Towards an Imaging Lattice for Magnetically Trapped Atoms

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Abstract

The imaging of neutral atoms confined in an optical lattice has been demonstrated by a number of groups, resolving single fluorescing atoms in samples of hundreds of atoms. To obtain a high signal the atoms remain confined in the wells of a deep optical lattice whilst fluorescence excitation light is applied for high resolution imaging. In most cases, the excitation light frequency is chosen so that there is a concomitant laser cooling effect. In existing implementations of an imaging lattice the preceding manipulation stage has used optical trapping and manipulation. In this thesis I describe work towards applying this imaging lattice technique on a magnetic trapping apparatus, in particular for carrying out quantum simulation experiments.

An introduction to the specific challenge of performing a quantum simulation of the fractional quantum Hall effect is given, with the link to and nature of the imaging lattice technique explained. The existing quantum-gas machine that has been developed to produce Bose-Einstein condensates of $^{87}\text{Rb}$ in this work is then described, including the new optical lattice subsystem and the associated diode laser system. An auxiliary project to develop a servo-control system for frequencies that are several GHz away from an atomic reference is presented.

We implement an optical lattice using laser light with a frequency detuning of $-20\,\text{GHz}$ below the D1-resonance line. The construction and geometry of the individual dipole trapping beams in combination with our existing apparatus is discussed as well as the alignment methods used. We confirm the presence of three independent 1D lattices through diffraction, and observe the decay of a BEC loaded into a lattice using absorption imaging. We observe good long-term alignment and operational stability of the optical lattice.

This system will enable us to ultimately detect single fluorescing atoms in the optical lattice, following the application of excitation light co-propogating with lattice beams, via an imaging system with a moderate numerical aperture of 0.27.
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Chapter 1

Quantum simulation with neutral atoms

This chapter gives a brief overview of quantum simulation using neutral atoms, in particular focusing on the fractional quantum Hall effect (FQHE). This has been a long-standing aim in Oxford and which is a major motivating factor for the development of single-atom imaging.

1.1 The 2D electron gas

![Figure 1.1: A schematic of the current $I$, magnetic field $B$ and measured longitudinal and Hall voltages with respect to a conducting slab, in a measurement of the Hall resistance. Taken from [1].](image)

The integer and fractional quantum Hall effects are condensed matter effects that, in their explanation and extension lead to a rich vein of interesting physical problems. Of these two effects, the FQHE is the more subtle and complex — there are a number of rather exotic fractional quantum Hall states that warrant further investigation. In
this introduction, the states of matter that exist in the FQHE are described first, closely following reference [1].

The Hall resistance is measured in a 3-dimensional electron gas (3DEG) when a static magnetic field is applied to a conductor in the direction perpendicular to the direction of flow of a current $I$, with the voltage $V_H$ measured across the conductor in the direction perpendicular to both: $R_H = V_H/I$. The geometry of this system is shown in Figure 1.1. Looking at this transverse resistance in a 2-dimensional electron gas (2DEG) system in 1980, von Klitzing discovered the integer quantum Hall effect by observing plateaus in the Hall resistance as a function of the magnetic field strength at precisely quantised values given by:

$$R_H = \frac{\hbar}{e^2} \frac{1}{n}$$

with $n$ an integer, and $\hbar$ and $e$ Planck’s constant and the electron charge. After further study the resistance was shown to be quantised to within 1 part in $10^9$, in units of the von Klitzing constant $\hbar/e^2$.

This quantisation can be understood in terms of Landau levels. In the presence of an applied magnetic field, the charged particles of this system enter into quantised cyclotron orbits with angular frequencies $\omega_c = eB/m_e$. The spacing between levels increases with increasing magnetic field, and the number of filled levels decreases until the only only occupied level is the lowest Landau level (LLL). The exact number of filled Landau levels depends on the electron density and the applied magnetic field and is denoted by the filling factor $\nu$.

The most well known signature of the integral Hall effect is the quantisation of the transverse Hall resistance — plateaus in $R_H$ — along with the coincident occurrence of a ‘superflow’ which is a drop in the longitudinal resistance $R_L = V_L/I$. This longitudinal resistance goes to zero in the limit of the temperature approaching 0 K. Figure 1.1 indicates the relative directions of these resistances. Together, these two signatures of the IQHE state are easily recognised on plots against the magnetic field strength as shown in Figure 1.2 — the longitudinal resistance (labeled $R_{xx}$) falls to zero at many values of the magnetic field (for example, at a field of 4 T where the filling factor is 1).

In 1998 the Nobel prize in physics was awarded to H. Störmer, D. Tsui and R. Laughlin, with the latter being responsible for a theoretical explanation, and the first two the experimental observation, of a quantum Hall state with a fractional filling factor of $1/3$ ([2]). This discovery was the first of many — whilst only the $1/3$ state
was known at the time Laughlin presented his now famous wave functions ([3]), a number of states with different fractional filling factors have since been discovered.

The fractional quantum Hall effect is seen in the existence of plateaus and dips in the Hall and longitudinal resistances, respectively, at values of $R_H$ given by;

$$R_H = \frac{\hbar}{e^2 f}$$  \hspace{1cm} (1.2)

where in this case $f$ is a rational fraction, rather than an integer. The most prominent plateaus in $R_H$ occur for fractional values below 1 where it has been more experimentally feasible to resolve the applied changes in magnetic field arising from the inverse response of the Landau level filling to the B-field. This can be seen in Figure 1.2. The largest dips come from the $f = 1$ integer quantum Hall state and the Laughlin $f = 1/3$ fractional quantum Hall state.

![Figure 1.2](image)

**Figure 1.2:** The longitudinal resistance as a function of magnetic field strength for a 2D electron gas at 35 mK. Note that some labeled filling factors do not have an associated FQHE state e.g. 1/2. Reproduced from [4].

Of these LLL fractional Hall signatures, the one occurring at $f = 1/3$ was the first to be observed due to its comparatively large extent. Improving experimental conditions with increased electron mobility, higher magnetic fields and lower temperatures
made it possible for further fractional states to be identified. It is now clear that there are a large number of fractions that manifest, each corresponding to a different quantum state with similar experimental signatures — signatures constrained by the nature of the 2DEG systems used, such as GaAs heterostructures and quantum wells. The resistance measurements are carried out with relatively standard condensed matter techniques and tools (such as dilution fridges), but a major effort in this work was obtaining samples of the highest possible purity (as characterised by a high mobility of the electrons). Even then, such measurements give an indirect picture of the nature of the 2DEG in a FQH state as only bulk properties of the electron gas are measured. This limitation can be seen as a motivation for exploring alternative FQH systems.

The inability to directly probe the distribution of particles within the 2DEG makes it difficult to verify one interesting prediction about the system, indicated by Figure 1.3, which shows the result of a simulation for 1000 electrons in a random distribution (left) and in the state $\Psi_3$ (right). $\Psi_3$ is referred to as the ‘Laughlin wavefunction’ and it describes the state of the system in the FQH state with filling factor $f = 1/3$;

$$\Psi_3 = \prod_{j<k} (z_j - z_k)^{1/3} \exp \left[ -\frac{1}{4} \sum_i |z_i|^2 \right]$$  \hspace{1cm} (1.3)

\textbf{Figure 1.3:} The difference between a random distribution of 1000 electrons (LHS) and those in the $\Psi_3$ state (RHS) where electrons have a preferred separation. Taken from Chapter 7, Figure 7.7, page 259 of [5].

The contrast between the two situations of Figure 1.3 is striking. The FQH state displays a near-uniform electron density. A Landau level structure is sufficient to understand the IQHE, however it is only with the addition of the inter-particle
correlations shown that the FQHE can be explained. Laughlin describes this feature of these states in the following way (in [5]): “The (FQH) effect occurs when electrons are at a particular density, determined by the magnetic field strength. The separation between neighbouring electrons locks in at particular values.” Further, “The effect occurs only in the cleanest samples.” Figure 1.3 demonstrates this density behaviour, and this quotation highlights the potential usefulness of working with alternative, naturally cleaner systems.

Figure 1.3 shows the predicted characteristic density distribution for electrons in a particular FQH state (in [6] distributions for other filling fractions are computed), but this has not been observed directly — and for the 2DEG system it is extremely challenging. The observation of strongly correlated states is the major issue of interest in quantum simulation, and a possible solution is outlined next in Section 1.2, that provides an extremely pure sample. Further, it should be expected that an alternative system will present new possibilities in terms of manipulation and direct detection.

1.2 The Bosonic FQHE

In our experiments we produce ultracold Bose gases and so the connection to the FQHE in a gas of electrons (fermions) is not obvious. However, in the same year as the Nobel prize (1998) for the discovery of the FQHE in solid-state systems, Cooper, Wilkin and Gunn ([7]) suggested that degenerate ultracold atoms are an alternative system in which to study FQHE states. This first paper highlights the analogy between a rotating bosonic system and the fermionic system in a magnetic field usually associated with the FQHE. They developed this idea in a series of subsequent papers ([8], [9] and [10]) with the prediction of the existence of “…novel phases in the dilute limit of rotating Bose-Einstein condensates” closely related to those seen in the FQHE.

The equivalence between an applied magnetic field on charged particles (electrons in a 2DEG) and an imposed rotation on neutral particles (bosonic atoms — $^{87}$Rb in our work) can be seen through the following analysis. Electrons with charge $e$ moving with velocity $v$ in an applied $B$ field (as in the FQHE) experience a Lorentz force $F_L = -e v \times B$. For neutral atoms of mass $m$ in a rotating frame, the Coriolis force is $F_C = 2mv \times \Omega$ with $\Omega$ the angular velocity. Therefore if we set $F_L = F_C$ for an equivalent force on the two different particle species, the condition for equivalent behaviour is that the electron cyclotron frequency is equal to twice the rotational
frequency: $\omega_c = eB/m_e = 2\Omega$. This classical argument can be made more rigorous by the following quantum mechanical treatment.

For the neutral atom system we consider the atoms (with mass $m$) in a harmonic trap with trapping frequencies $\omega_\perp$ and $\omega_z$, the tangential and axial oscillation frequencies respectively — perpendicular to and parallel to the axis of some applied rotation (with angular velocity $\Omega$) respectively. The energy arising from this rotation is the product of the angular velocity and the angular momentum: $-\Omega \cdot \mathbf{L} = -\Omega \cdot \mathbf{r} \times \mathbf{p}$ where $\mathbf{p}$ is the linear momentum, so that the full Hamiltonian for the rotating atomic system can be written as

$$H_\Omega = H_0 - \Omega \cdot \mathbf{r} \times \mathbf{p}. \quad (1.4)$$

Expanding the $H_0$ term for the linear momentum and trap energy (first step), and grouping terms (second step) gives the following for neutral atoms with an applied rotation:

$$H_\Omega = \frac{1}{2m} \mathbf{p}^2 + \frac{1}{2} m \omega_\perp^2 (x^2 + y^2) + \frac{1}{2} m \omega_z^2 z^2 - \Omega \cdot \mathbf{r} \times \mathbf{p} \quad (1.5)$$

$$= \frac{1}{2m} (\mathbf{p} - m\Omega \times \mathbf{r})^2 + \frac{1}{2} m (\omega_\perp^2 - \Omega^2) (x^2 + y^2) + \frac{1}{2} m \omega_z^2 z^2. \quad (1.6)$$

Consider Equation 1.6 when the rotation angular frequency approaches the confining harmonic trapping frequency in the opposing direction to the centripetal force. In the limit where $\Omega \to \omega_\perp$, the tangential confinement approaches zero and for $\Omega \gg \omega_\perp$ the rotational force expels atoms from the trap — this is referred to as the ‘deconfinement problem’, the presence of which makes the practical task of approaching this limit a delicate operation (the experimental challenge is discussed further below).

Returning to the 2DEG system with an applied B-field, the Hamiltonian is written in terms of the generalised momentum which includes the magnetic vector potential $\mathbf{A}$. In a stationary, uniform magnetic field as in the case of interest, the vector potential can be defined with the symmetric gauge $\mathbf{A} = \frac{1}{2} \mathbf{B} \times \mathbf{r}$;

$$H = \frac{1}{2m} (\mathbf{p} - q\mathbf{A})^2 = \frac{1}{2m} \left( \mathbf{p} + \frac{e}{2} \mathbf{B} \times \mathbf{r} \right)^2. \quad (1.7)$$

A comparison of Equations 1.7 and 1.6 confirms the equivalence between the two system Hamiltonians under the condition $\omega_c = eB/m_e = 2\Omega$. The trapping frequency term in Equation 1.6 proportional to $\omega_z$ can be neglected as $\Omega \to \omega_\perp$ since $\omega_z^2 \gg (\omega_\perp^2 - \Omega^2)$ so the atomic gas is essentially two-dimensional: the cloud spreads out tangentially so that the temperature and chemical potential $\mu$ both become lower than $\hbar \omega_z$, the harmonic oscillator energy in the tightly confined $z$ direction. In this regime the axial motion is ‘frozen out’. When the harmonic trapping terms are
neglected, Equation 1.6 has exactly equal structure to the Hamiltonian in Equation 1.7.

Further analysis, presented in [11] and elsewhere, gives an indication of the possible experimental conditions necessary to generate the desired FQHE atomic states. Estimated requirements give low atom numbers of $\sim 50$, which, in the context of our goal of observing the characteristic density distribution signature, guided the selection of the imaging method described in later chapters. The experimental requirement of very low atom numbers is expanded upon in Section 1.2.1.

### 1.2.1 Cold atom experiments

The experimental limit on the atom number is one of the factors that makes observing the atomic FQHE states very challenging, and which indicates a requirement for the novel imaging technique developed in this thesis. It should be noted that detecting the anti-correlated distribution shown in Figure 1.3 (and discriminating from the random distribution case) via direct single-particle imaging does not require large atom numbers — the difference should be observable with the $\sim 50$ atoms suggested above. Chapter 2 contains an examination of appropriate imaging methods.

One of the motivations to look for the FQHE with neutral atoms is the possibility of carrying out direct imaging of the particle density. In addition to this, as stated in [12], there is a high degree of controllability (and purity) available in a cold atom experiments. An *indirect* approach that has been attempted in [13] was to work with a large number of isolated small clusters of atoms and detect the presence of the characteristic uniform density FQHE states averaged across many clusters.

This as yet unpublished work is the only existing claim of creating FQHE analogue states with neutral atoms. That experiment formed many isolated clusters of $\sim 4$ atoms per site of an optical lattice, constructed in such a way that angular momentum could be imparted to each cluster through a time-averaged rotating potential. Their detection method relied upon a ‘photoassociation-based probe of short-range correlations’. Pairs of atoms confined in close proximity (as in a single optical lattice well) undergo inelastic light-induced collisions when exposed to near-resonant (as in the MHz detuning used in laser cooling) light, which causes them to be lost from the trap with a high probability. As such, the loss rate reflects the probability of finding pairs of atoms in close proximity, which is strongly suppressed in FQHE states. In [13] the total number of trapped atoms remaining after photoassociation is measured against rotation rate (applied prior to the photoassociation light pulse and the subsequent imaging pulse), and a reduction in the loss rate where the rotation rate $\Omega$
approaches the trap frequency $\omega_{\perp}$ was observed. The total atom number was $\sim 10^4$, spread across several thousand lattice sites.

Indirect detection methods such as this (averaged over an ensemble of samples) do not provide the level of detail that will give an unambiguous confirmation of the presence of bosonic FQHE states, and limit the scope of new physics that can be explored.

Typically, we make measurements of cold atomic gases by removing the trapping potential and allowing the sample to expand prior to imaging. In the low density case of FQHE states, it is an important question as to whether features of the in-trap FQHE state are preserved during this expansion. In [14] this question is analysed and the positive conclusion is that “...the particle distribution in coordinate space in a free expansion (time of flight) experiment is related to that in the trap at the time it is turned off by a simple rescaling and rotation.” Further, in this paper the authors point out that because there is no loss of distribution information in expansion, it is reasonable to consider a high-resolution ‘snapshot’ image, i.e. direct detection of the density at a single particle level. Since expansion does not lead to a loss of the distribution information about the correlations of interest it is therefore possible to increase the inter-particle separation prior to imaging in order to reduce the required resolution, i.e. we can ‘magnify’ the quantum states by a time-of-flight method rather than constructing a very high resolution optical system.

### 1.2.2 Our proposed neutral atom FQHE scheme

We have developed two techniques that in combination provide a promising system for generating and observing a neutral atom FQH state. The scheme is shown schematically with Figures 1.4, 1.5 and 1.6. The scheme makes use of two techniques demonstrated by our group — the first is a magnetic potential for atom trapping that possesses an unusually high degree of versatility (compared to other types of magnetic trap) in general as well as favourable properties for two-dimensional confinement, as demonstrated in [15]. The adiabatic potential arises from the application of RF radiation at MHz frequencies to atoms trapped in a static quadrupole field. The RF ‘dresses’ the bare Zeeman substates, the result of which is an ellipsoidal potential minimum along a contour of constant B-field. With gravity, atoms are confined at the bottom part of a surface with a high trapping frequency $\omega_z$ (calculations indicate that a trap frequency of $\omega_z/2\pi = 4 \text{kHz}$ could be achieved) in the axial direction. The second technique is a dipole trapping beam that can be accurately and rapidly moved in two directions with a two-axis acousto-optic deflector (AOD). By time-averaging
a Gaussian beam an array of potentials can be generated, as seen in [16]. We have also implemented the dipole beam manipulation shown in Figure 1.4 offline.

The specifics of these techniques are not covered in this thesis. For further details the references given, as well as the theses [17] and [18], can be studied.

A possible application of the adiabatic trapping potential and the moving dipole beam is illustrated. The series of images in Figure 1.4 show our scheme for introducing rotational angular momentum to a sample of ultracold atoms trapped with strong confinement in the radial direction (such that the atoms are 2-dimensional) from the adiabatic potential and strong confinement in the tangential direction \( \omega_\perp \) by tightly focusing the dipole beam.

\[ \text{Figure 1.4: } (a) \text{ Atoms are confined on the ellipsoidal adiabatic potential surface, at the bottom due to gravity. Strong confinement in the tangential direction comes from a tightly focused red-detuned dipole beam. (b) The dipole beam can be time-averaged through circular rotation to create a uniform intensity profile. (c) Similarly, an asymmetric elliptical potential can be generated by time-averaging. (d) This elliptical potential can rotated to impart angular momentum to the trapped atoms.} \]

Figure 1.5 shows schematically, in the spirit of Figure 1.3, the sought-after single-particle level density change occurring when transitioning to a FQHE state at high rotation frequencies induced by the rotating dipole beam. Figure 1.6 illustrates the final stage in the envisioned experimental sequence. Removal of the dipole beam giving tangential confinement begins an expansion along the surface of the adiabatic potential (from the ‘centrifugal force’), which is predicted to preserve the atomic distributions. The detection stage, following from Figure 1.6, is considered in the next chapter.

### 1.3 Summary

In this Chapter I have described how the FQHE can be realised with neutral atoms, which offers the possibility of observing features of the FQHE complementary to the solid-state experiments. Simulating an applied magnetic field with rotation allows
Figure 1.5: (e) At low angular momentum, a small sample of trapped neutral atoms has a random spacing (no well-defined inter-particle separation is visible). (f) High angular momentum states are a neutral atom analogy of a FQHE state, with a key property being the predicted emergence of observable anti-correlations.

Figure 1.6: (g) After removal of the tangentially confining dipole beam atoms move outwards along the surface of the adiabatic potential (side view). (h) Correlations in the particle distribution are preserved during expansion and the greater spacing permits imaging of individual atoms (view from above).
the direct observation of the uniform density and anti-correlations predicted to arise in these states. A possible technique for their generation has been presented, with the detection method to be considered. This is presented in the following chapter.

The benefits of studying the FQHE with neutral atoms can be summarised as:

1. Increased purity — whilst a solid-state system must be created with high electron mobility\(^1\) to observe the FQHE with an electron gas, the atomic system is naturally pure.

2. New observables — as made clear above, one of the central predictions about the nature of the FQHE state cannot be easily verified in the solid-state (but can for the atomic system).

3. Techniques for manipulating ultracold atoms are highly advanced, with some of these capabilities shown in our own group, and able to provide control over atomic FQHE states.

\(^1\) Perhaps surprisingly, a small but essential amount of disorder is required for an observable effect, as pointed out in [19].
Chapter 2

Advances in imaging atomic states

This chapter describes methods of capturing an image of atoms at the single-particle resolving level required to take a ‘snapshot’ such as that shown in Figure 1.6 and discussed in the previous chapter.

Whilst there is one recently developed all-optical detection method that is widely applicable and has favorable performance, it is worth noting the existence of some other techniques for single-atom detection that recreates the original positions. The first uses a scanning electron beam to selectively ionize individual atoms which are then drawn into an in-vacuum detector; [20]. In the second, a position sensitive ‘multi-channel plate detector’ is located in the vacuum chamber, onto which metastable rare-gas atoms are dropped (undergoing a time-of-flight expansion); [21]. Both of these techniques have a high hardware requirement that reduces the apparatus flexibility, and as such they are not appropriate in most cases where, for example, good optical access or a high magnetic field is required. An all-optical detection method as outlined in Section 2.1 below is therefore preferable.

2.1 Imaging lattice detection

A very powerful technique of resolving single neutral atoms in many-atom samples has emerged in recent years that has a range of potential applications but in particular the possibility of addressing the atomic FQHE detection problem. The application of this imaging technique in different labs is currently a very active area in the atomic physics community, and the different labs that have implemented it are listed in Table 2.1. The technique combines three elements;

1. An optical lattice with confinement in all spatial directions.
2. Excitation light that leads to fluorescence scattering from the atoms.
3. An imaging system to collect the fluorescence light.

Each of these elements is examined in more detail below before presenting our scheme in Section 2.2, which incorporates features from the successful experiments in Table 2.1. In particular, our scheme is most closely related to that used by the Greiner group at Harvard in [22].

2.1.1 Elements of an imaging lattice

With regards to the imaging system, in the pioneering experiments of the Bloch and Greiner groups it was essential to have an imaging system that could resolve atoms on neighbouring optical lattice sites (for example, in the experiment of [23] the dynamics of a single spin impurity was monitored, and in [24] site-resolved imaging was used for sensitive thermometry). This requires a numerical aperture (NA) large enough to satisfy the condition

\[ \frac{1.22}{\text{NA}} \frac{1}{\lambda_F} \lesssim d \quad (2.1) \]

for imaging atoms in a lattice with periodicity \( d \) with light of wavelength \( \lambda_F \). This resolution criterion is relaxed for regularly spaced atoms in an optical lattice where the periodicity is known and can be projected onto the observed fluorescence distribution — using this additional information to the problem, the diffraction limit defined above can be ‘beaten’ to a degree. Broadly speaking a missing atom, or hole, in an otherwise filled lattice corresponds to a gap of \( 2d \) where the localised reduction in intensity seen in imaging can be ascribed to a missing atom. The alternative approach is to image a dilute cloud where the probability of nearest-neighbour site occupation is low and hence a lower resolution is required. For example, an NA of 0.68 gives a resolution limit of 700 nm by this criterion, however in [25] they discern the presence of neighbouring atoms with this NA and a lattice spacing of 532 nm by applying an algorithm which convolves a (separately acquired) average single-atom point spread function with a reconstruction of the optical lattice structure.

The presence of an optical lattice for the imaging phase (element 1) comes naturally from the optical lattices used in the manipulation stage of the experiments listed in Table 2.1 (the exceptions being [26] and [27], where 3D and 1D lattices for fluorescence imaging are loaded directly from a magneto-optical trap).

Typically the manipulation stage of existing experiments takes place in a shallow optical lattice such that there is tunneling between sites (by atoms of mass \( m \)) in the

\footnote{Using Rayleigh’s criterion for overlapping Airy disks, with the transmitting medium refractive index being 1 and making the usual paraxial approximation.}
paradigm example of observing the Mott insulator transition. The required lattice
depths are on the order of tens of $E_R$ in units of the photon recoil energy $E_R = (\frac{\hbar}{\lambda})^2 \frac{1}{2m}$.
At the beginning of the detection stage the lattice depth is increased (to a depth of a
few thousand $E_R$), effectively ‘freezing’ the atoms in place by shutting off inter-lattice
site tunneling prior to the depth increase. The fluorescence images are then obtained.
The increase in the depth $U_{\text{dip}}$ required to go from the manipulation stage to the
detection stage was achieved in two different ways by the Bloch and Greiner groups;
the former simply increased the intensity $I$ of the confining light whilst the latter
switched on another lattice with a much lower frequency detuning $\Delta$; the depth of a
dipole trap is $U_{\text{dip}}(r) \propto I(r)/\Delta$ as given in Equation 3.15.

Increasing the lattice depth up to several hundred $E_R$ suppresses site hopping
at the temperature of atoms that have undergone a preceding stage of evaporative
cooling, but for in-situ fluorescence imaging the depth must be increased to several
thousand $E_R$. If the excitation light simply imparts $\sim 2E_R$ of energy for every
scattering event the atoms will heat up and be lost from the trap when their energy $\geq
U_{\text{dip}}$. This would impose a limit of scattering a maximum of several thousand photons
per atom, whereas for a good signal we want to collect (as in [26]) $\sim 3000$ photons per
atom during the exposure time when the collection efficiency is 10 % or less. Therefore,
to collect fluorescence light over a long duration, the excitation light parameters are
set so that it simultaneously has a laser cooling effect to allow unambiguous detection
of the trapped atoms above the background noise. It is informative to compare the
depth of the confining optical lattice, expressed in units of kelvin (K), to the Doppler
temperature $T_D$ (the lowest temperature expected from the application of optical
molasses laser cooling without a sub-Doppler cooling mechanism, as described in
[28]). For $^{87}\text{Rb}$, $T_D = 146 \mu\text{K}$, and in both the Bloch and Greiner $^{87}\text{Rb}$ imaging
lattices $U_{\text{dip}} \sim 2k_B T_D$. At this depth, inter-site tunneling and loss from the lattice
by atoms during the exposure time is below 10 %, i.e. this depth is sufficient to keep
atoms confined assuming they remain at or below the Doppler cooling limit.

Laser cooling techniques for a number of atomic species are well developed, and
applicable in optical lattices as well as free space. Perhaps the most well known are
those found in magneto-optical trapping of Rb — optical molasses cooling relies on the
Doppler effect and simple momentum transfer to cool atoms down to $T_D$, and taking
into account a more complex energy level structure allows for lower sub-Doppler
temperatures with polarisation gradient cooling. As an indication, in [29], a model of
the measured thermal hopping rate under molasses light indicates a temperature of the
atoms during laser cooling of 26 $\mu\text{K}$. Both of these methods can be applied to lattice
trapped atoms allowing for cooling (from absorption) with concomitant fluorescence (from emission). For fermionic species laser cooling techniques are more challenging. This is the main reason why the demonstration of single atom fluorescence imaging in optical lattices with fermions took a further six years following the first demonstration with bosons.

An exception to the need for simultaneous laser cooling is recent work in [30] on the element Yb (which has two valence electrons). Yb lacks hyperfine splitting so that polarisation gradient cooling methods for sub-Doppler cooling are not applicable. However, the heating incurred to obtain about $10^4$ photons per atom for imaging with Yb can be lower than an achievable deep lattice, so that laser cooling is not required from the excitation light.

The imaging system that collects some fraction of the scattered excitation light to form an image is the final element of a general imaging lattice system. Typically there are two key components: an objective lens that captures fluorescence light from the atoms and a sensitive CCD camera. When considering the collection of fluorescence photons by a lens, it is useful to convert the numerical aperture value into the fraction of the light collected from the full solid angle sphere of emission, given by $\frac{1}{2}(1 - \cos(\theta))$, where $\theta = \arcsin(NA)$ is the half-angle of the cone of light entering the objective. In the limit where the numerical aperture approaches unity, 50\% of emitted light is captured. When considering the required numerical aperture, the primary consideration should be the resolution of the objective, given above by Equation 2.1, as the ultimate experimental limit on the duration of the imaging process is more likely to come from the camera. If the fluorescing atoms have a small hopping and loss rate, as seen in other experiments, with the latter in the limit determined by background gas collisions on a timescale of 100 s — the lifetime measured for our vacuum system — then the imaging duration can be very long. This assumes that the noise level on the camera is low. Additionally, the camera quantum efficiency ranges from as high as 90\% down to perhaps 40\% in the types of cameras typically used in these experiments.

Beyond these elements (the lattice, excitation light and imaging system) of an imaging lattice, there is an additional common feature of all experiments. Where the number of atoms confined together with a high density at a single lattice site is more than one, the applied fluorescence excitation light causes pairs to undergo inelastic light-assisted collisions which imparts enough energy for both atoms to be lost on a timescale short compared to the exposure time, i.e. the image does not show the initial occupation number at a given site, rather this method gives a binary detection of the
lattice site occupation — the original occupation number \textit{modulo} 2. This condition can be defeated under certain conditions, as demonstrated in [31], where light induced collisions are manipulated to promote single atom loss from atom pairs, leading to an 83\% probability of a single atom remaining.

Table 2.1 summarises the different experiments that have demonstrated single-atom and \textit{single-site} resolving optical lattice based fluorescence detection. Beyond those experiments listed in Table 2.1, a number of further groups have stated they have optical lattice, single-atom resolving machines under construction, including \textit{a}) a $^{6/7}\text{Li}$ quantum gas microscope in the group of Sengstock in Hamburg, \textit{b}) a $^6\text{Li}$ quantum gas microscope in the group of Bloch in Munich, 	extit{c}) a $^{40}\text{K}$ machine in the group of Thywissen in Toronto\textsuperscript{2}, \textit{d}) a Sr quantum gas microscope in the group of Schreck in Amsterdam. In the first experiment by the Weiss group, lattice beams were overlapped at small angles to give a comparably large site spacing $d$ (the increase from a spacing $d = \lambda_L/2$ as used in some designs is given by Equation 3.21).

A very high $NA$ is desirable for increased resolution and detecting a large fraction of the fluorescence signal, and many of the experiments listed in Table 2.1 feature an imaging component positioned very close to the atoms, either with one face ([30]), or the entire (Greiner and Zwierlein designs), first lens within the vacuum system. This allows for a ‘solid-immersion’ effect which enhances the \textit{effective} numerical aperture to values $\geq 0.8$. The recently published Thywissen experiment [32] achieves a similarly high $NA$ through the use of a thin sapphire vacuum window, allowing them to bring a standard microscope objective with a short working distance very close to the atoms.

One of the most interesting examples (in the context of this thesis) from Table 2.1 is the use by Greiner of a \textit{near}-resonant lattice at 795 nm\textsuperscript{3} during the imaging phase, achieving the necessary lattice depth of thousands of $E_R$ with a moderate intensity, indicating that the increased scattering of lattice light is not detrimental.

As given in Equation 3.15, the scattering rate $\Gamma_{sc}$ from dipole trapping light is $\Gamma_{sc}(r) \propto I(r)/\Delta^2 = (I/\Delta) \times 1/\Delta$ so for an equivalent depth $U_{dip} \propto (I/\Delta) = \text{const.}$ thus in general there is a disadvantage in using light that is close to resonance for dipole trapping. However, this is of little consequence relative to the \textit{excitation} light scattering rate. A detuning of a few tens of GHz reduces the rate of scattering of trapping light to $\sim 20$ kHz compared to a molasses scattering rate which can range from 60 kHz ([33]) all the way up to, in the limit of high intensities, half of the excited

\textsuperscript{2} The Thywissen group has now published results from their working $^{40}\text{K}$ single atom resolving apparatus in [32].

\textsuperscript{3} The wavelength of the $^{87}\text{Rb}$ D1 line.
<table>
<thead>
<tr>
<th>PI</th>
<th>Location</th>
<th>Atoms</th>
<th>$\lambda_L, \ d$</th>
<th>Lattice depth</th>
<th>Cooling method</th>
<th>NA</th>
<th>Citation</th>
<th>Sample image</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weiss</td>
<td>Penn State</td>
<td>$^{133}$Cs (bosons)</td>
<td>846 nm, 4.9 $\mu$m</td>
<td>165 $\mu$K</td>
<td>Polarisation gradient</td>
<td>0.55</td>
<td>[26], 2007</td>
<td>![Image 1]</td>
</tr>
<tr>
<td>Greiner</td>
<td>Harvard</td>
<td>$^{87}$Rb (bosons)</td>
<td>795 nm, 640 nm</td>
<td>380 $\mu$K</td>
<td>Polarisation gradient</td>
<td>0.80*</td>
<td>[22], 2009</td>
<td>![Image 2]</td>
</tr>
<tr>
<td>Bloch</td>
<td>Munich</td>
<td>$^{87}$Rb (bosons)</td>
<td>1064 nm, 532 nm</td>
<td>300 $\mu$K</td>
<td>Polarisation gradient</td>
<td>0.68</td>
<td>[33], 2010</td>
<td>![Image 3]</td>
</tr>
<tr>
<td>Greiner</td>
<td>Harvard</td>
<td>$^6$Li (fermions)</td>
<td>1064 nm, 569 nm</td>
<td>2.4 mK</td>
<td>Raman sideband</td>
<td>0.87*</td>
<td>[34], 2015</td>
<td>![Image 4]</td>
</tr>
<tr>
<td>Zwierlein</td>
<td>MIT</td>
<td>$^{40}$K (fermions)</td>
<td>1064 nm, 541 nm</td>
<td>220 $\mu$K</td>
<td>Raman sideband</td>
<td>0.87*</td>
<td>[35], 2015</td>
<td>![Image 5]</td>
</tr>
<tr>
<td>Kozuma</td>
<td>Tokyo</td>
<td>$^{174}$Yb (bosons)</td>
<td>1082 nm, 544 nm</td>
<td>30 mK</td>
<td>No cooling</td>
<td>0.81*</td>
<td>[30], 2015</td>
<td>![Image 6]</td>
</tr>
<tr>
<td>Kuhr</td>
<td>Strathclyde</td>
<td>$^{40}$K (fermions)</td>
<td>1064 nm, 532 nm</td>
<td>245 $\mu$K</td>
<td>EIT</td>
<td>0.68</td>
<td>[25], 2015</td>
<td>![Image 7]</td>
</tr>
<tr>
<td>Meschede</td>
<td>Bonn</td>
<td>$^{133}$Cs (bosons)</td>
<td>866 nm, 433 nm</td>
<td>400 $\mu$K</td>
<td>Molasses</td>
<td>0.29</td>
<td>[27], 2009</td>
<td>![Image 8]</td>
</tr>
</tbody>
</table>

**Table 2.1:** Noteworthy experiments capable of resolving single atoms within a lattice array. $\lambda_L$ denotes the wavelength of the laser generating the confining light, $d$ denotes the periodicity of the resulting optical lattice. Numerical aperture: * indicates enhancement via the use of a solid-immersion lens.
state decay rate $\Gamma/2 = 2\pi \times 6.1 \text{MHz}/2 = 19 \text{MHz}$ (for the $^{87}\text{Rb}$ D2 line) — so that the equilibrium temperature, with laser cooling, is not increased significantly.

Prior to the optical lattice imaging stage for which parameters are given in Table 2.1, the majority of the experiments go through a precursor stage of optical lattice trapping, where the trapping geometry is the same as in the imaging stage (only lattice depths are varied). The more straightforward way in which to load atoms into an imaging lattice is to load directly from a MOT as in the Weiss [26] and Meschede [27] experiments. It should also be noted that individual optical lattice planes are ‘selected-out’ to derive an isolated 2D plane of atoms, one benefit of which is to remove light scattered by out-of-plane atoms. The nature of the preparation stage prior to the optical lattice fluorescence imaging technique described above is the defining difference between all the so far demonstrated experiments and our scheme. The long term goal is to apply the lattice to rotating samples of neutral atoms, as shown in Figure 2.1.

### 2.2 Our imaging lattice scheme for Rb

To begin, over the last decade our research group has performed laser and evaporative cooling, and magnetic and optical trapping of the single atomic species $^{87}\text{Rb}$. Having demonstrated techniques applicable to a possible scheme for generating the states of matter we would wish to probe with an imaging lattice in Rb, we chose to work with that atomic species on the apparatus that has previously yielded promising results around trapping and manipulation in RF-dressed magnetic potentials that could ultimately provide the foundation of a scheme to explore the physics described in Chapter 1 — these developments are outlined in [15], [36] and [37]. A future goal is to combine those tools with the imaging lattice detection method considered in this work, as shown in Figure 2.1. Single, two-dimensional planes of atoms perpendicular to the imaging axis are obtained in the sequence, however to demonstrate the imaging method we load into the trapping potential simple clouds of atoms that we can derive using less complex, robust techniques — specifically the TOP trap described in Section 3.2.

The vacuum system does not contain imaging components\(^4\) that would allow us to obtain very high NA values using the solid-immersion lens approach seen in some of the experiments listed in Table 2.1. Initially, we aim to use the objective that has

\(^4\) We did investigate a design for mounting and positioning an aspheric lens inside our glass vacuum cells for enhanced resolution imaging on a different quantum gas machine to our current one, but this was not brought through to implementation.
Figure 2.1: A schematic of the imaging lattice scheme for detecting FQHE states generated in the hybrid adiabatic potential and dipole trap shown in Figure 1.4 previously. (a) Following the extinction of the dipole beam used to induce rotation, atoms expand outwards along the adiabatic potential surface. (b) We apply a deep 3D optical lattice to the atoms suddenly in order to ‘lock-in’ their positions in space. (c) The deep optical lattice has a periodicity $d$ which is greater than the inter-particle separation following expansion. Further, the inter-particle spacing is larger than the resolution limit of the fluorescence imaging system. (d) Excitation light is applied to the lattice confined atoms to generate a fluorescence signal whilst also inducing heating.
been developed with a moderate NA of 0.27, which is already installed in the imaging system shown in Section 4.5.

The two choices remaining are the lattice wavelength, or alternatively the choice between red- and blue-detuned light (where the lattice confines using attractive or repulsive light, as shown in Section 3.4), and the geometry of the different beams (the result of which determines the lattice geometry and site spacing). The latter choice is discussed in depth in Chapter 6. The most common dipole trapping solution is to use 1064 nm light frequency detuned to the red (the D1 and D2 optical resonances of $^{87}$Rb are at 795 nm and 780 nm) which is derived from an Nd:YAG laser, as is the case in many of the examples listed in Table 2.1. In previous work in our group 532 nm blue-detuned light has been used to create a hybrid dipole and magnetic trap, and separately we have used red-detuned 830 nm in [38] to create an accordion optical lattice. An alternative is seen in the Greiner example of [22], where a (comparatively, very) near-resonant optical lattice, 30 GHz blue-detuned from the $^{87}$Rb D1 line, is used. Our choice of wavelength and detuning for trapping light is to follow this example, which as well as having favourable technology requirements, as described in Section 5.1, has the unique advantage of allowing us to easily move between red and blue detuning, which is useful for testing and alignment purposes, as shown in Chapter 6.

2.3 Structure of this thesis

In the preceding Chapters 1 and 2, we have identified an outstanding physical problem of interest and then examined an imaging technique that may form part of a solution, in conjunction with a method for generating the physical state of interest. Beyond this, it is expected that combining the imaging lattice technique with the types of magnetic trap available in our apparatus will open up a range of physics problems beyond what has been identified in the previous chapter.

Chapter 3 provides the background details of rubidium (the element which we work with), magnetic trapping, used in our quantum gas machine to create samples of ultracold neutral atoms, and optical dipole trapping and optical lattices, which relates to the new experimental subsystem developed in this project. Laser cooling, which is an integral part of other imaging lattice experiments, is also described.

Chapter 4 gives details of the quantum gas machine we work with and the sequence of fields and operations we apply to trap and cool down to quantum degeneracy rubidium gas, prior to the application of dipole trapping and optical lattice light.
The machine is continuously undergoing upgrades and so changes following previous theses are described, as well as a description of the key operations that result in rubidium BECs.

The new dipole trapping subsystem installed during this project is described in two parts. The laser system, located away from the vacuum system to generate and deliver the necessary lattice and molasses light, and a sub-project on deriving a laser locking signal at detunings tens of GHz away from resonance, is presented in Chapter 5. The part of the dipole trapping subsystem located around the vacuum system is presented in Chapter 6, which describes our solution to the beam geometry question above, as well as a discussion about our beam alignment technique. A complete and effective methodology for beam alignment is crucial for making an optical lattice subsystem like ours work, and so this topic is discussed in detail.

Chapter 7 presents initial results from our work with aligned dipole beams. We operate with red-detuned light so that we are able to demonstrate the capture of a BEC into a red-detuned optical lattice, as well as highlighting the lattice structure present in all 3 individual 1D lattices through resolving the momentum distribution. Specific features of the distribution of atoms trapped in our $Z$ lattice, which provides vertical confinement, are identified and explained.

Chapter 8 concludes this thesis with a discussion about the next stages of this project to implement an imaging lattice, which is centered around fluorescence imaging. Having demonstrated the ability to confine atoms in a 3D optical lattice (albeit with losses due to heating from the fully expected effects of scattering trap light) the application of laser cooling and the detection of the resulting scattered light are the next two imaging lattice elements for us to examine. Details of the detection problem are also given.

In Appendix A the derivation of the Doppler temperature limit for laser cooling of a two-level atom is given.

In Appendix B an interesting alternative scheme to consider which separates the frequencies of the fluorescence signal and the driving excitation light is presented. This is a possible extension to the scheme used here that might improve imaging in an alternative version of the system.
Chapter 3

Trapping and cooling rubidium

This chapter provides background details relevant to the later chapters concerning our quantum gas apparatus, the sequence followed to produce a Rb condensate and the new optical lattice subsystem.

3.1 Properties of rubidium

The element rubidium is an alkali metal with two naturally occurring isotopes, $^{85}\text{Rb}$ and $^{87}\text{Rb}$, found in proportions of about 70% to 30% respectively. Both are bosonic, and have a single valence electron in the $n = 5$ shell. We make use of only the $^{87}\text{Rb}$ isotope, although we see the presence of the $^{85}\text{Rb}$ isotope in our table-top vapour cell spectroscopy, as seen in Figure 3.3. In this work we only consider the lowest energy $5S \rightarrow 5P$ optical transition, shown in Figure 3.1, which is in the infra-red. There could be a use for higher energy optical transitions (either with the orbital angular momentum quantum number $L > 1$ or the principal electron quantum number $n > 5$) for background free fluorescence detection which we outline in Appendix B.

There are two broad optical transitions in the ‘D-line’ between the $L = 0$ ($S$) ground state and the $L = 1$ ($P$) excited states. These arise from the fine structure, where spin and orbital angular momentum of the $n = 5$ valence electron are coupled. The $5^2S_{1/2} \rightarrow 5^2P_{1/2}$ transition is referred to as the D1 line ($\lambda = 795$ nm) and the $5^2S_{1/2} \rightarrow 5^2P_{3/2}$ the D2 line ($\lambda = 780$ nm). On a much smaller energy scale the nuclear spin $I = 3/2$ interacts with the electrons total angular momentum $J$, giving rise to hyperfine structure, with the quantum number $F = J + I$. These hyperfine energy levels and their energy splittings are also shown in Figure 3.1.

With narrow bandwidth lasers and techniques (saturated absorption spectroscopy) that can suppress Doppler broadening on resonance, we can resolve this fine structure in room temperature vapour cells. We use two of these transitions for laser cooling,
Figure 3.1: The structure of the two main $^{87}$Rb optical transitions. The D$_2$ line at 780 nm contains the $|F = 2\rangle \rightarrow |F' = 3\rangle$ cycling transition (described in the main text), and which is the transition most commonly used for laser cooling and imaging.

the $|F = 2\rangle \rightarrow |F' = 3\rangle$ cooling transition and the $|F = 1\rangle \rightarrow |F' = 2\rangle$ repumping transition in the D$_2$ line. Selection rules for optical transitions permit $F$ to change by only $\Delta F = -1, 0, +1$, so the excited state $|5^2P_{3/2}, F' = 3\rangle$ can only decay to the $|5^2S_{1/2}, F = 2\rangle$ ground state. Thus the $|F = 2\rangle \rightarrow |F' = 3\rangle$ transition is a cycling transition and is used to laser cool and image atoms.

However there is a small probability of off-resonantly exciting atoms to the $|F' = 2\rangle$ level from where the $\Delta F$ selection rule allows decay to the $|F = 1\rangle$ ground state, so exiting the closed cycle. In this state atoms are detuned 6.8 GHz from resonance with the cooling light making the state ‘dark’. These atoms can be repumped back into the $|F = 2\rangle$ ground state through excitation on the $|F = 1\rangle \rightarrow |F' = 2\rangle$ transition, whereby atoms can decay to either of the $|F = 1\rangle$ and $|F = 2\rangle$ ground states. These two transitions are separated by 6.8 GHz and can be seen in the spectroscopic traces of Figure 3.2, as well as some of the accompanying crossovers. The D$_2$ line $|F = 1\rangle \rightarrow |F'\rangle$ transition is much weaker than the $|F = 2\rangle \rightarrow |F'\rangle$ transition. We typically need to heat the vapour cell to increase the vapour pressure in order to resolve the transition more clearly.
Figure 3.2: Saturated absorption spectroscopy traces of the D2 line of $^{87}\text{Rb}$ from $|F = 2\rangle$ (LHS) and $|F = 1\rangle$ (RHS). The $|F = 2\rangle \rightarrow |F' = 1, 2\rangle$ and the $C_{1/2}$ crossover, found just to the left of the $C_{1/3}$ crossover, are very weak and are not shown in the LHS plot.

Addressing the rubidium D2 line optical transitions for ultracold atomic physics has been standard practice for a long time. Less attention has been paid to the D1 line, as without a cycling transition it is not useful for laser cooling. The excited state hyperfine structure is simpler, as seen in the spectroscopy trace in Figure 3.3. In this thesis we present a near resonant optical lattice potential formed by a Stark shift of the $|S_{1/2}, F = 2\rangle$ ground state on the D1 line. For the red detuned lattices observed the detuning is given relative to the $|F = 1\rangle \rightarrow |F' = 2\rangle$ optical transition shown.

Figure 3.3: A saturated absorption spectroscopy trace of the $^{87,85}\text{Rb}$ D1 line covering the $\sim 7 \text{ GHz}$ hyperfine ground state splitting. Crossover peaks are visible between the labeled pairs of hyperfine transitions.

Having identified all of the optical transitions of interest, it is also important to describe the Zeeman substrates which are degenerate unless a magnetic field is
applied. Each hyperfine $F$ state has $2F + 1$ Zeeman substates, each labeled by a quantum number $m_F$. The Zeeman energy shifts for these levels are given, in the low $B$-field regime where we exclusively operate, by the linear expression

$$\Delta E = \mu_B g_F m_F B.$$  \hfill (3.1)

where the Bohr magneton $\mu_B = e\hbar/2m_e = 9.27 \times 10^{-24} \text{ J T}^{-1}$, $g_F$ is the Lande $g$-factor and $B$ is the magnitude of the applied magnetic field.

Figure 3.4 shows the structure for the two hyperfine ground states in our system, with the value of $g_F$ given for each state. It is convenient to express $\mu_B$ in units of MHz G$^{-1}$ (rather than the SI units of J T$^{-1}$) to give $\mu_B/h = 1.4 \text{ MHz G}^{-1}$, so that, with the values given in Figure 3.4 for the Zeeman substates, the frequency shifts of the $|F = 1, m_F = \pm 1\rangle$ states is $\mp 0.7 \text{ MHz G}^{-1}$.

![Figure 3.4: The degeneracy of magnetic Zeeman substates is lifted with an applied magnetic field of magnitude $B$. The states are labeled with $m_F$ quantum numbers and shift up or down in energy depending on the value of $g_F$ for the hyperfine state.](image)

### 3.2 Magnetic trapping

The magnetic field dependent energy level structure shown in Figure 3.4 makes it possible to confine atoms against gravity with an applied magnetic $B$-field. For the ‘low-field seeking’ $m_F$ states $|F = 1, m_F = -1\rangle$ and $|F = 2, m_F = 1, 2\rangle$ a minimum in space of $B$ gives a potential energy minimum according to Equation 3.1 where


atoms can be trapped. The other \( m_F \) states are either anti-trapped or untrapped in a similar \( B \)-field.

The simplest construction that generates a \( B \)-field that has a free-space local minimum is a quadrupole field, which can be created in the lab with a pair of coils with a shared symmetry axis, and opposite current flow. A field created in this way has \( B = 0 \) at the mid-point between the coils along the symmetry axis. Near this field zero the field is given by:

\[
B(r) = B'_q(x \mathbf{e}_x + y \mathbf{e}_y - 2z \mathbf{e}_z); \tag{3.2}
\]

where \( B'_q \) is the quadrupole field’s gradient along the direction the directions \( \mathbf{e}_x \) or \( \mathbf{e}_y \). For convenience we set the coordinate origin at the field zero. The quadrupole trap is cylindrically symmetric about the axis of the coils, as can be seen from Equation 3.2. The gradient along the vertical \( z \) symmetry axis is a factor of 2 higher than in the horizontal plane. Thus the contours of constant \( B \)-field are ellipsoids with their minor axis in the \( z \) direction, and the minor axis length \( 1/2 \) that of the major axis.

This type of quadrupole trap works best for hotter clouds, where atoms have a smaller probability of passing close to the field zero where they can be lost from the trap. In the lab frame the spin of an atom must rotate rapidly to follow the direction of the quadrupole field if the atom is to remain trapped as it passes close to the field zero. This rotation of the atoms’ spin must be slow compared to its Larmor frequency \( 1/(2\pi)\mu_B g_F m_F B \) otherwise a Majorana spin flip to an untrapped state is likely to occur. This mechanism limits the temperature which clouds can be evaporatively cooled to in the quadrupole trap (this is illustrated by Figure 4.8 in Chapter 4). This loss mechanism can be overcome by creating a time-averaged, orbiting potential (TOP), in which atoms can be evaporatively cooled efficiently to below the phase transition for the formation of a Bose-Einstein Condensate (BEC). By applying a bias field in the symmetric \( x - y \) plane with magnitude \( B_T \), the quadrupole field zero is shifted by \( r_0 = B_T/B'_Q \). Rotating \( B_T \) in the \( x - y \) plane causes the quadrupole field zero to orbit the \( z \)-axis at radius \( r_0 \).

The spin vector of the atom continues to follow the instantaneous magnetic field, but the mechanical motion of the massive rubidium atom instead responds to the time-averaged potential \( U_T(r) \) seen when the changing fields are averaged over a rotation:

\[
U_T(r) = \frac{\omega_T}{2\pi} \int_0^{2\pi/\omega_T} U(r, t) dt \tag{3.3}
\]

where

\[
U(r, t) = B'_q(x \mathbf{e}_x + y \mathbf{e}_y - 2z \mathbf{e}_z) + B_T(\cos \omega_T \mathbf{e}_x + \sin \omega_T \mathbf{e}_y). \tag{3.4}
\]
neglecting the prefactors from Equation 3.1. Expanding \( U_T(r) \) around \( r = 0 \), one finds the harmonic approximation for \( U_T(r) \) with trap frequencies given by

\[
\omega_r = \sqrt{\frac{\mu_B g F m_F B'_q}{2mB_T}} \quad (3.5)
\]

\[
\omega_z = \sqrt{8}\omega_r \quad (3.6)
\]

Time-averaging works when the oscillation frequency \( \omega_T \) of the bias field \( B_T \) falls within a range bounded by the TOP trap frequencies and the Zeeman splitting at the centre of the TOP trap, which from \( \Delta E/\hbar = \mu_B g F m_F B_T \) is given by 0.7 MHz G\(^{-1}\). For a typical TOP field rotating at 7 kHz with a magnitude of \( B_T = 9 \) G in a quadrupole field of \( B'_q = 202 \) G cm\(^{-1}\) the trap frequency \( \omega_r = 50 \) Hz and the Zeeman splitting at \( B_T \) is \( \omega_L = 6 \) MHz for which \( \omega_r < \omega_T < \omega_L \) is satisfied.

The TOP trap bypasses the quadrupole trap Majorana losses by lifting the energy of the trap centre up from zero. However, atoms located close to the circle of radius \( r_0 \) where the quadrupole zero orbits are still lost. Because of this we load the TOP trap by lowering the quadrupole gradient and turning on an initially high TOP bias field to move this ‘circle of death’ defined by \( r_0 \) beyond the extent of the atomic cloud. Then by reducing \( B_T \), we reduce \( r_0 \) and use the circle of death for an initial stage of evaporative cooling.

### 3.2.1 Field calibrations

The TOP field can be calibrated from RF spectroscopy of the Zeeman splitting of cold atoms located at \( r = 0 \) in the TOP trap or by measurement of the Zeeman shift of the \( |F = 2\rangle \rightarrow |F' = 3\rangle \) optical transition through absorption imaging of a fixed atom number cloud of atoms at two different bias fields (a full description is given in [18]). Trap frequencies can be determined by measuring the frequency of the centre-of-mass oscillation of a cloud in the TOP trap. Given \( B_T \) and \( \omega_r \) or \( \omega_z \), the quadrupole field \( B'_Q \) can be determined using Equation 3.5.

Using the methods mentioned above, the following field calibrations were obtained;

\[
B'_Q(z) = 2.6 \times I_Q \quad \text{in units of G cm}^{-1} \text{ A}^{-1} \quad (3.7)
\]

\[
B_T = 0.81 \times I_T \quad \text{in units of G A}^{-1} \quad (3.8)
\]

where it should be noted that the value for the quadrupole gradient applies in the \( z \) direction, where the gradient is twice as high as in the horizontal plane i.e. \( B'_Q(x, y) = 1.3 \times I_Q \).
### 3.3 Gaussian laser beams

It is possible to shape light fields to create diverse and dynamically variable optical potentials, through the use of devices such as spatial light modulators, acousto-optic deflectors (AOD), holographic masks and phase plates. Future work in our group will examine the use of a two-axis AOD to create elliptical and rotating dipole traps within our quantum gas machine — however, in this work all the laser light we use has a Gaussian spatial mode (the lowest order Hermite-Gaussian spatial mode).

The intensity $I(r,z)$ in a Gaussian beam for a beam propagating in the $z$ direction is relevant to the discussion of dipole trapping below, as the energy of the dipole potential seen by the atoms ($U_{dip}(r)$ in Equation 3.15) is proportional to the intensity $I(r,z)$. The intensity distribution can be written as

$$I(r,z) = I_0 \left( \frac{w_0}{w(z)} \right)^2 \exp\left( -\frac{2r^2}{w(z)^2} \right),$$

with the peak intensity $I_0$ found at the $(r, z) = (0, 0)$ point where the beam is focused. Here, $w_0$ characterises the width of the Gaussian beam at the focus and $w(z)$ the width at a distance $z$ along the beam path, away from the focus.

From Equation 3.9 we see that $w(z)$ is the radial distance at which the intensity has dropped to $1/e^2 \sim 0.135$ of its peak value on the propagation axis. The beam waist changes according to

$$w(z) = w_0 \sqrt{1 + \left( \frac{z}{z_R} \right)^2},$$

where $z_R = \pi w_0^2 / \lambda$ is referred to as the Rayleigh range, and $2z_0$ represents the depth-of-focus of the beam, the extent over which the beam can be said to be ‘in focus’. A collimated Gaussian beam of waist $w_0$ focused with a lens of focal length $f$ produces a beam focused to the width

$$w_0 = \frac{f \lambda}{\pi w_0},$$

a distance $f$ from the lens.

The relationship between a beam’s intensity and its optical power $P$, the quantity we measure in the lab and therefore used to describe real laser beams, is

$$I_0 = \frac{2P}{\pi w_0^2}.$$  

The waist $w(z)$ is determined\(^1\) by measuring the power remaining in the beam after it is partially obscured by a block with a sharp edge (usually a razor blade). Moving

---

\(^1\) A detailed guide to beam-waist fitting and the background theory can be found here: [http://massey.dur.ac.uk/resources/lab_resources.html](http://massey.dur.ac.uk/resources/lab_resources.html).
this block across the beam, we measure the power as a function of the position of the edge along the beam. For a Gaussian beam \( X_{10-90} = 1.28w(z) \), where \( X_{10-90} \) is the distance moved across the laser beam by the block that causes the power measured after the block to fall from 90\% to 10\% of its total power before the block. We repeat this for a range of \( z \) positions along the beam and fit the function for the beam shape as it propagates given by \( a \sqrt{1 + ((z - c)/b)^2} \) of the form given in Equation 3.10 (with \( a, b \) and \( c \) free parameters corresponding to \( w_0, z_R \) and \( c \) the position along the beam), it is possible to extract all of the three key beam parameters: 1. the exact distance from a fixed reference point to the beams focus, 2. the achieved beam waist at the focus \( w_0 \) and 3. the depth of focus \( z_0 \). This is relevant to the optical lattice beam configuration described in Chapter 6.

### 3.4 Dipole trapping

![Figure 3.5](image)

**Figure 3.5:** Relative light shifts across the D1 line hyperfine transitions are shown by blue lines. The splitting between the two pairs of vertical lines correspond to the 6.8 GHz splitting between the \( |F = 1\rangle \) and \( |F = 2\rangle \) ground states. The separation within the pairs is 0.8 GHz — the excited hyperfine states splitting. The highest energy \( |F = 1\rangle \rightarrow |F' = 2\rangle \) transition is on the far-right of the D1 line centre, the lowest energy \( |F = 2\rangle \rightarrow |F' = 1\rangle \) transition is on the far-left. For positive detunings \( \omega_L > \omega_0 \) the energy shift is positive (blue region) and at negative detunings \( \omega_L < \omega_0 \) the energy shift is negative (red region).

The simplest approach to evaluate dipole trap light shifts \( U_{dip} \) and the associated
scattering rates $\Gamma_{sc}$ is to adopt a formulism in which there are only two energy levels being addressed by the applied light (at frequency $\omega_L$). As such there is only one transition frequency to consider, $\omega_0$. Further, the spontaneous decay rate $\Gamma$, is applicable to the one decay path. For this idealised ‘two-level’ atom,

$$U_{dip}(r) = -\frac{3\pi c^2}{2\omega_0^3} I(r) \Gamma \left( \frac{1}{\omega_0 - \omega_L} + \frac{1}{\omega_0 + \omega_L} \right)$$  \hspace{1cm} (3.13)$$

$$\Gamma_{sc}(r) = \frac{3\pi c^2}{2\hbar \omega_0^3} I(r) \Gamma^2 \left( \frac{\omega_L}{\omega_0} \right)^3 \left( \frac{1}{\omega_0 - \omega_L} + \frac{1}{\omega_0 + \omega_L} \right)^2$$  \hspace{1cm} (3.14)$$

This simplification to a two-level system can be applicable in the regime where the detuning, defined as $\Delta = \omega_L - \omega_0$ (and with the standard terminology of ‘red’ frequency detuning for $\Delta < 0$ and ‘blue’ frequency detuning for $\Delta > 0$), is very large $\Delta \gg \Delta_{FS}$ such that the fine structure splitting is unresolved, effectively giving a single absorption line.

The terms in the Equations 3.13 for the frequencies can be written as $1/(−\Delta)$ and $1/(2\omega_0 + \Delta)$. The rotating wave approximation can be made when $|\Delta| \ll \omega_0$ by setting the second of the two detuning terms to zero in Equations 3.13 and 3.14 and approximating $(\omega_L/\omega_0)^3 \approx 1$ in Equation 3.14.

With this approximation, we have;

$$U_{dip}(r) = \frac{3\pi c^2}{2\omega_0^3} I(r) \Gamma \frac{1}{\Delta}$$  \hspace{1cm} (3.15)$$

$$\Gamma_{sc}(r) = \frac{3\pi c^2}{2\hbar \omega_0^3} I(r) \Gamma^2 \left( \frac{1}{\Delta} \right)^2$$  \hspace{1cm} (3.16)$$

The distinction between red and blue detuned dipole traps can be seen, with atoms attracted to, and repulsed by, regions of increasing intensity, respectively.

It should be noted that at this stage there is a potential contradiction in applying the rotating wave approximation alongside the assumption that $\Delta \gg \Delta_{FS}$, suggesting that care must be taken when considering arbitrary laser detunings. The approximations made in deriving Equations 3.13 and 3.15 must therefore be carefully examined. In line with the exposition in [39], we note that in our $^{87}\text{Rb}$ system the fine structure doublet is split by $\Delta_{FS} = 15 \text{ nm}$, the hyperfine ground state splitting is $\Delta_{HFS} = 6.8 \text{ GHz}$, and the D1 line excited state hyperfine structure splitting is $\Delta_{HFS'} = 0.8 \text{ GHz}$. A common approach to dipole trapping of $^{87}\text{Rb}$ is to use a long wavelength for the trapping light of 1064 nm laser, which is very far red detuned from both D-lines as the scattering rate $\Gamma_{sc}$ is then negligible compared to other experimental timescales at this wavelength, even for high optical powers of a few Watts. In
this case the laser detuning satisfies $\Delta \gg \Delta_{FS}, \Delta_{HFS}, \Delta'_{HFS}$, and it is reasonable to reduce the system to two levels with one detuning term defined with respect to the center of the D-line doublet.

In previous work in our group ([40]) dipole trapping light of wavelengths 830 nm and 865 nm were used. In this regime it is still true that the hyperfine structure is unresolved, but it is no longer appropriate to dismiss the fine structure — $\Delta \sim \Delta_{FS} \gg \Delta_{HFS}, \Delta'_{HFS}$. Accurate calculations require summing separate terms for the D1 and D2 line contributions, with each term containing a detuning and relative line-strength factor.

In this thesis we consider the dipole potential formed by 795 nm light red detuned by $\sim 20$ GHz from the D1 line. More general expressions for $U_{dip}$ and $\Gamma_{sc}$ applicable at this comparably low detuning are a modified version of Equation 18 (applicable to the case where each D line is considered) from [39], with the subscript notation $i$ for the particular electronic ground state of interest and $j$ for the coupled excited states. The light shift of ground state $|i\rangle$ is given by;

$$U_{dip}(r)_i = \frac{3\pi c^2}{2} \Gamma I(r) \times \sum_j \frac{1}{\omega_{ij}^3} \frac{c_{ij}^2}{\Delta_{ij}}$$

with a similar expression for the scattering rate obtained through inspection;

$$\Gamma_{sc}(r)_i = \frac{3\pi c^2}{2\hbar} \Gamma^2 I(r) \times \sum_j \frac{1}{\omega_{ij}^3} \frac{c_{ij}^2}{\Delta_{ij}^2}.$$ (3.18)

The sum is over all contributions from coupled excited states taking into account (i.e. weighted by) line strength factors $c_{ij}$ and transition specific detunings $\Delta_{ij}$. We will assume the applicability of the rotating wave approximation and neglect counter-rotating detuning terms. The $1/\omega_0^3$ term is moved inside the sum as this too will be dependent on the particular excited state being evaluated — this is the approach taken in [40] and [41].

For the $\sim -20$ GHz light considered here, the detuning $\Delta$ from the D1 line is only a few times larger than the hyperfine structure of this optical line. The detuning for the D2 line is significantly larger, and excitation on this line can be ignored ($1/\Delta$ for the D2 line $F = 1 \rightarrow F' = 2$ hyperfine transition is less than 0.5% of the value for the same D1 line transition). Thus we take $i = \{1, 2\}$, corresponding to the ground states $|1\rangle = |5^2S_{1/2}, F = 1\rangle$ and $|2\rangle = |5^2S_{1/2}, F = 2\rangle$. The index $j = \{1, 2\}$ similarly corresponds to the excited states $|1'\rangle = |5^2P_{1/2}, F' = 1\rangle$ and $|2'\rangle = |5^2P_{1/2}, F' = 2\rangle$. Figures 3.7 and 3.8 show the energy shift $U_{dip}$ calculated with Equation 3.17 in this way.
The other useful point is that the $\Gamma$ term remains outside of the summation for the hyperfine transitions. As shown in [42], all excited fine-structure state sublevels (hyperfine states) decay at the same rate $\Gamma$, and the population decays into different ground state sublevels according to the transition branching ratios.

### 3.4.1 Line strengths

![Diagram](image)

**Figure 3.6:** Values for $c_{ij}^2$ obtained using Equation 3.19 for the four different hyperfine transitions on the $^{87}$Rb D1 line.

The line strength coefficients are calculated by reducing the matrix elements of the dipole operator using the Wigner-Eckart theorem. We consider the case where the zero-magnetic is zero and therefore we do not need to account for the differences between the Zeeman substates. The following expression from [42] yields the hyperfine transition line strengths as multiples of the D-line reduced dipole operator matrix elements $|\langle J||e r||J'\rangle|^2$;

$$c_{ij}^2 = (2J_j + 1)(2F_j + 1) \left\{ \frac{J_i}{F_i} \frac{J_j}{F_j} \right\}^2$$

where $J_i, F_i$ are the electronic and total angular momentum quantum numbers respectively for a ground state indexed $i$ and $J_j, F_j$ are the quantum numbers of the excited states. $I$ is the nuclear spin quantum number. The final term on the right of Equation 3.19 is the Wigner 6-$j$ symbol for this system of coupled angular momenta. For the D1 line in $^{87}$Rb we have $F_i, F_j = \{1, 2\}$ for the total angular momentum $F = J + I$, $J_i, J_j = 1/2$ for the total electron angular momentum and $I = 3/2$ for the nuclear spin.
Figure 3.7: Differences in the ground state light shifts red-detuned from the $|F = 2\rangle \rightarrow |F' = 1\rangle$ line, calculated using Equations 3.17 and 3.18. Recoil energies are calculated using the intensity of a 20 mW, 100 μm waist Gaussian beam. The dotted line corresponds to the calculated shift without resolving the individual hyperfine states i.e. using Equation 3.15.

Figure 3.8: Differences in the ground state light shifts blue-detuned from the $|F = 1\rangle \rightarrow |F' = 2\rangle$ line.

Figures 3.7 and 3.8 indicate the size of the discrepancy between $U_{dip}$ calculated with Equation 3.15 (dashed line) and Equation 3.17 (solid lines) for the red and blue detuned cases, respectively. The Stark shift from Equation 3.15 for a two-level
system is significantly different to the result of Equation 3.17 at a 20 GHz detuning in either case however at a detuning of 50 GHz it becomes negligible. Since we consider detunings of 20 GHz in this thesis, it is appropriate to use Equations 3.17 and 3.18 to calculate the dipole potential and scattering rate, using the strength factors given in Equation 3.19. We anticipate optical pumping into the $F_i = 2$ hyperfine ground state in the scheme where laser cooling light is applied to dipole trapped atoms.

### 3.5 Formation of optical lattices

The light field that forms an optical lattice arises from the interference of two coherent laser beams resulting in a periodic potential with a length scale on the order of $\lambda/2$. The simplest way to form the necessary overlapping beams is by retro-reflection of one beam, thus forming a 1D lattice. Higher dimensional lattices can be created through the addition of further independent 1D lattices; cross interferences between the standing waves in separate directions can be suppressed by operating the 1D lattices at different frequencies or with orthogonal polarisations to one another. In practice both methods are usually used, as it can be difficult to remove cross interferences through polarisation alone. A 10 MHz shift is enough for cross interferences to time-average to zero when lattice wells have a characteristic frequency $< 1$ MHz. Such shifts are easily produced using an acousto-optic modulator. The same devices can also be used for amplitude control or fast switching of the lattice beams.

For a 1D lattice created by retro-reflected beams of equal intensities the light intensity is modulated by a $\cos(kz)^2$ term, with $k = 2\pi/\lambda$. Where the two counter-propagating beam amplitudes interfere constructively the peak intensity $I_0$ is increased by a factor of 4. The lattice produced by interfering two counter-propagating focused beams of the form given in Equation 3.9 is proportional to the intensity;

$$I(r, z) = 4I_0 \left( \frac{w_0}{w(z)} \right)^2 \exp\left( -\frac{2r^2}{w(z)^2} \right) \cos^2(kz + \phi) \quad (3.20)$$

where $\phi$ is determined from the relative phase of the two beams. Inserting this into Equation 3.17 gives the lattice potential.

Optical lattices formed by interfering more than two beams have been made. For example in [43] a 2D hexagonal lattice structure was created with three interfering beams in a plane, with equal angles between them. Generating a 2D lattice with 3 beams is potentially useful since it reduces the number of beams required. However,
such lattices lack the symmetry between red and blue detuning which gives equivalent lattice well structures with overlapping independent 1D lattices\(^2\). This makes such configurations unsuitable for blue-detuned trapping. We restrict ourselves to considering independent 1D lattices.

A standard approach to generating 1D lattice with a periodicity greater than \(\lambda/2\) is to decrease the angle between the beams. The periodicity \(d\) is increased according to

\[
d = \lambda/(2 \sin \theta)
\]

(3.21)

where the angle between the beams is \(2\theta\). The direction \(r\) of the optical lattice that is formed is at an angle of \(90^\circ - \theta\), as shown in Figure 3.9.

![Figure 3.9: The geometry of a 1D optical lattice formed from two beams (of waist \(w\)) crossing at an angle \(\theta < 90^\circ\). The lattice is formed in the direction \(r\) with a periodicity \(d > \lambda/2\).](image)

Figure 3.9 highlights an important practical difference between forming a 1D lattice through retroreflection and through crossing a pair of beams as shown. For a retro-reflected beam with intensity \(I_0\), the lattice depth is \(\propto 4I_0\). For the crossed beam configuration shown in Figure 3.9 to achieve the same depth requires an intensity \(I_0\) in each of the two beams. In practice the two beams are derived by beam splitting (so that they are coherent with each other), twice as much power.

### 3.5.1 Red versus blue frequency detuning

For a single focused beam, a 1D lattice or a 2D lattice there is a clear difference between the shape of the dipole potential in the blue and red detuned cases. For red

\(^2\) When more than 2 beams interfere, the standing wave pattern formed consists of regions of high intensity surrounded by regions of low / zero intensity, so that there are isolated potential wells for only the red-detuned case.
detuning, atoms can be trapped at the focus of a single beam due to the localised region of high intensity creating a potential minimum in all directions. For blue detuning an atom must be surrounded by light, which is practically more challenging — an example is the uniform box potential dipole trap created in [44]. For the following discussion we shall focus on the spatially symmetric case of a square optical lattice made from three independent 1D lattices, all perpendicular. In this case the regions of low intensity and high intensity look identical except for a translation by half a lattice period.

For our detuning of interest where $\Delta \sim 20 \text{ GHz}$ the scattering rate is not negligible and gives rise to heating. Thus another notable difference is that for equal detunings, an atom residing at the region of highest intensity, as in a red detuned, trap would be expected to scatter many more photons from the confining light fields compared to the blue detuned trap, where atoms are attracted to regions of low intensity. Equations 3.16 and 3.18 for scattering rate derived above only contain detuning in the form $1/\Delta^2$ and so do not indicate any difference in terms of scattering rate.

In [39] the question of heating by scattering dipole trapping light in each case is examined. An important difference between red and blue detuning originates from the different environments for thermal atoms which explore the region around the trap minimum due to their random motion. The parameter representing this is $U_{dip}$, the mean dipole potential experienced by the atoms in a cloud at temperature $T$. We characterise the two trapping scenarios, illustrated in Figure 3.10 by the trap depth (in the red detuned case) or the height of the repulsive walls (in the blue detuned case) $\hat{U}$. Following the approach in [39], we can obtain the mean scattering rate $\Gamma_{sc}$ in terms of the mean potential by combining Equations 3.15 and 3.16 to give;

$$\Gamma_{sc} = \frac{\Gamma}{\hbar \Delta} U_{dip}. \quad (3.22)$$

Heating is caused by an increase of the total thermal energy by $2E_R$ at a rate $\Gamma_{sc}$ where $E_R = (\hbar k)^2/2m$ is the recoil energy. Thus the heating rate $\dot{T}$ is given by

$$\dot{T} = \frac{1}{3} 2E_R \Gamma_{sc} = \frac{1}{3} 2E_R \frac{\Gamma}{\hbar \Delta} U_{dip} \quad (3.23)$$

where a harmonic trap potential has been assumed in deriving the factor of $1/3$ from the equipartition theorem. This assumption is valid in the deep trapping regime where $\hat{U} \gg k_B T$. 

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Assuming that this heating is a quasi-equilibrium process the mean dipole potential $\overline{U}_{dip}$ atoms experience is related to their temperature to give

$$\overline{U}_{dip} = U_0 + 3/2k_BT$$

(3.24)

where $U_0$ is the trap potential energy offset. For red detuned traps where atoms are trapped at regions with the largest light shift, $U_0 = -\tilde{U}$. For blue detuned traps where atoms are trapped in regions with the smallest light shift, $U_0 = 0$.

Inserting Equation 3.24 into Equation 3.23 for these two cases, provided $\tilde{U} \gg k_BT$, the heating rates for red and blue detuning are

$$\dot{T}_{red} = \frac{1}{3} \frac{2E_R}{\hbar|\Delta|} \frac{\Gamma}{k_BT} \tilde{U},$$

(3.25)

$$\dot{T}_{blue} = \frac{1}{2} \frac{2E_R}{\hbar|\Delta|} \frac{\Gamma}{k_BT}.$$

(3.26)

The ratio of the two heating rates $\dot{T}_{blue}/\dot{T}_{red} \propto k_BT/\tilde{U}$ shows that in the deep trap regime $\tilde{U} \gg k_BT$ blue detuning is favourable in terms of heating. In the red detuned case linear heating is expected, whilst in the blue detuned case there is an exponential increase in temperature.

However, it could be argued that differences in the scattering and heating rates are less significant in the specific scenario of trapping in an imaging lattice, due to the application of near-resonance light with a far higher scattering rate and heating effect. In that case, the important question is whether there are differences between the two cases that might interfere with the role of the excitation light, and in the case where
it has a laser cooling effect, whether this cooling is disrupted. In the analysis above we relate the average dipole trapping potential to the temperature of the atoms in Equation 3.24. The frequency of the applied excitation light in the red detuned case must be adjusted to account for the energy offset $|U_0|$, after which both cases appear equal on length scales smaller than $\sim \omega_0$. At larger length scales, where the intensity of the crossed Gaussian beams varies significantly there will be inhomogeneities in the Stark shift across the lattice. This is removed for the blue detuned case, so that the detuning of molasses light is applicable over larger length scales than in the red detuned case. This indicates that a blue detuning is preferable when imaging atom distributions over this scale, otherwise the two detuning cases are equivalent.

Finally, beyond effects around scattering and energy shifts from the light, there is one obvious practical difference between the red and blue detuned cases, suggested in [45]. Interference of the lattice light with undesired stray reflections creates disorder with a magnitude proportional to the lattice depth. In the red detuned case, where atoms reside at the intensity antinodes, the magnitude of this disorder is comparably large, whereas it is absent for atoms residing at the nodes as in the blue detuned case.

### 3.6 Laser cooling

The mechanism of laser cooling, in its simplest form, is to apply a net force to atoms through the selective absorption of photons with momentum $\hbar k$ i.e. a force of magnitude $F_{sc} = \hbar k \times \Gamma_{sc}$. The absorption process transfers momentum in a direction equal to the light propagation direction, which covers a narrow distribution in a collimated laser beam. The accompanying emissions, averaged over many events, have no well defined direction when the quantisation axis of the atom is not well defined as in a magneto-optical trap or optical molasses. Thus the net force is applied through absorption only.

Atoms experience a Doppler shift which depends on their velocity. Thus the detuning and subsequently the scattering force have a velocity dependence. When the light is red-detuned for stationary atoms, this force can be used to cool the cloud.

Considering only a single laser beam with the atoms motion in a single direction, the frequency of light seen by an atom moving with a velocity $v$ is shifted by $kv$ from its stationary value $\Delta = \omega_L - \omega_0$ to give an effective detuning of $\delta = \Delta + kv$. Provided that $|kv| < |\Delta|$, $\delta$ is reduced (a shift closer to resonance) in the case where the laser is red detuned and the velocity is positive (atom moving opposite to beam direction),
and vice-versa. The detuning term appears in the expression for $\Gamma_{sc}$, Equation 3.27 (following [28]);

$$\Gamma_{sc} = \frac{\Gamma_D^2}{2} \frac{I/I_{sat}}{1 + I/I_{sat} + 4(\delta/\Gamma_D)^2},$$  \hspace{1cm} (3.27)

and as such there is a velocity dependent scattering force which slows the atom down. (Note that the previous expression for $\Gamma_{sc}$, in Equation 3.15 diverges on resonance rather than approaching $\Gamma_D^2/2$ as expected for the two-level system under consideration — not unreasonable given the assumptions about detuning made in deriving the expression).

In a gas, trapped or otherwise, which has reached a thermal equilibrium, atoms move in all directions and as such need to be cooled in all directions, requiring a set of laser beams propagating, or at least, with components\textsuperscript{3} propagating, in all orthogonal directions. A cloud of atoms in such a configuration of red-detuned laser cooling beams that covers all 3 spatial directions is commonly referred to as an ‘optical molasses’.

Cooling of a simple two-level atom in an optical molasses has a lower limit, the Doppler temperature, which for $^{87}$Rb is

$$T_D = \frac{\hbar \Gamma_D^2}{2k_B} = 146 \mu K.$$ \hspace{1cm} (3.28)

The Doppler temperature is that at which the heating rate due to the random motion induced by spontaneous emission is balanced by the cooling rate. Appendix A gives a derivation of this equilibrium temperature.

\textsuperscript{3} Full 3D cooling may not be necessary where the motion of an atom in a confining potential in a direction perpendicular to a molasses beam is coupled into the beam direction — the applicability of this in our system is examined in Section 8.1.1.
Chapter 4

The rubidium quantum-gas apparatus

This chapter describes the quantum-gas apparatus used in our research group to produce samples of cold atoms and Bose-Einstein condensates in a range of magnetic traps, which are transferred into the dipole traps and optical lattice potentials described later in Chapters 6 and 7.

4.1 Introduction

The apparatus used to generate the samples of cold $^{87}$Rb atoms studied in this project was constructed during 2008 — 2009 by Benjamin Sheard, Benjamin Sherlock and Marcus Gildemeister. Their theses ([46], [18] and [17] respectively) describe the design, construction and testing of the apparatus to produce BECs in a time-averaged orbiting potential (TOP) trap from a pyramid magneto-optical trap (MOT), and the stage beyond this where the advanced magnetic adiabatic potential (AP) and time-averaged adiabatic potential (TAAP) traps are created.

The broad structure of the apparatus is a two-chamber vacuum system, connected by a differential pumping tube through which cold atom samples are transferred from the first chamber (housing the pyramid MOT) to the second chamber (the glass cell at the centre of the magnetic trapping array) using a transport mechanism that is a hybrid of moving magnetic coils and ramping the fields of a series of stationary coils. This is shown schematically in Figure 4.1.

In the original design, all confinement in this apparatus comes from magnetic trapping within the glass cell, with optical access through the surrounding coil array needed only for absorption and fluorescence imaging. The main goal in designing this apparatus was to facilitate the delivery of RF dressing and time-averaging fields
to atoms located a short distance away from high-field quadrupole coils above a microscope objective with a moderate numerical aperture (NA). The possibilities for manipulating and detecting trapped quantum gases with this apparatus are demonstrated in [37], for example, where a vortex array is generated in a TAAP trap with the individual vortices resolved. This result was the culmination of the (first experimental) demonstration in this machine of a TAAP trap, which was published in [15] and [36] and had been proposed shortly before by others in [47].

The machine has undergone a substantial amount of modification and improvement since that work. We have rebuilt and modified both the coil array (described further in Section 4.3) and the vertical imaging system (Section 4.5). We re-baked segments of the vacuum system (Section 4.2) and switched over to a new set of rubidium dispensers. The laser system has been completely remade as described in Section 4.4.

4.2 Vacuum system

The vacuum system has two regions connected through by a length of small-bore tubing that permits a pressure difference between the two allowing us to achieve lower pressures and hence reduced background particle collisions and losses for magnetically trapped atoms in the glass-cell. We continuously run two ion pumps to maintain the background pressure: one pump acts on the region of the glass cell giving ultra-high vacuum (UHV) pressures in that region, where we need minutes-long trapping lifetimes and the other pump acts on the MOT chamber.

Laser light enters the MOT chamber through a large diameter viewport and is incident on a cluster of four triangular mirrors which form the ‘pyramid’ (four 45° mirror surfaces, square to each other in an inverted four-sided pyramid structure) that generate the required light field of 6 laser beams — 3 counterpropagating pairs in each orthogonal spatial direction. Adjacent to these mirrors are four rubidium dispensers connected in two pairs in series (each pair is on a feedthrough out of the vacuum system). When a current is passed through these they heat up and emit a dilute vapour of $^{85}$Rb and $^{87}$Rb proportionally to their natural abundance. Following a disruptive period where the dispensers behaviour became unpredictable, we switched over to using the second set of dispensers. The first pair were reliable for $\sim 6$ years, of use at a current of 3 A during operation. We operate the new pair of dispensers differently — each evening a current of 4 A is applied for a duration of

1 Alkali Metal Dispensers from SAES group
**Figure 4.1:** A schematic of the vacuum system, looking down on the two chambers and the small bore tube connecting them. On the left is the pyramid MOT chamber, and on the right the rectangular glass cell. Not shown are the two ion pumps — with rates of $401\text{s}^{-1}$ and $551\text{s}^{-1}$ — which maintain UHV conditions in the MOT and glass cell regions respectively. The MOT cloud is actually formed above one of the four glass blocks, rather than above a vertex where there is a missing component from the molasses light. The coordinate system used in this thesis, which is referenced to the glass cell and gravity, is shown.

10 min, and the next day the MOT size is large enough throughout the day and the background pressure low enough for efficient transport\(^2\). The pressure is too high to run the experiment for several hours after firing the dispensers.

The pyramid MOT configuration, first laid out in [48], is an alternative to using a chamber with three orthogonal viewports to transmit three (retroreflected) beams. We need to transfer the atoms to the other end of the vacuum system and therefore one of the corners of the pyramid has been removed to allow the trapped cloud to pass out of the pyramid. The six required beams, in the correct circularly polarised states, are

\(^2\) It’s clear now that the behaviour of the dispensers is more complex than we previously understood. We believe that as they near depletion their emission changes dramatically, to the detriment of the background pressure. In the past, we have observed occasional major improvements in transport efficiency when the dispensers have been turned off following continuous operation, whilst the MOT atom number reduces only slightly. Those dispensers are no longer used. With the previous method of continuous running at 3 A, after switch off it took approximately 2 h for the MOT atom number to drop to a problematically low level, making periodic operation of the dispensers necessary. In earlier years continuous operation caused no problems.
obtained by single or double reflection of a single large diameter, circularly polarised beam. This construction is simple to align, though it does have the disadvantage of making the beam power balancing required for any sub-Doppler laser cooling virtually impossible. The pyramid used in this system is mounted with its symmetry axis parallel to gravity. Another common application for this type of pyramid construction is used by our research group to generate a beam of laser-cooled atoms traveling horizontally to load a second MOT in a lower pressure region.

The dimensions of the glass cell at the opposite end of the atom transport path were kept small to allow us to generate high magnetic fields in the vicinity of the atoms. The glass cell has a rectangular profile: 20 mm × 10 mm (interior walls have a thickness of 2 mm), with the short dimension in the vertical to allow the quad coils to sit within a few cm of their centre point, where atoms are trapped. No special treatment has been applied to the quartz — in particular, the walls of the glass cell are not anti-reflection coated (relevant to an assessment of our imaging lattice scheme, as described in Section 2.2).

4.3 Magnetic coils

There are several different types of magnetic coil used in this apparatus to apply DC and AC magnetic fields to the atoms. Of the DC coils (Figure 4.2) four are designed to handle large currents and so are water cooled to prevent overheating — these coils are situated in an array around the glass cell. There are two coils involved in creating and transporting atoms from the MOT, which requires three low field coils to cancel the background field. The water cooled coils are wound from square cross-section, hollow core wire insulated with kapton, as described in [46].

The AC coils are arranged in a compact array (Figure 4.3) around the atoms at the centre of the DC coil array. The coils of this array are wound and glued together in a unit that is carefully pushed into place encompassing the glass cell centre (all the fields applied through it are bias fields that are moderately insensitive to misalignment, unlike quadrupole fields), and held in place through compression between the high current coils above and below.

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3 We have constructed a separate vacuum apparatus with this design — a pyramid with a clear aperture at the apex funnels atoms in a stream to an intermediate MOT, from which atoms are transferred via a series of ramped quadrupole trapping fields to the glass cell. The increased complexity of the vacuum system is outweighed by the reduced experimental duty cycle in this design.
The original design and construction of these coil systems is described in [46]. Since then, there have been major changes. The AC coil array (shown in Figure 4.3) has been reconstructed twice to 1) repair possible damage due to a failure in the water cooling system and, then again 2) to change the radio-frequency (RF) dressing field coils for a set that have a higher self resonance frequency, allowing us to efficiently apply higher RF frequencies. The quadrupole coils, at the heart of the high current DC coil array, were also replaced due to a catastrophic water cooling failure.

4.3.1 DC coils

Around the MOT chamber, the spatial dependence on photon scattering is created with a coil pair generating a 14 G cm$^{-1}$ quadrupole field. The main MOT coils initially used to generate the low magnetic field gradient for the MOT serve a second purpose as the transport coils. They are mounted on the mechanical rail which runs alongside the axis of the vacuum system, and, following the MOT loading and capture stage, are used for the magnetic trapping during the mechanical transport (the magnetic field gradient is increased immediately prior to initiating the movement of the coils). Also attached to this moving coil mount is the lateral steering coil, which serves to manoeuvre the field zero of the MOT sideways (+y direction) so that the atomic cloud can pass through the gap in the inverted pyramid vertex before heading towards the glass cell. Three additional low-field coils located orthogonally around the MOT chamber null the small background fields, ($< 1$ G).

The MOT and lateral steering coils are not water cooled so they do heat up. The current bottleneck that demands some delay between multiple shots with the apparatus is the slow air cooling of these coils. To improve matters, small (computer) fans were installed to hasten cooling.

The coils that make up the DC coil array, shown in Figure 4.2, are water cooled through a channel at the centre for the cooling water to flow through. All coils are wound in house from large spools onto aluminium formers. The replacement quad

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4 In our first demonstration of atom trapping in an adiabatic potential at 1.4 MHz, the coil impedance grew rapidly above 2 MHz. We are now able to couple frequencies over 4 MHz into the RF coils.

5 Prior to this event in March 2013, the high-current power supply was not interlocked with regards to the flow of cooling water — and so the inevitable happened. A short between turns in one or both of the quad coils because of a breakdown of insulation was detected as an asymmetry between their voltages induced during the coil switch off. The quadrupole trap could no longer be switched on and off without atom heating being observed.

6 Normally these coils would be used to null out the background field from the Earth’s magnetic field, however we instead use them to bias the position of the field minimum slightly to a region of uniform intensity giving the maximum MOT cloud size and uniformity.
coils wound more recently were tested for faults and similar \( \text{G A}^{-1} \) performance (Tesla per Amp in SI units) with a gaussmeter, although deliberate and careful winding was ultimately enough to reliably meet the latter requirement. The selected pair, like the other coils, have metal push fit plumbing connectors soldered onto the ends for plumbing in alongside the high current connection. Apart from the push coil, these water cooled coils are rigidly housed in hard plastic mounts, positioned using threaded studding rods.

**Figure 4.2:** The DC coil array consisting of three coil pairs and one single bias coil. All these coils are water cooled. The Quad coils at the centre reside close to the glass cell and generate (in the \( z \) direction) a high field gradient approaching 900 G cm\(^{-1}\).

The push and auxiliary coils serve a role during the latter phase of the transport stage by shifting the magnetic trapping quadrupole field minimum from the MOT coils to the quad coils. The Helmholtz coil is somewhat separate and is currently unused.

The water cooling system and high-current system both serve only these coils. To generate the maximum quad coil field gradients in the horizontal plane, \( B'_{x,y} = \)
420 G cm\(^{-1}\), the coils carry a current of 330 A.

The exposed faces of the quad coils make physical contact with the flat top and bottom faces of the AC coil array, which slides around and over the glass cell. The pressure of the two quad coils keeps the AC coil array in place, and helps to provide cooling to the AC array through the water cooling within the quad coils. To achieve high field gradients with the quad coils, we seek to have the minimum distance between them — there is very little space in the vertical direction to ensure there is no pressure on the delicate glass cell, and great care must be taken when sliding the AC array over and off of the cell!

4.3.2 AC coils

The purpose of the AC coil array is to deliver 1) time-averaging fields to the atoms for TOP, or TAAP, trap generation and 2) RF dressing fields. Time-averaging fields in the \(xy\)- plane are delivered through the larger outer rectangular coils, with the coils connected up in pairs for bias fields. The smaller RF coils are arranged in the same way. There is an additional coil pair made up of two circular coils, as seen in Figure 4.3, that can be used to apply RF in the \(z\) direction.

![Figure 4.3](image-url)

**Figure 4.3:** A view of the AC coil array from above. The AC coil array has two pairs of time-averaging TOP coils and two pairs of RF dressing coils. In the centre are two circular coils that produce an RF magnetic field along the \(z\) direction.
Finally, there is a single small coil for delivering evaporative RF located in the aperture of the upper quad coil (separate to the array in Figure 4.2).

### 4.3.3 TOP circuitry

There are some unusual issues associated with working in the kHz frequency regime that, whilst appropriate for time-averaging in our experiments, is more commonly associated with audio technology. As such, it is worthwhile to discuss the system we use to deliver 7kHz oscillating bias fields to the atoms, shown below.

![Figure 4.4: A schematic of the TOP signal generation electronics. There are two channels, for the two orthogonal time-averaging fields. VCA = voltage controlled attenuator, TTL = transistor-transistor logic, in practice a +5V triggering signal. The analog and TTL signals are delivered by the experimental control system.](image)

A sinusoidal 7kHz signal is generated from a function generator (Agilent 33220A), and split into two channels, with one phase shifted by 90°. The function generator is triggered by the computer control as shown in Figure 4.4. Our ability to control the amplitude of the signal in each channel comes from additional computer control of the voltage at two voltage-controlled attenuators.

A ‘pre-amplifier’ module, designed by Will Heathcote and described in [49], contains an additional stage of attenuation control that we use to fine tune the balancing of the two channels. Additionally, and as a consequence of our incorporation of audio technology more commonly used in sound reinforcement systems at the final amplification stage, the time-averaging signal is duplicated, and one is then phase-inverted for the second conduction line in the three-conductor cable connecting the...
pre-amplification module to the audio power amplifier, which has XLR connector inputs designed to accept a balanced signal of this form.

In order to achieve currents in the TOP coils large enough for the bias fields we require during the TOP trap evaporation stage, we use an audio amplifier (*Crown XLS 5000*) prior to the transformer stage which is specified for 36.7 dB of voltage gain per channel into a 4 Ω load with the level controls (attenuators) at maximum i.e. zero attenuation.

The TOP coils themselves do not present an optimal 4 Ω impedance to the amplifier and so we have an intermediate transformer stage. The transformer that has been in use since 2009 presents an impedance (magnitude) of 5.6 Ω at 7 kHz.

Apart from the complications that come from incorporating audio technology in an application very different to that which it was intended for, the 7 kHz TOP circuit is relatively simple. One key concern with the circuitry is the maintenance of stable signal amplitudes. The final steps of the TOP trap evaporation sequence in which we create a BEC are sensitive to shifts in the field strength since these lead to shifts in the ‘bottom’ of the trap i.e. the magnetic field corresponding to atoms of zero kinetic energy (or zero-point energy strictly speaking).

### 4.4 D2-line laser system

The main laser system needs to provide light for the D2-line to address the \(|F = 2 \rightarrow |F' = 3\rangle\) cycling and \(|F = 1 \rightarrow |F' = 2\rangle\) repumping transitions identified in Figure 3.1 from Chapter 3. We apply laser light to the atoms in the MOT stage and for imaging atoms in the glass cell. For laser cooling in the MOT we require red-detuned light and in the imaging phase we use light at or near to resonance. At both stages light resonant with the repumping transition is also applied.

The cycling transition light is derived from ‘slave’ lasers injection locked to a ‘master’ laser — this standard terminology is used in the following description.

#### 4.4.1 Design features

Throughout changes to the laser system design, it has remained the case that all laser light is delivered to the atoms inside the vacuum system via single-mode polarisation maintaining optical fibres. We physically separate the laser system from the vacuum system via fibres to decouple alignment drifts and also to give a clean spatial laser mode — only a single transverse mode is supported by the optical fibres so the output beam profile is an undisturbed Gaussian, whereas transmission of laser beams through
the many optical components involved in frequency and path splitting and shifting cause significant beam profile degradation.

In the original design, the vacuum and transport systems sat atop the same optical table as the laser system, and the movement of the transport rail unavoidably generated vibrations that perturbed the lasers, which made frequency stabilisation challenging. For both the repumping and master lasers we use the *Toptica PID 110* module to control the feedback loop for locking to the generated error signals, generated in the current design via modulation transfer spectroscopy (described in [50]) for the master⁷ and beat-note offset locking to the master (described below) for the repumper ⁸. Previously, the repumper laser was independently locked via frequency-modulation (FM) directly to the repumping line.

Whilst this PID module can easily keep lasers on resonance under ambient conditions, the transport vibrations presented a greater challenge. Locating the narrow range of parameters for the feedback loop to keep a laser locked under these conditions is difficult, and this is one reason why we altered the system, locating the lasers requiring active frequency stabilisation on a separate optical table, with greater use of optical fibres. In the current system we account for the additional power loss through fibre coupling with an additional slave laser.

We did attempt to improve the stability of the repumping laser when it was subject to moving transport mechanical vibrations in the following way: The $|F = 1\rangle \rightarrow |F' = 2\rangle$ repumping line is not the most prominent in a saturated absorption spectroscopy trace, so that the error signal derived through FM does not have as steep a slope (and hence as large a bandwidth) as that of the crossover transition between the $|F = 1\rangle \rightarrow |F' = 1\rangle$ and $|F = 1\rangle \rightarrow |F' = 2\rangle$ lines, detuned by about 80 MHz from the repumping line. This frequency gap can be bridged by an acousto-optic modulator (AOM) allowing us to make use of the crossover feature for locking.

In our current system, shown schematically in Figure 4.5, we use offset locking to frequency stabilise the repumping external-cavity diode laser (ECDL) relative to the master ECDL, itself locked to the cooling transition. The specific details of our offset locking scheme, which follows that in [51], will be presented in the future thesis of Elliot Bentine — briefly, a high-frequency photodiode (with practical resolution up to $\sim 7$ GHz) detects the beat note frequency of cooling light and repumping light

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⁷ A Toptica DL pro extended-cavity diode laser.
⁸ A Toptica DL 100 extended-cavity diode laser.
combined, which is then compared to the frequency of a generated reference signal. The derived error signal is fed into a Toptica module, with a sharp transition across the locking set-point voltage where the phase difference between these two signals goes through 0°.

We use a large number of optical fibres in our system, as indicated in Figure 4.5 — each module is coupled to the others by optical fibres to allow us to make repairs and replacements without disrupting the alignment in ‘downstream’ parts of the system. It also allows us to locate the mechanically sensitive repumping and cooling master lasers in a separate room to the vacuum system, where environmental conditions are more stable. The cost is in laser power due to the extra losses from fibre coupling, which is resolved by an additional slave laser (‘Slave laser 1’ in Figure 4.5) that was not present in our old system.

We require a higher power level for the pyramid MOT than can be achieved with normal diode lasers, whose maximum output level is limited by the cross section of the chips gain medium — this is made small enough that higher-order transverse output modes (in this case Hermite-Gaussians modified by the rectangular cross-section of the gain region) are suppressed. Higher power can be achieved using a tapered amplifier (TA) chip to amplify the combined repumping and cooling light up to a level suitable for laser cooling in the large volume of the pyramid MOT, as indicated in Figure 4.5. The TA has a large cross-sectional area of the gain medium at the output, allowing for high powers in the emitted light. The Gaussian beam output of the TA is of a lower quality than the diode lasers generate and gives approximately 300 mW at the fibre output above the MOT chamber.

For absorption imaging we apply spatially separated repumping and cycling transition beams at the glass cell, with the ability to apply the latter beam in either the horizontal or vertical directions.

4.5 Imaging system

The imaging system used for detecting atomic gas samples is shown in Figure 4.6. We use a simple arrangement in the horizontal plane to carry out absorption imaging. A lens pair equidistant between the atoms and the camera CCD provide ×1 magnification.

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9 We supply a reference frequency of 6476 MHz — the frequency difference between the cooling and repumping lines is 6568 MHz, and with the inclusion of an AOM operating at 92 MHz for rapid shuttering this gives the required frequency reference.
Figure 4.5: A schematic of the current D2-line laser system for laser cooling in the pyramid MOT and absorption imaging in the glass cell. The optical power required for the large beam size in the MOT is achieved via a tapered amplifier (TA). The master extended-cavity diode laser (ECDL) is frequency stabilised via modulation transfer spectroscopy, and the repumping ECDL is stabilised relative to the master laser using an offset locking arrangement. All lasers and spectroscopy setups are built on separate breadboards and are connected together by optical fibre.
Figure 4.6: A schematic of the imaging pathways on the apparatus. Light is delivered to the glass cell region via optical fibre, in two orthogonal directions. Below the glass cell, light is directed onto one of two cameras. A dichroic beamsplitter is used rather than a mirror in the vertical imaging pathway to allow us to insert an additional dipole trapping beam through the objective to the atoms. The repumping light that accompanies imaging is delivered along a separate path, not shown.

It should be noted that this optical system gives a geometrical image but does not re-collimate the Gaussian probe beam before the CCD.

This absorption imaging system is used routinely for almost all of our measurements, which do not require a high resolution and do not generate structure in the horizontal plane. However, there are certain states of the atomic gas that require observation from above — either because of the orientation of the structure of interest (for example, a vortex lattice produced by rotation perpendicular to gravity) or because the length scale of the structure is smaller than the resolution limit in the horizontal imaging. In these cases we use the more complex vertical imaging system to carry out absorption imaging, with the goal to also develop fluorescence imaging for very low and single-atom detection (the subject of this thesis).

The vertical imaging system has a ×12.5 magnification factor with the combination of lenses shown in Figure 4.6, including the four-lens compound objective. The

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10 Verified through fitting to the motion of a cloud falling under gravity, which is accurately known from experimental timing.
four-lens objective used in the vertical imaging system is a variation of the design presented in [52], which previous testing in our group has been shown to have a numerical aperture of 0.27, giving a resolution of 1.74 µm and depth of field of 21 µm at a working distance of 37 mm. This numerical aperture gives a collection efficiency of 1.9%.

Another important factor in the overall efficiency of photon detection is the quantum efficiency, or QE of the camera itself. The Andor front-illuminated electron-multiplying CCD (EMCCD) has a specified QE of 43% at 780 nm (higher than the QE of the Unibrain camera that we more commonly use in absorption imaging). However, the camera features a ‘UV optimisation’ coating which is expected to reduce the QE slightly, and, further, the data provided is for an operating temperature of −20°C. A further reduction in temperature, whilst reducing the unwanted dark current signal, would reduce the QE also. Taking these two factors into account, we estimate a QE of ~30% to 35% at −80°C. The Andor camera makes use of an EMCCD which is a specialist scientific-grade CCD, allowing it to amplify the number of electrons accumulated on each pixel prior to the electronic readout stage, thus increasing sensitivity — an essential feature found in single-atom fluorescence imaging experiments elsewhere. It is possible to achieve a QE of > 80% at 780 nm with a back-illuminated EMCCD, but this factor of 2 advantage as compared with our front-illuminated device is not insurmountable. The pixel size on the Andor camera is 16 µm, more than twice as large as on the Unibrain camera (6.45 µm).

To implement the optical lattice configuration shown later in Figure 6.6 we cannot avoid having one of the laser beams at 795 nm lattice beams directed towards the camera. Without filtering, this beam would overwhelm any fluorescence signal. In future applications the background scatter may also include light from a separate, single-beam dipole trap. For these reasons it is essential to use 780 nm interference notch filters\textsuperscript{11} which have a stated transmission of >98% across 780 nm with a typical FWHM width of ~3 nm and an optical density (OD) of > 6.

4.6 Condensate creation

Figure 4.7 shows the stages of cooling we go through to produce a BEC in a TOP trap, which we then load into the optical lattice dipole trap discussed in Chapters 6 and 7. It is a simple matter to curtail the final evaporation stage to produce warmer

\textsuperscript{11} Semrock LL01-780-25.
or larger rubidium samples that have not reached the BEC phase transition when such samples are required.

Figure 4.7: The sequence of events for laser cooling, transferring, evaporating and imaging a sample of ultracold atoms from capture in a MOT through to the BEC phase transition in our apparatus. The total duration of this sequence is 85 s. The duration of each stage is represented by the length of the corresponding bar. The key shows the length of a bar representing a 10 s duration.

These stages are described in detail below;

1. **MOT loading stage (20 s)**

   To load the MOT a shutter blocking the light (repumping and cooling) is opened, and the quadrupole field generated by the moving coil pair is applied (at the level of \( \sim 15 \text{ G cm}^{-1} \)). With the magnetic and optical fields applied rubidium atoms from the background gas inside the chamber collect at the centre of the magnetic quadrupole.

   In a past version of the experimental sequence, the subsequent compressed MOT (CMOT) stage was triggered when the rising MOT atom number, monitored through
the level of fluorescence emitted by the MOT cloud, reached a set level. In the current sequence, we simply trigger the CMOT stage after a fixed time of 20 s for MOT loading, without an observable increase in the fluctuations of the number of atoms following evaporative cooling.

2. CMOT stage (65 ms)

Once the cloud in the MOT has reached an equilibrium atom number we wish to increase the density prior to loading into a purely magnetic trap to start the ensuing transport stage. We do this with a frequency sweep of the cooling light away from resonance — the equilibrium value of \(-22\) MHz (red frequency detuning) is increased to \(-74\) MHz over 50 ms; the scattering rate of scattering photons within the MOT decreases, which gives a reduction in the outward radiation pressure in the optically thick cloud that causes a reduced density at equilibrium. The repumping light is then extinguished 1500 \(\mu\)s ahead of the cooling light so that all atoms are pumped into the \(F = 1\) state which we use for magnetic trapping.

We apply the CMOT detuning sweep with a single AOM in double pass configuration, the output of which is used to injection lock a slave laser. Whilst in a correctly aligned double-pass AOM there is no beam alignment shift as a function of frequency (which would immediately cause the slave laser to cease following the seeding light), the efficiency of an AOM (the fraction of the input light diffracted into the first order) will drop with the changing frequency. The mode-hop free frequency range over which a slave follows scales in proportion to the seeding light power. As such, the CMOT sweep causes the mode-hop free tuning range (MHFTR) of the slave to decrease, placing greater demands on the performance of the slave laser if mode-hops are to be suppressed throughout the sweep. An alternative scheme would use an ECDL which is offset locked to a reference laser to deliver the cooling light used in the MOT. By applying the CMOT frequency sweep with a sweep in the beat-note locking frequency it would be possible to remove the issue of AOM efficiency and alignment shifts. We are currently investigating this approach as a simple extension of the technique we already use to lock the repumping laser to the correct frequency.

The technique of moving from a MOT through a CMOT stage, where the density of the atoms is increased before being transferred into a magnetic trap, followed by transport using a moving coil pair, is a popular and robust method. It has been reported in work from JILA in 2003 in [53] and in from Durham in 2012 [54], though in our method and those two there is variation in the exact details of the CMOT
stage — for example, we do not alter the repumping light at all, in contrast to those references.

3. Rail transfer stage (2.3 s)

We use a hybrid transport method that begins with a magnetic transport stage where a single coil pair is moved by a mechanical rail most of the way towards the final high-gradient quadrupole field within the glass cell. Figure 4.1 in Section 4.2 gives an indication of the fraction of the total movement carried out by the rail transfer (43 cm, or 83% of the total distance). We use a Parker 404XR rail, which has demonstrated a high degree of reliability, only requiring servicing (cleaning and greasing) after 5 years of operation. The movement of the transport rail is defined in a proprietary software interface. During an experimental sequence we apply a digital trigger signal to initiate the rail movement.

At the end of the MOT and CMOT laser cooling stage, the light is extinguished, and the current in the coils is sharply increased to generate a quadrupole magnetic field gradient strong enough to suspend the atoms against gravity. An additional lateral steering coil is ramped on — this generates a bias field to shift the atom position in the Y direction a small distance to move them onto the open transport path down the axis of the differential pumping tube. We cannot apply optical pumping to drive atoms into the low-field seeking $|F = 1, m_F = -1\rangle$ state in the pyramid MOT arrangement, and therefore when we transfer to a purely magnetic trap at the start of the atom transport, we only trap $\sim 1/3$ of the atoms collected in the MOT.

4. Magnetic transfer stage (4 s)

The second stage of the hybrid transfer is necessary because of the position of the four-lens objective. This objective forms part of our vertical absorption imaging setup (shown earlier in Figure 4.6) and needs to be co-linear with the symmetry axis of the quad coils that generate the quadrupole field that is the final destination of the atom cloud in transit; therefore it sits just below the glass cell, blocking the transport coils from going past. Given this obstruction, we transport atoms over the remaining distance with a series of appropriate current ramps in the transport, ‘push’, ‘auxiliary’ and ‘quad’ coils (shown above in Figure 4.2), causing the confining potential to move in space as the fields shift, with the end point being a field generated purely by the quad coils, and the transport coils having moved back to their starting position.
The initial design and optimisation of the full transport procedure is described in the thesis of Ben Sheard [46], however there has been considerable improvement of these ramps since then. As discussed in [55], where a magnetic transfer procedure using three overlapping quadrupoles was developed, minimisation of aspect ratio variation is desirable to reduce heating from the transport. This parameter has not been fully optimised on our machine — rather, atom number alone has been maximised in the transport stage. From Figure 4.7, it is clear that the majority of an experimental sequence is spent increasing the phase-space density (PSD) by TOP trap evaporation and we expect that further optimisation of the transport ramps in the future can increase our starting PSD and so reduce the required duration of the evaporation stage.

5. Quadrupole evaporation stage (12s)

We make use of two different magnetic traps during a standard evaporation procedure and the advantages and disadvantages of each have already been mentioned in Section 3.2. As stated, it is not possible to evaporate to arbitrarily high phase space densities (PSD) in the quadrupole trap as is illustrated in Figure 4.8.

In a TOP trap there is an intrinsic evaporation mechanism because the trap has a finite depth and we can apply an additional field to drive evaporation. We make use of both types in the full evaporation sequence, however the quadrupole trap has a depth that is too great for any useful loss rate of the atoms with above average energy in the thermal distribution. Indeed, in a quadrupole it is the atoms with below average energy that have a higher probability of being lost from the trap, leading to heating. The effective depth of the quadrupole and TOP traps can be controlled by applying RF radiation that couples the low-field seeking and untrapped Zeeman substates — atoms in the higher energy part of the thermal distribution come into resonance with this field (RF in the range from 120 MHz decreasing over the evaporative ramp to 20 MHz) and are expelled so that the PSD increases. As the temperature decreases, the cloud size also decreases and the rate of collisions goes up leading to runaway evaporation.

If we remain in the quadrupole trap past the point where we would normally transfer atoms into the TOP trap, the PSD and atom number initially tracks towards our final BEC parameters (points up and left of the bottom right in Figure 4.8) but this progression quickly falls off until there is only atom loss — precisely as expected for Majorana transitions of below-average energy atoms. In practice, we move into
the TOP trap (next stage, described below) at the earliest possibility, determined by reaching a maximum cloud size for the trap loading, as the transfer to the TOP trap causes heating that tends to undo preliminary cooling in the quadrupole trap.

![Figure 4.8: The progression of evaporative cooling in two different scenarios. Empty circle data points show a typical progression during the evaporative cooling process, where the cloud is loaded into a TOP trap after a short stint in the quadrupole potential. The filled circles show the progression when the atoms remain in a quadrupole potential, clearly showing that the PSD is limited by atom loss at the trap centre where the field is zero.](image)

6. TOP trap evaporation stage (44 s)

We make plots of the PSD (plotted on a logarithmic scale) such as that shown in Figure 4.8 to track the progression towards BEC in the TOP trap (empty circles). The trap minimum energy in the TOP trap is lifted up from zero, preventing Majoranna transitions, by the rotating bias field $B_T$ applied through the AC coils identified in Section 4.3.2. Whilst the magnetic dipole of the atom is able to follow the rotating field adiabatically, the centre-of-mass of the atom responds to the force applied by the time-averaged potential, so that for understanding the spatial resonance of an applied RF for forced evaporative cooling the rotating quadrupole picture is the correct one.

Solving Equation 3.3 for the rotating quadrupole gives the energy of the trap bottom as simply $g_F m_F \mu_B B_T$ located at the trap centre as expected from symmetry (not
accounting for gravitational sag, which does not shift the quadrupole field zero location but does shift the TOP trap minimum location). In contrast to the quadrupole trap, the TOP trap possesses an intrinsic evaporation mechanism as described in Section 3.2. We make use of both evaporation using this intrinsic mechanism and forced RF evaporation during the progression towards a maximum of the PSD where the trapped gas undergoes the BEC phase transition — the energy of the intrinsic mechanism is also \( \propto B_T \) and thus cannot be controlled independently. This mechanism is used in the first 20 s of the TOP trap evaporation stage, with forced RF evaporation used for the remaining cooling. We aim for a large BEC at the end of this evaporation. Prior to some of the changes to the apparatus mentioned at the start of this chapter, BECs of \( 5 \times 10^5 \) were created. Following reconstruction of parts of the machine, \( 2 \times 10^5 \) was the average BEC number. A long period of optimisation and testing has increased this to \( \sim 3 \times 10^5 \) in the last year. However, recent work on evaporation of a thermal cloud transferred early in the TOP evaporation into the optimised vertically offset TAAP trap (described in [15]) promises to dramatically supersede the usual TOP trap evaporation. With investigations ongoing, it appears that substantially increased trap frequencies are part of the reason why BEC sizes approaching \( 1 \times 10^6 \) have been observed. It seems likely that in the near future the default evaporation sequence for producing BECs will incorporate trapping and evaporation in the adiabatic potential.

The images of cloud density shown in Figure 4.9, obtained using the imaging process described below, show the emergence of a BEC through the final cuts of forced RF evaporation in the TOP trap.

7. Condensate imaging stage (2.2 s)

Most imaging of BEC is carried out after a suitable time-of-flight expansion by switching off the magnetic confinement rapidly. Gravity causes the cloud to fall, and in our system this gives us \( \sim 27 \text{ms} \) of free fall. Of course, simply dropping the cloud is not useful. We apply some time-of-flight in order to expand the cloud, lowering the integrated column density along the imaging direction so that we can resolve with light at or on resonance all of the atoms within the condensate, which is optically dense in-trap. By switching off the trap the clouds expansion makes this possible. Additionally, the in-trap aspect ratio is inverted during expansion from the interaction energy of the trapped condensate (which is proportional to the trap frequency and
thus stronger in the vertical direction — strikingly different to the isotropic expansion seen above the phase transition).

We usually capture three images, the first two being 15 μs bursts of imaging light switch by an AOM, with the atoms present in only the first image. In this first image the intensity of the probe beam is attenuated according to the Beer-Lambert law such that the transmitted light intensity is related exponentially to the atomic column density. The background probe beam light level is subtracted using the second image, and the third image is captured without either the probe beam or the atoms to allow for removal of residual dark counts.

The quantisation axis of the atoms is defined by the application of a bias field collinear with the probe beam, using the TOP bias field. It is convenient to use this field despite the fact that it is rotating at 7 kHz. The field direction is well defined over the short timescale of an imaging pulse, and the phase of the rotation is accounted for by locating a maximum in observable atom number against the phase of the oscillating field. The light is circularly polarised and resonant (accounting for the applied Zeeman shift) so that the light drives the closed transition between the maximally stretched states $|F = 2, m_F = +2⟩ → |F' = 3, m_F = +3⟩$.

**Figure 4.9:** A sequence of images showing the phase transition to BEC. Each image has a square area of 460 μm × 460 μm. The sequence of images are for final RF cut frequencies in the TOP trap of 7.1 MHz, 7.0 MHz and 6.9 MHz. The central picture shows a BEC forming at high density within a surrounding cloud of atoms where the characteristic Thomas-Fermi distribution of the BEC contrasts with the Gaussian distribution of the remaining thermal atoms.
Chapter 5

D1-line laser system

Dipole trapping of rubidium has been successfully carried out many times in the past, and so there are well understood experimental solutions. Most dipole traps use far-detuned light and require relatively high-power laser types (e.g. Ti:Sapph, Nd:Yag etc.) to produce traps from a few µK to several mK in depth. More recently, dipole traps have been demonstrated with smaller detunings of ~30 GHz, consequently requiring much lower intensities to produce traps of similar depth. We demonstrate that a system of diode lasers is sufficient to produce a deep dipole potential.

The Greiner Rb experiments use a near resonant lattice similar to ours for single atom resolved imaging([22]. They use a Ti:Sapphire laser system\(^1\) to generate the D1-line-detuned lattice light, in conjunction with a wavemeter for rough wavelength determination. That laser system is stable enough to stay 20 GHz detuned from resonance over very long time scales without any active stabilisation. In this chapter I describe our diode laser approach in more detail and describe a method to lock the laser frequency 10s of GHz away from a spectroscopic line.

5.1 Lattice light generation

We have become accustomed to using laser diode devices over the past 5 years in Oxford due to their small footprint and the relative ease with which the cavity is aligned and their inexpensive components replaced.

A brief calculation using Equation 3.15 from Chapter 3 for the magnitude of the AC Stark shift gives an indication of the optical power required to obtain an optical lattice, of depth comparable to those used in previous relevant optical lattice experiments (shown in Table 2.1 from Chapter 2) at the previously mentioned low

\(^1\) Based on personal communications.
detuning of 30 GHz from the $^{87}$Rb D1-line i.e. a trap depth of $\sim 400 \mu K$. Optical power is converted\(^2\) to intensity using Equation 3.12 for a beam waist of $w_0 = 100 \mu m$.

Given the above, a 400 $\mu K$ lattice requires less than 4 mW of laser power at this small detuning (assuming retro-reflection). Similarly, 20 mW generates a 2 mK deep trap. Such optical powers are easily obtained from diode lasers, suggesting that in the near-resonant regime it is not necessary to use a high-power laser — if more power were required, one could decrease the detuning further, decrease the beam waists, or otherwise make use of either further slave lasers (currently one is used, shown in Figure 5.2) or a tapered amplifier (of which we use one in the 780 nm laser system as shown in Figure 4.5). As a comparison, in [33] they use 10 W of 1064 nm trapping light for a 300 $\mu K$ deep trap.

The intrinsic frequency stability of the laser is another consideration, as (thus far) we do not actively stabilise it, and can be understood in the following way. Frequency shifts large enough to shift the lattice spacing by half of the site spacing are considered unacceptably large. The relative shift is given by

$$\frac{\lambda'}{\lambda} = \frac{f}{f'} = \frac{n + 1/2}{n} = \frac{2L/\lambda + 1}{2L/\lambda},$$

(5.1)

where $n$ is the number of lattice sites in the distance $L$ over which the interfering beam travels to the retro-reflection mirror, where the phase is fixed by an anti-node. We take $f$ and $\lambda$ for the D1-line and $L = 0.2 m$. The lattice standing wave has a spacing $\lambda/2$ and so $n = 2(L/\lambda)$. Rewriting $f' = (f - \Delta f)$, and evaluating the last term in Equation 5.1 as 0.999 998, we obtain $\Delta f = 750 MHz$. Frequency stability at this level over the course of an experimental image exposure (at the highest limit 1 s for fluorescence imaging) is confirmed via the etalon. Therefore, our diode laser is acceptably mechanically stable.

A long search for a laser diode compatible with our standard Toptica DL100 external-cavity diode laser (ECDL) modules that can operate correctly at 795 nm yielded a diode with a high extracavity power of up to 150 mW (Axcel Photonics M9-795-0150-S5P, $\sim \text{£} 160 / \text{unit}$). We typically operate the master ECDL at 160 mA and the slave at 170 mA. Figure 5.1 shows typical performance for one of these diodes in an aligned ECDL (aligning the diode with feedback from the grating typically lowers the threshold lasing current from the free-running case - for Diode 2 the free-running threshold was measured as 52 mA). Losses in the laser system require such high laser output powers to ultimately emit $\sim 10$ to 20 mW towards the glass cell after the

\(^2\) Noting that there is a factor of 4× increase in the intensity at a lattice anti-node compared to the intensity in a single non-interfering beam.
Figure 5.1: I/P performance for the Axxel laser diodes used to generate master and slave 795 nm light. Diodes 0 and 1 were from the same batch, displaying similar thresholds, however Diode 1 had a lower gradient. Diode 2 is from a different batch, and displays a significantly higher threshold current, but with similar gradient performance to diode 0.

No diodes were found that were specified for 780 nm but could be coerced to run at 795 nm, so we had to explore a different range of products.

We use two of these diodes in our laser system — one in a master ECDL that we can tune to the desired wavelength. The frequency range of the ECDL gives us the flexibility to operate either blue- or red-detuned from the D1 line, or on resonance for testing. This is a very useful advantage of operating close to resonance. A second laser is used in a slave configuration that is front injection locked with a low power (100 µW) seeding beam, as shown in Figure 5.2. We found it important to use high quality isolators in this system, especially for the slave laser, due to the direct retro-reflection of the horizontal confinement lattice beams back down their optical fibre. The fibre input labeling X, Y, Z corresponds to the 3 different dipole trapping beam directions, indicated in Figures 6.3 and 6.6.

In Section 5.2 I describe a method we explored for generating a spectroscopic signal away from the D1 line resonances should greater frequency stability be required in an alternative version of the laser system. Thus far, with such a signal absent, we have verified single-mode operation at the desired frequency of the two lasers using

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3 There may be some new products available that are cheaper than the Axxel diodes whilst still working well, but this seems unlikely. An expensive diode that has not been tested but would be very likely to work with an ECDL design is the Eagleyard EYP-RWE-0810 (~ £750 / unit).
Figure 5.2: The optical lattice laser system, with two lasers generating 795 nm trapping light that is sent to the vacuum system and to diagnostic devices via optical fibres, combined with 780 nm excitation / cooling light. Not shown are paths to absorption (for slave light) and saturated absorption (for master light) spectroscopy, used for on-resonance testing and optimisation. Mechanical shutters are used to control which beams are used during alignment and optimisation (requiring e.g. a crossed $X - Y$ dipole trap).

a Fabry-Perot etalon and a wavemeter ($Burleigh$ WA-10) as follows. The location in frequency space of the master ECDL is determined by the wavemeter. Single-mode operation is then verified by the transmission signal from the etalon with a small amount of master laser light inserted (the etalon cavity length is scanned to give a trace across frequency space larger than the free spectral range of the etalon itself). Within one free spectral range the presence of other modes is obvious. We then swap over to coupling slave laser light into the etalon (master and slave beams are overlapped before a fibre which brings them to the device) and similarly can verify
Table 5.1: Frequencies of the three AOMs used in the D1-line laser system. The sign of the frequency shift indicates the diffraction order used, determined only by the AOM alignment.

<table>
<thead>
<tr>
<th>AOM</th>
<th>Frequency shift</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (X lattice)</td>
<td>-80MHz</td>
</tr>
<tr>
<td>B (Y lattice)</td>
<td>+90MHz</td>
</tr>
<tr>
<td>C (Z lattice)</td>
<td>-100MHz</td>
</tr>
</tbody>
</table>

Figure 5.3: The normalised response of the three AOMs to RF power, with the amplitude controlled by a voltage controlled attenuator. 10 V corresponds to zero attenuation.

single-mode operation. Finally, by observing that etalon transmission peaks are on top of each other on the etalon trace we verify the following of the slave laser to the front-injection locking light. The wavemeter has a resolution of 0.000 001 µm ≈ 2 GHz, and by noting its reading for on-resonance saturated absorption spectroscopy the absolute detuning from a hyperfine transition can be determined.

Three acousto-optic modulators (AOMs) (Crystal Technology 3110-140) offset the beams from each other by 10’s of MHz to mitigate against cross interferences in the optical lattice, as well as providing fast switching control. We do not currently have intensity stabilisation for the lattice beams through the AOMs, though this is relatively easy to implement in the future if desired using a recently developed PID feedback control. Table 5.1 gives the current frequency (which is fixed) configuration
for these devices, which was chosen to coincide with the region of highest efficiency performance for the model of AOM used, but with differences to give a minimum frequency separation of the lattice beams in accordance with relative detunings seen in other experiments. Figure 5.3 shows the response of the AOM diffraction amplitude through the driver circuit shown in Figure 5.4 via a voltage-controlled attenuator (VCA).

![Figure 5.4: The AOM circuit used in the 795 nm laser system.](image)

We use a configuration where the master laser light is used for the Z lattice beams. With no retro-reflection for the vertically confining Z beams the total optical power needs to be twice as large due to the initial path splitting. The retro-reflected X and Y beams draw their light from the slave laser.

Finally, it should be noted that care should be taken with waveplates. A multiple order half-wave plate for 780 nm has a strong wavelength dependence such that it will no longer be half-wave for 795 nm. Therefore, zero-order waveplates are used everywhere that these two wavelengths are superimposed.

### 5.2 Off resonant laser locking

In this section I describe a technique used to observe the off-resonance Faraday effect in an atomic vapour and to derive a spectroscopic signal away from the hyperfine transitions on the $^{87}$Rb D1 line, which could be used for laser frequency stabilisation.

#### 5.2.1 Introduction

There are generally two regimes when it comes to laser frequency stabilisation in quantum gas experiments. The near-resonance regime in this context can be thought of as the range around the atomic absorption lines where you can stabilise a laser to an
atomic reference and obtain different frequencies using frequency shifting devices i.e. AOMs. 200 MHz would be a very high frequency for an AOM used at our wavelengths, which, in a double-pass configuration, might give you access to a frequency shifted range around the D-lines of less than 0.5 GHz. As such, an alternative approach must be used in order to move into the far-detuned regime.

There are many alternative methods that have been developed — offset locking is a robust method that utilises high-speed electronics, in particular fast photodiodes to detect a beat-note between two lasers, one of which is locked to a known reference, as in [51]. The highest bandwidths attainable are $\sim 10$ GHz — close to being, but not, large enough for our application. This method is particularly attractive in the alkali gases for setting a laser to the repumping transition via offsetting from a laser locked to the cooling line (or vice versa). It is also possible to generate optical sidebands in the microwave regime as a means to deriving repumping light from cooling light.

Atomic and molecular references other than those of the species being studied can also be used. Rich molecular spectra have been tabulated and offer a wide range of potential laser locking references that come with the benefit, as with atomic species, of having absolute positions in frequency space. Away from using atoms and molecules as references, mechanical reference structures are used, most commonly etalons and cavities. In this case the stability of the reference is no longer absolute, but relies on the structural integrity of the reference.

We identified a method that seemed like a convenient alternative solution to the problem of laser stabilisation in the off-resonance regime in [56]. Whilst it suffers from the same issue as a cavity, in that the stability of a reference point relies on the stabilisation of other secondary parameters, it does not require any electronic techniques, expensive equipment or access to alternative chemical references. Accessing the 10’s of GHz detunings of interest is easily achievable, and no secondary reference laser is required as in offset locking.

5.2.2 The off-resonance Faraday effect

The magneto-optical Faraday effect is well known in physics, and regularly utilised for creating optical isolators. It can be understood in the following way: A linear polarisation state can be seen as a superposition of left- and right-handed circularly polarised states, and the relative phase between these two oppositely handed polarisation states determines the angle of the composite linear polarisation state. The detailed physical effects that give rise to birefringence in an atomic medium applicable
to the case of rubidium vapour can be found in further publications by the Durham group: [57], [58] and [59].

Figure 5.5: For linearly polarised light propagating from left to right through the birefringent medium, the polarisation state is rotated by an angle $\beta$. The medium is defined by the parameters of length $d$, Verdet constant $v$ and the applied magnetic field of magnitude $B$, which defines the quantisation axis and introduces an asymmetry into the propagation of opposite handedness of circularly polarised light states.

A birefringent atomic medium with a finite retardation difference between the two circular polarisation states (i.e. one where the specific refractive indices $n_{RH}$ and $n_{LH}$ for each state are not equal) manifests the Faraday effect as a rotation of the linear polarisation state, as indicated in Figre 5.5. The rotation angle $\beta$ is given by

$$\beta = vBd$$ (5.2)

where the Verdet constant is itself a function of the relative detuning $\delta$ of the light from an optical transition and the atomic density, which varies with temperature $T$ i.e $v = v(T, \delta)$. The experimental parameters available to us for adjustment are $\delta$ by adjusting the laser frequency and $T$ and $B$, through the cell design, described below in Section 5.2.2.1.

The dependence of $\beta$ on these variables is somewhat complex. Phenomenologically, the effect of temperature is easiest to understand. As $T$ is increased, the vapour pressure of the confined rubidium gas increases, increasing the refractive index leading to greater circular birefringence. Figure 5.6 shows the dependence of the rotation angle (measured through balanced polarimetry to give the signal $(I_x - I_y)/(I_0)$ using a setup similar to ours, shown in Figure 5.8) on temperature. The degree of absorption by the vapour is intrinsically related to the magnitude of the birefringence, and is greatest on resonance. Both absorption and birefringence (or, dispersion) fall as the detuning is increased. Close to the atomic transition there is complete absorption and no light is transmitted, and the extent of this range in frequency space increases with temperature, as seen by the increasing detuning at which the measured signal rises.
from zero in Figure 5.6. The amplitude of the signal decreases towards this point as absorption increases.

![Figure 5.6: The temperature dependence of the Faraday signal. Detuning is measured with respect to the $^{85}\text{Rb} \ D2$-line $|F = 2\rangle \rightarrow |F' = 3\rangle$ transition. Coloured dots indicate zero crossings of the polarisation differencing signal, acquired by detecting on two photodiodes perpendicular polarisation state components of transmitted light. Taken from [56] with permission.](image)

The effect of changing $B$ is similar, without the accompanying increase in absorption. The strongest dependence is on $\delta$, as evidenced by the large number of observed oscillations (each a full rotation of the polarisation state) seen in a typical Faraday spectroscopy trace. These oscillations, bounded towards resonance by increasing absorption, die down at increasing detunings due to the commensurate fall in the dispersive refractive index. Thus the most rapid oscillation is seen close to resonance. Transmission increases as detuning is increased, leading to an increase in the signal amplitude, however the refractive index decreases and falls to zero (it falls more slowly with detuning than the absorption). Together, these effects at a given field $B$ and temperature $T$ explain the shape of the traces observed. The strong dependence on frequency is of most interest for frequency stabilisation applications, and from the argument above it is clear how it can be necessary to increase $B$ and $T$ to generate polarisation oscillations at increasing detunings from resonance.

### 5.2.2.1 Cell design

A robust and compact housing for a standard rubidium vapour cell was sought in order to carry out polarisation spectroscopy. The previous investigations of polarimetry in
give indicative values of $B$ and $T$ required for reaching the $\sim 30\,\text{GHz}$ detuning regime of interest to us (using a similar glass vapour cell) — specifically, that “... detunings of 25 and 50\,\text{GHz} would require 155\,^\circ\text{C} and 200\,^\circ\text{C operating temperatures, respectively.”} The corresponding $B$ field was 270 G. It was also made clear that the primary challenge was reaching the required temperatures, so this was our primary design focus. The simple solution was to construct a solenoid which doubled as an oven — current carrying wires coiled around to form a cylinder directly generate a magnetic field, and indirectly through resistive heating raise the temperature of the surrounding structure. A hollow interior region contains the rubidium cell, and small apertures allow laser light to pass along the symmetry axis. Figure 5.7 shows the final construction.

A standard cylindrical vapour cell, as is often used in table-top spectroscopy for laser stabilisation, is housed inside a brass tube that has been modified to accommodate the glass nipple in the centre of the cylindrical exterior. Brass end-caps close off the ends, except for an aperture for a laser beam to pass through. Because of the cell nipple the structure is created in two halves, with each halve having aluminium mounting that supports the tube as well as acting as end pieces for the coil winding.
process. Recognising the need to reach nearly 200°C, the construction was all metal, with high temperature wire used. The complete cell has 6 layers of 31 turn wire wound on each of the two halves, each 36 mm long. In tests the temperature of this cell has been raised to 180°C without issue.

With the coil configuration given, the calculated interior field $B = \mu_0 n I \times 6 \times 10^4$ can easily reach a field of 400 G, which in tests corresponded to an applied current of $I = 6.1$ A with a resulting external temperature of 125°C.

5.2.2.2 Signal generation

![Diagram of optical system](image)

**Figure 5.8:** The optical system for detecting frequency dependent polarisation changes through the Faraday cell (pictured in Figure 5.7). The input beam polarisation is cleaned to give a fixed linear state with a polarising beam splitter. The output beam is split into two orthogonal linear polarisation states, each detected on a separate photodiode. The state with even splitting is set by the preceding half-wave plate. The difference between the two photodetector signals displays the characteristic oscillations (seen in Figure 5.9) when the light frequency is scanned.

Figure 5.8 shows the apparatus we use to generate the Faraday rotation signal. We ensure a clean linear polarisation input state with a polarising beamsplitting cube (PBS) before the Faraday cell. The photodiode differencing is made up of two photodiodes, providing the signals $I_x$ and $I_y$ with opposite polarities connected to a voltage adder.

To produce a signal that depends on the rotation angle $\beta$ the light is split into the $s$ and $p$ polarisation states by a PBS after the cell. Each component is focused onto a photodiode. In the configuration shown, the cube transmits the $p$ state, and reflects the $s$ state. With no rotation by the rubidium, the cube preceding the cell transmits $p$ state light that the $\lambda/2$ half-wave plate rotates by an angle of 45°. This sets the measured intensities without any induced birefringence — $(1/2) I_0$ on each detector.

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4 Assuming the fields from each of the six layers add, and with a factor of $10^4$ to convert to the non-SI unit of Gauss. The turns per unit length $n = \frac{31}{0.036}$. 

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Rotation of the polarisation state by an angle $\beta$ then changes the measured intensities as $I_x = (1/2) I_0 (\cos(\beta)+1)$ and $I_y = (1/2) I_0 (\sin(\beta)+1)$. Thus the measured $(I_x - I_y)$ oscillates with a frequency $\beta(\delta)$, and can be used as a discriminant for $\delta$ at a given $B$ and $T$.

Before switching on the solenoid, the half-wave plate is adjusted to give equal signal levels on each photodiode — this gives balanced Faraday oscillations during operation (i.e. symmetric around $(I_x - I_y) = 0$), however to simply observe the off-resonance birefringence this is of course not a requirement. We do not normalise our signal to the total transmitted intensity.

Before the coils are turned on, each photodiode displays, on resonance, the characteristic absorption features of the $^{87/85}$RB D1 line with equal amplitudes. Following switch on, the temperature begins to rise towards a point where there is no measurable transmitted light on resonance. Beyond this point, oscillations emerge to the right and left of the D1 line (and also in the 7 GHz region between the two pairs of hyperfine transitions) as seen in Figures 5.6 and 5.9. In Figure 5.9 a saturated absorption spectroscopy signal derived separately is shown alongside the Faraday spectroscopy to show the suppression of signal near one of the D1 line optical resonances. The characteristic behaviour of signal amplitude increasing with $\delta$ due to decreasing absorption and simultaneously oscillation frequency decreasing due to decreasing refractive index away from resonance is visible.

![Figure 5.9](image)

**Figure 5.9:** The highest frequency D1 line hyperfine transition can be seen on the left. A GHz to the right of the transition, where the probe beam absorption is not total, frequency dependent Faraday rotation of the polarisation state can be observed, using the photodetector arrangement shown in Figure 5.8.
Following testing for high temperature tolerance, I examined the extent of the Faraday oscillations using our wavemeter. (This was necessary in our case, though not in [56], due to our more limited mode-hop free tuning range preventing us from covering rubidium spectroscopy reference features as well as the end points of the Faraday oscillations.\footnote{One common technique for increasing the mode-hop free tuning range of a diode laser is to ‘feed-forward’ some fraction of the voltage scan applied to the piezo behind the external cavity diffraction grating to the laser current, in order to stay on the lasing mode over the full range. The very large frequency range scans seen in [56] require not only a large movement of the piezo but also a large current ramp to be applied as feed-forward. For comparison, a large current feed-forward in our scans can, at the limit, cover the entire 7 GHz Rb D lines and would require a 2 mA amplitude feed-forward. In [56] this overlayed diode current ramp is 20 mA.}) It was possible to obtain a zero crossing of the difference signal at a (blue) detuning of 30 GHz with a cell current of 7.5 A, for which the steady state temperature is about 170°C. This surpasses the 14 GHz detuning at which oscillations were observed in [56], where the temperature was 124°C. It should be noted that the most extreme 0 V, $(I_x - I_y) = 0$ crossing of the Faraday signal is not the best lock point — it has the shallowest gradient of all crossings. So, depending on other parameters of the locking feedback system, it may be that a steeper preceding crossing provides the required bandwidth for a feedback error signal.
Chapter 6
Optical lattice configuration

Our goal is to uniquely combine in the experimental apparatus the dynamic magnetic traps demonstrated in previous work with an optical imaging lattice for quantum gas microscopy of novel quantum states. Compared to other imaging lattice constructions, particularly in [22] and [33] where the quantum gas is prepared using almost exclusively optical means, we have restricted optical access for the required additional beams needed to generate an optical lattice. Useful features, specifically reflective coatings on glass cell surfaces and in-cell optical elements, are absent in our apparatus. Section 6.1 describes the various options explored for our 3D optical lattice, as well as details of our final design. Section 6.2 provides the analysis required to convert the experimentally accessible powers incident on the glass cell into transmitted powers. Section 6.3 presents a discussion of how to align a laser beam in general and our methodology for alignment.

6.1 Beam geometries

6.1.1 Horizontal confinement

Typical lattice implementations require a clear beam path through a vacuum system, along which the lattice beam can be retro-reflected. Optical access to the trapping region at the centre of our glass cell is restricted due to the presence of the surrounding AC and DC coil arrays and the optics for the vertical and horizontal imaging systems. Consequently, there are no clear beam paths that go through the centre of the cell which are incident normal to a face of the cell\(^1\).

\(^1\) This may not be desirable anyway. For example, the horizontal imaging beam is tilted slightly from normal incidence to suppress interference fringes from the cell walls. Ideally such unwanted reflections are dealt with by either coating the relevant surfaces or by being incident with the transmitted polarisation state at Brewster’s angle.
Moving off the normal to the cell face, it is possible to find paths passing through the cell and out the other side in the horizontal plane. Two ideally orthogonal pairs of lattice beams are required in this plane to give confinement in two of the three dimensions for the 3D lattice. This was achieved with the beam paths shown in Figure 6.1. Vertical confinement is considered separately in Section 6.1.2.

**Figure 6.1:** Schematic of the initial optical arrangement in the horizontal plane around the glass cell. Polarisation and focusing optics for the horizontal imaging beam are not shown. The intermediate mirror is required for the shallow angle of incidence $X$ beam due to the presence of the moving transport rail.

The low resolution imaging optics in the horizontal direction leaves some room for a beam to go through the AC coil array in the $y$-direction. At a small angle to the $x$-axis of the apparatus there is also a clear path through the glass cell — but the presence of the moving transport rail complicates this, so that an intermediate mirror is required, as shown in Figure 6.1. In this geometry, the $X$ beam is focused over a long distance of 600 mm and the relative angle between the two lattice beams is very close to 90°.

Indirect observations of the $X$ beam position via the alignment method described in Section 6.3.2.2 suggested poor day-to-day stability, which was verified by setting up a CCD camera to monitor the beam position after the glass. Analysis of this data yielded the plot shown in Figure 6.2, where the positional shift on the vertical axis should be compared to the size of the beam, roughly 100 µm at the time. The
dramatic misalignments that occur as part of normal operation of the quantum gas machine, indicated by the movement during Jan 18th, could not be overcome through any compensation. The long beam path and use of an intermediate mirror, which had to be mounted on a thin arm, were the obvious causes of this behaviour, and so we altered the beam mounting of the X beam to give the configuration in the horizontal plane seen in Figure 6.3.

Figure 6.2: Drift in the X lattice beam reflecting off the intermediate mirror prior to the glass cell over time. Orange and blue spots show the beam integrated centre-of-mass $x_0$ in the horizontal and vertical directions. On Jan 17th (a Sunday) beam movement was within an acceptable fraction of the beam waist. On Monday morning experiments began and the associated thermal and mechanical perturbations led to shifts in the beam position beyond acceptable levels so that over one day the beam became misaligned to a degree comparable to a beam waist.

Redirecting the X beam in this way resolved the stability issues, however the structure of the 2D lattice formed through overlaying the X and Y 1D lattices is altered. A contour plot in Figure 7.1 shows the resulting structure for these two beams intersecting at an angle less than the ideal 90° — the increase in the spacing of lattice wells in the horizontal plane was found to be tolerable given the expected use of expansion to increase inter-particle spacing in the proposed small atom number experiments.

Each lattice beam can carry both 795 nm lattice light and, if desired, 780 nm cooling light, combined on a PBS before the fibre input as shown previously in Figure 5.2. The perpendicular linear polarisations of these two beam components are maintained through the coupling of each to either the fast or slow birefringence axis of the single
mode, polarisation maintaining fibres used. Each optical fibre’s output collimator is held in an adaptor, which, combined with a lens tube to hold the beam focusing lenses, is mounted in a robust mirror mount (Thorlabs KC1-T/M). The fibre output collimators and focusing lenses were selected to meet the two key constraints of, firstly, giving a beam waist of 100µm or slightly higher, and secondly a distance from the output mount optics to this focus which fits on the table, practically about 20 cm to 35 cm. These constraints were then considered when applying Equation 3.11 for focusing a Gaussian beam from a waist $\omega'_0$ down to $\omega_0$ over $f$, alongside the specified output beam diameter for the collimator (shown in Figure 6.5) and the available lens focal lengths. Table 6.1 shows the resulting beam waists, measured using the technique described in Section 3.3.

Given adequate laser power for the confinement of laser cooled atoms as calculated in Section 5, the choice of 100µm goal beam waists was guided primarily by desired alignment ease and stability. Recent previous trapping with a different apparatus in our group, described in [60], used a ‘light-sheet’ dipole trap made up of two sheets of blue-detuned light separated by a dark region of 12µm, and the experience aligning

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**Figure 6.3:** A schematic of the horizontal beams following repositioning of the X beam to direct it through the same clearing in the AC coil array as the Y beam. For clarity the angle of the X beam is shown larger than it is in the lab — the real angle between the two beams is about 35°.
Table 6.1: Final beam waists measured at the focus of the four primary lattice laser beams. The $X$ beam waist was deliberately increased from its original size of 100 $\mu$m to add additional effective beam stability following redirecting of the beam as shown in Figure 6.3. The same is true for the $Z'$ beam (for which the optics were also rebuilt recently), which has a relatively long beam path as shown in Figure 6.6.

those beams suggested that a beam size of 100 $\mu$m would be large enough to be reasonably easy to align and keep aligned.

Figure 6.4, a computer-aided design (CAD) drawing, shows the optical lattice output mount construction we used for the two horizontal beams. The design goal was to achieve good mechanical stability for long-term beam alignment through a compact structure. An important aspect of the design, needed in order to carry out the precise alignment stage described below, was to replace the original horizontal and vertical actuators in these mounts with micrometer differential adjustors (Thorlabs DM22), allowing for precise beam adjustment with a numerical readout of the actuator length (translated into sideways shift at the atoms by trigonometry with the kinematic mount structure and a measure of the mount - atom distance). Figure 6.5 shows the inside parts of this mount relevant to the collimation of the Gaussian beam out of the fibre prior to focusing and the rotation of the polarisation state.

From a practical standpoint of realising our design goal for these fibre output mounts, we appear to have achieved a level of stability where the drift day-to-day is small enough to be corrected using the quick alignment technique presented in Section 7.3.1. If higher stability is desired, there are two possible upgrades. The first would be to upgrade the mounting hardware shown in Figure 6.4 — there are well known low drift kinematic mounts available which have better performance than the mount used here, such as the Thorlabs Polaris series of mounts. These mounts are not compatible with the manual actuators we use\textsuperscript{2}. The ultimate solution would be to use a low drift mount with piezoelectric adjustors. The use of piezo adjustment would make lattice beam realignment more precise due the the removal of the need for manual handling. The second way to improve stability would be to have all lattice

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
Beam & Waist \\
\hline
$X$ & 135 $\mu$m \\
$Y$ & 110 $\mu$m \\
$Z$ & 102 $\mu$m \\
$Z'$ & 144 $\mu$m \\
\hline
\end{tabular}
\end{table}

\textsuperscript{2} Besides compromising the intrinsic design of the Polaris mount, the actuator thread pitch is 100 TPI whilst the DM22 adjustors have an external thread pitch of 80 TPI.
**Figure 6.4:** A CAD model of the lattice beam output mounts for the horizontal beams. A receptacle-style fibre connector fixes the fibre output into a collimator, which is held together and in alignment with the focusing lens inside a short 25.4 mm lens tube. The tube is fixed in the adjustable front plate of a Thorlabs kinematic mount, with a rotatable half-wave plate mounted in front separately. The original kinematic mount adjustors have been swapped for micrometer adjustors with a numerical scale on the two corners for horizontal and vertical pointing adjustment.

**Figure 6.5:** A CAD cross-section of the beam optics within the mounting arrangement of Figure 6.4. The single-mode Gaussian output beam expands freely from the fibre tip and is collimated (approximately) by a small aspheric lens. The focusing lens which brings the beam to a ∼100 μm focus over ∼25 cm is mounted coaxially with this collimator. The half-wave plate is mounted separately.
beam optics mounted on a single breadboard, wrapping around the vacuum system. Both of these changes would at this stage require a complex rebuilding of the system. Having confirmed by the same method that provided Figure 6.2 that the stability of each beam is not so large as to give positional shifts larger than the beam waist, we will continue with the current equipment.

For interference to occur between the outgoing and retro-reflected lattice beams they must have the same polarisation. However, for \text{lin.Llin} molasses, the retro-reflected 780 nm beam must have its polarisation rotated through an angle of 90°. To achieve this wavelength selective rotation of the polarisation, we use dichroic mirrors (\textit{Semrock LPD01-785RU-25}) to separate the two wavelengths, with the 795 nm light being retro-reflected without polarisation change, whilst the 780 nm passes through a (zero-order) quarter-wave plate twice with retro-reflection as shown in Figure 6.1. The dichroics used provide good splitting ratios at a convenient angle around 45°: relative wavelength transmissions of 98% and 0.2% have been measured. The best performance is seen for the perpendicular polarisation state — a reduction of a few % is seen for the state orthogonal to this.

\subsection*{6.1.2 Vertical confinement}

Generating confinement in the vertical direction, where optical access is tightly restricted by the coil arrays and four-lens objective of the vertical imaging system, is more challenging than in the horizontal plane. With no one option clearly better than others, we have considered many different approaches. These are listed and discussed next.

\begin{itemize}
\item[a)] Initially, cage mount components were constructed to split on a PBS the power derived from one beam, and direct the resulting pair of beams onto the atoms to form a lattice standing wave. One beam passed along the vertical \(-z\)-direction of the apparatus, through the apertures of the auxiliary and quad coils to the glass cell interior and the other along the \(-x\)-direction, forming a 1D lattice in the \(x - z\) plane at close to 45°. The lattice spacing is increased from \(\lambda/2\) by a factor of \(1/\sin(\theta/2) = \sqrt{2}\). Compared to using a retro-reflected beam, this has the disadvantage that alignment is more demanding. In the first version of this configuration, the beam inserted adjacent to the vertical imaging beam was carried over the top of the DC coil array from the side in cage mount — there was a concern that the stability of the beam position would be compromised.
by this long path, and so a breadboard was placed at the level of the top of the
DC coil array to mount optics directly.

b) It is possible to generate a 2D optical lattice using three mutually interfering
beams, as demonstrated in [61], and as discussed in Section 3.5. We explored
the option of creating a 2D optical lattice in the $x - z$ plane by interfering the
near-vertical beam generated for the scheme in item a) with the $\sim x$-direction
lattice beams. This would have the advantage of reduced alignment demands,
but as stated previously has a drawback if we wish to trap in regions at nodes of
the 3D lattice as in the blue-detuned case. For blue-detuned confinement, which
may prove to have better performance than the red-detuned case, a 3-beam, 2D
lattice is not an option.

c) An alternative configuration where two independent beams interfere to generate
a 1D lattice has both beams delivered through the end face of the glass cell.
There are a number of constraints that limit the angle that can be realised
between the two beams, and hence the lattice spacing is increased. A rough
maximum acceptable spacing is given by the depth of focus of the four-lens
objective — we want this range to contain a single well in the vertical — the
depth of focus is 21 $\mu$m. The mounts of the quad coils ultimately limits the
maximum spanning angle to 18$^\circ$, which would give a lattice spacing of 2.5 $\mu$m,
well below this threshold.

One delivery method we considered was a device wherein one beam is split into
two parallel, collimated beams that are passed through a single large lens to
simultaneously bring them to a focus and cause them to overlap. This can
potentially reduce alignment work, as ensuring parallel propagation is sufficient
for ensuring good overlap of the two beams. The usual task of lateral alignment
to the atoms remains, but this can be done for both beams together by mounting
everything on one large mirror mount. This then places a very strict requirement
on longitudinal alignment, since the angle between the beams remains fixed.
Additionally, the relative angle between the beams is further reduced due to
constraints in available large lens focal lengths.

d) In one obvious configuration the vertical confinement is realised with a retro-
reflected beam that passes through the four-lens objective$^3$. Any configuration

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$^3$ Inconveniently, there is no straight line that passes through the apertures in the DC coil mount,
the atoms and also through the small space between the upper edge of the objective and the lower
quad coil mount - you have to go through the objective.
where the light passes through the atoms and into the objective naturally overlaps the lattice beam with the vertical imaging path, so dichroic mirrors would be required to separate the different wavelengths.

A key consideration is that molasses light propagating with the lattice light could not be separated from any fluorescence. As such, we could never in our system realise a fully perpendicular lattice: Cooling in the horizontal plane would be completely decoupled from the vertical due to the 90° angle, but a cooling beam in the vertical would go straight into the camera without an option to separate it. The relationship between lattice geometry and the required molasses beams is discussed further in Section 8.1.1.

e) One less obvious solution we considered was to project a speckle pattern onto the atoms to provide additional confinement, in addition to a 2D lattice in the horizontal plane. Speckle patterns have been previously applied in cold atom physics for their use in, for example, observing Anderson localisation ([62]). In our case we would seek to achieve the $\sim 3000E_R$ depth in the vertical direction through the contrast between dark and light speckles. The speckle generating phase plate would have to be carefully selected to give a favourable speckle distribution, and some work would be needed to characterise it prior to inclusion in the experiment. With study, this method of generating a confining structure could be useful, but we did not pursue it further in this project.

The vertical confinement solution we settled on was option a) that may intrinsically help us solve the dilemma presented with option d). Whilst we do have a long beam path (which implies a higher risk of mechanical instability and so both potential long term misalignment and short term unwanted movement) the greater structural stability from using cage-mount mechanics and a breadboard will help reduce this. A beam stability measurement has been carried out for one of the $Z$ lattice beams and has indicated good stability. A 1D lattice formed through the interference of the $Z$ and $Z'$ beams is made up of planes at an angle to the vertical as indicated in Figure 7.1, which raises some interesting questions about both atom trapping and imaging that are discussed in Section 8.1 later. The chosen beam configuration is shown in Figure 6.6.

We positioned the $Z$ lattice beam at the maximum possible angle to the vertical (constrained by the upper DC coil mount aperture as seen in Figure 4.2) so that it is at the very edge of the objective and the imaging path, in order to minimise overlap
with the vertical imaging path. Any 795 nm light propagating towards the vertical imaging camera is blocked by a filter.

### 6.2 Glass cell transmission

Since the horizontal beams are not incident to the walls of the glass cell, we carefully consider what fraction of the light is transmitted through the cell, and how this is affected by the angle of incidence and polarisation. The transmission is described by the Fresnel equations (here, drawn from [63]). For the first air-glass interface, the transmittances $T_\perp$ and $T_\parallel$ (the experimentally observed quantity, rather than the amplitude transmission coefficients, $t$) are given by:

\begin{align}
T_\perp &= \frac{n_t \cos \theta_i}{n_i \cos \theta_i} t_\perp^2, \\
T_\parallel &= \frac{n_t \cos \theta_i}{n_i \cos \theta_i} t_\parallel^2,
\end{align}  

where the subscripts $i$ and $t$ denote the incident and transmitted beams respectively, and $n$ is the refractive index. The subscripts $\perp$ and $\parallel$ denote the polarisation states perpendicular and parallel to the plane of incidence respectively. The amplitude...
coefficients are in turn given by;

\[ t_\perp = \frac{2n_i \cos \theta_i}{n_i \cos \theta_i + n_t \cos \theta_t}, \]  
\[ t_\parallel = \frac{2n_i \cos \theta_i}{n_i \cos \theta_t + n_t \cos \theta_i}. \]  

(6.3)

(6.4)

Refraction at the interface causes a deflection of the transmitted beam towards the normal at the first interface (where the refractive index increases) and back at the second interface (where the beam enters the vacuum inside the cell). These angles are calculated using Snell’s Law;

\[ \theta_t = \arcsin \left( \frac{n_i}{n_t \sin \theta_i} \right). \]

(6.5)

The transmittances are given by Equations 6.1 and 6.2 and follow the well known behaviour of light entering uncoated glass whereby at normal incidence both polarisation states have the same transmittance of 0.96, but behave differently at other angles. In these calculations \( n_{\text{glass\ cell}} = 1.458 \). At the Brewster angle of \( \theta_i = 56^\circ \) \( T_\perp \) is equal to 1, while \( T_\parallel \) is much lower.

Of course, in the system we are interested in, a laser beam passes through two interfaces at each encounter with the 2 mm thick wall of the glass cell. Transmittances in this case are just the product of two \( T \) values (here I take as equal the refractive indices of air and the inter-system vacuum);

\[ T_{\text{cell\ wall}} = T(n_i = n_{\text{air}}, n_t = n_{\text{glass\ cell}}, \theta_i) \times T(n_i = n_{\text{glass\ cell}}, n_t = n_{\text{air}}, \theta_t) \]  

(6.6)

Figure 6.7 shows the transmittance through a cell wall for the two polarisation states.

Figure 6.7 indicates that at a steep angle of incidence (such as we had for the X lattice beam in the original beam configuration) it is preferable to have the beam in the perpendicular polarisation state to maximise transmission and thus intensity and trap depth. Practically this is obtained by adjusting the half-wave plate after the focusing lens to maximise the power transmitted. In the new beam configuration the angle of incidence is reduced. During the early stages of this project these calculated transmittances were confirmed to be accurate for a real glass cell by experiments on a spare glass cell measuring transmitted laser power for different angles of incidence.

6.3 Alignment

One of the key challenges is to achieve and maintain the alignment of the laser beams that form the optical lattice. To give a sense of the challenge, we create BECs that
Figure 6.7: Transmittance through a wall of the glass cell for varying angle of incidence, for parallel and perpendicular light polarisations. The perpendicular polarisation rises to total transmittance at the Brewster angle. The two pink bands indicate the angles physically achievable in our apparatus, given the presence of the AC coil array.

have a Thomas-Fermi diameter inside the glass cell of approximately 30µm, smaller in tighter traps. The closest physical structure is the glass cell, which has an internal cross section of approximately 10 mm, so there is a \( \sim 300 \times \) difference in scale! After preliminary alignment by eye, further alignment involves interactions with an atomic sample. Below, I give an overview of alignment methods found in other work, and then describe our approach.

The tools at our disposal for alignment are the 780 nm and 795 nm fields directed into the lattice fibres as shown in Figure 5.2. The 780 nm light can be tuned around resonance and be applied as either cooling light from the TA part of the main D2-line laser system, or temporarily borrowed from the imaging beam subsystem (advantageous as it is AOM switchable for very short pulses). We can also apply resonant or near-detuned repumping light similarly. The 795 nm light can be blue- or red-detuned from resonance as mentioned previously. Further, we have access to clouds ranging in size from filling the field of view down to BECs. In initial stages we move between using a very large, thermal cloud to using the cloud at the end stage of the quadrupole evaporation, which is itself much larger than a BEC. We have the ability to alter the size of the atom cloud ‘target’ through the existing quadrupole and TOP trap evaporation sequence.
6.3.1 Lattice beam alignment methods

Direct imaging

a) As seen in [64], and also for the lattice used in [33]4, one possible method would be to generate an absorption image of atoms using resonant light propagating along the path of a lattice beam as the probe beam, analogously with the absorption imaging we carry out using the large diameter horizontal absorption imaging beam seen in Figure 6.3. This would require a dedicated CCD camera, which would output in a single shot the relative positions of a BEC with dimensions smaller than the lattice beam waist. This method would be advantageous in providing a clear indication in a single shot of positions for a coarse alignment of the beam. However, constructing an additional imaging system for the lattice beams that can coexist with the required retro-reflection optics is challenging in our setup.

Detuned dipole beam effects

b) In [65] a blue-detuned dipole beam is used during alignment, by observing the displacement effects on a cloud of atoms via standard imaging. Specifically, using a cloud larger than the dipole beam waist, they look a symmetric splitting of the cloud due to the beams repulsion, which indicates optimal alignment. In our experiments, the relative dimensions of the dipole beam and an in-trap BEC do not permit such an observation. One alternative is to look at lateral displacement effects on clouds released in time-of-flight — a red- or blue-detuned beam will pull or push the position of a cloud seen in time-of-flight around depending on relative alignment. We colloquially call this method ‘cloud shifting’.

c) By loading atoms into a red-detuned dipole beam it is possible to visualise the spatial position of the beam as a narrow line of atoms via standard absorption imaging. Alignment is optimised by aligning the line of atoms to the pixel position of an in-trap BEC as observed in a separate image. A typical image captured with this approach is seen in Figure 6.8. It is also possible to progressively attenuate this beam and look at atom number for a BEC loaded into the dipole beam against beam position as a more precise method for the next level of accuracy. We colloquially call this method ‘dipole channeling’.

4 Based on personal communications.
Figure 6.8: Atoms loaded into a red-detuned dipole beam in the x-direction propagating close to perpendicular to the horizontal imaging direction. Image size: $\sim 7.5 \text{ mm} \times 1.5 \text{ mm}$.

d) A well-used technique is to induce diffraction of a BEC with the lattice and to observe the diffracted orders in time-of-flight using standard absorption imaging. This effect is discussed in more detail in Section 7.2.1. The fraction of atoms diffracted into non-zero momentum modes will increase for a deeper lattice i.e. when the lattice beam overlap is increased.

Resonant beam effects

Figure 6.9: Localised transparency to the imaging light along the path of the Y lattice beam is seen in this absorption image of a cloud, obtained mid-way through the quadrupole evaporation stage. The two spots derive from the primary outgoing beam and the first reflection from the wall of the glass cell. Image size: $\sim 9 \text{ mm} \times 3.5 \text{ mm}$.

e) Another method uses resonant light to indicate the location in space of the lattice beam by saturating the atoms along the path of the lattice beam during an absorption image capture. This method produces dark spots in the absorption images where the lattice beams penetrate the cloud, as seen in Figure 6.9. For this method the 780 nm light used for absorption imaging, on resonance with the $|F = 2\rangle \rightarrow |F' = 3\rangle$ transition, is simultaneously emitted from the lattice fibre. The small beam waist of the lattice beam at the atoms means that a few $\mu$W of light is enough to saturate the atoms, making them transparent to the horizontal absorption imaging beam. This method has a high resolution for beams propagating at a small angle to the horizontal imaging direction through small clouds, as seen by the clarity of the spots in Figure 6.9, and poor
resolution for beams perpendicular to this direction with large clouds, as the localised reduction in the integrated cloud density becomes imperceptible. We colloquially call this method ‘780 blasting’.

f) Repumping light resonant with the $|F = 1\rangle \rightarrow |F' = 2\rangle$ transition can be applied to atoms in a magnetic trap to transfer atoms into the $|F = 2\rangle$ ground state. Atoms are predominantly repumped into the $|F = 2, m_F = -2, -1, 0\rangle$ states and are thus lost from the trap — Equation 3.1 gives the $m_F$ state dependence of the Zeeman shift used for magnetic trapping. The beam is best aligned to the cloud of trapped atoms when the loss from a pulse of this light is maximised. Such a method is employed in the Spielman / JQI group\(^5\), and our implementation yields plots such as that shown in Figure 6.10. We colloquially call this method ‘destructive repumper’.

![Figure 6.10](image-url)

**Figure 6.10:** Atom number plotted against actuator position of the alignment mount of a lattice beam. Blue dots are atom number for a BEC acquired through absorption imaging, with the red line a fitted function. 7 divisions of the actuator scale correspond to a $\approx 100 \mu m$ transverse shift of the beam position.

6.3.2 Our alignment method

To align the optical lattice of this thesis, we use methods (e), ‘780 blasting’, and (c), ‘dipole channeling’, for coarse alignment, and previously used method (f), ‘destructive repumper’, for precise alignment of the lattice beams without retro-reflection. We have recently implemented method (b), ‘cloud shifting’, using red-detuned light as an alternative method for precise alignment, described in more detail in Section 7.3.1.

\(^5\) Based on personal communications.
For the final precise alignment of the retro-reflected beam we rely simply on the directional sensitivity of coupling back into the optical fibre. Specific aspects of these alignment stages are discussed below.

The optical lattice alignment carried out initially aligns the lattice to a BEC produced in the TOP trap used for confinement in the final stages of TOP trap evaporation. This is relevant due to the presence of gravitational sag in the TOP trap, where the location of the trap minimum in space is displaced downwards with gravity, and also when considering the spatial shift of the trap minimum for adiabatic potentials. The TOP trap we align to has $B_Q' = 295 \text{ G cm}^{-1}$ and $B_T = 9 \text{ G}$, and for these parameters has a gravitational sag of $32 \mu\text{m}$.

6.3.2.1 Longitudinal alignment

The sensitivity of the longitudinal alignment of the lattice beams is determined by the Rayleigh range, or depth-of-focus $z_R$ of the beam, given in Section 3.3 as a function of the beam waist, and also obtained during beam waist measurements. For a typical beam waist of $w_0 = 110 \mu\text{m}$ the depth-of-focus distance is $z_0 = 40 \text{ mm}$. It is possible to locate the central axis of the quadrupole coils and the position of the reference point on the beam output optomechanics to within a few mm, which is small compared to the depth-of-focus. As such longitudinal alignment was a matter of carefully measuring the distance between those two locations around the vacuum system. For the $Z$ and $Z'$ beams the focusing lens, located on its own between the fibre output and the glass cell, is positioned to give the correct separation.

The remaining discussion concerns only the alignment of the optical lattice beams in the plane perpendicular to the direction of propagation.

The Rayleigh range in a typical beam is 400 times greater than the beam waist $w_0$, and hence we can consider the lattice beams to be collimated cylinders, still with their Gaussian intensity cross-section, at the atoms. This is true even into the brightest 10% of the beam — the divergence of the beam as it propagates can be ignored for these parameters. The lack of confinement along the beam means that the lattice scattering force is more than enough to cause atoms in a single red-detuned beam to move along the beam in the direction of light propagation, which is what we see during the single red-detuned beam alignment method described below. Over time the cloud shown in Figure 6.8 is seen to move off to the left, the direction of propagation of the dipole beam confining it against gravity.
6.3.2.2 Coarse alignment

The cloud of atoms we trap in the glass cell, at the centre of the quad coil pair, has a volume of (following some time-of-flight) approximately $4 \text{mm}^3$, so that with some care taken in the initial positioning, the beam passes through part of the large thermal cloud. The beam’s position is identified in a standard absorption image using the ‘780 blasting’ method (e). This method works well for the $y$-direction lattice beam which has a shallow angle in the plane with respect to the horizontal imaging direction, so that the ‘hole’ created in the cloud is viewed almost end-on, as evident in Figure 6.9. In smaller thermal clouds where one can ‘see’ all the way through the ‘hole’ formed by this method, the hole is aligned to the pixel position of where an in-trap BEC would sit (determined separately).

This method works less well for the $Z$ and $Z'$ beams, being near perpendicular to the horizontal imaging direction — hopefully a localised effect is visible and can be used for some initial alignment due to the depletion along the path of the beam being faintly visible. Better contrast is available when atoms from a cloud that has undergone some evaporation are loaded into a dipole potential produced by a red-detuned beam of 795 nm light, as in method (c) above. After the magnetic trap is turned off, excess atoms are dispersed in time-of-flight, and an absorption image shows the remaining atoms in the dipole trap as a thin line of atoms as seen in Figure 6.8.

As with method (e) the inferred beam position can be compared to the position of an in-trap BEC. Maximising atom number in the dipole trap for a small cloud or the condensate gives an indication of the optimal transverse relative alignment of the two.

6.3.2.3 Precise alignment

Probing colder, smaller clouds of atoms increases the sensitivity of the alignment. For the most precise alignment of the outgoing lattice beams we use the ‘destructive repumper’ method (f). A weak 15 µs to 30 µs, $\sim 10 \mu\text{W}$ pulse is applied to a BEC held in the TOP trap before the trap is turned off. The BEC is then imaged following time-of-flight greater than 25 ms to resolve all of the atoms and its atom number recorded. In order to plot a change in atom number as shown in Figure 6.10, it is necessary to attenuate the light significantly in order to only partially reduce the

\[ \text{It is not correct to call a single red-detuned beam a trap for these beams with very long Rayleigh lengths — the confinement along the beam is negligible compared to the lattice light scattering force.} \]
BEC atom number — $\sim 50 \mu W$ will remove all atoms from the BEC. We constructed a laser system specifically for this application\textsuperscript{7} where the detuning and pulse time could be carefully controlled. A schematic of this system is shown in Figure 6.11.

![Schematic of the lattice beam alignment repumping laser system](image)

**Figure 6.11:** A schematic of the lattice beam alignment repumping laser system. Fibre 1 carries light to a standard saturated absorption spectroscopy setup that allows us to resolve (and frequency stabilise the laser to one of) the $^{87}\text{Rb} \ |F = 1\rangle \rightarrow |F'\rangle$ transition hyperfine lines. Fibre 2 carries light to an AOM aligned in a double pass configuration for (increasing) frequency shifting and fast shuttering. Fiber 3 is the lattice fibre (a different fibre for each beam direction) that separately carries 795 nm trapping and 780 nm cooling light through to the glass cell and the atoms therein.

To obtain the curve shown in Figure 6.10, of atom number versus actuator position, this laser was locked to the $|F = 1\rangle \rightarrow |F' = 1\rangle$ hyperfine transition, which is 157 MHz below the repumping line in frequency. The AOM was set to 78.5 MHz to shift the light into resonance with the repumping transition following a double-pass. The applied 15 $\mu$s pulse was observed to destroy the BEC found in the final TOP trap. By reducing the RF power driving the AOM, the light level is reduced to give the required partial atom loss needed.

These ‘destructive repumping’ pulses are applied at different actuator positions in order to locate the position which gives the minimum atom number. Atom number variation follows the expected form of the intensity cross-section of a Gaussian laser beam — for each data set the function $-d \exp(-2(x-a)^2/b^2) + c$ is fitted to the

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\textsuperscript{7} Recently it has become possible to use the repumping light readily available as part of the main D2-line laser system, rather than derive it separately as described here.

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data and the free parameter $a$ for the position of the curves minimum is extracted and used to set the position of the micrometer screw.

6.3.2.4 Retro-reflected beam alignment

Optical fibre alignment is sensitive to both the Poynting vector of the light beam being coupled, as well as the waist of the (ideally) Gaussian mode of the beam. The sensitivity of the coupling efficiency to the alignment of the beam is used for the alignment of the retro-reflected beams $X'$ and $Y'$ shown in Figure 6.3. By rotating the linear polarisation state of the returning beam through an angle of $90^\circ$ using a quarter-wave plate the power coupled back down the fibre can be measured off a polarising beam-splitting cube at the other end of the fibre. Thus, maximising the coupling efficiency of the retro-reflected beam back down the lattice fibres ensures these beams are well overlapped with the outgoing beams.
Chapter 7

Lattice performance and optimisation

The previous chapter describes some important aspects of setting up and aligning the optical lattice beams on our apparatus. In this chapter we verify that atoms can be trapped in the near-resonant optical lattice. In Section 7.1, we present the structure of the full 3D lattice, which requires some work to visualise given that the individual 1D lattices are not perpendicular.

We then evaluate the performance of the three 1D lattices, looking at the momentum transferred to the atoms by a short pulse of the lattice standing wave light in Section 7.2.1 and the lifetime of the trapped atoms in Section 7.2.2. In Section 7.2.3 we examine the structure of the Z lattice, which differs in performance from the X and Y lattices due to its different geometry.

In Sections 7.3.1 and 7.3.2 we present two techniques we use to verify the overlap of the three 1D lattices (having applied the precise alignment technique described in the previous chapter). Finally, we show a cloud trapped by our near resonant 3D lattice in Section 7.3.3. For the 3D lattice we find the atom number decays substantially faster than for the 1D lattices. We suggest mechanisms which cause this rapid decay, and how these may be overcome when using the lattice to confine and image diffuse clouds.

7.1 Lattice structure

The X and Y beams shown in Figure 6.3 and their retro-reflections X’ and Y’ generate a pair of 1D lattices in the horizontal plane. The structure of a 1D lattice formed by retro-reflection of, and intersection of non-parallel, beams is given in Section 3.5. The frequency of the Y lattice light is offset by 180 MHz from the X lattice light, so
Figure 7.1: (a) The light intensity structure of the 2D optical lattice in the horizontal plane formed by the overlapping X and Y 1D lattices in the directions indicated by the two arrows. (b) The structure of the 1D optical lattice in the vertical plane formed by the overlapping and interfering Z and Z’ beams. The legend indicates normalised light intensity — in a the intensity at an anti-node is twice the intensity at an anti-node of the Z lattice in b.

Figure 7.2: The 3D structure of the optical lattice formed by overlapping the 2D horizontal and 1D vertical lattices shown in Figure 7.1. The blue surfaces are contours of constant intensity. The orange planes indicate the structure of the Z lattice only. The region has a volume of $(3\lambda)^2$.
that the interference pattern of the 2D lattice resulting from their overlap in space can be derived through the addition of the intensity of the two independent 1D standing waves. The structure of the 2D optical lattice formed by these beams is shown in Figure 7.1 (a). Since the \( X \) and \( Y \) beams are not orthogonal, the 2D lattice is not square. Furthermore, the axes of this 2D lattice are not coincident with the perpendicular plane, indicated by the red dashed line, containing the \( Z \) lattice beams. The \( Z \) lattice is formed by the pair of beams shown previously in Figure 6.6. These beams intersect at an angle as shown in Figure 7.1 (b) and form another 1D lattice, with planes at 39° to the vertical and separated by \( d = 0.8 \lambda \). As with the \( X \) and \( Y \) lattices, the frequency of the \( Z \) lattice is offset such that the interference pattern is independent of the \( X \) and \( Y \) lattice beams. Figure 7.2 illustrates the structure of the wells in the 3D lattice which results from the overlap of all three of these 1D lattices. The ‘tilted’ structure of this lattice has a potential advantage for laser cooling and fluorescence imaging as outlined in Section 8.1.1.

7.2 Understanding the three 1D lattices

We have studied individually the three 1D optical lattices: \( X \) and \( Y \), formed through retro-reflection, and \( Z \) formed by intersecting two beams as a precursor to trapping in all directions with all lattices applied at once. This section presents three specific investigations.

7.2.1 Observing the momentum distribution

To verify the existence and quality of the lattices formed by each pair of beams, we look for diffraction of cold atoms by each lattice. We observed the atomic distribution in momentum space through time-of-flight imaging after exposure of a BEC to each individual 1D lattice for a short time — the results of this are shown in Figure 7.3. We pulse the optical lattices on for a duration of 20 \( \mu \)s at high power (\( \sim 10 \text{ mW per beam} \)) and observe the distribution of atoms after \( \sim 10 \text{ ms} \) of free fall. In a standing wave light field, momentum is transferred to the atoms in quanta of \( 2\hbar k \). In order to observe this effect a short duration pulse is necessary to minimise the random motion induced by scattering lattice photons, which causes the momentum distribution to decohere, reducing the contrast between the diffracted clouds.
During the time-of-flight the different momentum components of the cloud move apart such that the spacing between different orders $\Delta x$ after time $t$ is given by

$$\Delta x = \frac{2\hbar k}{m_{Rb}} t.$$  \hspace{1cm} (7.1)

In the images in Figure 7.3, obtained through horizontal absorption imaging, the multiple diffracted orders appear as clouds of atoms separated out along the same direction as the lattice wave vector. In the case of the X and Y lattices which are formed by retro-reflection, this direction is along the path of the beams. In the case of the Z lattice the standing wave which results from the two beams is an at angle of about 39° to the vertical. This angle is seen in Figure 7.3 for the Z lattice diffraction pattern.

For the X lattice\(^1\) diffraction shown in Figure 7.3 the time-of-flight was 20 ms, from which we calculate an expected spacing from Equation 7.1 between the momentum states of $\Delta x = 230 \mu$m. The cloud positions in the X image have a spacing of 200 $\mu$m, which is equal to our expected $\Delta x$ when divided by a factor of $\cos 30^\circ = 0.87$ to account for the 30° angle between our observation direction and the lattice beams.

Further information can be gained by looking at the diffraction of atoms by the lattice. Primarily, one can estimate the lattice depth from the relative populations in the diffracted orders [66, 67, and references therein]. This can be used to calibrate the depth of the optical lattices.

7.2.2 Lifetime measurements

One of the key aspects of our scheme is the use of near-resonant dipole light to form the deep optical lattices in which laser cooled atoms may be confined. For atoms confined in a conservative potential, the primary loss mechanism is typically background gas collisions. In our glass cell at UHV pressures the measured lifetime is $\sim 110$ s [46]. However, we observe much shorter lifetimes of $\sim 50$ ms for atoms confined in each of the red-detuned 1D lattice potentials. Being only 20 GHz from resonance, atoms scatter lattice light which results in heating and loss from the trap.

Figure 7.4 shows measurements of the numbers of atoms remaining in the X lattice trap (old configuration) versus time, from which the trap lifetime is determined. The detuning of the lattice light for the data in Figure 7.4 was 20 GHz to the red of the lowest energy $|F = 2\rangle \rightarrow |F' = 1\rangle$ transition of the D1-line. The power incident on the glass cell was 1.5 mW. In tests we found a minimum power of 1.0 mW was

\(^1\) In its original configuration at 90° to the Y lattice, as shown in Figure 6.1.
Figure 7.3: Momentum distributions for a BEC exposed to a short burst of light from each individual 1D optical lattice. In the $X$ and $Y$ images, the time of flight is 20 ms. In the $Z$ image, it is about 10 ms. The different angles between direction of imaging and the pairs of counter propagating lattice beams in the $X$ and $Y$ cases causes the apparent change in momentum state spacing.
required to suspend the atoms against gravity for each 1D lattice, and for higher powers the lifetime was reduced.

In order to accurately determine the atom number for a lifetime measurement such as this, we require a small time-of-flight prior to absorption imaging to allow the optically thick cloud to expand and its density to decrease. The in-trap density of the lattice confined cloud is far too high for an accurate measure of the atom number (and similarly in the TOP trap). Prior to imaging the lattice was switched off and the cloud was allowed to expand for 300 µs. The red data point in Figure 7.4 at $t = 0$ shows the number of atoms when no lattice potential is applied. The blue data points show the atom number that remains in the lattice after various times which decays exponentially with a lifetime of 49 ms.

### 7.2.3 Z-lattice behaviour

Figure 7.5 shows the difference in shape of a cloud of atoms confined in the Z lattice compared to the cloud shape observed in the X lattice, where a large BEC has
Figure 7.5: (a) A BEC loaded into and held in the $Z$ lattice for 22 ms. The white bar indicates the extent of the observed distribution of atoms. The legend indicates the normalised atom density. (b) A BEC of $2 \times 10^5$ loaded into and held in the $X$ lattice for 26 ms.

been loaded into the red-detuned lattice in each case. We explain how the observed differences may arise from the different geometry of the $Z$ lattice. We highlight the features of the $Z$ lattice cloud that can be discerned in Figure 7.5 (a) as follows;

1. The elongated nature of the cloud along the direction labeled $s$.
2. The appearance of stripes in the atomic density along the direction labeled $s$.
3. The asymmetry between the upper left and lower right parts of the cloud.
4. The faint streak of atoms falling downwards in roughly the direction of gravity (marked $g$).

In Section 3.5 we describe the 1D optical lattice potentials formed by two intersecting beams as we have in the $Z$ lattice case. The potential energy landscape the atoms experience is a combination of the optical lattice standing wave in the $r$ direction and the larger scale spatial intensity profile of the two intersecting Gaussian laser beams $Z$ and $Z'$. Furthermore, we must take into account the effect of gravity, and the scattering force $F_{sc} = \hbar k \times \Gamma_{sc}$ from each beam. Together these factors can explain the $Z$ lattice cloud distribution in the following way;
Feature 1: The elongated cloud shape is along the direction $s$, which is perpendicular to the direction of the lattice standing wave. Thus the confinement along the direction $s$ is weak, and atoms rapidly spread out in this direction. Along the direction $r$ indicated in Figure 7.5 (a) the cloud is approximately $40\,\mu m$ wide, which is similar to the diameter of the BEC in the TOP trap prior to lattice loading.

Feature 2: The stripes observed along $s$ are not caused by the lattice fringes, which are separated by an indiscernible $d = 0.8\lambda$ as stated above in Section 7.1. Instead, they can be reasonably explained as a diffraction of the imaging probe beam due to the high gradient in optical density along the direction $r$. Such patterns are common when observing small dense clouds and prevent us from imaging the $40\,\mu m$ wide BEC in the TOP trap.

Feature 3: The asymmetry of the main cloud along $s$ can be ascribed to the net scattering force, which from the two lattice beams is along $s$. This tilts the potential over in this direction, and as the atoms heat up they flow out of the lower right hand side of the trap. The component of gravity along this direction contributes to this.

Feature 4: A similar effect gives rise to the small stream of atoms seen flowing out of the bottom of the cloud in a direction similar to gravity, marked by $g$ in Figure 7.5. In this case the gravitational tilt causes the hottest atoms to spill out along this direction. It should be noted that we observe similar behaviour in atom loss from other traps where the very hottest atoms simply fall downwards. However, in the $Z$ lattice case, the atoms are channeled by the single $Z$ beam dipole potential from the vertical lattice beam which is below the cloud.

7.3 Overlapping the 1D lattices

To reliably form a 3D lattice, we must ensure that the three 1D lattices are well aligned and overlapped. We next present two simple ways in which we check the alignment of the beams that form these lattices to the BEC’s position in the TOP trap and to each other (it should be noted that these methods are equally applicable to the case of a BEC in the more versatile adiabatic potential).
7.3.1 Cloud shifting alignment technique

The precision alignment technique described in Section 6.3.2.3 has proved reliable, however it is time-consuming as the horizontal and vertical directions must both be examined, requiring multiple data points before an indication of which direction a misalignment is in. Further, even if a beam is perfectly aligned, a few data points are needed to confirm this. We therefore sought an alignment method that could be applied each day quickly, and so developed one following discussions with the team behind the imaging lattice in [25].

Immediately after the TOP trap is switched off, we deliver an impulse to a BEC by pulsing on the beam we wish to align. We image the cloud and measure any shift in the BEC’s position this causes, after a period of free fall. By looking at the direction and amplitude of this shift we determine the amplitude and direction of the misalignment of the beam. This can then be corrected for if necessary.

With this method it is possible to determine the degree of misalignment and direction with respect to the unperturbed cloud position for both the horizontal and vertical directions of each beam in a single shot. For a precise description of the position shift, we begin with the expression for a dipole potential $V(x)$ from a Gaussian beam, with the size of the beam described by the standard deviation $\sigma = 2w_0$;

$$V(x) = -V_0 \exp\left(-\frac{x^2}{2\sigma^2}\right).$$  (7.2)

The trap potential depth $V_0$ is given by Equation 3.15. Here we consider red-detuned dipole beam, although the method can be applied to the blue-detuned case by a change of sign.

Assuming the cloud to be small compared to $\sigma$, its centre of mass $x(t)$ experiences a force which is the partial derivative of $V(x)$ i.e. $-\partial_x V(x)$ when exposed to this potential, causing it to oscillate through the region of highest intensity. The momentum $p(t)$ oscillates out of phase with $x(t)$. If the dipole beam is pulsed on for a duration $t$ and has its centre displaced from the clouds starting position by a distance $x_0$, the final position is $x(t) - x_0$, and at the end of this duration the cloud has a velocity $p(t)/m$ for the atoms mass $m$. Following a period of free fall $t_{tof}$, the clouds position relative to its unperturbed position will be shifted by

$$\Delta x = x(t) - x_0 + p(t)t_{tof}/m.$$  (7.3)

The motion of the cloud can be found by writing down the Hamiltonian for the atomic cloud’s total centre-of-mass mechanical energy, which is the sum of kinetic
and potential energy terms:

\[ H = \frac{p^2}{2m} - V_0 \exp \left( -\frac{x^2}{2\sigma^2} \right), \]  

(7.4)

and then solving Hamilton’s equations with the initial conditions \( x(0) = x_0 \) and \( p(0) = 0 \):

\[ \frac{\partial p(t)}{\partial x} = -x(t) \frac{V_0}{\sigma^2} \exp \left( -\frac{x(t)^2}{2\sigma^2} \right), \]  

(7.5)

\[ \frac{\partial x(t)}{\partial p} = \frac{p(t)}{m}. \]  

(7.6)

We solve these equations numerically for a given set of beam parameters. Figure 7.6 shows \( \Delta x \) for a beam position \( x_0 \) with beam power 1.5 mW, the RMS width \( \sigma = 67 \) µm and a 20 GHz detuning from resonance. The atomic parameters are for the D1-line of \( ^{87}\text{Rb} \).

![Figure 7.6](image)

**Figure 7.6:** Calculated cloud shift \( \Delta x \) in units of camera pixels (6.45 µm) against beam centre offset \( x_0 \). The two grey lines indicate the RMS beam width of 67 µm.

The ‘S’ shape of the shift response curve in Figure 7.6 highlights the advantages of this alignment method over the destructive repumper method of Section 6.3.2.3. The destructive repumper method gives a peak response at the optimal alignment, while the cloud shifting method’s ‘S’ shaped response gives an error signal proportional to the derivative of the peak (this is analogous to many spectroscopic locking schemes for lasers). This makes this alignment method intrinsically faster, relying on only the ability to begin with the beam position \( x_0 \) within one \( \sigma \) of the zero position, which we observe to be true day-to-day.
Figure 7.7: An ‘S’ curve for the vertical actuator on the horizontal Z beam. The cloud position is taken by fitting a Gaussian in each direction to the summed intensity counts to give the effective centre-of-mass of the cloud. The actuator adjustments used to adjust the beam are given for the x-axis rather than beam displacement as in Figure 7.6. The dotted line indicates the cloud position with no perturbation. The red line is an unconstrained fit of the derivative of the Gaussian exponential of the form in Equation 7.5.

A further advantage is the ability to distinguish, using the horizontal absorption imaging direction, the cloud shift in both the horizontal and vertical directions simultaneously for the X and Y lattice beams using the horizontal imaging system. We have taken data for this shift for each beam versus the actuator position of the beam directing kinematic mount. We use this data to calibrate the ‘S’-shaped response for each direction of each beam, as shown in Figure 7.7. These calibrations give the gradient of the curve near the zero crossing where the beam is best aligned, in units of pixels per actuator division. With these calibrations we determine the actuator adjustment required if we detect that the beam is misaligned. We now use this rapid alignment technique to check each beams alignment on a daily basis, improving our ability to maintain good alignment of all beams.

7.3.2 Crossed dipole traps

The method above allows us to verify the lattice beams are well aligned to the position of the BEC in the TOP trap. To verify the intersection of beams at this position we load the BEC into crossed dipole traps formed by combinations of the X, Y and two Z lattice beams. Figure 7.8 shows a crossed dipole formed by the 20GHz red-detuned X and Y beams (with the retro-reflections blocked). To load this trap we
turn the \(X\) and \(Y\) beams on as the TOP trap is switched off, so that gravity does not shift the cloud position. The trap was loaded with a BEC of \(2.5 \times 10^5\) atoms. The beam powers incident on the glass cell were 1.95 mW and 1.5 mW for the \(X\) and \(Y\) beams respectively — the imbalance in powers takes into account the larger beam waist of the \(X\) beam, as given previously in Table 6.1. Following a period where the cloud remains suspended in the crossed dipole trap we image the remaining cloud in the horizontal imaging. When the beams are well aligned to both the cloud and trap centre, as well as each other, we observe a symmetric atom distribution, and further the atoms being lost from the trap through heating fall away under gravity symmetrically. Both features of good alignment can be seen in Figure 7.8.

Figure 7.8: Atom loss from a BEC loaded into a crossed dipole trap formed at the intersection of the red-detuned \(X\) and \(Y\) beams. Trap hold times of 10 ms and 20 ms are indicated. The colour scale on both images is the same.

In a crossed dipole ‘trap’ formed from beams which are not well aligned, we observe a diffuse, asymmetric cloud whose centre-of-mass position and shape do not line up with the in-trap position and symmetry of the BEC. In some cases, we observe a stream of atoms escaping along one of the beams.

### 7.3.3 3D lattice

We have demonstrated the formation of 1D optical lattices and evaluated their performance. Further, by combining this knowledge with the ability to rapidly align and check the alignment of each of the beams using the methods described above, we can confidently apply all of the lattice beams to the atoms to form a fully confining 3D optical lattice. As in the preceding experiments, the lattice light detuning was 20 GHz
Figure 7.9: Decay of a BEC loaded into the full 3D lattice over 12 ms, observed through horizontal absorption imaging. Each subsequent image is taken after an additional 1 ms hold time. Each image has dimensions of 190 µm × 650 µm. The colour scale on each image is applied independently for clarity.

red-detuned from the D1-line $|F = 2⟩ \rightarrow |F' = 1⟩$ hyperfine optical transition. The beam powers incident on the glass cell were set to a power just above the threshold for each individual 1D lattice to confine the atoms against gravity, which is close to 1.5 mW for each beam. Figure 7.9 shows the behaviour over time for a BEC loaded directly from the TOP trap into and held in this lattice by switching on all beams simultaneously. Images are captured through horizontal imaging.

One notable feature of the cloud confined in the lattice that can be seen in Figure 7.9 is that the extent of the region of highest density corresponding to the BEC after only 1 ms of hold time does not increase appreciably. The density gradient across the cloud and at the edges falls, however there is no ‘spreading’ of the cloud in any of the visible directions, which is the behaviour we observe in the 1D lattices — most strikingly in the Z lattice. This is consistent with the model of the 3D lattice structure presented in Section 7.1 where with all lattice beams illuminating the atoms confinement is in all spatial directions. It should be noted that for approximately the first 4 ms we observe the very high optical densities in the centre of the cloud as a reduction in the total measured counts — we are familiar with this effect for in-trap imaging of the BEC, and see it again in Figure 7.9. The red data points in Figure 7.10 highlight this observed behaviour as an apparent rise in the number of atoms in the trapped cloud for the first part of the lattice hold. Without any optical confinement the cloud would have fallen 700 µm over 12 ms, off the bottom of the final image in the series shown, where the total height of each image is 650 µm.

For clarity the images in Figure 7.9 are coloured to highlight the density profile within each individual image, however the total counts in each image falls rapidly over the hold time shown, as indicated by Figure 7.10. The atom number reduction
is rapid in comparison to the previously measured lifetime of a BEC loaded into a 1D lattice of 49 ms for 1.5 mW of trapping light in each beam.

The lifetime of a BEC loaded into the 3D lattice is several times shorter than the lifetime observed in the 1D lattice case. One difference is that the intensity at the atoms is increased by a factor of three when all six beams are present rather than just one pair as was the case for the lifetime measurement of Figure 7.4. We observe behaviour consistent with this for the 1D lattice where the lifetime drops with increasing intensity (above the threshold intensity needed to suspend a cloud against gravity). Naively, one might think that this is due to the higher heating rate due to the higher scattering rate. The scattering rate $\Gamma_{sc} \propto I$ and so this rate is increased for the 3D lattice which consequently increases the heating rate $\dot{T} = \frac{1}{3}TR\Gamma_{sc}$ (Equation 3.23). When atoms reach a temperature comparable to the trap depth they are lost from the trap.

However, this intensity dependence of the lifetime does not account for the fact that the trap depth itself increases with intensity, i.e. $U_{dip} \propto I$, so that

$$\frac{\Gamma_{sc}}{U_{dip}} = \text{const.}$$

with respect to intensity $I$. This suggests that increased intensity should not result in the lifetime reductions seen. Therefore there is an additional mechanism which

**Figure 7.10**: Total counts in a 180 $\mu$m$^2$ region-of-interest around the cloud for each image in Figure 7.9. The red points indicate the images where the high BEC density reduces the visible atom number.
causes atoms to be lost from a dipole trap of the type used here. For the detuning of
tens of GHz used in this work the scattering rate is low compared to molasses light
as emphasised previously but not insignificant in isolation.

The first effect to consider is rescattering, or reabsorption, where spontaneously
emitted lattice light interacts with multiple atoms within the cloud. In [68] is it
pointed out that a spontaneously emitted photon can be reabsorbed by another atom.
This is particularly relevant when the cloud is optically thick, as in the case here
of a BEC loaded into and confined in the deep wells of an optical lattice. They
find that this reabsorption causes heating, increasing the equilibrium temperature
with increasing density. Rescattering effects have been studied in magneto-optical
traps where the limiting density is around $10^{11}$ cm$^{-3}$ [69]. We have comparable or
higher density in the 3D lattice trapping shown and the clouds of cold atoms are
initially optically thick as can be seen directly in absorption images. Thus any photon
scattered on resonance from an atom deep inside the cloud will be rescattered many
times before escaping.

All of the single atom resolving imaging lattice experiments listed previously in
Table 2.1 determine the site occupation modulo 2 because of light-assisted collisions
which eject pairs of atoms from lattice wells. This mechanism will be important even
for the moderate scattering rate from lattice light because the atoms are confined at
high densities in individual wells of the optical lattice. In summary, the scattering
of lattice light by atoms in our near-resonant optical lattice drives loss mechanisms
more commonly associated with the application of resonant light in the context of
laser cooling. In our scheme for detecting fractional quantum Hall states of rotating
neutral atoms at low densities atoms are confined in 2D and the total atom number
is $\sim100$ before illumination by either lattice or laser cooling light. Therefore these
secondary heating and collisional processes will not be important.
Chapter 8

Next steps and conclusion

8.1 Next steps

This section described aspects of laser cooling in our lattice and fluorescence imaging more generally. As mentioned in Section 7.1, our ‘off-square’ 3D lattice structure with beams intersecting non-orthogonally has a potential advantage over the orthogonal case, as discussed in Section 8.1.1 below. Possible drawbacks, considered in the wider context of creating a working imaging lattice, are discussed in Section 8.1.2. Separately, I discuss the question of loading the optical lattice potentials we have been applying.

One aspect of loading the constituent 1D optical lattices which we investigated was the difference between a sharp switch on and ramping up the lattice depth (over 1700 µs) while the magnetic trap is switched off. An intensity ramp of the form shown in Figure 5.3, truncated to end at the maximum efficiency was created by a linear voltage ramp applied through the AOM VCA control in the circuit of Figure 5.4. We observed a negligibly small improvement in the density of clouds transferred to a 1D optical lattice using a gradual loading ramp as compared to a sudden switch on.

This somewhat counter-intuitive is an avenue of future investigation as we transition to trapping smaller numbers of atoms. In the simple picture, when a lattice is turned on suddenly, the atoms are spatially distributed across nodes and anti-nodes, or (for red detuning), potential minima and maxima respectively. Atoms that start on top of a peak have a potential energy equal to the trap depth so that they could leave the lattice. As such, we would expect a significant increase in atom loss compared to when the lattice is adiabatically ramped on. The adiabaticity condition can be written as

$$\frac{d\omega_r}{dt} \ll \omega_r^2,$$

(8.1)
or equivalently $\dot{\omega}_r/\omega_r \ll \omega_r$ which can be interpreted as a statement that the change in frequency over one time period $(1/\omega_r)$ must be a small fraction of the frequency. An exponential ramp of the form we apply satisfies this criteria.

### 8.1.1 Cooling in the horizontal plane

When the excitation light for fluorescence imaging also needs to laser cool the atoms there are additional demands on the configuration of the laser beams. Whereas for excitation a single beam might be adequate, laser cooling is usually carried out with a full 6-beam optical molasses, with perpendicular beams.

In the imaging lattice of [33], and others, the standard beam configuration does not work as the wavelengths of the cooling and fluorescence signal light are equal and a 6-beam orthogonal molasses would direct light straight into the imaging system, thus burying the signal from the atoms. One solution would be to use separate wavelengths for cooling light and the fluorescence signal, which (in rubidium) would require additional lasers as indicated in Appendix B, and filter out the molasses light from the fluorescence signal. This solution is attractive as with easily available filters it would be possible to isolate the signal from the atoms to a high degree, in the same way as the background lattice light can be filtered out with a 780 nm transmission filter.

The square lattice geometry used in many optical lattice experiments (the Bloch design being an example) has three 1D lattices that are perpendicular and as such the vertical direction could not be left without any cooling as motion (and heating) in that direction would be uncoupled from the molasses beams in the horizontal plane. They solve the problem of cooling in the vertical direction by injecting a single beam from below and off to one side through their microscope objective, with no counter-propagating component (described in [29]). Reflections within the imaging system provide a significant background level to the fluorescence signal that must be dealt with. Even without balancing by a counter-propagating beam, they observe sufficient cooling in the vertical direction\(^1\). In the same way as no counter-propagating beam is required for the vertical cooling beam, they observe\(^2\) only a small reduction in the cooling efficiency by blocking the counter-propagating horizontal molasses beams (perpendicular to the imaging direction). This is because the atoms, strongly confined

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\(^1\) From [29]: “The light pressure imbalance can be absorbed by the tight confinement in the lattice sites.”

\(^2\) Based on personal communications.
in the wells of a deep optical lattice, experience a strong restoring force that turns motion away from the beam back towards it.

Our optical lattice configuration is not made up of three perpendicular 1D lattices — rather, the vertically confining lattice, formed by the pair of beams shown in Figure 6.6, has an angle of roughly 45° to the vertical as shown previously in Section 3.5. This can circumvent the problem of cooling in the vertical direction since cooling in the horizontal plane should cool in all spatial directions due to coupling of motion in the vertical direction (indicated by a black line in Figure 8.1) into the horizontal plane (in which the two laser cooling beams propagate as in Figure 8.1). We do not have the ability to insert a cooling beam from below in the current setup, making this an attractive idea.

Figure 8.1: (a) The ellipsoidal (blue) surface is a contour of fixed intensity in our optical lattice structure, or similarly the shape of an optical lattice well. The two X and Y molasses beam (which co-propagate with the lattice beams) directions in the horizontal plane are shown, propagating in a plane approximately perpendicular to the vertical imaging direction indicated by the black line. (b) A 2D contour plot in the vertical plane perpendicular to the plane of the molasses beams seen in a. The vertical line indicates the direction in which there is no cooling effect from the X and Y molasses beams. Motion in this direction is coupled into the horizontal plane by the force vectors, shown by arrows.

Figure 8.1 (b) shows the vector force field for the potential energy landscape shown by the intensity contours, indicating that any motion along the vertical line is coupled into the horizontal direction in which the laser cooling beams reside. This supports the hypothesis that our tilted optical lattice allows cooling of all spatial directions with only molasses beams in the horizontal plane.

This design is further supported by work on laser cooling trapped ions. The trap depths in such systems are much greater than we could achieve, however the physics of laser cooling a tightly bound ion in a Paul trap is similar to that for bound neutral
atoms. There is cooling of an ion’s motion in all three directions by the radiative force of a single laser beam as long as there are components of this force along each degree of freedom ([28]). This is achieved simply by ensuring that the laser beam does not go through the trap along any of the symmetry axes.

8.1.2 Fluorescence imaging

Inducing and detecting a fluorescence image is the next stage of this project. The imaging system has already been described as it pertains to detection of BECs through absorption imaging in Section 4.5, which we hope to use for fluorescence imaging with no modifications — the addition of notch filters to block 795 nm light being the only requirement specific to the imaging lattice technique. The imaging system can be considered in the following way: firstly, the system is able to collect and detect a finite amount of light, which impacts the required exposure time for the formation of a fluorescence image. The rate at which signal light is collected needs to be greater than the rate of increase of background noise contributions on the camera (from e.g. dark current) in a time short enough that there is not significant site-hopping and loss of atoms. Secondly, and intrinsically tied to the first aspect, there is a minimum resolvable separation for fluorescing atoms. As stated earlier, one way to tackle this is to expand the separation between atoms beyond the resolution limit if needed.

Regarding the issue of sensitivity, our microscope NA corresponds to 1.8% of a full solid angle, and an estimated QE of 35% is expected for the Andor EMCCD camera. The total scattering rate for lattice confined atoms under excitation with molasses light is the sum of scattering from both light fields $\Gamma_{sc} = \Gamma_{sc, lat} + \Gamma_{sc, mol}$ however we detect only the latter. Our detected signal level (not accounting for an additional few percent loss from reflections of glass surfaces in the imaging path) is given by;

$$0.018 \eta \Gamma_{sc} \tau$$  \hspace{1cm} (8.2)

where $\eta$ is the QE and $\tau$ is the exposure time. With the stated QE this gives an expected rate of 1900 counts per second per atom, where $\Gamma_{sc}$ for the molasses component has been calculated using Equation 3.27 for the D1-line at the saturation intensity$^3$. For an atom confined in an area of $(\lambda/2)^2$, with the magnification factor of the vertical imaging system being $\times 10$ and a pixel area of $16 \mu m^2$, the average fraction of a pixel illuminated is $(10\lambda/2)^2/16 \mu m^2 = 6\%$, so we would expect all of

$^3$ In [33] a scattering rate of 60000 Hz was obtained, so it seems likely that we will be able to achieve higher scattering rates than this.
those counts to be on one pixel. Previously (in [70]) dark current induced counts have been measured at 2.1 counts per pixel per second at 0°C, dramatically reduced to 0.0066 counts per pixel per second at −55°C. As such, we do not expect dark counts to be a significant noise factor. The Andor EMCCD effectively eliminates readout noise, so that the main source of noise is expected to come from the background light. Preliminary experiments indicate that with no molasses light propagating in the vertical Z beam this is also negligibly small, and for the reasons given above in Section 8.1.1 we believe we may be able to do without this beam. If the beam is on it will easily swamp any other effect as expected.

One noise source not mentioned is shot noise, by which CCD counts follow a Poisson distribution. Given the count rate calculated above, in an example exposure time of 100 ms the $\sqrt{N}$ statistical noise would be at the 7% level, and over 50 ms in would be at the 10% level. The experimental impact of this noise source, and of course the exposure time we can realise, remains to be seen.

On the topic of detection, an important consideration is the presence of atoms outside of a single plane of the optical lattice, and, as indicated in Section 7.1, the fact that we do not have a clear plane of lattice wells in the horizontal, perpendicular to the imaging direction. The capturing of a 2D sample of atoms into the optical lattice will require some consideration as the resulting distribution of atoms relationship to the original distribution will depend strongly on the average distance between an atom and a lattice trap minimum following manipulation and possible expansion stages. For now it is interesting to consider upcoming experiments where a 3D cloud is loaded into the lattice. In the near term, we expect to fill the lattice with a density greater than the sought after low densities where the imaging lattice contains a density of one atom per $1.8 \mu m^2$, i.e. one atom per resolvable area with our current microscope objective, and then harness light induced collisions to turn a potentially arbitrary initial density into a density of 0.5 atoms per lattice site (one atom at each site that initially has an odd number).

In other optical lattice experiments the atoms are loaded into a single plane of lattice sites — in [22] and [33] a single 2D plane of atoms is prepared prior to imaging so that out-of-plane atoms are not a concern. In the recent paper from the Greiner group [71] they are able to image two lattice planes and extract images for each, but again this relies on the ability to address an individual lattice plane. In [26] the lattice plane spacing in the imaging direction is $5 \mu m$ which, when imaged with a depth of field $2.8 \mu m$, means that out-of-plane atoms contribute to a background light level but are sufficiently out of focus so that atoms in a single plane are clearly distinguishable.
Apart from magnetic trapping methods, such as the preparation of a 2D sample in the RF-dressed adiabatic potential suggested as a route to the bosonic FQHE regime in Chapter 1, we do not have the same capability for removal of out of plane atoms seen in those other experiments.

The depth of focus of our objective is 21 \( \mu m \). The separation between two of the planes in our lattice structure that lie closest to horizontal is roughly \( 4\lambda/3 \sim 1 \mu m \). This is helpful since we want to reduce the probability of having more than one atom per site when working with small atom numbers to be less than 1\%. By having \( \sim 20 \) visible planes of sites available to distribute atoms over we can reduce the probability of multi-atom occupancy of a well.

Finally, it is expected that optimising the imaging system will require an exploration of the 2D parameter space of molasses intensity and detuning. For an optical lattice with a frequency detuning below resonance (red detuned) the atoms are at anti-nodes where the Stark shift \( \Delta_{lat} \) of the cooling transition is a maximum and thus needs to be included in the total detuning of the molasses light, as in \([29]\) where the total optimised detuning \( \Delta = \Delta_{free} + \Delta_{lat} = -40 \text{ MHz} - 40 \text{ MHz} = -80 \text{ MHz} \). This also applies to the simultaneously applied repumping light that keeps atoms on the cooling transition. We are able to achieve this offset for the cooling light using the already installed double-pass AOMs, and for the repumping light, where the frequency of the light is set through the beat-note locking circuit indicated in Figure 4.5. Both of these frequency shifts have already been applied independently of the rest of the system and confirmed to work.

8.2 Conclusion

Aspects of the imaging lattice that remain to be implemented are discussed above, specifically the combined laser cooling and excitation process with detection through our vertical imaging system. Work on these two aspects of our imaging lattice is ongoing at the time of writing to capture images of fluorescing, confined atoms in the near future. Beyond that, there remain several steps to implement the grand scheme laid out at the start of this thesis in Chapter 1 to detect highly correlated states of rotating neutral atoms — the experimental techniques needed to prepare the quantum states of interest are also progressing but are not the topic of this thesis.

To conclude, the key results of this thesis are as follows;

1. We have set up a diode laser system and saturated absorption spectroscopy on the D1-line of \(^{87}\text{Rb} \), as described in Chapter 5.
2. We have designed and robustly constructed lattice beam optics around the existing experimental system, as described in Chapter 6, and have brought the lattice beams into alignment with a trapped BEC.

3. We have generated optical dipole traps capable of confining atoms, examples of which are seen in Chapter 7. Further, we have confined atoms in an optical lattice structure formed from the interfering dipole beams.
Appendix A

Doppler temperature derivation

The Doppler cooling limit can be understood in the following way. The two processes of absorption and spontaneous emission, when averaged over many events, are, respectively, the scattering force we make use of for cooling, and zero — i.e. $\mathbf{F}_{\text{abs}} = \mathbf{F}_{\text{sc}}$ and $\mathbf{F}_{\text{spont}} = 0$. However, there are statistical fluctuations in each process, $\delta \mathbf{F}_{\text{abs}}$ and $\delta \mathbf{F}_{\text{spont}}$, that need to be accounted for. Considering a single spatial dimension, e.g. the $z$-axis, we can relate the mean kinetic energy, in which the mean square velocity $v^2_z$ is used, to the temperature in that direction by the equipartition theorem i.e. $1/2 m_{\text{Rb}} v^2_z = 1/2 k_B T$. Looking at the quanta of velocity imparted through photon interactions (the recoil velocity, $v_r = \hbar k/m_{\text{Rb}}$, for which there is a recoil (kinetic) energy $E_r = 1/2 m_{\text{Rb}} v^2_r = \hbar^2 k^2 / 2 m_{\text{Rb}}$), spontaneous emission causes the mean square velocity to increase as

$$\langle v^2_z \rangle_{\text{spont}} = \frac{1}{3} v_r^2 \Gamma_{\text{sc}} \times t$$

(A.1)

where the factor of $1/3$ comes from the angular average for isotropic emission. For the absorption process in a single direction, the averaged velocity is

$$\langle v^2_z \rangle_{\text{abs}} = v_r^2 \Gamma_{\text{sc}} \times t.$$  

(A.2)

These two terms are added to the simple damping (Equation A.3 with only the first of the three terms) equation, where a coefficient $\alpha$ contains information about the light intensity and detuning, as follows;

$$\frac{1}{2} m_{\text{Rb}} \frac{dv^2_z}{dt} = -\alpha \overline{v^2_z} + \frac{1}{2} m_{\text{Rb}} \left( \langle v^2_z \rangle_{\text{abs}} + \langle v^2_z \rangle_{\text{spont}} \right)$$

(A.3)

$$= -\alpha \overline{v^2_z} + \left( 1 + \frac{1}{3} \right) \frac{1}{2} m_{\text{Rb}} v_r^2 (2 \Gamma_{\text{sc}})$$

$$= -\alpha \overline{v^2_z} + \left( 1 + \frac{1}{3} \right) E_r (2 \Gamma_{\text{sc}})$$

(A.4)
The factor of 2 in front of the $\Gamma_{sc}$ term accounts for having now not a single beam but a counter-propogating pair of beams in the $z$ direction. Equation A.4 describes the balance between the damping of energy by a frictional force of the form $F_{damp} = -\alpha v$ and heating. To describe the typical arrangement of 3 independent (i.e. neglecting saturation effects) 1D molasses we multiple by 3 the $(\bar{v}_z^2)_{spont}$ term, so that the numerical factor in brackets becomes 2.

We can set the time derivative in Equation A.4 to zero to find the equilibrium temperature (now in a six beam optical molasses) and use the equipartition theorem relation from above;

$$\bar{v}_z^2 = 2E_r \frac{2\Gamma_{sc}}{\alpha}$$  \hspace{1cm} (A.5)

$$k_B T = \frac{1}{m_{Rb}} 2E_r \frac{2\Gamma_{sc}}{\alpha}$$  \hspace{1cm} (A.6)

$$= \frac{\hbar \Gamma_{sc}}{4} \left(1 + \left(\frac{2\delta}{\Gamma_D^2}\right)^2\right)$$  \hspace{1cm} (A.7)

where in the final stage a substitution for the damping coefficient $\alpha$ has been made. The function $\frac{1+x^2}{x}$ has a minimum at -1, which when applied to Equation A.7 give the result that the minimum temperature that can be obtained in this system is at the detuning $\delta = -\Gamma_D^2/2$, which gives the result of the Doppler temperature as above in Equation 3.28 — $T_D = \frac{\hbar \Gamma_D^2}{2k_B}$. For our $^{87}$Rb atom $T_D = 146\mu K \approx 400 \times T_R = 362\,\text{nK}$, where the recoil temperature is the temperature associated with 2 recoil energies, as scattering recoils must come in pairs.
Appendix B

Background free fluorescence detection

Throughout this thesis, and in the majority of experiments elsewhere on alkali metals (or other systems with one valence electron), the only optical transitions we consider are components of the two D-lines. In rubidium these have wavelengths of 780 nm and 795 nm which connect the 5S1/2 ground state to the 5P3/2 and 5P1/2 excited states, respectively. Laser cooling is more efficient for the cycling transitions within the hyperfine structure of the D2 line, which is therefore also used for absorption imaging. Dipole trapping commonly uses far-detuned laser light that can be easily be blocked by appropriate filters. Our scheme uses laser light at 795 nm near to the D1-line resonance to give strong dipole trapping, but we can filter out this trapping light from fluorescence at 780 nm.

Resonance fluorescence on the D2-line cycling transition is convenient in any scenario where laser cooling and imaging need to be combined - as in determining MOT atom numbers or the single atom resolving imaging in [22] and [33]. However, as is discussed in [72] where, analogously to the approach in 87Rb, laser cooling, fluorescence imaging (and repumping) are utilised for trapped ions, light scattered off of surroundings can increase the background signal.

The obvious way to reduce the background signal is to observe non-resonant fluorescence where no applied light field has the same wavelength as the imaging light. This could be achieved for 87Rb as shown by the level scheme in Figure B.1. However, collecting fluorescence from a separate transition leads to a reduction in the scattering rate, and a commensurate increase in imaging time required to achieve a similar signal-to-noise ratio compared to resonance fluorescence. In [73] two-photon spectroscopy is performed to directly excite the 5S1/2 to 5D5/2 transition. Whilst this approach is convenient in that it does not require the use of two lasers ∼ 4 nm apart,
here we are interested in conducting background free detection from the blue decay channel whilst simultaneously laser cooling the atoms, which is an added complexity.

The branching ratios given in [74] show that the proportion of excitations to $5D_{5/2}$ that lead to a blue fluorescence photon is only 7.5%. In contrast, every excitation on the cycling transition follows the same closed decay path. Working with blue light gives an increase in resolution that comes from decreasing the wavelength of the imaging light from 780 nm to 420 nm. In some optical lattice experiments the site-to-site spacing has been below the resolution of the imaging system but the crucial information can be recovered by incorporating knowledge about the structure of the lattice potential and the binary nature of atom number in the images.

An interesting approach that could be used to address the above two issues is making use of the 420 nm decay path, indicated in Figure B.1 to generate shorter wavelength fluorescence light that can be trivially separated from the molasses light required for cooling during the imaging phase.
References


Nomenclature

2DEG  2-dimensional electron gas, page 2
3DEG  3-dimensional electron gas, page 2
AOD  acousto-optic deflector, page 8
AOM  acousto-optic modulator, page 49
AOM  acousto-optic modulator, page 65
BEC  Bose-Einstein Condensate, page 26
CAD  computer-aided design, page 78
CMOT  compressed MOT, page 54
ECDL  external-cavity diode laser, page 49
ECDL  external-cavity diode laser, page 62
EMCCD  electron-multiplying CCD, page 53
FM  frequency modulation, page 49
FQHE  fractional quantum Hall effect, page 1
IQHE  integer quantum Hall effect, page 2
LLL  lowest Landau level, page 2
MHFTR  mode-hop free detuning range, page 55
MOT  magneto-optical trap, page 40
NA  numerical aperture, page 13
OD  optical density, page 53
PBS polarising beamsplitting cube, page 71
PSD phase-space density, page 57
QE quantum efficiency, page 53
RF radio-frequency, page 44
TA tapered amplifier, page 50
TAAP time-averaged adiabatic potential, page 40
TOP Time-averaged, orbiting potential, page 26
TOP time-averaged orbiting potential, page 40
UHV ultra-high vacuum, page 41
VCA voltage-controlled attenuator, page 66
VCO voltage-controlled oscillator, page 66