START OF LECTURES PROPER

We are in a fortunate situation with EM: Maxwell, based on earlier great work by Ampere, Gauss, Faraday, ..., taught us the fundamental eqns – and these are relatively simple!

$$\boxed{\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0} \quad (1)$$

"Gauss' Law" (also Coulomb) subsumed

"div"

electric field (a vector function of space and time)

charge density

constant dependent on system of units we employ. In SI units $\epsilon_0 \simeq 8.85\ldots \times 10^{-12}\ C^2/Nm^2$

Using Gauss' Thm from 1st yr maths we may convert above differential eqn to an equivalent integral form

$$\boxed{\int_{S=\partial V} \vec{E}\cdot d\vec{S} = \frac{Q_V}{\epsilon_0}}$$

total charge within volume V

surface bounding volume V ("∂V" means boundary of V)

2 $$\boxed{\vec{\nabla} \cdot \vec{B} = 0}$$

"no magnetic charges"

magnetic field (a vector function of space and time)

The fact that this is 0 is the mathematical statement of the observed fact that a <u>magnetic charge</u> (sometimes called a "magnetic monopole") has never been seen.

The equivalent integral form is

$$\boxed{\int_{S=\partial V} \vec{B} \cdot d\vec{S} = 0}$$

3

$$\boxed{\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

"Faraday's Law of Induction"

"curl of $\vec{E}$"

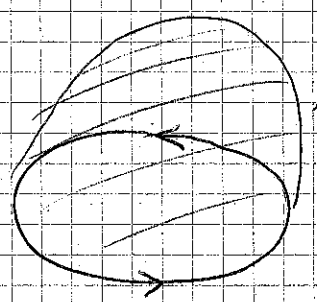
with equivalent integral form by applying Stoke's Thm

$$\boxed{\int_{C=\partial S} \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\underbrace{\int_S \vec{B} \cdot d\vec{s}}}$$

often called the flux, Φ, of $\vec{B}$ across surface S

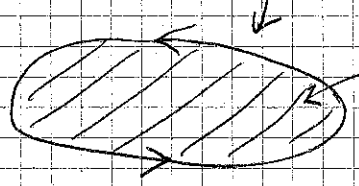
a closed curve that is the boundary of any "disc-like" surface (with boundary C)

note time derivative is outside integral — so can get non-zero RHS even if $\partial\vec{B}/\partial t = 0$ if curve C is moving (and thus S is changing)



a surface that has bdy C

curve C (with standard orientation)



another surface with same bdy C

4

$$\boxed{\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}}$$

"Ampere's Law with Maxwell's correction"

"permeability of free space"

$\mu_0 = 4\pi \times 10^{-7}\ N/A^2$

(exactly by def'n)

current density with units $C/s\,m^2$

The integral form of this is

$$\boxed{\frac{1}{\mu_0} \int_{C=\partial S} \vec{B} \cdot d\vec{l} = \underbrace{\int_S \vec{J} \cdot d\vec{s}} + \underbrace{\epsilon_0 \frac{d}{dt} \int_S \vec{E} \cdot d\vec{s}}}$$

the current flowing through surface S (bounded by closed curve C)

Maxwell's correction – the "displacement current" term

This set of equations describes how charges and currents, as well as changing $\vec{E}$ and $\vec{B}$ fields, <u>source</u> $\vec{E}$ and $\vec{B}$.

However it's not yet complete as they don't specify how charges respond to fields

— this is given by the "Lorentz force law"

$$\boxed{\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})} \quad (5)$$

velocity of particle (of charge q)

<u>Comments</u>

(a) Usually in elementary texts the above five equations are said to be a complete specification of EM (at least in vacuum). However there are 3 other very important facts about our world

(i) Charge is <u>quantized</u> — there is a <u>smallest unit of charge</u>, that of the proton (or positron), $e \simeq 1.60\ldots \times 10^{-19}$ C. No charge smaller than this has ever been observed for a 'free' particle.

('free' means not permanently bound inside a composite object – the main example of a non-free particle is a quark inside a proton or neutron. Quarks have charges $\pm e/3$ or $\pm 2e/3$ – so smaller in magnitude than that of electron – but the principle of charge quantization is still true – no one has ever observed charges smaller than this 'elementary' unit.)

In fact this quantization of charge is incredibly important for chemistry and biology (and physics of many-particle systems in general). If the charge of the electron $-e$ failed to exactly balance the charge of the proton by even 1 part in 10^{20} our world would be startingly different. (Why? – think about it!)

Why is charge quantized?

– we don't know! It may be the consequence of a 'grand unification of forces' or, as Dirac argued, the consistency of quantum EM if a magnetic monopole exists. But this is an open problem...

(ii) Both <u>positive</u> <u>and</u> <u>negative</u> charges exist.

Often people make the analogy between Coulomb's Law for charges

$$\vec{F}_C = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}$$

and Newton's Law of Gravity for masses

$$\vec{F}_N = -G_N \frac{m_1 m_2}{r^2} \hat{r}$$

Indeed they both are conservative forces varying as $\sim 1/r^2$ that are linear in the charges or masses.

However there is a big difference

- <u>charges</u> are both + and −
- <u>masses</u> are <u>only positive</u> (gravity is <u>always attractive</u>)

If it were not for the existence of both + and − charges, atoms (and thus all of chemistry, biology, ...) would be impossible.

We <u>do</u> understand why this is so!

In short (this is not meant to be understandable), the reason is

the force carrier of EM is "spin 1" — the photon
while the force carrier of gravity is "spin 2" — the graviton

<u>ASIDE</u> This explains why the antiparticle of a particle of charge q and mass m has charge $-q$ and mass m (still) → ie, antiparticles fall <u>down</u> like particles!

(ii) Differential (or integral) eqns are not fully specified unless we also give <u>boundary</u> <u>conditions</u> for how the fields behave. Sometimes these b.c.'s contain very significant facts
— we'll see an example when we discuss radiation from an accelerating charge

<u>Comment (b)</u>

One of the most important features of Maxwell's eqns and the Lorentz force law is that they are

<u>LINEAR EQNS</u>

⟹ Allows us to use <u>superposition</u> <u>principle</u> for solutions