Michaelmas Term 2019 1st Year Relativity - Problems 2

Special Relativity: Professor K M Blundell

SECTION A – Collision problems, threshold energies, decays, recoils

1. The K^0 meson at rest can decay into a pair of charged π mesons. Given that the rest masses of the K and π are 498 and 140 MeV/c^2 respectively, show that the speeds of the pions are 0.83c in the rest frame of the K^0.

2. An electron accelerated from the Stanford Linear Accelerator (SLAC) in California has a total energy of 50 GeV. (a) How much of this is kinetic energy? (b) What is the momentum of the electron? (c) What is its speed?

[Ans: (a) 0.99999E, (b) 2.7×10^{-17} kg m s^{-1}, (c) $(1 - (5 \times 10^{-11}))c$]

3. A π^0 meson whose rest mass is 135 MeV/c^2 is moving with a kinetic energy of 1 GeV. It decays in flight into two photons whose paths are along the direction of motion of the meson. Find the energies of the two photons.

[Ans: 4 and 1131 MeV.]

4. Neutral π^0 mesons can be formed by the reaction

$$p + p \rightarrow p + p + \pi^0.$$

What is the threshold kinetic energy for π^0 production by this process if the target proton is stationary? (Masses of π^0 and proton are 135 and 938 MeV/c^2.)

[Ans: 280 MeV.]

5. What is the threshold $K.E.$ for production of an electron pair by collision of an incident electron with a stationary electron *i.e.* for

$$e^- + e^- \rightarrow e^- + e^- + (e^+ + e^-)?$$

[Ans: $6m_e c^2$, where m_e is the rest mass of an electron.]

6. The Tevatron at the Fermi National Accelerator Laboratory in the USA carries two beams of 1 TeV (1000 GeV) protons moving in opposite directions, so that for proton-proton collisions the centre of mass frame is also the laboratory frame. What proton energy must an accelerator provide to get the same centre of mass energy when bombarding a stationary hydrogen target?

[Ans: 2.13×10^3 TeV.]

7. (i) Show that a free electron moving in a vacuum at velocity v cannot emit a single photon. (ii) A hydrogen atom in an excited state can emit a single photon, why is this different from (i) above?

8. An atom in an excited state of energy Q_0 (as measured in its rest frame) above the ground state moves towards a scintillation counter with speed v. The atom decays to its ground state by emitting a photon of energy Q (as recorded by the counter), coming completely to rest as it does so. If the rest mass of the atom is M, show that $Q = Q_0[1 + (Q_0/2Mc^2)]$.

9. A driver was caught running a red light. His defence is that he saw the light as green, as a result of the Doppler shift. He is arrested. What for? Estimate the seriousness of his transgression.

10. Light of frequency ν_0 is emitted from a source moving away from an observer with speed βc. If the light is emitted along the line of the motion show that the frequency measured by the observer is $\nu = \nu_0 \gamma(1 - \beta)$. An absorption line in Ca^+ has a wavelength of 394 nm as measured for a stationary source. The same line in the spectrum of Hydra occurs at 475nm. What can be deduced about the motion of Hydra with respect to the Earth?

SECTION B – Some past Prelims questions

11. State the *Lorentz transformations* which relate the space and time coordinates in two inertial frames, S and S', travelling with relative speed v along the $x(x')$ axis. Use the transformations to derive expressions for the phenomena of *length contraction* and *time dilation*.

A rocket passes close to the earth with relative velocity $0.8c$, at which time observers on the earth and the rocket reset their clocks $(t_E = t_R = 0)$.

(a) After 30 minutes as measured by the rocket's clock, the rocket passes a space-station which is fixed relative to the earth. What time is recorded by the observer on the earth?

(b) How far is the space-station relative to the earth

(i) in earth coordinates,

(ii) in rocket coordinates?

(c) When the rocket passes the space-station, the rocket sends a radio signal back to earth. At what time (by earth time) does the signal arrive?

(d) The station on earth replies immediately. At what time (by rocket time) is the reply received by the rocket?

12. Write down the relativistic expressions for momentum p and energy E of a particle of rest mass m_0 in terms of its velocity v and show that they satisfy the equation

$$E^2 = p^2c^2 + m_0^2c^4$$

(a) An electron of energy 9.0 GeV and a positron of energy E collide head-on to produce a B^0 meson and an anti-B^0 meson, each with a mass of 5.3 GeV/c^2. What is the minimum positron energy (threshold energy) required to produce the B^0-meson pair? [You may neglect the rest-mass energies of the electron and positron.]

(b) Electrons and positrons collide head-on in the laboratory with beam energies as in part (a), producing B^0 and anti-B^0 mesons at the threshold energy. The B^0 mesons undergo decay with a mean proper lifetime of 1.5×10^{-12} s. Calculate the mean distance that the B^0 mesons travel before decay, as observed in the rest frame of the laboratory.

13. State the *Lorentz transformations* which relate the space and time coordinates in two inertial frames, S and S', where S' is travelling with velocity v relative to S along the positive x direction. Use the transformations to derive an expression for the phenomenon of *length contraction*.

An observer in S measures the velocity of a body travelling in the positive x direction to be u_x. Show that the velocity of the body measured by an observer in S' is given by

$$u'_x = \frac{u_x - v}{1 - vu_x/c^2}$$

An observer on the earth measures a spaceship to be approaching at a speed of $0.7c$. The same observer measures a second spaceship to be approaching in the opposite direction at a speed of $0.8c$. Each spaceship has a length of 100 m (as measured by observers in the respective spaceships).

(a) What are the lengths of each spaceship, as measured by the observer on the earth?

(b) What is the length of each spaceship, as measured by the observer on the other spaceship?

14. Write down the relativistic expressions for momentum p and energy E of a particle of rest mass m_0 in terms of its velocity v and show that they satisfy the equation

$$E^2 = p^2c^2 + m_0^2c^4$$

A D^0 meson decays at rest to a $\bar{K}^0$ meson and a π^0 meson. Calculate the energy and velocity of the π^0.

The π^0 produced in the D^0 decay subsequently decays into a pair of photons. Calculate the maximum and minimum energies of each photon in the D^0 rest frame.

[The rest masses of the D^0, $\bar{K}^0$ and π^0 mesons are 1.86 GeV/c^2, 0.50 GeV/c^2 and 0.13 GeV/c^2, respectively.]