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Superconductivity
Lecture 2

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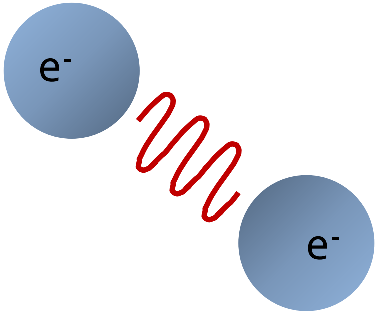
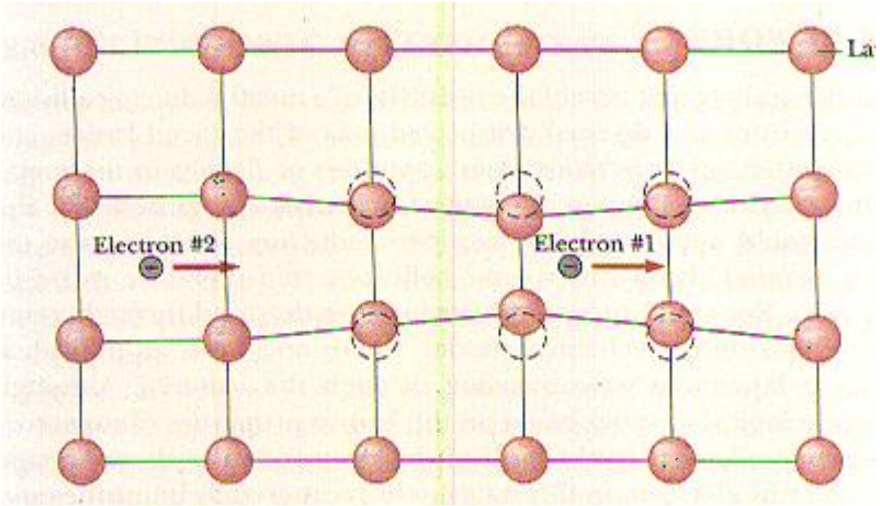
Superconducting electrons are bound in a **condensate** of **pairs**

What is the pairing mechanism?





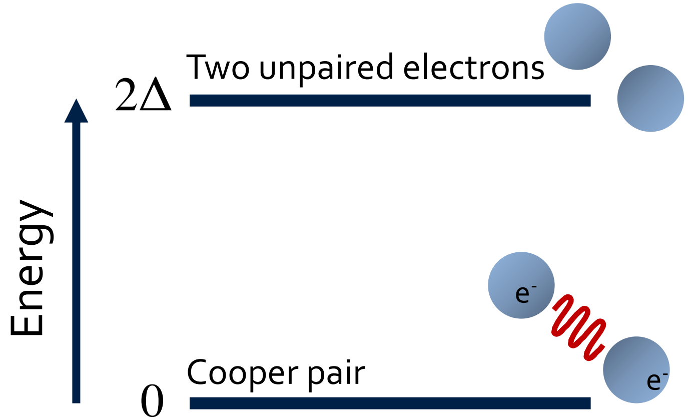
Electrons cause instantaneous distortion of ionic lattice and leave "trail" of positive charge.

BCS theory, 1957

John Bardeen (1908–1991) | Leon Cooper (1930–) | Robert Schrieffer (1931–)

Nobel Prize, 1972

(Bardeen also won the Nobel Prize in 1956 with William Shockley)

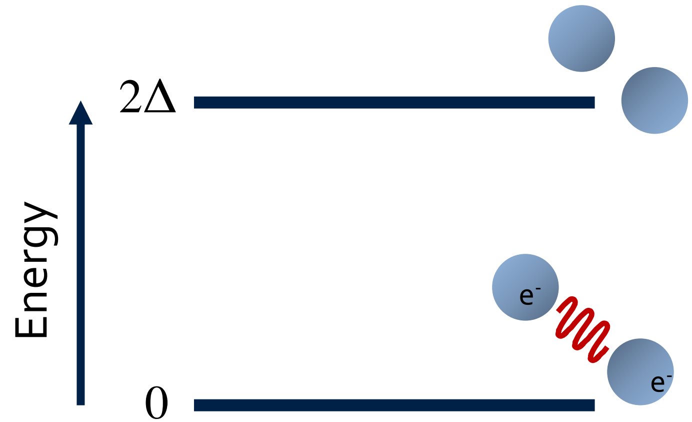


Binding energy of a Cooper pair is 2Δ

Thermal energy at temperature T:

$< \mathrm{KE} > \sim k_B\ T$ per particle
$k_B = 1.38 \times 10^{-23}$ J K^{-1}

Cooper pairs unstable when



$$\sim k_B T > 2\Delta$$

Transition from **superconducting** to **normal** state occurs at a critical temperature. From BCS theory:

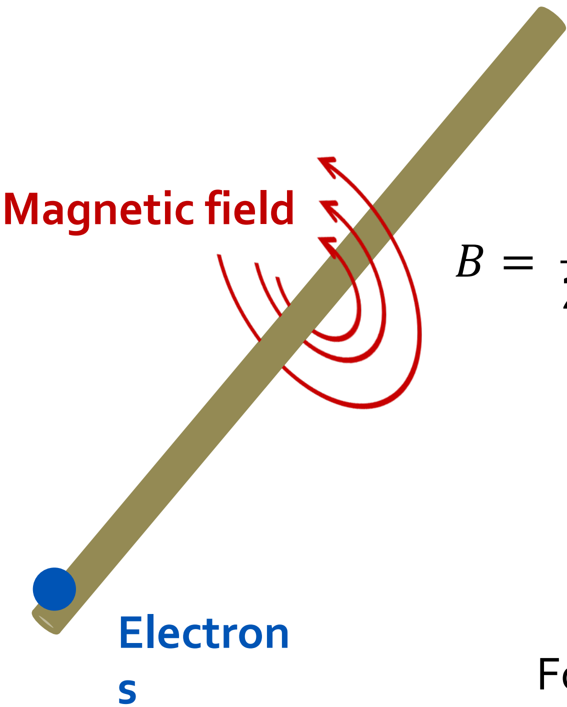
$$3.52 k_B T_c = 2\Delta$$

Cooper pairs destroyed when energy transferred during a collision exceeds 2Δ.

Critical current density: $j_c \approx ne\mathrm{k_B}T_c/m_e v$

Typically, $j_c \sim 10^{11}$ A m^{-2}

Magnetic fields above some critical value B_c will induce a current density in excess of j_c and destroy superconductivity.

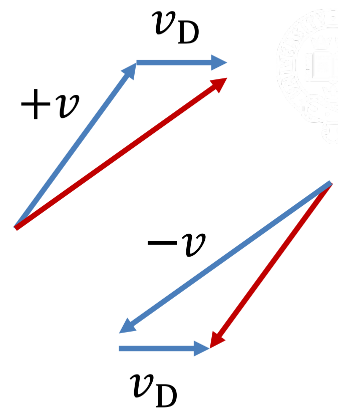


$$B = \frac{\mu_0 I}{2\pi r}$$

For a cylindrical wire of radius a carrying a uniform current

$$B_c = a\mu_0 j_c/2.$$

Electrons in Cooper pair have opposite momenta



$$\mathrm{KE_i} = \frac{1}{2}m(-v+v_\mathrm{D})^2 + \frac{1}{2}m(v+v_\mathrm{D})^2$$
$$= m(v^2 + v_\mathrm{D}^2)$$

Least KE after scattering if v and v_D are in opposite directions

$$\mathrm{KE_f} = 2 \times \frac{1}{2}m(-v+v_\mathrm{D})^2$$
$$= \mathrm{KE_i} - 2mvv_\mathrm{D} \qquad (1)$$

Possible to "break" the Cooper pair if

$$2\Delta + \mathrm{KE_f} \le \mathrm{KE_i}$$
$$2\Delta \le 2mvv_\mathrm{D} \qquad (2)$$

Critical current density: if we approximate $\Delta \approx k_\mathrm{B}T_\mathrm{c}$

Since $j = nev_\mathrm{D}$, substituting from (2), $j = ne\Delta/m_\mathrm{e}v$ and

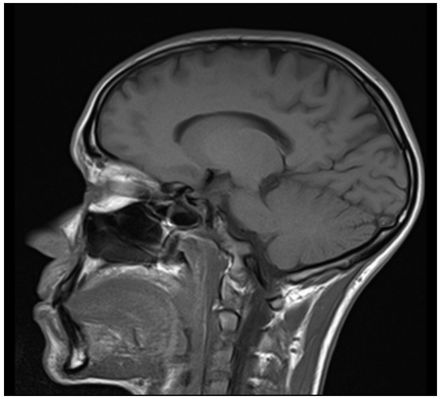
$$j_\mathrm{c} \approx \frac{nek_\mathrm{B}T_\mathrm{c}}{mv}$$

Superconducting magnets

- For a given field, require much less power to run than conventional electromagnets
- Achieve much higher fields: >30T continuous
- In many cases, much more compact than conventional electromagnets
- No ohmic heating

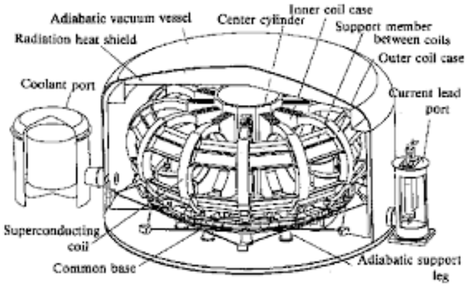
Applications

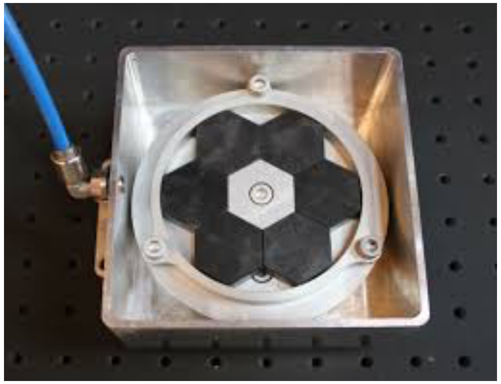
- Bending magnets in particle accelerators
- Fundamental research into the properties of matter
- Medical devices

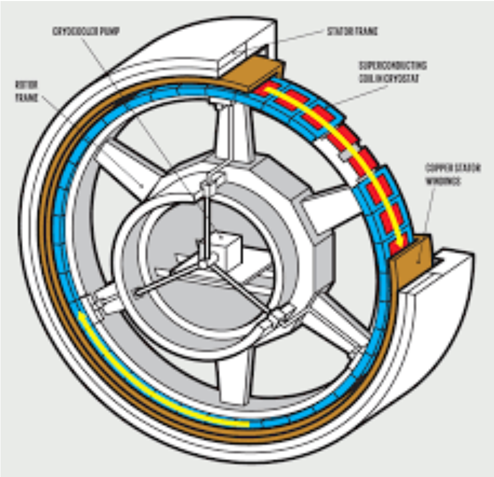


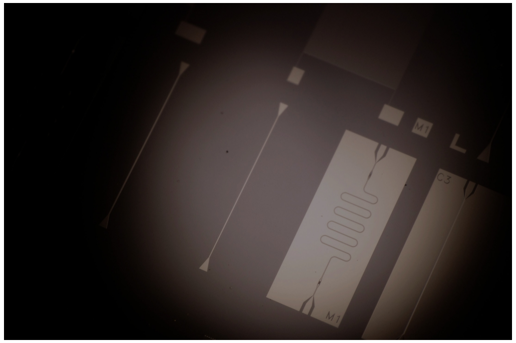
Other applications of superconductors

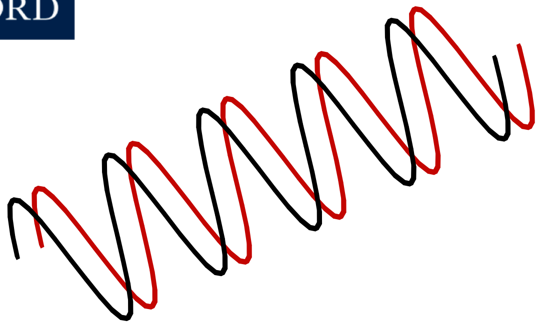
- Superconducting cables for power and signal distribution
- Generators and transformers
- Energy storage (superconducting magnetic energy storage = SMES)
- Motors and propulsion
- Space instrumentation
- Quantum computing devices
- SQUIDS
- Frictionless bearings
- …



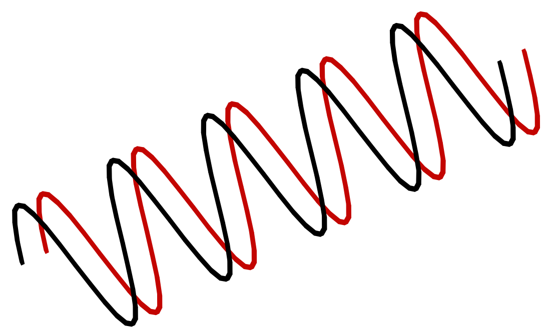


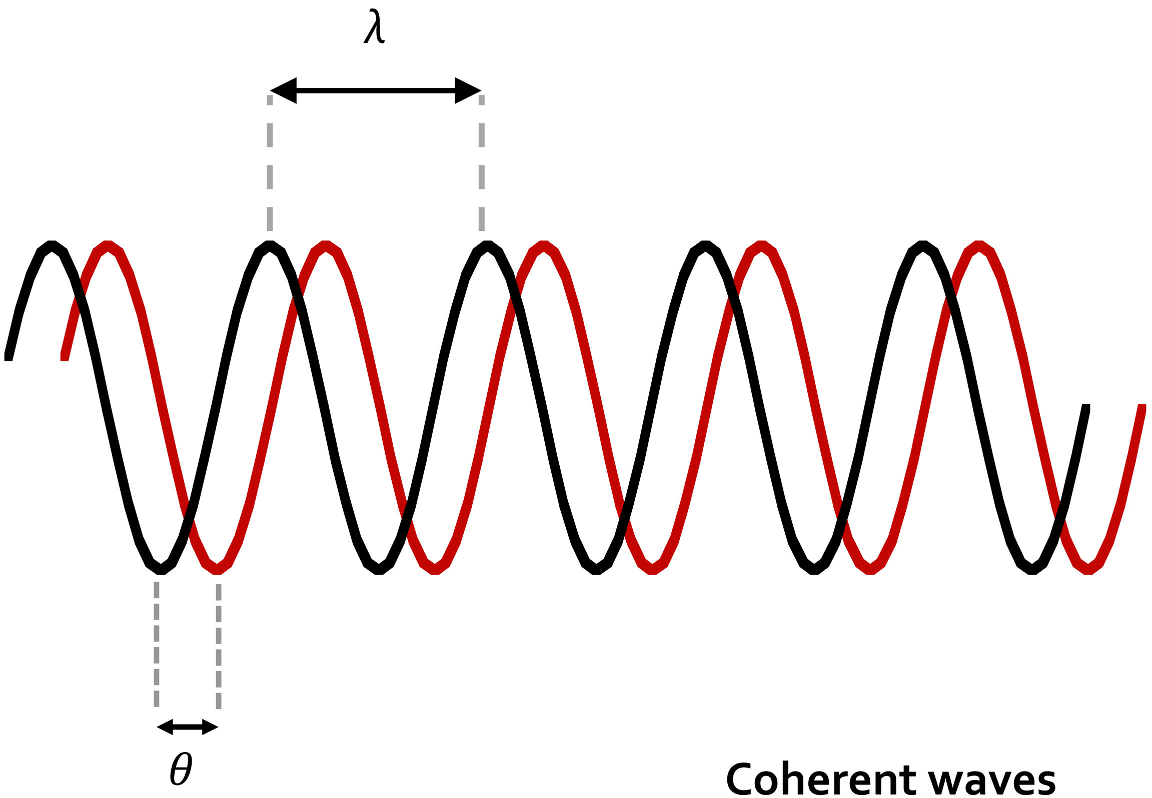






- What is coherence?
- Constructive and destructive interference
- Interference of matter waves
- Macroscopic quantum coherence in superconductors



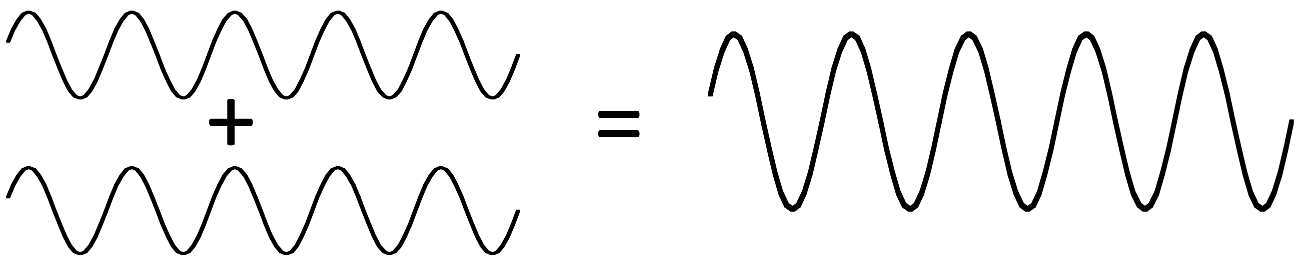


Coherent waves

- Fixed relative phase
- Same frequency

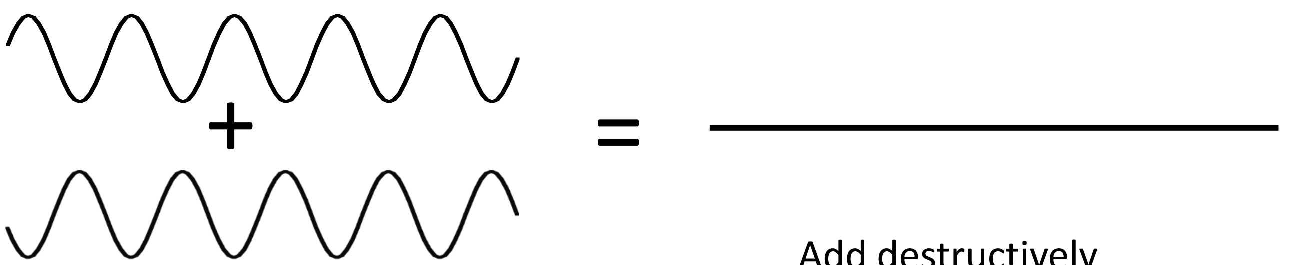
Superposition of coherent waves

In-phase ($\theta = 0$)



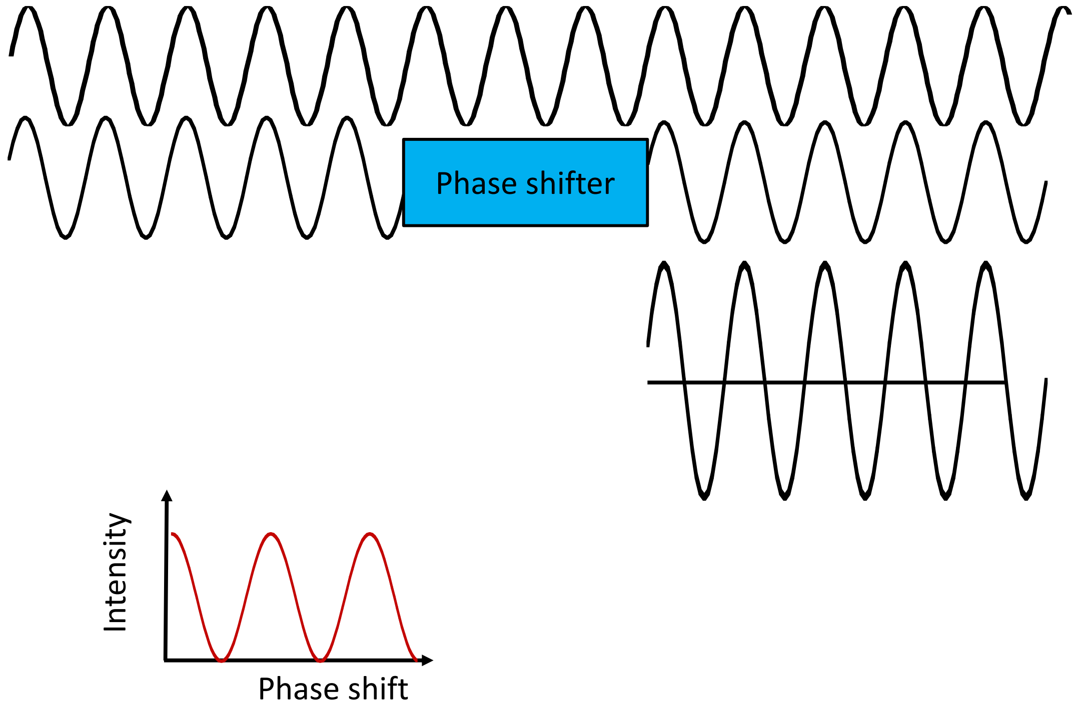
Add constructively

Out-of-phase ($\theta = \pi$)

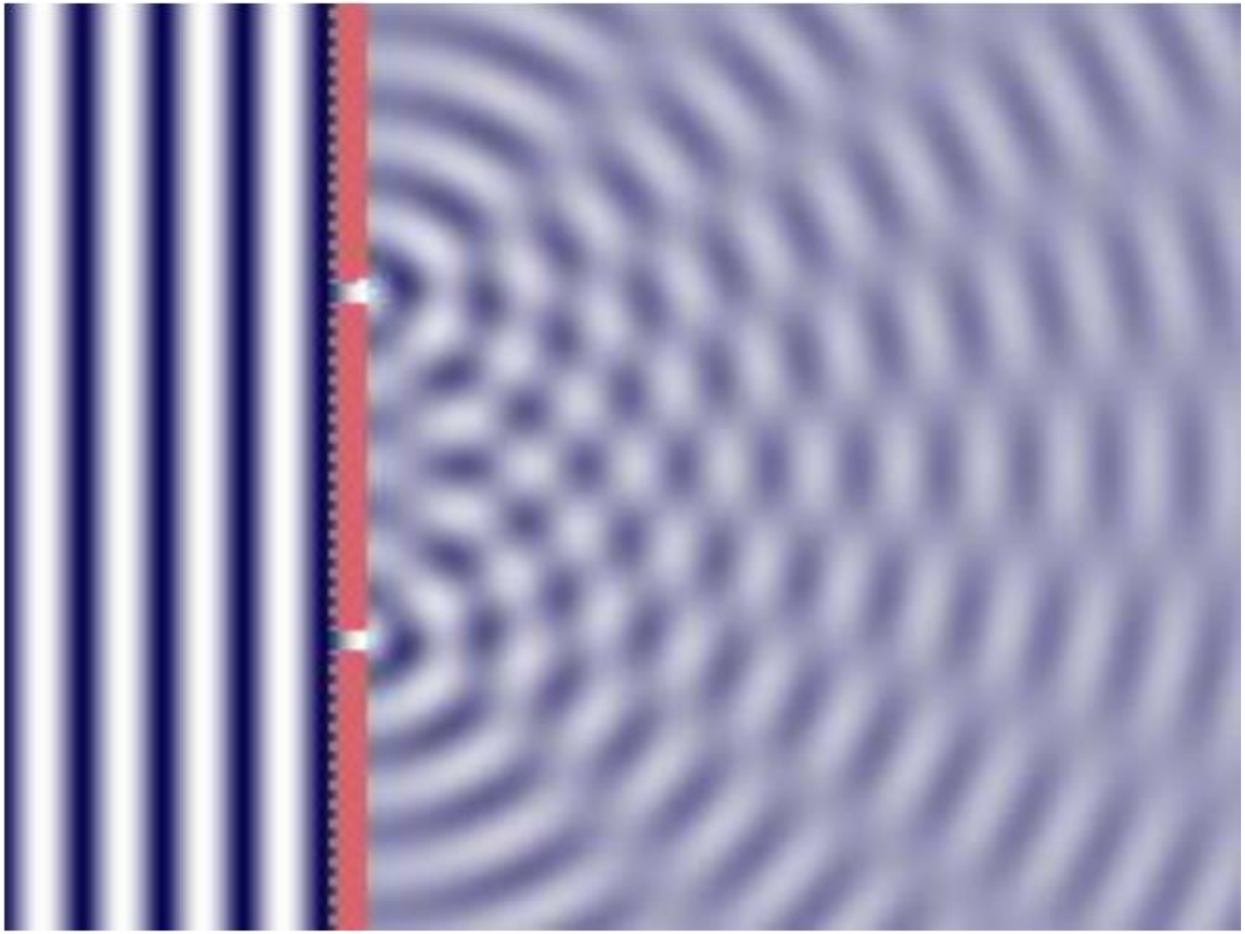


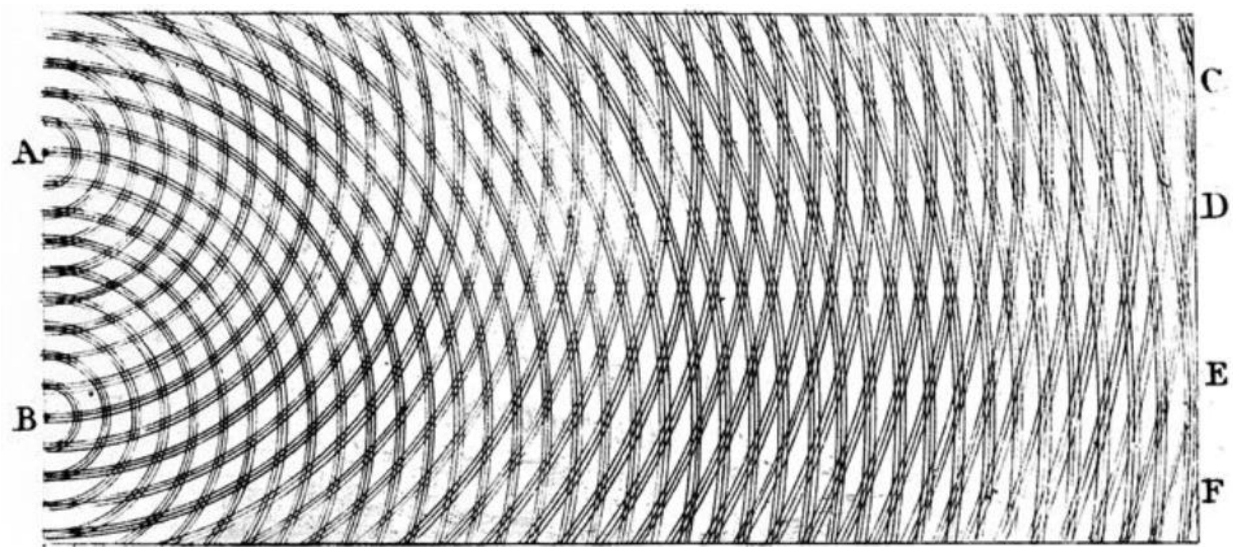
Add destructively

Interference fringes



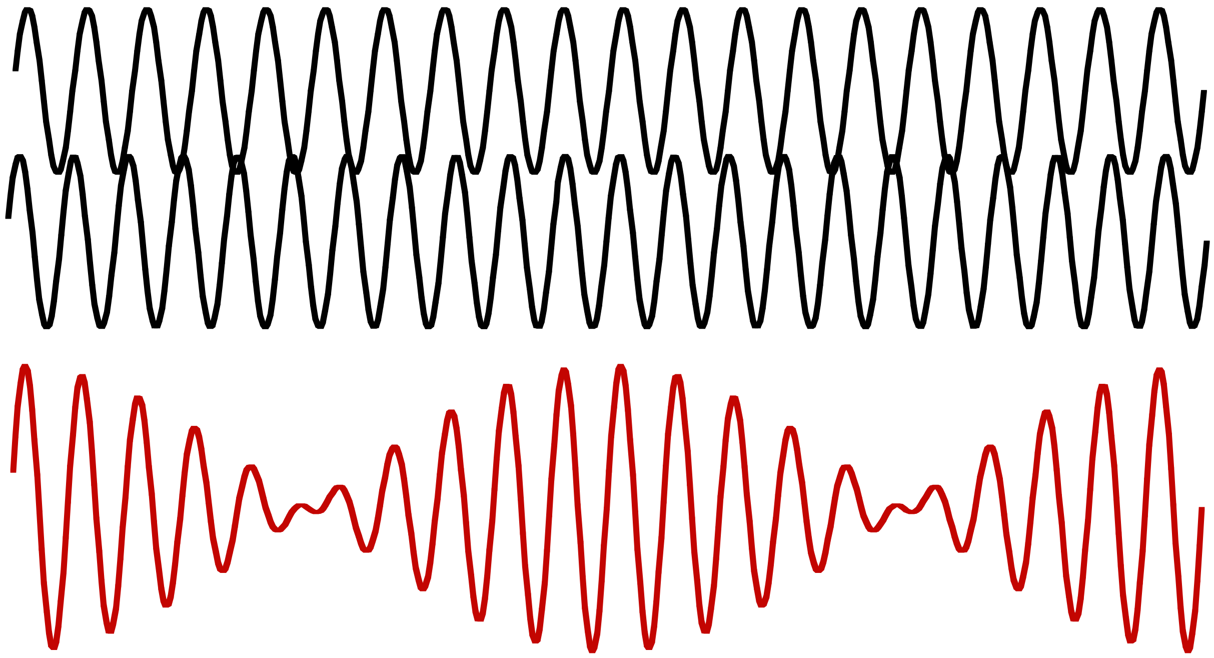
Young's slits experiment



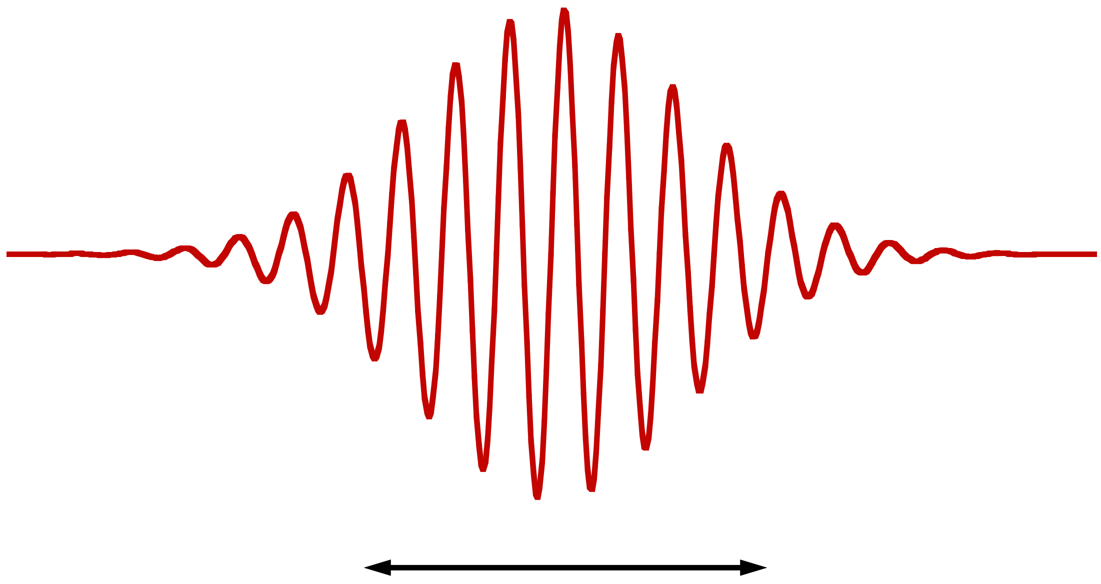


Thomas Young's sketch to explain the interference pattern from two slits. Presented to the Royal Society in 1803.

Interference between waves of slightly different wavelengths



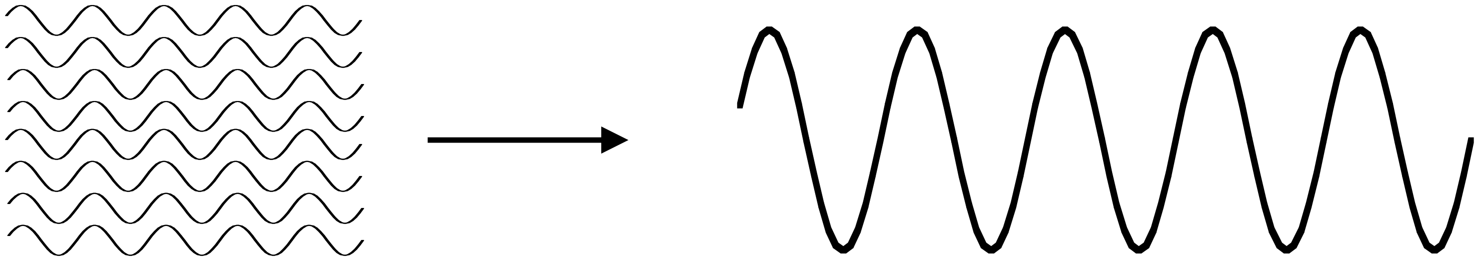
Interference between many waves of slightly different wavelengths



Coherence length

Interference between many waves

Case 1: All waves coherent and in phase



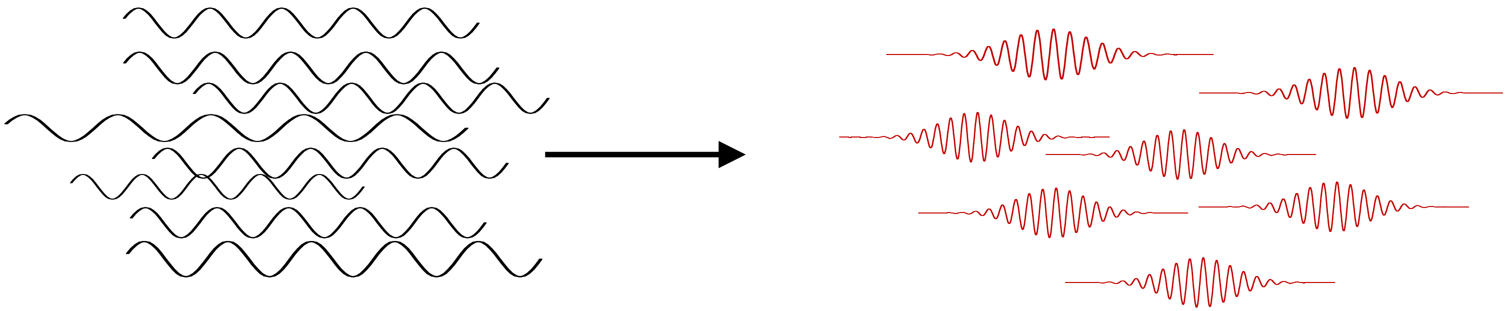
N waves each of amplitude a

Amplitude = Na
Intensity $\propto N^2 a^2$

e.g. laser light

Interference between many waves

Case 2: All waves incoherent and phase and wavelength vary randomly



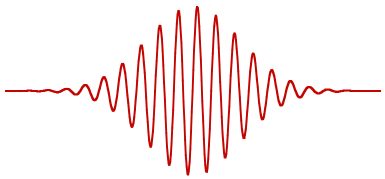
N waves each of amplitude *a*

Intensity $\propto Na^2$

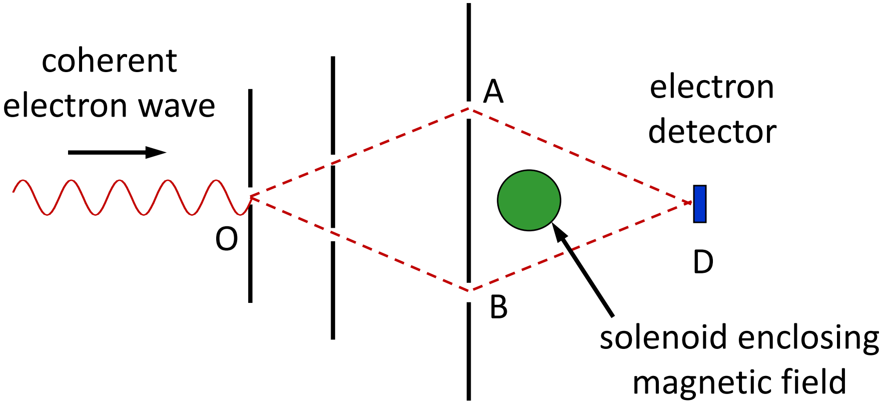
e.g. lightbulb, waves in the sea

Electron waves

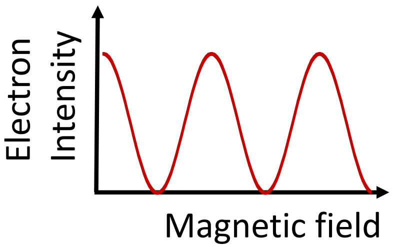
- Quantum mechanics: electrons have a wave-particle duality
- Can observe interference effects in so-called matter waves



- Aharonov–Bohm effect:

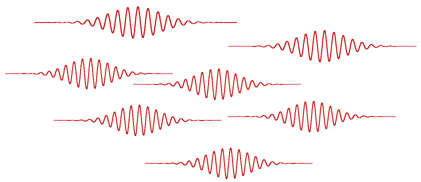


Phase difference between paths OAD and OBD due to magnetic field enclosed



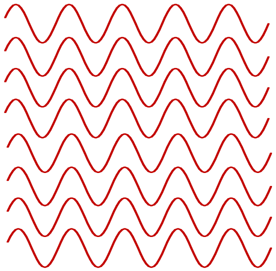
- Electrons carrying current in a normal metal are incoherent

Phase changes irregularly due to scattering e.g from defects and impurities



- Electrons (Cooper pairs) carrying current in a superconductor have phases that are locked together forming a macroscopic quantum state

Phase fixed



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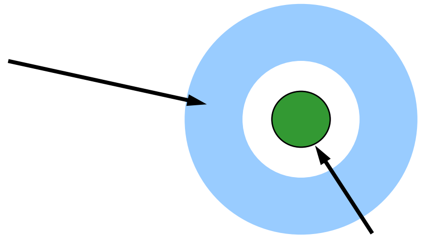
Superconducting properties are a consequence of quantum coherence

1. **Zero resistance** — the cooperative behavior of the Cooper pairs allows them to carry current without experiencing any resistance.

2. **Meissner-Ochsenfeld effect** — the application of a magnetic field induces superconducting eddy currents that oppose the applied field and perfectly cancel it. Since there is no resistance, this state persists indefinitely.

3. Magnetic flux quantisation



Perform circuit around ring:

Electron wave is single-valued

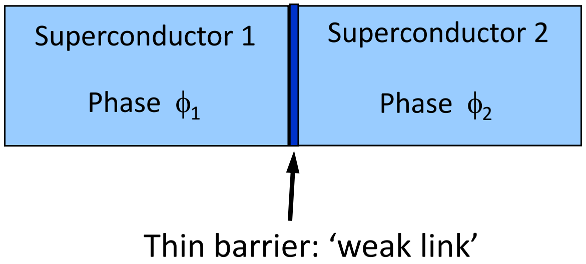
→ Phase can change by 0, 2π, 4π, 6π, …
i.e. $\Delta\phi = 2\pi \times n$ around ring

Now, $\Delta\phi \propto$ magnetic flux Φ enclosed (e.g. Aharonov–Bohm)

→ Magnetic flux is quantized!

Magnetic flux quantum $\Phi_0 = 2.07 \times 10^{-15}$ Wb

4. Josephson effect



Current flows across barrier even in absence of applied voltage:

$I = I_0 \sin(\phi_1 - \phi_2)$ dc Josephson effect

Apply voltage V across junction:

$\phi_1 - \phi_2 = 4\pi eVt/h$ (h = Planck’s constant)

$\longrightarrow$ $I = I_0 \sin(4\pi eVt/h)$ ac Josephson effect ($f = 2eV/h$)

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Summary

an amazing!!

- Superconductivity is ~~a~~ macroscopic coherent state of electron waves.
- Zero resistance and the Meissner-Oschenfeld effect are consequences of the cooperative behavior of the Cooper pairs.
- Magnetic flux trapped in a superconductor is quantized.
- Josephson currents flow between two superconductors separated by a thin barrier. Applications include SQUIDs, voltage standard, quantum bits (qubits) in superconducting quantum computers.

Slides adapted with enormous gratitude from those of Prof. Andrew Boothroyd