Problem Set 2

MMathPhys: The Standard Model

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Quantum Chromodynamics in deep-inelastic scattering and hadron-hadron collisions

Problems marked with * are more advanced.
To be completed and handed in by **Tuesday Week 6**.

1. **Splitting functions**

   (a) The plus prescription is defined by

$$\int_0^1 dz f(z)[g(z)]_+ = \int_0^1 [f(z) - f(1)]g(z). \tag{1}$$

   Show that

$$\left(\frac{1+z^2}{1-z}\right)_+ = \frac{\alpha}{(1-z)_+} + \beta\delta(1-z) + y(z), \tag{2}$$

   where $y(z)$ is an ordinary function, and α and β are rational numbers.

   (b) Consider the splitting functions

$$P_{q\leftarrow q} = C_F\left(\frac{1+z^2}{1-z}\right)_+ \tag{3}$$

$$P_{q\leftarrow g} = T_R(z^2 + (1-z)^2). \tag{4}$$

   The anomalous dimensions are given by the moments of the splitting functions,

$$\gamma_{ij}(N, a_s) = \sum_{n=0}^{\infty} \gamma_{ij}^{(n)}(N)\left(\frac{a_s}{2\pi}\right)^{n+1}, \tag{5}$$

$$\gamma_{ij}^{(0)}(N) = \int_0^1 dz\, z^{N-1} P_{ij}(z). \tag{6}$$

Show that

$$\gamma^{(0)}_{q\leftarrow q}(N) = C_F\left[-\frac{1}{2} + \frac{1}{N(N+1)} - 2\sum_{k=2}^{N}\frac{1}{k}\right] \tag{7}$$

$$\gamma^{(0)}_{g\leftarrow q}(N) = T_R\frac{2+N+N^2}{N(N+1)(N+2)}. \tag{8}$$

2. **Parton distribution functions and Higgs production in gluon-gluon fusion**

For this exercise, you will need to use the virtual machine that you have configured in the previous problem set. The first part of the exercise is meant to get familiar with parton distribution functions (PDFs) and their uncertainties. In the second part of the exercise you will study Higgs production in gluon fusion at LHC in order to quantify the impact of PDF uncertainties in precision measurements at hadron colliders.

There are several collaborations which provide PDFs. A PDF set generally contains several 'members': the 'central' member corresponds to the best-fit to the data used to extract the PDFs, whereas the other members are used to parametrize the PDF uncertainty. The way the uncertainty is parametrized may differ from one collaboration to the other. We will briefly discuss these strategies in the tutorial.

In this exercise you will calculate the PDF uncertainty using PDFs from the NNPDF collaboration, in particular using the set `NNPDF30_nnlo_as_0118`. This PDF set contains 101 members. Members 1-100 have all been fitted to the data: this means that each member provides a valid description of the data. Let O be a generic observable: this can be the PDF, but it could be something more generic, such as a total cross section. We want to evaluate its central value O_0 and the associated PDF uncertainty. In the NNPDF approach, one can obtain the central value by computing the mean value over the members

$$O_0 = \langle O\rangle_{\text{rep}} = \frac{1}{100}\sum_{j=1}^{100} O_j \tag{9}$$

and the (symmetric) error is obtained by computing the standard deviation

$$\sigma = \left[\frac{1}{99}\sum_{j=1}^{100}(O_j - O_0)^2\right]^{1/2}. \tag{10}$$

If one is not interested in computing the error, but is interested in the central value only, the computation performed with member 0 should be rather close to the one performed by averaging over all the members.

(a) In the folder `sm/problem_set_2` you will find the file `PDF.cc`. Have a look at the file, and understand some of the basic calls of `LHAPDF`. You will find a more detailed documentation at the website `http://lhapdf.hepforge.org`. Type `make` to produce an executable. To begin with, compute $g(x)$, $u(x)$, $d(x)$ as a function of x for $Q = 2, 10, 100, 1000$ GeV using member 0 of the PDF set `NNPDF30_nnlo_as_0118`. With the help of `PDFplotter.py` plot $xg(x)$, $xu(x)$, $xd(x)$ as a function of x at different values of the energy.

(b) Now we will introduce PDF errors. Compute $g(x)$, $u(x)$, $d(x)$ as a function of x for the same values of point (a) including PDF uncertainties and using the average over the different members for the central value. Typing now in the terminal `./PDFplotter.py -b` plot $xg(x)$, $xu(x)$, $xd(x)$ showing also the PDF uncertainty bands. Comment your results.

(c) In the last part of the exercise you will compute the cross section for Higgs production in gluon fusion at NNLO with the program `ggHiggs`. In the folder `sm/problem_set_2/Higgs` you will find the program `ggHiggs.cc` and the associated `makefile`.

- You can obtain the predictions for Higgs production with E_{CM} = Energy [GeV] and a given PDF set by typing in the terminal

  ```
  ./ggHiggs -p PDF_SET_NAME -E Energy.
  ```

  Obtain the LO predictions for `NNPDF30_nnlo_as_0118` at 13 and 14 TeV.
- Now you will have to compute the PDF uncertainties following the recipe of point (b). Compute central value and uncertainties for `NNPDF30_nnlo_as_0118` at 13 and 14 TeV.
- Now compute the cross section at LO for Higgs boson production in gluon fusion as a function of E_{CM} between 7 and 100 TeV, including the PDF error. Print your results in the files `gg_PDF_SET_NAME.dat`. Then type in the terminal

  ```
  ./Higgsplotter.py.
  ```

  Using the option `-b` you will also plot the PDF uncertainties. You should obtain a file `plots/gg_predictions.pdf`. Have a look at the figure produced. What is the behaviour of the cross section as a function of the energy? What is the behaviour of the PDF uncertainty? Can you relate what you have obtained to the behaviour of the gluon PDF that you have studied in point (a) and (b)?

3*. **Photoproduction of heavy quarks**

Consider the process of heavy quark pair photoproduction, $\gamma + p \to Q\bar{Q} + X$, for a heavy quark of mass M and electric charge Q. If M is large enough, any diagram contributing to this process must involve a large momentum transfer; thus a perturbative QCD analysis should apply. Work out the cross-section to the leading order in QCD. Choose the parton subprocess that gives the leading contribution to this reaction, and write the parton-model expression for the cross section. You will need to compute the relevant subprocesses cross section in order to do so, but you might find useful to take the results from the analogous QED calculations (e.g. Peskin-Schroeder Chapt. 5). Use the result to write an expression for the cross section for γ-proton scattering.