Faculty:
Derek Stacey, Andrew Steane, David Lucas.
Post-docs:
Post-grads:
Jonathan Home, Simon Webster.

Past students:
J-P Stacey, Charles Donald, David Stevens.
Summary

1. Some issues for phase gate by pushing ions
2. Designing traps for fast ion displacement
3. Experiments:
   1. Spin-state detection
   2. Rabi flopping of the qubit
   3. Cooling to near the ground state of motion
Push gate, wobble gate

Dipole force in moving standing wave: 

a) Ion trajectories

Slow oscillation of force: "push"

Near-resonant oscillation of force: "wobble"

Aim: phase gate: \( \theta = \begin{array}{cccc}
0 & \pi/2 & \pi/2 & 0
\end{array} \)


Phase gate:

\[
\begin{pmatrix}
1 & 1 \\
1 & -1
\end{pmatrix}
= 
\begin{pmatrix}
i & i \\
i & 1
\end{pmatrix}
\]
Geometrical Argument

Phase acquired
\[ \theta = \text{area in phase space in rotating frame} \]
**Dynamical Argument:** \( \theta = \exp( s_0 t - H \, dt/\tau) \)

**Push:** Coulomb interaction when ion pushed by distance \( x_F \)

\[
\frac{q^2}{4\pi\varepsilon_0} \begin{pmatrix} \frac{1}{d} & \frac{1}{d+x_F} & \frac{1}{d-x_F} \end{pmatrix} \approx \frac{q^2}{4\pi\varepsilon_0 d} \begin{pmatrix} 1, 1 + \frac{x_F}{d}, 1 - \frac{x_F}{d} \end{pmatrix}.
\]

Hence \( \theta_p \approx \frac{q^2}{4\pi\varepsilon_0 d} \frac{2x_F^2 \tau}{d^2 \hbar} \).

**Wobble:** Raman transition causes light shift.

- \( \left| \begin{array}{c} 11 \end{array} \right\rangle \)  \( n+1 \)
- \( \left| \begin{array}{c} 10 \end{array} \right\rangle \)  \( n \)
- \( \left| \begin{array}{c} 01 \end{array} \right\rangle \)  \( n-1 \)
- \( \left| \begin{array}{c} 00 \end{array} \right\rangle \)

Rabi freq.: \( \Omega_D = \eta \Omega_{\text{Raman}} \)

Level \( n \) shifted up by \( n-1 \) and down by \( n \)  

\( \rightarrow \) net effect ind. of \( n \)

\( \theta_p \approx 16\sqrt{3} \frac{\Omega_D^2}{\omega_0} \tau. \)

(Choose \( \delta = 2 \Omega_D \))

**COM freq:**

\( \theta_w = 2 \frac{\Omega_D^2}{\delta} \tau. \)

***c.f.***
Fidelity

Photon Scattering

\[ N \sim \frac{\Omega^2}{2\Delta^2} \frac{\Gamma \tau_g}{\eta_L \Delta} = \frac{3\omega_0 \Gamma}{4\eta_L^2 \Omega_0^2} \]

\[ \sim \frac{\pi \Gamma}{\eta_L \Delta} \]

\sim 1/\text{intensity}

\sim 1/\text{detuning}

Thermal

Non-uniform force
= deviation from
Lamb-Dicke approx

\begin{align*}
\text{Coupling to other mode} & \quad P \sim 2\pi \eta_L^4 \left( \frac{k_B T}{\hbar \omega_0} \right)^2 \\
\text{Debye-Waller factor} & \quad P \quad = \eta_L^4 \left( \frac{\pi^2}{4} \right) \bar{n} (\bar{n} + 1)
\end{align*}

The total thermal effect is similar for the two cases:

\[ P \simeq (\bar{n} + 1) \left( 0.3\pi^2 \eta_L^4 \bar{n} + 1.6 \frac{\Omega_D^2}{\tilde{\omega}_0^2} \right) \]

Discussion

- Wobble uses resonance to displace ions further for a given force → requires less laser intensity (for given photon scattering $N$)

- Laser intensity noise typically falls with freq. ⇒ better to oscillate the force.

- However, for a faster gate require $\delta$ not too small → adopt $\delta \approx \omega_{\text{com}}/2$
  i.e. intermediate between “push” and “wobble”.

- Fine structure → limit on $\Delta$

  \[
  N_{\text{scattering}} = \frac{2\sqrt{2\pi\Gamma}}{\eta\omega_{\text{fine}}} = \frac{2(\lambda\Gamma)\sqrt{m\omega_z/\hbar}}{\omega_{\text{fine}}}
  \]

  but $\omega_{\text{fine}} \sim m^2$ so speed $\omega_z \sim m^3N^2 = \text{mass}^3\text{error}^2$
Designing traps for fast ion displacement
Bringing ions together

\[ \omega_{\text{COM}}^2 = \frac{(2\alpha_z + 3\beta d^2)q}{m} \]

\[ \omega_{\text{stretch}}^2 = \omega_{\text{COM}}^2 (1 + \tilde{\epsilon}) \]

where \( d \) = separation of the ions, and

\[ \tilde{\epsilon} = \frac{q^2}{\pi \varepsilon_0 m \omega_{\text{COM}} d^3} \]

) To maintain \( \omega \) large when \( \alpha_z \) goes through 0, require large \( \beta \).
Electric Octopole Potential

Require large $\beta$ at small $\alpha_z$

- increase voltages
  - get large $|\alpha_x|$, $|\alpha_y|$, $|\alpha_z|$?
  - can’t confine the ions
- require $|\alpha_x|$, $|\alpha_y|$, $|\alpha_z|$ all small but with large $\beta$

Electric octopole trap

(small) d.c. quadrupole + octopole

\[
V(x, y, z, t) \simeq \alpha \left( z^2 - \frac{1}{2}(x^2 + y^2) \right) + \beta V_4(x, y, z) + Q_{ac} \cos(\Omega t)(x^2 - y^2)
\]

+ r.f. quadrupole for radial confinement
Geometric factors & electric field

\[ V(x, y, z, t) \approx \alpha \left( z^2 - \frac{1}{2}(x^2 + y^2) \right) + \beta V_4(x, y, z) + Q_{ac} \cos(\Omega t)(x^2 - y^2) \]

Assume limited by electrical breakdown, i.e. there is a maximum allowed electric field at an electrode surface.

Then:

Geometric factors \( \gamma, \mu \) defined by

\[
\text{for } Q_{ac} = 0, \quad \beta = \frac{\gamma E_{\text{max}}}{\rho^3},
\]

\[
\text{for } \beta = 0, \quad Q_{ac} = \frac{\mu E_{\text{max}}}{\rho}.
\]
Example structures

Some standard octopole configurations:

a)  

b)  

c)  

d)  

e)  

f)  

g)  

h)  

1. “Two hands”

\[ \gamma = 0.027 \]
\[ \mu = 1.54 \]

2. “hands” inverted

\[ \gamma = 0.0065 \]
\[ \mu = 0.62 \]

3. “Railway track”

\[ \gamma = 0.0064 \]
\[ \mu = 0.48 \]

4. Geometric octopole

\[ \gamma = 0.0033 \]
\[ \mu = 0.072 \]
Scaling with mass and $\rho$

$$\frac{\omega_{\text{COM}}(\text{octopole})}{2\pi} \approx \frac{840 (\gamma E_{\text{max}})^{3/10}}{\sqrt{A} \rho^{9/10}}.$$  

$$\frac{\omega_{\text{radial}}}{2\pi} \approx \frac{1105}{\sqrt{A}} \left( \frac{q_r \mu E_{\text{max}}}{\rho} \right)^{1/2}.$$  

$A =$ mass no. of ion, $q_r =$ Mathieu $q$ parameter  
$E_{\text{max}} =$ $V \mu \text{m}^{-1}$ in $\mu \text{m}$.  
$\rho =$ small $\approx \omega_{\text{COM}}(\text{octopole})$  
$\rho =$ large $\approx \omega_{\text{radial}}$  

Define $\omega_0 = (\omega_{\text{radial}} + \omega_{\text{COM}})/2$.

At $E_{\text{max}} = 10^8 \text{V/m}$, $q_r = 0.027$.  

<table>
<thead>
<tr>
<th>Ion</th>
<th>$\omega_0/2\pi$ (kHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^9\text{Be}$</td>
<td>201.8</td>
</tr>
<tr>
<td>$^{25}\text{Mg}$</td>
<td>121.1</td>
</tr>
<tr>
<td>$^{43}\text{Ca}$</td>
<td>92.3</td>
</tr>
<tr>
<td>$^{67}\text{Zn}$</td>
<td>73.9</td>
</tr>
<tr>
<td>$^{87}\text{Sr}$</td>
<td>64.9</td>
</tr>
<tr>
<td>$^{111}\text{Cd}$</td>
<td>57.5</td>
</tr>
<tr>
<td>$^{135}\text{Ba}$</td>
<td>52.1</td>
</tr>
<tr>
<td>$^{171}\text{Yb}$</td>
<td>46.3</td>
</tr>
<tr>
<td>$^{199}\text{Hg}$</td>
<td>42.9</td>
</tr>
</tbody>
</table>

1. $\gamma = 0.027$  
$\mu = 1.54$  

2. $\gamma = 0.0064$  
$\mu = 0.48$
Experiments
Physical system: Calcium ions in a trap

**Present**

- **Ion:** 40 Ca
- **Qubit:** M = +1/2, -1/2 spin state
- **A single linear trap.** $v_{\text{ion}} \sim 1$ MHz

**Future**

- **43 Ca**
- **Hyperfine levels**
- **Multiple traps**

### Diagram Details

- **397 nm**
- **866 nm**
- **850 nm**
- **393 nm**

- **$P_{3/2}$**
- **$P_{1/2}$**
- **$D_{5/2}$**
- **$D_{3/2}$**

- **Qubit:** $M = \pm 1/2$ spin state

- **7.2 mm**

- **λ** (lambda) in nm:
  - 854 nm
  - 850 nm
  - 866 nm
  - 393 nm
  - 397 nm

- **S** (S-state) and **P** (P-state) levels
Apparatus summary

Readout

Deshelving

Cooling

Ion trap in UHV (9 MHz helical resonator)

atomic beam

magnetic field direction

AOM

photo-ionization

Reference cavities

AOM

AOM

AOM
Spin-state detection (qubit readout)
Principle of spin detection

We want to detect: is the spin state $|\rightarrow\rangle$ or $|\rightarrow\rangle$?

Cycling: $S_{1/2} \rightarrow P_{3/2}$?

but optical pumping $\rightarrow$ only $\sim 1$ photon.

Problem:
No matter what type of transition, fluorescence is always accompanied by optical pumping between $|\rightarrow\rangle$ and $|\rightarrow\rangle$.

$\rightarrow$ Spin state is caused to relax before a detectable signal is obtained.
Principle of spin detection

Solution: Suppress the unwanted excitation by electromagnetically induced transparency (EIT).

\[
\text{Ratio wanted/unwanted excitation} = \frac{(\Omega_{\text{pump}})^2}{\Gamma \gamma} \gg 1
\]

where \( \gamma = \text{laser linewidths} + D \text{ linewidth} \)
Experiment in calcium

$|p_{3/2}\rangle \rightarrow |p_{-1/2}\rangle \rightarrow |p_{1/2}\rangle \rightarrow |p_{3/2}\rangle$

Pump = 850 nm, 5 mW

$|d_{-3/2}\rangle \rightarrow |d_{-1/2}\rangle \rightarrow |d_{1/2}\rangle \rightarrow |d_{3/2}\rangle$

$D_{5/2}$ = ‘shelf’, 1 s lifetime

Probe = 393 nm, 1 µW

(B=0)
First step: transfer to shelf, using EIT for selectivity.

Second step: detect fluorescence using the cooling lasers.
EIT spin state readout: results

Experimental sequence:

(Prepare $-1/2$ or $+1/2$ spin state by optical pumping.)
Rabi flopping of the qubit
Magnetic resonance

~10 mG magnetic field oscillating at the Larmor frequency drives Rabi oscillations of the spin state.

Static $B = \text{a few Gauss}$

$1 \text{ Amp, 4 MHz}$

$4 \text{MHz}$

$\frac{1}{2}$

$\frac{-1}{2}$
Rabi Oscillations

$t_{\text{coherence}} \sim 0.5 \text{ ms}$
Rabi Oscillations with 50 Hz line trigger

$t$ coherence ~ 1.2 ms
Ramsey fringes

Identify centre to ± 1 kHz

π/2 pulse time=27µs, pulse separation=214µs

Normalized population vs rf. Frequency (kHz)
Cooling to near the ground state
Continuous Raman Sideband Cooling

Typical values

- Trap $\nu_z = 812$ kHz
- Lamb-Dicke $\eta = 0.2$
- Raman $\Omega_{rsb} = 80$ kHz
- Repumping $R = 100$ kHz

Lindberg & Javanainen, JOSAB 3,1008 (1986)
G. Morigi et al. PRL 85,4458 (2000)
Cooling rate and steady-state temperature

- Cooling rate (vib. quanta per second) is given by Rabi frequency on the red sideband $\Omega_{rsb}$

For our Raman + repumping process, Linewidth $\Gamma = \max(R, \Omega_{rsb}) \sim 100 \text{ kHz}$

- So we expect $\langle n \rangle \sim (100/812)^2 \sim 0.01$
Sideband cooling – Results

3 MHz light shift due to pump beam

\[ \Delta = 130 \text{ MHz} \]

\[ \Omega = 80 \text{ kHz} \]
Sideband cooling – Results

\[ \Delta = 130 \text{ MHz} \]
\[ \Omega = 80 \text{ kHz} \]

812 kHz trap freq.
Sideband cooling – Results

\[ \Delta = 130 \text{ MHz} \]
\[ \Omega = 80 \text{ kHz} \]

170 kHz lightshift of sidebands by carrier
\[ \Delta = | n + 1 - n | = 130 \text{ MHz} \]
\[ \Omega = 80 \text{ kHz} \]

Mean \( n \) vs. probe detuning (MHz)
Upper bound on $\langle n \rangle$ and heating rate

Data implies $\langle n \rangle < 0.5$

$\frac{dn}{dt} < 10$ per ms
→ Suggests the 1st exp. was cold

\[ \Delta = 130 \text{ MHz} \]
\[ \Omega = 80 \text{ kHz} \]
Data gives upper bound

\[ <n> < 0.5 \]

\Rightarrow \quad \text{Ground state population } P_0 > 0.7

And indirect evidence for

\[ <n> \sim 0.1 \]

\Rightarrow \quad \text{Ground state population } P_0 \sim 0.9
Conclusions

- A really thorough grasp of the pushing methods is needed to aim for fidelity 0.9999, and is also a good starting point for understanding faster methods.

- Electrode designs: we would welcome discussion of this.

- Experimentally, we have preparation, readout, single-bit rotation and cooling
  → next stage is to diagnose the temperature better, and then entanglement.