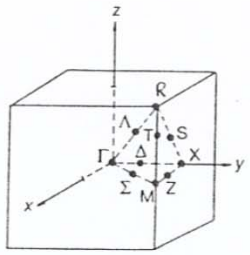
Symmetry in Condensed Matter Physics

Exercises 5: Translational symmetry

1. What are the little groups at the special points Δ, Σ, Λ, X, M and R in the Brillouin zone (shown below) of the monatomic, simple cubic crystal structure? Obtain the irreducible representations at the special points Γ, Δ, and X for the electron energy bands formed from overlapping *p*-orbitals.



Indicate the symmetry-induced degeneracies of *p* electron bands along the closed contour Γ-X-M-R-Γ. Label the symmetry of the orbital wavefunction for all bands and all high-symmetry points.

2. $SrTiO_3$ has a simple cubic lattice and a basis: Sr at 000, Ti at $\frac{1}{2}\frac{1}{2}\frac{1}{2}$ and O at $\frac{1}{2}\frac{1}{2}0$, $\frac{1}{2}0\frac{1}{2}$, and $0\frac{1}{2}\frac{1}{2}$. The point group of the structure is $m3m$.

Evaluate for a general k the characters of the representation formed by transforming the atomic displacements onto one another, and hence find the symmetries of the lattice vibrations for $k = 0$ and $k = (\frac{1}{2}, \frac{1}{2}, \frac{1}{2})\, 2\pi/a$.