Condensed Matter Physics Major Option (C3) — Outline for 2017/2018

`https://www2.physics.ox.ac.uk/students/course-materials/c3-condensed-matter-major-option`

The 4th year Condensed Matter Physics option builds on all the topics introduced in the 3rd year course, but treats them at a much more intellectually satisfying level. The course is primarily aimed at those interested in pursuing a research career, and is designed to take you to a level where you can comprehend research publications over a wide range of areas.

Experimental and theoretical condensed matter physics are both particularly active (and well funded) areas of research. Consequently, there are many locally and internationally funded opportunities for postgraduate research. Topics of research vary from the very pure, such as exotic forms symmetry breaking, to the very applied, such as the application of nanotechnology for the development of clean energy sources. In many areas of condensed matter physics a single researcher can work on very pure research, and then find real world applications of their work!

Syllabus:

Symmetry. Crystal structure, reciprocal lattice, Brillouin zones - general treatment for non-orthogonal axes. X-ray, neutron and electron diffraction. Disordered materials.

Lattice dynamics. Measurement of phonon dispersion. Thermal properties of crystals. Phase transitions. Soft modes.

Electronic structure of solids. Semiconductors. Transport of heat and electrical current. Quasiparticles, Fermi surfaces and interactions between electrons and magnetic fields. Low-dimensional structures.

Lorentz oscillator model. Optical response of free electrons and lattice. Optical transitions in semiconductors. Excitons.

Isolated magnetic ions. Crystal field effects. Magnetic resonance. Exchange interactions. Localized and itinerant magnets. Magnetic ordering and phase transitions, critical phenomena, spin waves. Domains.

Conventional and unconventional superconductors. Thermodynamic treatment. London, BCS and Ginzburg-Landau theories. Flux quantization, Josephson effects, quantum interference.

Lectures:

Topic		*2016/17 Lecturer*
Crystal Structure & Dynamics	10 lectures	Dr Roger Johnston
Band Theory	10 lectures	Prof Michael Johnston
Magnetism	7 lectures	Prof Radu Coldea
Optical Properties	6 lectures	Prof Laura Herz
Superconductivity	7 lectures	Dr Peter Leek and Dr Amalia Coldea

You will be divided into groups of 6 or 7 students and assigned a tutor for the year. There will be 8 tutorial classes covering the material presented in the lectures and revision.

C3 Web site

There is a well organised website which contains lecture notes, problem sets and a detailed synopsis for each part of the course. It also contains links to additional external resources. `https://www2.physics.ox.ac.uk/students/course-materials/c3-condensed-matter-major-option`.

Do have a glance at this site and see what was on offer in the past academic year.

Preparatory reading:

The first set of lectures in Michaelmas Term will cover crystal symmetry and structure determination. The first few problems on the next sheet review some topics covered in the third-year course and extend them in various directions. You will probably find it helpful to combine these problems with some reading around the subject, for example:

- Ashcroft and Mermin, *Solid State Physics*, Saunders, 1976.
- Dove, *Structure and Dynamics*, OUP 2003.
- Kittel, *Introduction to Solid State Physics*, Wiley 2004.

In addition to these, background reading for later parts of the course might include:

- Chaikin & Lubensky, *Principles of Condensed Matter Physics*, CUP, 2000.
- Singleton, *Band Theory and Electronic Properties of Solids*, OUP 2001.
- Blundell, *Magnetism in Condensed Matter*, OUP 2001.
- Annett, *Superconductivity, Superfluids and Condensates*, OUP 2004.
- Fox, *Optical Properties of Solids*, OUP 2002.

Vacation Problem Set:

The following is a set of exercises which you can attempt over the vacation. As this is primarily revision and introductory work there will not be a tutorial class on it (i.e. you do not need to hand in your solutions). Class time instead will be devoted to the interesting new material to be presented in C3 lectures. Outline solutions will be uploaded to the C3 website in early September so that you can check your solutions.

Good luck in your Part B exams, and have a good summer!

Prof. Andrew Boothroyd
(C3 Option Coordinator)
September 21, 2017

Condensed Matter Physics Major Option (C3) 2015/2016

Vacation Problems:

1. In a **cubic system**, the unit cell can be described by $a = b = c$ and $\alpha = \beta = \gamma = 90°$ where a, b, and c are the lengths of the sides of the conventional unit cell and α is the angle between $\mathbf{b}$ and $\mathbf{c}$, etc. In the third-year course, you showed that the spacing d_{hkl} of the (hkl) set of lattice planes was given by

$$d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}. \tag{1}$$

   (a) Remind yourself of where this result comes from.

   (b) Prove that the direction $[hkl]$ lies normal to the plane (hkl) for this cubic system.

   Now, consider an **orthorhombic** system. This is defined by by $a \neq b \neq c$ and $\alpha = \beta = \gamma = 90°$. For this system, show the following:

   (c) The direction $[hkl]$ is not normal to the plane (hkl) in general.

   (d) The direction $[UVW]$ lies in the plane (hkl) if $hU + kV + lW = 0$.

   (e) The spacing d_{hkl} of the (hkl) set of lattice planes is given by

$$d_{hkl} = \frac{1}{\sqrt{(h/a)^2 + (k/b)^2 + (l/c)^2}}. \tag{2}$$

2. What is a *reciprocal lattice vector*?

   Real space and reciprocal space are related by the Fourier transform. Why does a point in reciprocal space correspond to a plane wave in real space? How is a given reciprocal lattice vector connected to the set of planes (hkl) in real space? How does this help in solving diffraction problems?

   Bragg's law states that $\lambda = 2d \sin\theta$. How can this law be reformulated in terms of a reciprocal lattice vector $\mathbf{G}$?

3. This question is about group theory. You can do some background reading on this subject in many books on solid-state physics or on mathematical physics. There are also some good sources of information on the internet.

   A **group** is a set of elements and a binary operation to combine them which obeys certain rules. [The rules are: (1) *closure* (any combination of group elements is a group element); (2) *associativity* ($a(bc) = (ab)c$); (3) the existence of an *identity* element (so that $ae = a$ where e is the identity); (4) every element has an *inverse* ($a^{-1}a = aa^{-1} = e$).]

   An example of a group would be the set of integers 0,1,2,3 together with the binary operation being addition modulo 4. (This means $1 + 1 = 2$, $1 + 2 = 3$, $1 + 3 = 0$ etc.) In the jargon of this subject, the group is described as "the set of integers 0,1,2,3 *under* addition mod 4". It is useful to consider the group table which shows how all the elements combine with each other:

	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

   This demonstrates that "0" plays the role of an identity. Each element has an inverse (e.g. the inverse of 1 is 3 because $1 + 3 = 0$). This group is also a special type of group, called an *Abelian* group (after the mathematician Abel), because the group operation is commutative (i.e. $a + b = b + a$). Not all groups are Abelian.

Symmetry is a property of an object in which the object is brought into apparent self-coincidence by a certain motion or operation. That is, after an object is moved in some way, the object appears to be in the exact same position as before the movement. A symmetry operation represents the motion of the object. Symmetry operations form a group. The symmetry operations which act through a single point are called point symmetries. What are the point symmetries of the Eiffel tower? (You can take the relevant "point" to be the centre of mass of the Eiffel tower.) Identify the point group. By writing down the group table, confirm that the symmetry operations form a group under combination. Is the group Abelian?

4. The **Fermi energy** and **density of states** in 1,2 & 3 dimensions:
   a) A sample contains N electrons, which behave as free particles with energy $E = \hbar^2 k^2/2m^*$, where m^* is an effective mass. Derive formulae for the Fermi energy (E_F) in the following cases (you may assume that the number of k-states in V, a p-dimensional volume of r-space, and V_k, a p-dimensional volume of k-space, is given by $(\frac{1}{2\pi})^p V V_k$):

   (i) The sample is one dimensional (1D), and is of length L.

   (ii) The sample is two dimensional (2D), and is of area A.

   (iii) The sample is three dimensional (3D), and is of volume V.

   In each case, show that the density of states at E_F is of the form

   $$g(E_\mathrm{F}) = \xi \frac{n}{E_\mathrm{F}} \tag{3}$$

   with ξ a number ~ 1 and

   $$n = \begin{cases} N/L & (1\mathrm{D}) \\ N/A & (2\mathrm{D}) \\ N/V & (3\mathrm{D}) \end{cases} \tag{4}$$

   Hence show that, irrespective of the number of dimensions, $g(E_\mathrm{F})$ is proportional to m^*. (This observation will be important later on).

   b) Calculate E_F in both eV (electron volts) and K (kelvins) for

   (i) Copper, which has a fcc structure with $a = 0.361$ nm and is monovalent;

   (ii) an InAs surface "accumulation layer" containing $2 \times 10^{12}\mathrm{cm}^{-2}$ electrons with $m^* \approx 0.024 m_\mathrm{e}$ (never mind what an accumulation layer is; look at the carrier density to see its dimensionality) and

   (ii) a one-dimensional organic conductor whose unit cells each contribute one mobile electron ($m^* \sim m_\mathrm{e}$) and are 6 Å long.

5. Thermodynamics of the Fermi surface.
   a) Show that the mean energy of each electron in a 3D free electron system at $T = 0$ is $\frac{3}{5}E_\mathrm{F}$. Hence or otherwise show that the bulk modulus B of such an electron system is given by

   $$B = -V \left(\frac{\partial P}{\partial V}\right)_N = \frac{2}{3} n E_\mathrm{F}, \tag{5}$$

   where all the symbols have the same meaning as in Question 4. (Hint: remember that $\mathrm{d}U = T\mathrm{d}S - P\mathrm{d}V$.)

   b) Substitute values of n and E_F for copper (see Question 4) into your expression for B and compare with the actual bulk modulus of the metal itself. The two figures should be of the same order of magnitude, *i.e.* metals would probably be pools of gunge lying on the floor if they didn't have a Fermi surface.

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