parameters does it contain? Need the parameters appearing in the potential be real? Is the combination $d = \varphi^\dagger \psi$ $SU_L(2) \times U_Y(1)$ invariant?

(d) Suppose the parameters of the potential are such that it is minimized when

$$\varphi = \varphi_{\text{min}} = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \tag{2}$$

$$\psi = \psi_{\text{min}} = \begin{pmatrix} \frac{1}{\sqrt{2}}(u + i\omega) \\ 0 \end{pmatrix} \tag{3}$$

u, v, w all real. Do these values break the electromagnetic group $U_{\text{em}}(1)$ generated by the electric charge $Q = T_3 + Y$? Identify the terms in the Lagrangian that are quadratic in the gauge fields and find their masses in terms of u, v, and w. Call the mass eigenstates $W^\pm_\mu = 1/\sqrt{2}(W^1_\mu \mp iW^2_\mu)$, $Z_\mu = W^3_\mu \cos\theta - B_\mu \sin\theta$, and $A_\mu = B_\mu \cos\theta + W^3_\mu \sin\theta$. Express $\cos\theta$ in terms of the gauge couplings g_1 and g_2. Is the standard model mass relation $M_W = M_Z \cos\theta$ also true for this model?

(e) What are the possible Yukawa couplings of the spin zero fields, φ and ψ, to the fermions? Suppose the Lagrangian is required to be invariant under the symmetry

$$P_R E_m \to e^{i\theta} P_R E_m, \qquad P_R U_m \to e^{i\theta} P_R U_m, \qquad P_R D_m \to e^{i\theta} P_R D_m \tag{4}$$

$$\varphi \to e^{-i\theta} \varphi \qquad \psi \to e^{-i\theta} \psi \tag{5}$$

with θ a real constant and all other fields being invariant. What are the resulting restrictions on the Yukawa couplings and Higgs potential, $V(\varphi, \psi)$?