Oblique

No. 1

Patterson symmetry $p_2$

**Origin** arbitrary

**Asymmetric unit** $0 \leq x \leq 1; \ 0 \leq y \leq 1$

**Symmetry operations**

(1) 1

**Generators selected** (1); $t(1,0)$; $t(0,1)$

**Positions**

<table>
<thead>
<tr>
<th>Multiplicity</th>
<th>Coordinates</th>
<th>Reflection conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$a \ 1$</td>
<td>(1) $x, y$</td>
</tr>
</tbody>
</table>

**Maximal non-isomorphic subgroups**

I  none
IIa none
IIb none

**Maximal isomorphic subgroups of lowest index**

IIc $[2] \ p \ 1 \ (\alpha' = 2a \text{ or } b' = 2b \text{ or } \alpha' = a + b, b' = -a + b) \ (1)$

**Minimal non-isomorphic supergroups**

I  $[2] \ p \ 2 \ (2); [2] \ pm \ (3); [2] \ pg \ (4); [2] \ cm \ (5); [3] \ p \ 3 \ (13)$
II none

---


Copyright © 2006 International Union of Crystallography
Oblique

Patterson symmetry $p2$

\begin{align*}
\text{Origin} & \text{ at } 2 \\
\text{Asymmetric unit} & 0 \leq x \leq \frac{1}{2}; \ 0 \leq y \leq 1 \\
\text{Symmetry operations} & (1) \ 1 \quad (2) \ 2 \ 0,0 \\
\text{Generators selected} & (1); \ t(1,0); \ t(0,1); \ (2) \\
\text{Positions} & \\
\text{Multiplicity, Wyckoff letter, Site symmetry} & \text{Coordinates} \quad \text{Reflection conditions} \\
2 \quad e \quad 1 \quad (1) \ x, y \quad (2) \bar{x}, \bar{y} \\
1 \quad d \quad 2 \quad \frac{1}{2}, \frac{1}{2} \\
1 \quad c \quad 2 \quad \frac{1}{2}, 0 \\
1 \quad b \quad 2 \quad 0, \frac{1}{2} \\
1 \quad a \quad 2 \quad 0, 0 \\
\text{Maximal non-isomorphic subgroups} & \\
\text{I} \quad [2] \ p1 \ (1) \ 1 \\
\text{IIa} \quad \text{none} \\
\text{IIb} \quad \text{none} \\
\text{Maximal isomorphic subgroups of lowest index} & \\
\text{IIc} \quad [2] \ p2 \ (a' = 2a \text{ or } b' = 2b \text{ or } a' = a + b, b' = -a + b) \ (2) \\
\text{Minimal non-isomorphic supergroups} & \\
\text{I} \quad [2] \ p2mm \ (6); \ [2] \ p2mg \ (7); \ [2] \ p2gg \ (8); \ [2] \ c2mm \ (9); \ [2] \ p4 \ (10); \ [3] \ p6 \ (16) \\
\text{II} \quad \text{none}
\end{align*}
### Plane group $pm$  

**Rectangular**  

**No. 3**  

$ p 1 m 1 $  

Patterson symmetry $p2mm$

---

**Origin** on $m$

**Asymmetric unit**  

$0 \leq x \leq \frac{1}{2}; \ 0 \leq y \leq 1$

**Symmetry operations**

(1) 1  

(2) $m \ 0, y$

**Generators selected**  

(1); $t(1, 0); t(0, 1); (2)$

**Positions**

<table>
<thead>
<tr>
<th>Multiplicity</th>
<th>Coordinates</th>
<th>Reflection conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>General:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$c \ 1$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$(1) \ x, y$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$(2) \ \bar{x}, y$</td>
<td></td>
</tr>
</tbody>
</table>

| 1            | $b \ .m.$ |                       |
|              | $\frac{1}{2} \ y$|                       |

| 1            | $a \ .m.$ |                       |
|              | $0 \ y$|                       |

**Maximal non-isomorphic subgroups**

| I  | $[2] \ p 1 \ (1) \ 1$ |
| Ia | none |
| Ib | $[2] \ p g (b' = 2b) (4); [2] \ c m (a' = 2a, b' = 2b) (5)$ |

**Maximal isomorphic subgroups of lowest index**

| IIc | $[2] \ p m (a' = 2a) (3); [2] \ p m (b' = 2b) (3)$ |

**Minimal non-isomorphic supergroups**

| I   | $[2] \ p 2 m m (6); [2] \ p 2 m g (7)$ |
| II  | $[2] \ c m (5)$ |
Rectangular

Patterson symmetry $p\overline{2}mm$

$\begin{array}{c}
\begin{array}{c}
\text{Origin on } g \\
\text{Asymmetric unit } \quad 0 \leq x \leq \frac{1}{2}; \quad 0 \leq y \leq 1
\end{array}
\end{array}$

Symmetry operations

(1) $1$
(2) $b$; $0, y$

Generators selected

(1); $t(1,0)$; $t(0,1)$; (2)

Positions

\begin{tabular}{lll}
Multiplicity, & Coordinates & Reflection conditions \\
Wyckoff letter, & & \\
Site symmetry & & \\
\hline
2 & $a$ & 1 \\
\hline
(1) & $x, y$ & (1) \\
(2) & $\bar{x}, y + \frac{1}{2}$ & (2) $ar{k}$: $k = 2n$
\end{tabular}

Maximal non-isomorphic subgroups

I

IIa none

IIb none

Maximal isomorphic subgroups of lowest index

IIc

Minimal non-isomorphic supergroups

I

II


Copyright © 2006 International Union of Crystallography

\begin{equation*}
cM \quad \begin{array}{c}
\text{Rectangular} \\
\text{No. 5} \\
c1m1
\end{array}
\end{equation*}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
\textbf{Origin} & on \(m\) & & \\
\hline
\textbf{Asymmetric unit} & \(0 \leq x \leq \frac{1}{2}; \quad 0 \leq y \leq \frac{1}{2}\) & & \\
\hline
\textbf{Symmetry operations} & & & \\
For \((0,0) + \) set & (1) & \(m\) & 0, \(y\) \\
& (2) & & \(0, y\) \\
For \((\frac{1}{2}, \frac{1}{2}) + \) set & (1) & \(t(\frac{1}{2}, \frac{1}{2})\) & \\
& (2) & \(b\) & \(\frac{1}{2}, y\) \\
\hline
\textbf{Generators selected} & (1) & \(t(1,0); \quad t(0,1); \quad t(\frac{1}{2}, \frac{1}{2}); \; (2)\) & \\
\hline
\textbf{Positions} & Multiplicity, Wyckoff letter, Site symmetry & Coordinates & \multicolumn{2}{c}{\text{Reflection conditions}} \\
& & & & General: \\
\hline
& & \((0,0) + \) & \( (\frac{1}{2}, \frac{1}{2}) + \) & \(hk: \; h+k = 2n\) \\
\hline
& & & & \(b0: \; h = 2n\) \\
\hline
& & & & \(0k: \; k = 2n\) \\
\hline
& & & & Special: no extra conditions
\end{tabular}
\end{table}

\begin{align*}
\text{Maximal non-isomorphic subgroups} \\
\text{I} & \quad [2] \; c1 \; (p1, 1) \; 1+ \\
\text{IIa} & \quad [2] \; pg \; (4) \; 1; \; 2 + (\frac{1}{2}, \frac{1}{2}) \\
\text{IIb} & \quad [2] \; pm \; (3) \; 1; \; 2 \\
\end{align*}

\begin{align*}
\text{Maximal isomorphic subgroups of lowest index} \\
\text{IIC} & \quad [3] \; cm \; (a' = 3a) \; (5); \; [3] \; cm \; (b' = 3b) \; (5) \\
\end{align*}

\begin{align*}
\text{Minimal non-isomorphic supergroups} \\
\text{I} & \quad [2] \; c2mm \; (9); \; [3] \; p3m1 \; (14); \; [3] \; p31m \; (15) \\
\text{II} & \quad [2] \; pm \; (a' = \frac{1}{2}a, b' = \frac{1}{2}b) \; (3)
\end{align*}

Copyright © 2006 International Union of Crystallography
Rectangular

Patterson symmetry \( p2mm \)

\[ \begin{align*}
\text{Origin at } & 2mm \\
\text{Asymmetric unit } & 0 \leq x \leq, \ 0 \leq y \leq
\end{align*} \]

Symmetry operations

(1) 1 (2) 2 0,0 (3) \( m, 0, y \) (4) \( m, x,0 \)

Generators selected

(1): \( t(1,0); t(0,1); (2); (3) \)

Positions

<table>
<thead>
<tr>
<th>Multiplicity, Wyckoff letter, Site symmetry</th>
<th>Coordinates</th>
<th>Reflection conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>4  i 1</td>
<td>(1) ( x,y ) (2) ( \bar{x},\bar{y} ) (3) ( \bar{x},y ) (4) ( x,\bar{y} )</td>
<td>General: no conditions Special: no extra conditions</td>
</tr>
<tr>
<td>2  h . ( m ).</td>
<td>( \frac{1}{2},y ) ( \frac{1}{2},\bar{y} )</td>
<td></td>
</tr>
<tr>
<td>2  g . ( m ).</td>
<td>0,0 ( 0,\bar{y} )</td>
<td></td>
</tr>
<tr>
<td>2  f . ( m ).</td>
<td>( x,\frac{1}{2} ) ( \bar{x},\frac{1}{2} )</td>
<td></td>
</tr>
<tr>
<td>2  e . ( m ).</td>
<td>( x,0 ) ( \bar{x},0 )</td>
<td></td>
</tr>
<tr>
<td>1  d ( 2mm )</td>
<td>( \frac{1}{2}, \frac{1}{2} )</td>
<td></td>
</tr>
<tr>
<td>1  c ( 2mm )</td>
<td>( \frac{1}{2},0 )</td>
<td></td>
</tr>
<tr>
<td>1  b ( 2mm )</td>
<td>0,0</td>
<td></td>
</tr>
<tr>
<td>1  a ( 2mm )</td>
<td>0,0</td>
<td></td>
</tr>
</tbody>
</table>

Maximal non-isomorphic subgroups

\( \text{I} \)

\[ [2] p1m1 (pmm, 3) 1:3 \]
\[ [2] p11m (pmm, 3) 1:4 \]
\[ [2] p111 (p2, 2) 1:2 \]

\( \text{IIa} \) none

\( \text{IIb} \)

\[ [2] p2mg (a' = 2a) (7) \]
\[ [2] p2gm (b' = 2b) (p2mg, 7) \]
\[ [2] c2mm (a' = 2a, b' = 2b) (9) \]

Maximal isomorphic subgroups of lowest index

\( \text{IIc} \)

\[ [2] p2mm (a' = 2a \text{ or } b' = 2b) (6) \]

Minimal non-isomorphic supergroups

\( \text{I} \)

\[ [2] p4mm (11) \]

\( \text{II} \)

\[ [2] c2mm (9) \]
\( p2mg \)

Rectangular

\( 2mm \)

No. 7

\( p2mg \)

Patterson symmetry \( p2mm \)

**Origin** at 2 1 g

**Asymmetric unit** \( 0 \leq x \leq \frac{1}{2}; \quad 0 \leq y \leq 1 \)

**Symmetry operations**

(1) 1  \quad (2) 2 0,0  \quad (3) \( m \frac{1}{4}, y \)  \quad (4) \( a x, 0 \)

**Generators selected** \((1); \quad (2 )); \quad (3)\)

**Positions**

<table>
<thead>
<tr>
<th>Multiplicity, Wyckoff letter, Site symmetry</th>
<th>Coordinates</th>
<th>Reflection conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>4  ( d ) 1  ((1) x, y )  ((2) \bar{x}, \bar{y} )  ((3) \bar{x} + \frac{1}{2}, y )  ((4) x + \frac{1}{2}, \bar{y} )</td>
<td>General:</td>
<td>( h0: \quad h = 2n )</td>
</tr>
<tr>
<td>2  ( c ) . ( m ) .  ( \frac{1}{2}, y )  ( \frac{1}{2}, \bar{y} )</td>
<td>Special: as above, plus no extra conditions</td>
<td>( hk: \quad h = 2n )</td>
</tr>
<tr>
<td>2  ( b ) 2 . . 0, ( \frac{1}{2} )  ( \frac{1}{2}, \frac{1}{2} )</td>
<td>( hk: \quad h = 2n )</td>
<td></td>
</tr>
<tr>
<td>2  ( a ) 2 . . 0,0  ( \frac{1}{2}, 0 )</td>
<td>( hk: \quad h = 2n )</td>
<td></td>
</tr>
</tbody>
</table>

**Maximal non-isomorphic subgroups**

**I**

\( [2] p1 1 g (p g, 4) 1: 4 \)

\( [2] p1 m 1 (p m, 3) 1: 3 \)

\( [2] p2 1 1 (p 2, 2) 1: 2 \)

**IIa** none

**IIb**

\( [2] p2 g g (b' = 2b) (8) \)

**Maximal isomorphic subgroups of lowest index**

**Ic**

\( [2] p2 mg (b' = 2b) (7); \quad [3] p2 mg (a' = 3a) (7) \)

**Minimal non-isomorphic supergroups**

**I** none

**II**

\( [2] c2 mm (9); \quad [2] p2 mm (a' = \frac{1}{2}a) (6) \)
Rectangular

Patterson symmetry $p\overline{2}mm$

Origin at $2$

Asymmetric unit $0 \leq x \leq \frac{1}{2}; \quad 0 \leq y \leq \frac{1}{2}$

Symmetry operations

(1) 1 \hspace{10pt} (2) 2 \hspace{10pt} 0,0 \hspace{10pt} (3) $b \ \downarrow, y \hspace{10pt} \frac{1}{2}\hspace{10pt} (4) a \hspace{10pt} \downarrow, y$

Generators selected \hspace{10pt} (1); \hspace{10pt} t(1,0); \hspace{10pt} t(0,1); \hspace{10pt} (2); \hspace{10pt} (3)

Positions

<table>
<thead>
<tr>
<th>Multiplicity, Coordinates</th>
<th>Reflection conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wyckoff letter, Site symmetry</td>
<td>General:</td>
</tr>
<tr>
<td>4 $c$ 1 $(1) \hspace{5pt} x, y \hspace{5pt} (2) \hspace{5pt} \bar{x}, \bar{y} \hspace{5pt} (3) \hspace{5pt} x + \frac{1}{2}, y + \frac{1}{2} \hspace{5pt} (4) \hspace{5pt} x + \frac{1}{2}, \bar{y} + \frac{1}{2}$</td>
<td>$h0: \hspace{5pt} h = 2n$</td>
</tr>
<tr>
<td>2 $b$ 2 . . $\frac{1}{2}, 0 \hspace{5pt} 0, \frac{1}{2}$</td>
<td>$0k: \hspace{5pt} k = 2n$</td>
</tr>
<tr>
<td>2 $a$ 2 . . $0, 0 \hspace{5pt} \frac{1}{2}, \frac{1}{2}$</td>
<td>Special: as above, plus</td>
</tr>
</tbody>
</table>

$hk: \hspace{5pt} h + k = 2n$ |

Maximal non-isomorphic subgroups

I \hspace{10pt} [2] $p\overline{1}g1 (pg, 4) \hspace{10pt} 1; \hspace{5pt} 3$

[2] $p\overline{1}g1 (pg, 4) \hspace{10pt} 1; \hspace{5pt} 4$

[2] $p\overline{2}1g (p2, 2) \hspace{10pt} 1; \hspace{5pt} 2$

IIa none

IIb none

Maximal isomorphic subgroups of lowest index

IIc \hspace{10pt} [3] $p2gg (a' = 3a \hspace{5pt} \text{or} \hspace{5pt} b' = 3b) \hspace{5pt} (8)$

Minimal non-isomorphic supergroups

I \hspace{10pt} [2] $p4gm (12)$

II \hspace{10pt} [2] $c\overline{2}mm (9); \hspace{5pt} [2] p2mg (a' = \frac{1}{2}a) \hspace{5pt} (7); \hspace{5pt} [2] p2gm (b' = \frac{1}{4}b) \hspace{10pt} (p2mg, 7)$
Origin at 2mm

Asymmetric unit \(0 \leq x \leq \frac{1}{4}; 0 \leq y \leq \frac{1}{2}\)

Symmetry operations

For \((0,0)\) + set

(1) 1
(2) 2, 0, 0
(3) \(m\), 0, y
(4) \(m\), x, 0

For \((\frac{1}{2}, \frac{1}{2})\) + set

(1) \(t(\frac{1}{2}, \frac{1}{2})\)
(2) 2, \(\frac{1}{2}, \frac{1}{2}\)
(3) \(b\), \(\frac{1}{2}, y\)
(4) \(a\), \(x, \frac{1}{2}\)

Generators selected

(1); \(t(1,0)\); \(t(0,1)\); \(t(\frac{1}{2}, \frac{1}{2})\); (2); (3)

Positions

<table>
<thead>
<tr>
<th>Multiplicity</th>
<th>Wyckoff letter, Site symmetry</th>
<th>Coordinates</th>
<th>Reflection conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 (f) 1</td>
<td>((0,0) + (\frac{1}{2}, \frac{1}{2}) +)</td>
<td>((1), x, y)</td>
<td>General: (hk): (h+k = 2n) (h0): (h = 2n) (0k): (k = 2n)</td>
</tr>
<tr>
<td>4 (e) . (m)</td>
<td>0, y</td>
<td>0, (y)</td>
<td>Special: as above, plus (hk): (h = 2n)</td>
</tr>
<tr>
<td>4 (d) . (m)</td>
<td>(x, 0)</td>
<td>(x, 0)</td>
<td>no extra conditions</td>
</tr>
<tr>
<td>4 (c) 2 .</td>
<td>(\frac{1}{2}, \frac{1}{2})</td>
<td>(\frac{1}{2}, \frac{1}{2})</td>
<td>(hk): (h = 2n)</td>
</tr>
<tr>
<td>2 (b) 2 (mm)</td>
<td>0, (\frac{1}{2})</td>
<td></td>
<td>no extra conditions</td>
</tr>
<tr>
<td>2 (a) 2 (mm)</td>
<td>0, 0</td>
<td></td>
<td>no extra conditions</td>
</tr>
</tbody>
</table>

Maximal non-isomorphic subgroups

I    \[[2]\] \(c 1 m 1 (c m, 5)\) \((1); (3) +\)
\[[2]\] \(c 1 1 m (c m, 5)\) \((1); (4) +\)
\[[2]\] \(c 2 1 1 (p 2, 2)\) \((1); (2) +\)

IIa  \[[2]\] \(p 2 g g (8)\) \((1), 2; (3); 4 + (\frac{1}{2}, \frac{1}{2})\)
\[[2]\] \(p 2 g m (p 2 m g, 7)\) \((1); (2); 3 + (\frac{1}{2}, \frac{1}{2})\)
\[[2]\] \(p 2 m g (7)\) \((1), 3; (2); 4 + (\frac{1}{2}, \frac{1}{2})\)
\[[2]\] \(p 2 m m (6)\) \((1), 2; 3; 4\)

IIb  none

Maximal isomorphic subgroups of lowest index

IIc \([3]\) \(c 2 m m (a' = 3a \text{ or } b' = 3b) (9)\)

Minimal non-isomorphic supergroups

I    \[[2]\] \(p 4 m m (11)\); \[[2]\] \(p 4 g m (12)\); \[[3]\] \(p 6 m m (17)\)

II   \[[2]\] \(p 2 m m (a' = \frac{1}{2}a, b' = \frac{1}{2}b) (6)\)

Square 4  \[ p4 \]  p4  

Patterson symmetry \( p4 \)

**Origin** at 4

**Asymmetric unit**  
\[ 0 \leq x \leq \frac{1}{2}; \quad 0 \leq y \leq \frac{1}{2} \]

**Symmetry operations**

(1) 1  
(2) 2 0,0  
(3) 4 + 0,0  
(4) 4 − 0,0

**Generators selected**  
(1); \( t(1,0) \); \( t(0,1) \); (2); (3)

**Positions**

<table>
<thead>
<tr>
<th>Multiplicity</th>
<th>Wyckoff letter</th>
<th>Site symmetry</th>
<th>Coordinates</th>
<th>Reflection conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 d 1</td>
<td>( x,y )</td>
<td>(1)</td>
<td>(1) ( x,y )</td>
<td>General: no conditions</td>
</tr>
<tr>
<td>2 c 2</td>
<td>( \frac{1}{2},0 )</td>
<td>Special:</td>
<td>(2) ( \bar{x},\bar{y} )</td>
<td>( \bar{h}k: h+k=2n )</td>
</tr>
<tr>
<td>1 b 4</td>
<td>( \frac{1}{2},\frac{1}{2} )</td>
<td></td>
<td>(3) ( \frac{1}{2},x )</td>
<td>no extra conditions</td>
</tr>
<tr>
<td>1 a 4</td>
<td>0,0</td>
<td></td>
<td>(4) ( y,\bar{x} )</td>
<td>no extra conditions</td>
</tr>
</tbody>
</table>

**Maximal non-isomorphic subgroups**

I  
[2] \( p2 \) (2)  
1; 2

IIa none

IIb none

**Maximal isomorphic subgroups of lowest index**

Ic  
[2] \( c4 \) (\( a'=2a, b'=2b \)) \( p4,10 \)

**Minimal non-isomorphic supergroups**

I  
[2] \( p4mm \) (11); [2] \( p4gm \) (12)

II none
Origin at 4mm

Asymmetric unit
\[ 0 \leq x \leq \frac{1}{4}; \quad 0 \leq y \leq \frac{1}{4}; \quad x \leq y \]

Symmetry operations

(1) \( 1 \)
(2) \( m \), \( 0, y \)
(3) \( 4^+ \), \( 0, \bar{y} \)
(4) \( 4^- \), \( 0, y \)
(5) \( m \), \( x, 0 \)
(6) \( m \), \( x, x \)
(7) \( m \), \( x, \bar{x} \)
(8) \( m \), \( x, \bar{x} \)

Generators selected

(1); \( t(1,0) \); \( t(0,1) \); \( (2); \( (3); \( (5) \)

Positions

Multiplicity, Wyckoff letter, Site symmetry

\[
\begin{array}{cccc}
8 & g & 1 & (1) x, y \\
& & & (2) \bar{x}, \bar{y} \\
& & & (3) \bar{y}, x \\
& & & (4) y, \bar{x} \\
& & & (5) \bar{x}, y \\
& & & (6) x, \bar{y} \\
& & & (7) y, x \\
& & & (8) \bar{y}, \bar{x} \\
\end{array}
\]

\[
\begin{array}{cccc}
4 & f & . & m \\
& & & x, x \\
& & & \bar{x}, \bar{x} \\
& & & \bar{x}, x \\
& & & x, \bar{x} \\
\end{array}
\]

\[
\begin{array}{cccc}
4 & e & . & m \\
& & & x, \frac{1}{2} \\
& & & \bar{x}, \frac{1}{2} \\
& & & \frac{1}{2}, x \\
& & & \frac{1}{2}, \bar{x} \\
\end{array}
\]

\[
\begin{array}{cccc}
4 & d & . & m \\
& & & x, 0 \\
& & & \bar{x}, 0 \\
& & & 0, x \\
& & & 0, \bar{x} \\
\end{array}
\]

\[
\begin{array}{cccc}
2 & c & 2 & mm \\
& & & \frac{1}{2}, 0 \\
& & & 0, \frac{1}{2} \\
\end{array}
\]

\[
\begin{array}{cccc}
1 & b & 4 & mm \\
& & & \frac{1}{2}, \frac{1}{2} \\
\end{array}
\]

\[
\begin{array}{cccc}
1 & a & 4 & mm \\
& & & 0, 0 \\
\end{array}
\]

Maximal non-isomorphic subgroups

I

[2] \( p411(p4,10) \)

[2] \( p21m(c2mm,9) \)

[2] \( p2m1(p2mm,6) \)

IIa

none

IIb

[2] \( c4mg(a' = 2a, b' = 2b)(p4gm,12) \)

Maximal isomorphic subgroups of lowest index

IIc

[2] \( c4mm(a' = 2a, b = 2b)(p4mm,11) \)

Minimal non-isomorphic supergroups

I

none

II

none
Square  

Patterson symmetry $p4mm$

### Origin
at $41g$

### Asymmetric unit
$0 \leq x \leq \frac{1}{2}; \quad 0 \leq y \leq \frac{1}{2}; \quad y \leq \frac{1}{2} - x$

### Symmetry operations

<table>
<thead>
<tr>
<th>No.</th>
<th>Operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>$1$</td>
</tr>
<tr>
<td>(2)</td>
<td>$2$ $0,0$</td>
</tr>
<tr>
<td>(3)</td>
<td>$4^*$ $0,0$</td>
</tr>
<tr>
<td>(4)</td>
<td>$4^*$ $0,0$</td>
</tr>
<tr>
<td>(5)</td>
<td>$b \frac{1}{4},y$</td>
</tr>
<tr>
<td>(6)</td>
<td>$a x,\frac{1}{4}$</td>
</tr>
<tr>
<td>(7)</td>
<td>$g\left(\frac{1}{4},\frac{1}{4}\right) x,x$</td>
</tr>
<tr>
<td>(8)</td>
<td>$m x,\frac{1}{4},\bar{x}$</td>
</tr>
</tbody>
</table>

### Generators selected
(1); $t(1,0)$; $t(0,1)$; (2); (3); (5)

### Positions

<table>
<thead>
<tr>
<th>Multiplicity</th>
<th>Wyckoff letter</th>
<th>Site symmetry</th>
<th>Coordinates</th>
<th>Reflection conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$8$</td>
<td>$d$</td>
<td>$1$</td>
<td>$(1) x,y$</td>
<td>$h0: h = 2n$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$(5) \bar{x},\bar{y} + \frac{1}{4}$</td>
<td>$0k: k = 2n$</td>
</tr>
<tr>
<td>$4$</td>
<td>$c$</td>
<td>$..m$</td>
<td>$x, x + \frac{1}{4}$</td>
<td>$h0: h = 2n$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\bar{x}, \bar{x} + \frac{1}{4}$</td>
<td>no extra conditions</td>
</tr>
<tr>
<td>$2$</td>
<td>$b$</td>
<td>$2..mm$</td>
<td>$\frac{1}{2},0$</td>
<td>$hk: h + k = 2n$</td>
</tr>
<tr>
<td>$2$</td>
<td>$a$</td>
<td>$4..mm$</td>
<td>$0,0$</td>
<td>$hk: h + k = 2n$</td>
</tr>
</tbody>
</table>

### Maximal non-isomorphic subgroups

- **I**
  - $[2] p411 (p4, 10)$
  - $[2] p21m (c2mm, 9)$
  - $[2] p2g1 (p2gg, 8)$

- **IIa**
  - none

- **IIb**
  - none

### Maximal isomorphic subgroups of lowest index

- **IIc**
  - $[9] p4gm (a' = 3a, b' = 3b)$ (12)

### Minimal non-isomorphic supergroups

- **I**
  - none

- **II**
  - $[2] c4gm (p4mm, 11)$

---

**Hexagonal No. 13**

Patterson symmetry $p6$

**Origin** at 3

Asymmetric unit  
$0 \leq x \leq \frac{1}{3}; \quad 0 \leq y \leq \frac{1}{3}; \quad x \leq (1+y)/2; \quad y \leq \min(1-x, (1+x)/2)$

Vertices  
$0, 0 \quad \frac{1}{3}, 0 \quad \frac{1}{3}, \frac{1}{3} \quad 0, \frac{1}{3}$

**Symmetry operations**

(1) 1  
(2) $3\cdot 0, 0$  
(3) $3\cdot 0, 0$

Generators selected  
(1); $t(1,0)$; $t(0,1)$; (2)

**Positions**

<table>
<thead>
<tr>
<th>Multiplicity</th>
<th>Coordinates</th>
<th>Reflection conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>$d$</td>
<td>General: no conditions</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>Special: no extra conditions</td>
</tr>
</tbody>
</table>

Maximal non-isomorphic subgroups

| I  | [3] $p1$ (1) | 1 |
|    |              |   |
| IIa | none |
| IIb | none |

Maximal isomorphic subgroups of lowest index

| Ic | [3] $h3$ ($a' = 3a, b' = 3b$) ($p3, 13$) |

Minimal non-isomorphic supergroups

|    |                                               |
| II | none |

---

Hexagonal

Patterson symmetry \( p6mm \)

\[ \vcenter{\hbox{\begin{tikzpicture}[scale=0.5]
    \foreach \x in {0,1,2,3,4,5}
    \foreach \y in {0,1,2}
    \fill [black, opacity=0.5] (\x,\y) circle (0.5mm);
    \end{tikzpicture}}} \]

\[ \vcenter{\hbox{\begin{tikzpicture}[scale=0.5]
    \foreach \x in {0,1,2,3,4,5}
    \foreach \y in {0,1,2}
    \fill [black, opacity=0.5] (\x,\y) circle (0.5mm);
    \end{tikzpicture}}} \]

**Origin** at \( 3m1 \)

**Asymmetric unit**

\[ 0 \leq x \leq \frac{\pi}{3}; \quad 0 \leq y \leq \frac{2\pi}{3}; \quad x \leq 2y; \quad y \leq \min(1-x,2x) \]

**Vertices**

\[ 0,0; \quad \frac{\pi}{3}, \frac{\pi}{3}; \quad \frac{\pi}{6}, \frac{\pi}{6} \]

**Symmetry operations**

\[
\begin{align*}
(1) & \quad 1 \\
(2) & \quad 3 \cdot 0,0 \\
(3) & \quad 3 \cdot 0,0 \\
(4) & \quad m \ x, \bar{x} \\
(5) & \quad m \ x, 2x \\
(6) & \quad m \ 2x, x
\end{align*}
\]

**Generators selected**

(1); \( t(1,0); t(0,1); (2); (4) \)

**Positions**

\[
\begin{array}{|c|c|c|}
\hline
\text{Multiplicity} & \text{Wyckoff letter} & \text{Site symmetry} \\
\hline
6 & e & 1 \\
3 & d & m \\
1 & c & 3 m \\
1 & b & 3 m \\
1 & a & 3 m \\
\hline
\end{array}
\]

**Coordinates**

\[
\begin{align*}
(1) & \quad x, y \\
(2) & \quad \bar{y}, x - y \\
(3) & \quad \bar{x} + y, \bar{x} \\
(4) & \quad \bar{y}, \bar{x} \\
(5) & \quad \bar{x} + y, y \\
(6) & \quad x, x - y
\end{align*}
\]

**Reflection conditions**

General:

no conditions

Special: no extra conditions

**Maximal non-isomorphic subgroups**

I

\[
\begin{align*}
[2] & \quad p311 \ (p3, 13) & \quad 1; \ 2; \ 3 \\
[3] & \quad p1m1 \ (cmm, 5) & \quad 1; \ 4 \\
[3] & \quad p1m1 \ (cmm, 5) & \quad 1; \ 5 \\
[3] & \quad p1m1 \ (cmm, 5) & \quad 1; \ 6
\end{align*}
\]

II

none

IIb

\[ [3] \quad h3m1 \ (a' = 3a, b' = 3b) \ (p3m1, 15) \]

**Maximal isomorphic subgroups of lowest index**

IIc

\[ [4] \quad p3m1 \ (a' = 2a, b' = 2b) \ (14) \]

**Minimal non-isomorphic supergroups**

I

\[ [2] \quad p6mm \ (17) \]

II

\[ [3] \quad h3m1 \ (p3m1, 15) \]

---


Copyright © 2006 International Union of Crystallography
**p31m**

No. 15  

**p31m**

Hexagonal

Patterson symmetry \(p6mm\)

**Origin** at \(31m\)

**Asymmetric unit**  

\[
0 \leq x \leq \frac{1}{3}; \quad 0 \leq y \leq \frac{1}{3}; \quad x \leq (1+y)/2; \quad y \leq \min(1-x, x)
\]

**Vertices**  

\[
0, 0; \quad \frac{1}{3}, 0; \quad \frac{1}{3}, \frac{1}{3}; \quad \frac{1}{3}, \frac{1}{3}
\]

**Symmetry operations**

\[
(1) \quad 1 \quad (2) \quad 3 \cdot 0, 0 \quad (3) \quad 3 \cdot 0, 0
\]

\[
(4) \quad m \quad x, x \quad (5) \quad m \quad x, 0 \quad (6) \quad m \quad 0, y
\]

**Generators selected**

(1); \(t(1,0); t(0,1); (2); (4)

**Positions**

<table>
<thead>
<tr>
<th>Multiplicity</th>
<th>Wyckoff letter</th>
<th>Site symmetry</th>
<th>Coordinates</th>
<th>Reflection conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 (d) 1</td>
<td>((1)) (x, y)</td>
<td>((2)) (\bar{y}, x-y)</td>
<td>((3)) (\bar{x}+y, \bar{x})</td>
<td>General: no conditions</td>
</tr>
<tr>
<td></td>
<td>((4)) (y, x)</td>
<td>((5)) (x-y, \bar{y})</td>
<td>((6)) (\bar{x}, \bar{x}+y)</td>
<td>Special: no extra conditions</td>
</tr>
</tbody>
</table>

| 3 \(c\) . . \(m\) | \(x, 0\) | \(0, x\) | \(\bar{x}, \bar{x}\) |
| 2 \(b\) 3 . . | \(\frac{1}{3}, \frac{1}{3}\) | \(\frac{1}{3}, \frac{1}{3}\) |
| 1 \(a\) 3 . \(m\) | 0, 0 |

**Maximal non-isomorphic subgroups**

| I  \[2\] | \(p311\) \((p3, 13)\) | 1; 2: 3 |
| I \[3\] | \(p11m\) \((cm, 5)\) | 1; 4 |
| I \[3\] | \(p11m\) \((cm, 5)\) | 1; 5 |
| I \[3\] | \(p11m\) \((cm, 5)\) | 1; 6 |

| IIa | none |

| IIb \[3\] | \(h31m\) \((a' = 3a, b' = 3b)\) \((p3m1, 14)\) |

**Maximal isomorphic subgroups of lowest index**

| IIc \[4\] | \(p31m\) \((a' = 2a, b' = 2b)\) \((15)\) |

**Minimal non-isomorphic supergroups**

| I  \[2\] | \(p6mm\) \((17)\) |
| II \[3\] | \(h31m\) \((p3m1, 14)\) |
Hexagonal  \( p6 \)  

Patterson symmetry \( p6 \)  

\[ \text{No. 16} \]

**Origin** at 6

**Asymmetric unit**

\[
0 \leq x \leq \frac{1}{3}; \quad 0 \leq y \leq \frac{1}{2}; \quad x \leq (1+y)/2; \quad y \leq \min(1-x,x)
\]

**Vertices**

\(0,0\) \(\frac{1}{3},0\) \(\frac{1}{2},\frac{1}{2}\) \(\frac{1}{2},1\) \(1,\frac{1}{2}\) \(1,0\)

**Symmetry operations**

\begin{align*}
(1) & \quad 1 \\
(2) & \quad 3^+ 0,0 \\
(3) & \quad 3^- 0,0 \\
(4) & \quad 2 0,0 \\
(5) & \quad 6^- 0,0 \\
(6) & \quad 6^+ 0,0
\end{align*}

**Generators selected**

(1); \(t(1,0); t(0,1); (2); (4)

**Positions**

\[
\begin{array}{cccc}
\text{Multiplicity} & \text{Wyckoff letter} & \text{Site symmetry} & \text{Coordinates} \\
6 & d & 1 & (1) x,y \\
 & & & (2) \bar{y},x-y \\
 & & & (3) \bar{x}+y,\bar{x} \\
 & & & (4) \bar{x},\bar{y} \\
 & & & (5) y,\bar{x}+y \\
 & & & (6) x-y, x \\
3 & c & 2 & \frac{1}{3},0 \\
 & & & 0,\frac{1}{3} \\
2 & b & 3 & \frac{1}{3},\frac{1}{3} \\
1 & a & 6 & 0,0
\end{array}
\]

**Reflection conditions**

General:

no conditions

Special: no extra conditions

**Maximal non-isomorphic subgroups**

I  \[ [2] p3 (13) 1; 2; 3 \]

IIa  \[ [3] p2 (2) 1; 4 \]

IIb  \[ \text{none} \]

**Maximal isomorphic subgroups of lowest index**

IIc  \[ [3] h6 (a' = 3a, b' = 3b) (p6, 16) \]

**Minimal non-isomorphic supergroups**

I  \[ [2] p6mm (17) \]

II  \[ \text{none} \]
p$6mm$  \hspace{1cm} 6mm  \hspace{1cm} \text{Hexagonal}

No. 17  \hspace{1cm} p$6mm$

\text{Patterson symmetry } p6mm

\begin{align*}
\text{Origin at } & 6mm \\
\text{Asymmetric unit} & : 0 \leq x \leq \frac{1}{2}; \quad 0 \leq y \leq \frac{1}{2}; \quad x \leq (1 + y)/2; \quad y \leq x/2 \\
\text{Vertices} & : (0,0) \quad \frac{1}{2},0 \quad \frac{1}{2},\frac{1}{2}
\end{align*}

\text{Symmetry operations}

\begin{align*}
(1) & \quad  1 \\
(2) & \quad  3^* 0,0 \\
(3) & \quad  3^* 0,0 \\
(4) & \quad  2 0,0 \\
(5) & \quad  6^* 0,0 \\
(6) & \quad  6^* 0,0 \\
(7) & \quad  m \ x,\bar{x} \\
(8) & \quad  m \ x,2x \\
(9) & \quad  m \ 2x,x \\
(10) & \quad  m \ x,x \\
(11) & \quad  m \ x,0 \\
(12) & \quad  m \ 0,y
\end{align*}
Generators selected  
(1); t(1, 0); t(0, 1); (2); (4); (7)

Positions

<table>
<thead>
<tr>
<th>Multiplicity</th>
<th>Coordinates</th>
<th>Reflection conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>General: no conditions</td>
</tr>
</tbody>
</table>

Maximal non-isomorphic subgroups

I  
[2] p6 11 (p6, 16) 1; 2; 3; 4; 5; 6
[2] p3 11 m (15) 1; 2; 3; 10; 11; 12
[2] p3 m 1 (14) 1; 2; 3; 7; 8; 9

IIa none
IIb none

Maximal isomorphic subgroups of lowest index

IIc  
[3] h6 mm (a' = 3a, b' = 3b) (p6 mm, 17)

Minimal non-isomorphic supergroups

I none
II none