Generalised Spin-Echo to Protect Against Changing Magnetic Field

The Hahn spin-echo is a well-known method to improve the coherence of a Ramsey experiment, if the frequency of the atom's free precession is slightly different to the driving radiation. A π pulse in the middle of the gap can "undo" the excess phase acquired from this offset, and restore the fringe contrast.

This method is however limited by the requirement for a π pulse in the middle of the gap, as this requires the frequency uncertainty to be small compared to the Rabi frequency. If this is not so (in particular for a range of frequencies) then a π/2 pulse can be used, in order to avoid the π/2-π-π/2 echos becoming a Ramsey experiment.

To generalise further, suppose the detuning δ(τ) is an (n−1)th order polynomial with time: δ(τ) = α_{n−1}τ^{n−1} + α_{n−2}τ^{n−2} + ... + α_1τ + α_0. Perfect, instantaneous π pulses occur at times α_1/2, α_2/2, ..., α_n/2. The spurious accumulated phase φ_{error} is given by integrating the detuning with respect to time, so demanding that φ_{error} vanishes requires us to solve:

\[ 0 = \phi_{error} = \int_0^{\alpha_{n-1}} \delta(t) dt + \int_{\alpha_{n-1}}^{\alpha_{n-2}} \delta(t) dt + \ldots + \int_{\alpha_2}^{\alpha_1} \delta(t) dt + \int_{\alpha_1}^{\alpha_0} \delta(t) dt \]

\[ = \left( \frac{\alpha_0 t + \frac{\alpha_1}{2} t^2}{n} + \frac{\alpha_2}{3} t^3 + \ldots + \frac{\alpha_{n-1}}{n} t^n + \ldots + (−1)^n \right) \frac{\alpha_0 t + \frac{\alpha_1}{2} t^2 + \ldots + \frac{\alpha_{n-1}}{n} t^n}{n} \]

\[ = \sum_{j=0}^{n-1} \frac{\alpha_j}{j+1} \left( 2 \sum_{j=1}^{n} (−1)^j \right) \]

This equation must be independently true for each polynomial coefficient α_j, because they can take any (real) value. So we obtain a set of simultaneous equations:

\[ (−1)^n + 2 \sum_{j=1}^{n} (−1)^j \alpha_j' = 0 \quad \forall j = 1, 2, \ldots, n. \]

These simultaneous equations are solved when the pulse times take the values:

\[ t = \tau \left( \frac{i}{2} - \frac{1}{n} \right) \]

This pulse sequence is illustrated below for n = 4.

An n pulse sequence precisely cancels out the spurious accumulated phase when the detuning is an (n−1)th order polynomial in time.