A Precise Measurement of the W Boson Mass at CDF
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Spontaneous Symmetry Breaking

- **2008 Nobel Prize in Physics**
  "for the discovery of the mechanism of spontaneously broken symmetry in subatomic physics"

  Yoichiro Nambu

- **The mass of the W boson is linked to the mechanism of Electroweak Symmetry Breaking**
Spontaneous Symmetry Breaking

- Is the mechanism of Electroweak Symmetry Breaking, the Standard Model Higgs mechanism?
Motivation for Precision Measurements

- The electroweak gauge sector of the standard model, defined by \((g, g', \nu)\), is constrained by three precisely known parameters

  - \(\alpha_{\text{EM}}(M_Z) = 1 / 127.918(18)\)
  - \(G_F = 1.16637 (1) \times 10^{-5} \text{ GeV}^{-2}\)
  - \(M_Z = 91.1876 (21) \text{ GeV}\)

- At tree-level, these parameters are related to other electroweak observables, e.g. \(M_W\)

  - \(M_W^2 = \pi \alpha_{\text{EM}} / \sqrt{2} G_F \sin^2 \vartheta_W\)

  - where \(\vartheta_W\) is the Weinberg mixing angle

\[ \cos \vartheta_W = M_W / M_Z \]
Motivation for Precision Measurements

- Radiative corrections due to heavy quark and Higgs loops and exotica

Motivate the introduction of the $\rho$ parameter: $M_W^2 = \rho \left[ M_W(\text{tree}) \right]^2$

With the predictions $\Delta \rho = (\rho - 1) \sim M_{\text{top}}^2$ and $\Delta \rho \sim \ln M_H$

- In conjunction with $M_{\text{top}}$, the $W$ boson mass constrains the mass of the Higgs boson, and possibly new particles beyond the standard model
Detecting New Physics through Precision Measurements

- Willis Lamb (Nobel Prize 1955) measured the difference between energies of $^2S_{\frac{1}{2}}$ and $^2P_{\frac{1}{2}}$ states of hydrogen atom
  - 4 micro electron volts difference compared to few electron volts binding energy
  - States should be degenerate in energy according to tree-level calculation

- Harbinger of vacuum fluctuations to be calculated by Feynman diagrams containing quantum loops
  - Modern quantum field theory of electrodynamics followed (Nobel Prize 1965 for Schwinger, Feynman, Tomonaga)
Contributions from Supersymmetric Particles

- Radiative correction depends on chiral structure of SUSY sector and mass splitting ($\Delta m^2$) between squarks in SU(2) doublet

- After folding in limits on SUSY particles from direct searches, SUSY loops can contribute $\sim 100$ MeV to $M_W$
Uncertainty from $\alpha_{EM}(M_Z)$

- $\delta\alpha_{EM}$ dominated by uncertainty from non-perturbative contributions: hadronic loops in photon propagator at low $Q^2$
- equivalent $\delta M_W \approx 4$ MeV for the same Higgs mass constraint
  - Was equivalent $\delta M_W \approx 15$ MeV a decade ago
From the Tevatron, $\Delta M_{\text{top}} = 0.9$ GeV $\Rightarrow \Delta M_\text{H} / M_\text{H} = 8\%$

equivalent $\Delta M_W = 6$ MeV for the same Higgs mass constraint (and further improvements possible from Tevatron and LHC)

2011 world average $\Delta M_W = 23$ MeV
  - progress on $\Delta M_W$ has the biggest impact on Higgs constraint
Motivation II

- SM Higgs fit: $M_H = 94^{+29}_{-24} \text{ GeV}$ (LEPEWWG)
- Direct searches: $M_H \sim 125 \text{ GeV}$ (ATLAS, CMS)

In addition to the Higgs, is there another missing piece?

$A_{FB}^b \, vs \, A_{LR}: \sim 3\sigma$

Must continue improving precision of $M_W, M_{\text{top}}$...

other precision measurements constrain Higgs, equivalent to $\delta M_W \sim 15 \text{ MeV}$

Motivate direct measurement of $M_W$ at the 15 MeV level and better
Motivation III

- Generic parameterization of new physics contributing to W and Z boson self-energies through radiative corrections in propagators

Motivation III

- Generic parameterization of new physics contributing to W and Z boson self-energies: $S$, $T$, $U$ parameters (Peskin & Takeuchi)

Additionally, $M_W$ is the only measurement which constrains $U$

$M_H \sim 120$ GeV
$M_H > 600$ GeV

(from P. Langacker, 2012)

$M_W$ and Asymmetries are the most powerful observables in this parameterization.
Motivation III

- An example: extending the Higgs sector to two SU(2) doublets (required in SUSY) predicts additional neutral scalar and pseudo-scalar, and charged Higgs bosons


$T$ parameter responds strongly to 2HDM parameters
1998 Status of $M_W$ vs $M_{top}$

![Graph showing the status of $M_W$ vs $M_{top}$ in 1998 with experimental errors at 68% CL.](image)

- **LEP2/Tevatron (1998)**
- Regions for light SUSY, heavy SUSY, SM, MSSM, and both models are indicated.

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Heinemeyer, Hollik, Stockinger, Weber, Weiglein
2012 Status of $M_W$ vs $M_{\text{top}}$

![Graph showing $M_W$ vs $M_{\text{top}}$ with experimental errors and theoretical models.]

Heinemeyer, Hollik, Stockinger, Weiglein, Zeune '12
### Previous CDF Result (200 pb\(^{-1}\))

**Transverse Mass Fit Uncertainties (MeV)**


Total uncertainty of 48 MeV on W mass

<table>
<thead>
<tr>
<th>Source</th>
<th>electrons</th>
<th>muons</th>
<th>common</th>
</tr>
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<tr>
<td>W statistics</td>
<td>48</td>
<td>54</td>
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<tr>
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<td>Recoil energy scale</td>
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<td>Parton dist. Functions</td>
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<td>QED rad. Corrections</td>
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</tr>
<tr>
<td><strong>Total systematic</strong></td>
<td><strong>39</strong></td>
<td><strong>27</strong></td>
<td><strong>26</strong></td>
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<tr>
<td><strong>Total</strong></td>
<td><strong>62</strong></td>
<td><strong>60</strong></td>
<td></td>
</tr>
</tbody>
</table>

Systematic uncertainties shown in green: statistics-limited by control data samples
W Boson Production at the Tevatron

Quark-antiquark annihilation dominates (80%)

Lepton $p_T$ carries most of $W$ mass information, can be measured precisely (achieved 0.01%)

Initial state QCD radiation is $O(10 \text{ GeV})$, measure as soft 'hadronic recoil' in calorimeter (calibrated to ~0.5%)

dilutes $W$ mass information, fortunately $p_T(W) \ll M_W$
Quark-antiquark annihilation dominates (80%)

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Quadrant of Collider Detector at Fermilab (CDF)

EM calorimeter provides precise electron energy measurement

COT provides precise lepton track momentum measurement

Calorimeters measure hadronic recoil particles

Select W and Z bosons with central ( | \( \eta \) | < 1 ) leptons
Event Selection

- **Goal**: Select events with high $p_T$ leptons and small hadronic recoil activity
  - to maximize $W$ mass information content and minimize backgrounds

- **Inclusive lepton triggers**: loose lepton track and muon stub / calorimeter cluster requirements, with lepton $p_T > 18$ GeV
  - Kinematic efficiency of trigger $\sim 100\%$ for offline selection

- **Offline selection requirements**:
  - Electron cluster $E_T > 30$ GeV, track $p_T > 18$ GeV
  - Muon track $p_T > 30$ GeV
  - Loose identification requirements to minimize selection bias

- **$W$ boson event selection**: one selected lepton, $|\mathbf{u}| < 15$ GeV & $p_T(\nu) > 30$ GeV
  - $Z$ boson event selection: two selected leptons
W & Z Data Samples

<table>
<thead>
<tr>
<th>Sample</th>
<th>Candidates</th>
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<tbody>
<tr>
<td>$W \rightarrow e\nu$</td>
<td>470126</td>
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<tr>
<td>$W \rightarrow \mu\nu$</td>
<td>624708</td>
</tr>
<tr>
<td>$Z \rightarrow e^+ e^-$</td>
<td>16134</td>
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<tr>
<td>$Z \rightarrow \mu^+ \mu^-$</td>
<td>59738</td>
</tr>
</tbody>
</table>

- **Integrated Luminosity (collected between February 2002 – August 2007):**
  - Electron and muon channels: $L = 2.2 \text{ fb}^{-1}$
  - Identical running conditions for both channels, guarantees cross-calibration
- **Event selection gives fairly clean samples**
  - Mis-identification backgrounds $\sim 0.5\%$
Analysis Strategy
Strategy

Maximize the number of internal constraints and cross-checks

Driven by two goals:

1) Robustness: constrain the same parameters in as many different ways as possible

2) Precision: combine independent measurements after showing consistency
Outline of Analysis

Energy scale measurements drive the W mass measurement

- **Tracker Calibration**
  - alignment of the COT (~2400 cells) using cosmic rays
  - COT momentum scale and tracker non-linearity constrained using $J/\psi \rightarrow \mu\mu$ and $\Upsilon \rightarrow \mu\mu$ mass fits
  - Confirmed using $Z \rightarrow \mu\mu$ mass fit

- **EM Calorimeter Calibration**
  - COT momentum scale transferred to EM calorimeter using a fit to the peak of the $E/p$ spectrum, around $E/p \sim 1$
  - Calorimeter energy scale confirmed using $Z \rightarrow e e$ mass fit

- **Tracker and EM Calorimeter resolutions**

- **Hadronic recoil modelling**
  - Characterized using $p_T$-balance in $Z \rightarrow ll$ events
Drift Chamber (COT) Alignment

COT endplate geometry
Internal Alignment of COT

- Use a clean sample of ~400k cosmic rays for cell-by-cell internal alignment

- Fit COT hits on both sides simultaneously to a single helix (A. Kotwal, H. Gerberich and C. Hays, NIM A506, 110 (2003))
  - Time of incidence is a floated parameter in this 'dicosmic fit'
Residuals of COT cells after alignment

CDFII preliminary

Before alignment

Cell number ($\phi$)

Residual (microns)

Final relative alignment of cells $\sim2 \mu$m (initial alignment $\sim50 \mu$m)
Cross-check of COT alignment

- Cosmic ray alignment removes most deformation degrees of freedom, but “weakly constrained modes” remain
- Final cross-check and correction to beam-constrained track curvature based on difference of $<E/p>$ for positrons vs electrons
- Smooth ad-hoc curvature corrections as a function of polar and azimuthal angle: statistical errors $\Rightarrow \Delta M_W = 2$ MeV
Signal Simulation and Fitting
Signal Simulation and Template Fitting

- All signals simulated using a Custom Monte Carlo
  - Generate finely-spaced templates as a function of the fit variable
  - perform binned maximum-likelihood fits to the data
- Custom fast Monte Carlo makes smooth, high statistics templates
  - And provides analysis control over key components of the simulation

- We will extract the W mass from six kinematic distributions: Transverse mass, charged lepton $p_T$ and missing $E_T$ using both electron and muon channels
Generator-level Signal Simulation

- Generator-level input for W & Z simulation provided by RESBOS (C. Balazs & C.-P. Yuan, PRD56, 5558 (1997) and references therein), which
  - Calculates triple-differential production cross section, and $p_T$-dependent double-differential decay angular distribution
  - Calculates boson $p_T$ spectrum reliably over the relevant $p_T$ range: includes tunable parameters in the non-perturbative regime at low $p_T$

- Multiple radiative photons generated according to PHOTOS (P. Golonka and Z. Was, Eur. J. Phys. C 45, 97 (2006) and references therein)
Validation of QED Calculations

- Extensive comparisons between PHOTOS and HORACE (C.M. Carloni Calame, G. Montagna, O. Nicrosini and A. Vicini, JHEP 0710:109, 2007) programs
  - Comparing multi-photon final state radiation algorithms
  - Including multi-photon radiation from all charged lines (HORACE), and consistency with exact one-photon calculation

Validations confirm systematic uncertainty due to QED radiation of 4 MeV
Uncertainties in QED Calculations

- Extensive studies performed on uncertainties arising from
  - leading logarithm approximation
  - Multi-photon calculation
  - higher order soft and virtual corrections
  - Electron-positron pair creation (included at LO)
  - QED/QCD interference
  - dependence on electroweak parameters/scheme

- Total systematic uncertainty due to QED radiation of 4 MeV on W mass
Constraining Boson $p_T$ Spectrum

- Fit the non-perturbative parameter $g_2$ and QCD coupling $\alpha_s$ in RESBOS to $p_T(ll)$ spectra:

  $\Delta M_W = 5 \text{ MeV}$

Position of peak in boson $p_T$ spectrum depends on $g_2$

Tail to peak ratio depends on $\alpha_s$
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    - Confirmed using \( Z \rightarrow \mu \mu \) mass fit

- EM Calorimeter Calibration
  - COT momentum scale transferred to EM calorimeter using a fit to the peak of the E/p spectrum, around E/p ~ 1
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- Tracker and EM Calorimeter resolutions

- Hadronic recoil modelling
  - Characterized using \( p_T \)-balance in \( Z \rightarrow ll \) events
Custom Monte Carlo Detector Simulation

- A complete detector simulation of all quantities measured in the data
- First-principles simulation of tracking
  - Tracks and photons propagated through a high-resolution 3-D lookup table of material properties for silicon detector and COT
  - At each material interaction, calculate
    - Ionization energy loss according to detailed formulae and Landau distribution
    - Generate bremsstrahlung photons down to 0.4 MeV, using detailed cross section and spectrum calculations
    - Simulate photon conversion and Compton scattering
    - Propagate bremsstrahlung photons and conversion electrons
    - Simulate multiple Coulomb scattering, including non-Gaussian tail
  - Deposit and smear hits on COT wires, perform full helix fit including optional beam-constraint
Custom Monte Carlo Detector Simulation

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- First-principles simulation of tracking
  - Tracks and photons propagated through a high-resolution 3-D lookup table of material properties for silicon detector and COT

\[
\begin{align*}
\text{Calorimeter} & \\
\gamma & \xrightarrow{\text{interaction}} e^- \\
& \xrightarrow{\text{multiple Coulomb scattering}} e^- \\
& \xrightarrow{\text{simulated hits}} e^- 
\end{align*}
\]
3-D Material Map in Simulation

- Built from detailed construction-level knowledge of inner tracker: silicon ladders, bulkheads, port-cards etc.

- Tuned based on studies of inclusive photon conversions

- Radiation lengths vs $(\phi,z)$ at different radii shows localized nature of material distribution

- Include dependence on type of material via Landau-Pomeranchuk-Migdal suppression of soft bremsstrahlung
Tracking Momentum Scale
Tracking Momentum Scale

Set using $J/\psi \rightarrow \mu\mu$ and $\Upsilon \rightarrow \mu\mu$ resonance and $Z \rightarrow \mu\mu$ masses

- Extracted by fitting $J/\psi$ mass in bins of $1/p_T(\mu)$, and extrapolating momentum scale to zero curvature
- $J/\psi \rightarrow \mu\mu$ mass independent of $p_T(\mu)$ after 4% tuning of energy loss

\[ \Delta p/p = (-1.284 \pm 0.024) \times 10^{-3} \]
\[ \chi^2/{\text{dof}} = 95/86 \]

$\int L \cdot dt = 2.2 \text{ fb}^{-1}$

$\langle 1/p_T(\mu) \rangle$ (GeV$^{-1}$)
Tracking Momentum Scale

$\gamma \rightarrow \mu\mu$ resonance provides

- Momentum scale measurement at higher $p_T$
- Validation of beam-constraining procedure (upsilons are promptly produced)
- Cross-check of non-beam-constrained (NBC) and beam-constrained (BC) fits

$$\int L \, dt = 2.2 \, fb^{-1}$$

Data

Simulation

$$\Delta p/p = (-1.335 \pm 0.025_{\text{stat}}) \times 10^{-3}$$

$$\chi^2/\text{dof} = 59/48$$

NBC $\gamma \rightarrow \mu\mu$ mass fit
Tracking Momentum Scale Systematics

Systematic uncertainties on momentum scale

<table>
<thead>
<tr>
<th>Source</th>
<th>$J/\psi$ (x10^{-3})</th>
<th>NBC-$\Upsilon$ (x10^{-3})</th>
<th>common (x10^{-3})</th>
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</thead>
<tbody>
<tr>
<td>QED</td>
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<td>B field non-uniformity</td>
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<td>Ionizing material</td>
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<tr>
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<tr>
<td>Total</td>
<td>0.092</td>
<td>0.072</td>
<td>0.058</td>
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</table>

$\Delta M_{W,Z} = 6$ MeV

Uncertainty dominated by QED radiative corrections and magnetic field non-uniformity
Tracking Momentum Scale

$\gamma \rightarrow \mu\mu$ resonance provides

- Cross-check of non-beam-constrained (NBC) and beam-constrained (BC) fits
- Difference used to set additional systematic uncertainty

\[ \int L \, dt \approx 2.2 \, \text{fb}^{-1} \]

$\Delta p/p = (-1.185 \pm 0.02_{\text{stat}}) \times 10^{-3}$

$\chi^2/\text{dof} = 48 / 38$

BC $\gamma \rightarrow \mu\mu$

mass fit
Using the $J/\psi$ and $\Upsilon$ momentum scale, performed “blinded” measurement of $Z$ mass

- $Z$ mass consistent with PDG value (91188 MeV) (0.7$\sigma$ statistical)
- $M_Z = 91180 \pm 12_{\text{stat}} \pm 9_{\text{momentum}} \pm 5_{\text{QED}} \pm 2_{\text{alignment}}$ MeV

\[ \int L \, dt = 2.2 \text{ fb}^{-1} \]
Tracker Linearity Cross-check & Combination

- Final calibration using the J/ψ, ϒ and Z bosons for calibration

- Combined momentum scale correction:

\[
\Delta p/p = \left( -1.29 \pm 0.07_{\text{independent}} \pm 0.05_{\text{QED}} \pm 0.02_{\text{align}} \right) \times 10^{-3}
\]

\[
\Delta M_W = 7 \text{ MeV}
\]
EM Calorimeter Response
Calorimeter Simulation for Electrons and Photons

- Distributions of lost energy calculated using detailed GEANT4 simulation of calorimeter
  - Leakage into hadronic calorimeter
  - Absorption in the coil
  - Dependence on incident angle and $E_T$

- Energy-dependent gain (non-linearity) parameterized and fit from data
- Energy resolution parameterized as fixed sampling term and tunable constant term
  - Constant terms are fit from the width of $E/p$ peak and $Z\rightarrow\text{ee}$ mass peak
EM Calorimeter Scale

- E/p peak from $W \rightarrow ev$ decays provides measurements of EM calorimeter scale and its ($E_T$-dependent) non-linearity

$$\Delta S_E = (9_{\text{stat}} \pm 5_{\text{non-linearity}} \pm 5_{x0} \pm 9_{\text{Tracker}}) \times 10^{-5}$$

Setting $S_E$ to 1 using E/p calibration from combined $W \rightarrow ev$ and $Z \rightarrow ee$ samples

$\int L \, dt = 2.2 \, \text{fb}^{-1}$

$\Delta M_W = 13 \, \text{MeV}$

$\chi^2/\text{dof} = 18 / 22$

Tail of E/p spectrum used for tuning model of radiative material
Consistency of Radiative Material Model

- Excellent description of E/p spectrum tail
- Radiative material tune factor: $S_{X0} = 1.026 \pm 0.003_{\text{stat}} \pm 0.002_{\text{background}}$

Achieves consistency with E/p spectrum tail.
Measurement of EM Calorimeter Non-linearity

- Perform E/p fit-based calibration in bins of electron $E_T$
- GEANT-motivated parameterization of non-linear response:
  \[ S_E = 1 + \beta \log\left(\frac{E_T}{39 \text{ GeV}}\right) \]
- Tune on W and Z data: $\beta = (5.2^{+0.7}_{-0.7} \text{ stat}) \times 10^{-3}$
  \[ \Rightarrow \Delta M_W = 4 \text{ MeV} \]
• Checking uniformity of energy scale in bins of electron pseudo-rapidity
Z→ee Mass Cross-check and Combination

- Performed “blind” measurement of Z mass using E/p-based calibration
  - Consistent with PDG value (91188 MeV) within 1.4σ (statistical)
  - $M_Z = 91230 \pm 30_{\text{stat}} \pm 10_{\text{calorimeter}} \pm 8_{\text{momentum}} \pm 5_{\text{QED}} \pm 2_{\text{alignment}}$ MeV

- Combine E/p-based calibration with Z→ee mass for maximum precision

$\Delta M_W = 10$ MeV
Lepton Resolutions

- Tracking resolution parameterized in the custom simulation by
  - Radius-dependent drift chamber hit resolution $\sigma_h \sim (150 \pm 1_{\text{stat}}) \mu m$
  - Beamspot size $\sigma_b = (35 \pm 1_{\text{stat}}) \mu m$
  - Tuned on the widths of the $Z \rightarrow \mu\mu$ (beam-constrained) and $\Upsilon \rightarrow \mu\mu$ (both beam constrained and non-beam constrained) mass peaks
  
  $$\Delta M_{W} = 1 \text{ MeV}$$ (muons)

- Electron cluster resolution parameterized in the custom simulation by
  - 12.6% / $\sqrt{E_T}$ (sampling term)
  - Primary constant term $\kappa = (0.68 \pm 0.05_{\text{stat}}) \%$
  - Secondary photon resolution $\kappa_\gamma = (7.4 \pm 1.8_{\text{stat}}) \%$
  - Tuned on the widths of the $E/p$ peak and the $Z \rightarrow ee$ peak (selecting radiative electrons)

  $$\Delta M_{W} = 4 \text{ MeV}$$ (electrons)
Hadronic Recoil Model
We remove the calorimeter towers containing lepton energy from the hadronic recoil calculation.

- Lost underlying event energy is measured in $\phi$-rotated windows.

$$\Delta M_W = 2 \text{ MeV}$$

**Electron channel W data**

**Muon channel W data**
Constraining the Hadronic Recoil Model

Exploit similarity in production and decay of $W$ and $Z$ bosons

Detector response model for hadronic recoil tuned using $p_T$-balance in $Z \rightarrow ll$ events

Transverse momentum of Hadronic recoil ($u$) calculated as 2-vector-sum over calorimeter towers
Hadronic Recoil Simulation

Recoil momentum 2-vector $u$ has

- a soft 'spectator interaction' component, randomly oriented
  - Modelled using minimum-bias data with tunable magnitude
- A hard 'jet' component, directed opposite the boson $p_T$
  - $p_T$-dependent response and resolution parameterizations
  - Hadronic response $R = u_{\text{reconstructed}} / u_{\text{true}}$ parameterized as a logarithmically increasing function of boson $p_T$ motivated by Z boson data

\[ \int L \, dt \sim 2.2 \text{ fb}^{-1} \]

\[ \chi^2 / \text{DoF} = 22.5 / 29 \]
Tuning Recoil Response Model with Z events

Project the vector sum of $p_T(ll)$ and $u$ on a set of orthogonal axes defined by boson $p_T$

Mean and rms of projections as a function of $p_T(ll)$ provide information on hadronic model parameters

$$\int L \, dt = 2.2 \, \text{fb}^{-1}$$

$$\chi^2 / \text{DoF} = 8 / 9$$

Hadronic model parameters tuned by minimizing $\chi^2$ between data and simulation

$$\Delta M_W = 4 \, \text{MeV}$$
Tuning Recoil Resolution Model with $Z$ events

At low $p_T(Z)$, $p_T$-balance constrains hadronic resolution due to underlying event.

At high $p_T(Z)$, $p_T$-balance constrains jet resolution.

$\Delta M_W = 4$ MeV
Testing Hadronic Recoil Model with $W$ events

Compare recoil distributions between simulation and data

Recoil projection (GeV) on lepton direction

Recoil projection (GeV) perpendicular to lepton

CDF II

$\int L \, dt = 2.2 \, \text{fb}^{-1}$

MC

Data

$\mu = -0.387 \, \text{GeV}$

$\sigma = 4.631 \, \text{GeV}$

$\mu = -0.388 \pm 0.007 \, \text{GeV}$

$\sigma = 4.628 \pm 0.005 \, \text{GeV}$

$\mu = 0 \, \text{GeV}$

$\sigma = 5.054 \, \text{GeV}$

$\mu = 0 \pm 0.006 \, \text{GeV}$

$\sigma = 5.063 \pm 0.005 \, \text{GeV}$

Data

Simulation
Recoil model validation plots confirm the consistency of the model.
Parton Distribution Functions

- Affect W kinematic lineshapes through acceptance cuts
- We use CTEQ6 as the default PDF
- Use ensemble of 'uncertainty' PDFs
  - Represent variations of eigenvectors in the PDF parameter space
  - Compute $\delta M_W$ contribution from each error PDF
- Using MSTW2008 PDF ensemble defined for 68% CL, obtain systematic uncertainty of 10 MeV
- Comparing CTEQ and MSTW at 90% CL, yield similar uncertainty (CTEQ is 10% larger)
  - Cross-check: default MSTW2008 relative to default CTEQ6 yields 6 MeV shift in W mass
Backgrounds in the W sample

### Muons

<table>
<thead>
<tr>
<th>Background</th>
<th>% of $W \rightarrow \mu\nu$ data</th>
<th>$\delta m_W$ (MeV)</th>
<th>$m_T$ fit</th>
<th>$p_T^\mu$ fit</th>
<th>$p_T^\nu$ fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z \rightarrow \mu\mu$</td>
<td>$7.35 \pm 0.09$</td>
<td>2</td>
<td>4</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>$W \rightarrow \tau\nu$</td>
<td>$0.880 \pm 0.004$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
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<tr>
<td>QCD</td>
<td>$0.035 \pm 0.025$</td>
<td>1</td>
<td>1</td>
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</tr>
<tr>
<td>DIF</td>
<td>$0.24 \pm 0.08$</td>
<td>1</td>
<td>3</td>
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<tr>
<td>Cosmic rays</td>
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### Electrons

<table>
<thead>
<tr>
<th>Background</th>
<th>% of $W \rightarrow e\nu$ data</th>
<th>$\delta m_W$ (MeV)</th>
<th>$m_T$ fit</th>
<th>$p_T^e$ fit</th>
<th>$p_T^\nu$ fit</th>
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<tbody>
<tr>
<td>$Z \rightarrow e\bar{e}$</td>
<td>$0.139 \pm 0.014$</td>
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<tr>
<td>$W \rightarrow \tau\nu$</td>
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<td>Total</td>
<td></td>
<td>4</td>
<td>3</td>
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<td></td>
</tr>
</tbody>
</table>

Backgrounds are small (except $Z \rightarrow \mu\mu$ with a forward muon)
W Mass Fits
blind analysis technique

- All W and Z mass fit results were blinded with a random [-75, 75] MeV offset hidden in the likelihood fitter

- Blinding offset removed after the analysis was declared frozen

- Technique allows to study all aspects of data while keeping Z mass and W mass result unknown within 75 MeV
$W$ Transverse Mass Fit

CDF II

\[ \int L \, dt \approx 2.2 \text{ fb}^{-1} \]

**Muons**

- **Data**
- **Simulation**

$M_W = (80379 \pm 16_{\text{stat}}) \text{ MeV}$

$\chi^2/\text{dof} = 58/48$
$W$ Mass Fit using Lepton $p_T$

CDF II

$\int L\,dt \approx 2.2\, fb^{-1}$

$M_W = (80393 \pm 21_{\text{stat}})\, \text{MeV}$

$\chi^2/\text{dof} = 60 / 62$

Electrons

Data

Simulation
# Summary of $W$ Mass Fits

<table>
<thead>
<tr>
<th>Charged Lepton</th>
<th>Kinematic Distribution</th>
<th>Fit Result (MeV)</th>
<th>$\chi^2$/DoF</th>
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<tr>
<td>Electron</td>
<td>Transverse mass</td>
<td>80408 ± 19</td>
<td>52/48</td>
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<tr>
<td>Electron</td>
<td>Charged lepton $p_T$</td>
<td>80393 ± 21</td>
<td>60/62</td>
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<tr>
<td>Electron</td>
<td>Neutrino $p_T$</td>
<td>80431 ± 25</td>
<td>71/62</td>
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<tr>
<td>Muon</td>
<td>Transverse mass</td>
<td>80379 ± 16</td>
<td>57/48</td>
</tr>
<tr>
<td>Muon</td>
<td>Charged lepton $p_T$</td>
<td>80348 ± 18</td>
<td>58/62</td>
</tr>
<tr>
<td>Muon</td>
<td>Neutrino $p_T$</td>
<td>80406 ± 22</td>
<td>82/62</td>
</tr>
</tbody>
</table>

CDF III: $\int L \, dt = 2.2 \, \text{fb}^{-1}$

- Muons: $p_T^\gamma$ = 80406 ± 22
- Muons: $p_T^l$ = 80348 ± 18
- Muons: $m_T$ = 80379 ± 16
- Electrons: $p_T^\gamma$ = 80431 ± 25
- Electrons: $p_T^l$ = 80393 ± 21
- Electrons: $m_T$ = 80408 ± 19
Combined Results

- Combined electrons (3 fits): $M_W = 80406 \pm 25$ MeV, $P(\chi^2) = 49\%$

- Combined muons (3 fits): $M_W = 80374 \pm 22$ MeV, $P(\chi^2) = 12\%$

- All combined (6 fits): $M_W = 80387 \pm 19$ MeV, $P(\chi^2) = 25\%$
<table>
<thead>
<tr>
<th>Source Description</th>
<th>Electrons</th>
<th>Muons</th>
<th>Common</th>
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</thead>
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<td>Parton dist. Functions</td>
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<td>QED rad. Corrections</td>
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<tr>
<td><strong>Total systematic</strong></td>
<td><strong>39</strong></td>
<td><strong>27</strong></td>
<td><strong>26</strong></td>
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<tr>
<td><strong>Total</strong></td>
<td><strong>62</strong></td>
<td><strong>60</strong></td>
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</table>

Systematic uncertainties shown in green: statistics-limited by control data samples
New CDF Result (2.2 fb\(^{-1}\))

Transverse Mass Fit Uncertainties (MeV)

<table>
<thead>
<tr>
<th></th>
<th>electrons</th>
<th>muons</th>
<th>common</th>
</tr>
</thead>
<tbody>
<tr>
<td>W statistics</td>
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<tr>
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<tr>
<td>Lepton resolution</td>
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</tr>
<tr>
<td>Recoil energy scale</td>
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<td>Selection bias</td>
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<td>0</td>
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<td>Lepton removal</td>
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<tr>
<td>Backgrounds</td>
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<tr>
<td>pT(W) model</td>
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<td>Parton dist. Functions</td>
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<td>QED rad. Corrections</td>
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<td>4</td>
<td>4</td>
</tr>
<tr>
<td><strong>Total systematic</strong></td>
<td><strong>18</strong></td>
<td><strong>16</strong></td>
<td><strong>15</strong></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>26</strong></td>
<td><strong>23</strong></td>
<td></td>
</tr>
</tbody>
</table>

Systematic uncertainties shown in green: statistics-limited by control data samples
Combined W Mass Result, Error Scaling

- Lepton Scale
- Lepton Resn
- Recoil Scale
- Recoil Resn
- Lepton Removal
- Background
- $p_T(W)$
- PDF
- QED
- Stats

Error (MeV)

- 200 pb$^{-1}$ Error
- 2.2 fb$^{-1}$ Error
2012 Status of $M_W$ vs $M_{\text{top}}$

experimental errors 68\% CL:

- LEP2/Tevatron: today

- light SUSY

- MSSM

- heavy SUSY

$M_H = 114$ GeV

$M_H = 127$ GeV

SM

Heinemeyer, Hollik, Stockinger, Weiglein, Zeune '12
W Boson Mass Measurements from Different Experiments

Previous world average
= 80399 ± 23 MeV

new CDF result more precise than other measurements

ArXiv: 1204.0042

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Mass (MeV) ± Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>D0 I</td>
<td>80483 ± 84</td>
</tr>
<tr>
<td>CDF I</td>
<td>80433 ± 79</td>
</tr>
<tr>
<td>DELPHI</td>
<td>80336 ± 67</td>
</tr>
<tr>
<td>L3</td>
<td>80270 ± 55</td>
</tr>
<tr>
<td>OPAL</td>
<td>80416 ± 53</td>
</tr>
<tr>
<td>ALEPH</td>
<td>80440 ± 51</td>
</tr>
<tr>
<td>D0 II (PRL 108, 151804)</td>
<td>80375 ± 23</td>
</tr>
<tr>
<td>CDF II (PRL 108, 151803)</td>
<td>80387 ± 19</td>
</tr>
<tr>
<td>World Average</td>
<td>80385 ± 15</td>
</tr>
</tbody>
</table>
Improvement of $M_W$ Uncertainty with Sample Statistics

Non-scaling floor (11 MeV) dominated by PDF uncertainty (10 MeV)
Future $M_W$ Measurements at Tevatron and LHC

- Factor of 2-5 bigger samples of W and Z bosons available
- Huge samples at LHC
- For most of the sources of systematic uncertainties, we have demonstrated that we can find ways to constrain them with data and scale systematic uncertainties with data statistics
- Exception is the PDF uncertainty, where we have not made a dedicated effort to constrain the PDFs within the analysis

- We need to address specific PDF degrees of freedom to answer the question:
  - Can we approach total uncertainty on $M_W \sim 10$ MeV at the Tevatron? 5 MeV at the LHC?

PDF Uncertainties – scope for improvement

- Newer PDF sets, *e.g.* CT10W include more recent data, such as Tevatron W charge asymmetry data

- Dominant sources of W mass uncertainty are the $d_{\text{valence}}$ and $\bar{d}-\bar{u}$ degrees of freedom
  - Understand consistency of data constraining these d.o.f.
  - PDF fitters increase tolerance to accommodate inconsistent datasets

- Fermilab/Seaquest, Tevatron and LHC measurements that can further constrain PDFs:
  - Drell-Yan, Z boson rapidity distribution
  - $W \rightarrow l\nu$ lepton rapidity distribution
  - $W$ boson charge asymmetry
Improvement of $M_W$ Uncertainty with $W$ Asymmetry data

G. Bozzi et al, PHYSICAL REVIEW D 83, 113008 (2011)

ATLAS and CMS measurements of $W$ charge asymmetry ($\sim 35$ pb$^{-1}$) with 7% uncertainty $\Rightarrow$ pseudo-data with 1% uncertainty
Summary

- The W boson mass is a very interesting parameter to measure with increasing precision.

- New Tevatron W mass results are very precise:
  - \( M_W = 80387 \pm 19 \text{ MeV} \) (CDF)
  - \( M_W = 80375 \pm 23 \text{ MeV} \) (D0)
  - \( M_W = 80385 \pm 15 \text{ MeV} \) (world average)

- New global electroweak fit \( M_H = 94^{+29}_{-24} \) GeV @ 68% CL (LEPEWWG)
  - SM Higgs prediction is pinned in the low-mass range
  - Consistent with mass of Higgs-like boson \( \sim 125 \) GeV

- Looking forward to \( \Delta M_W < 10 \) MeV from full Tevatron dataset
  - goal of \( \Delta M_W \sim 5 \) MeV from LHC data
Z → ee Mass Cross-check using Electron Tracks

-Performed “blind” measurement of Z mass using electron tracks
  - Consistent with PDG value within $1.8\sigma$ (statistical)

- Checks tracking for electrons vs muons, and model of radiative energy loss

\[ \int L \, dt = 2.2 \text{ fb}^{-1} \]

\[ M_Z = (91268 \pm 47_{\text{stat}}) \text{ MeV} \]

\[ \chi^2/\text{dof} = 62 / 46 \]
$W$ Transverse Mass Fit

CDF II

$\int L \, dt \approx 2.2 \text{ fb}^{-1}$

Electrons

$M_W = (80408 \pm 19_{\text{stat}}) \text{ MeV}$

$\chi^2/\text{dof} = 52 / 48$

Data

Simulation
$W$ Lepton $p_T$ Fit

CDF II

$\int L \, dt \approx 2.2 \text{ fb}^{-1}$

$M_W = (80348 \pm 18_{\text{stat}}) \text{ MeV}$

$\chi^2/\text{dof} = 54 / 62$

Muons

Data

Simulation
$W$ Missing $E_T$ Fit

CDF II

$\int L \, dt \approx 2.2 \text{ fb}^{-1}$

$M_W = (80431 \pm 25_{\text{stat}}) \text{ MeV}$

$\chi^2/\text{dof} = 71 / 62$

- Electrons

- Data
- Simulation
$W$ Missing $E_T$ Fit

CDF II

\[ \int L \, dt \approx 2.2 \, fb^{-1} \]

Muons

$M_W = (80406 \pm 22_{\text{stat}}) \, \text{MeV}$

$\chi^2/\text{dof} = 79 / 62$

- Data
- Simulation
$W$ Mass Fit Residuals, Muon Channel
$W$ Mass Fit Residuals, Electron Channel
$W$ Mass Fit Window Variation, $m_T$ Fit
$W$ Mass Fit Window Variation, $p_T(l)$ Fit

**upper**

CDF II preliminary

$\int L \, dt = 2.2 \, fb^{-1}$

**lower**

CDF II preliminary

$\int L \, dt = 2.2 \, fb^{-1}$

- $W \rightarrow e\nu$
- $W \rightarrow \mu\nu$
$W$ Mass Fit Window Variation, $p_T(\nu)$ Fit
$W$ Mass Fit Results

- Electron and muon $m_T$ fits combined
  \[ m_W = 80390 \pm 20 \text{ MeV}, \chi^2/\text{dof} = 1.2/1 \ (28\%) \]
- Electron and muon $p_T$ fits combined
  \[ m_W = 80366 \pm 22 \text{ MeV}, \chi^2/\text{dof} = 2.3/1 \ (13\%) \]
- Electron and muon MET fits combined
  \[ m_W = 80416 \pm 25 \text{ MeV}, \chi^2/\text{dof} = 0.5/1 \ (49\%) \]
- All electron fits combined
  \[ m_W = 80406 \pm 25 \text{ MeV}, \chi^2/\text{dof} = 1.4/2 \ (49\%) \]
- All muon fits combined
  \[ m_W = 80374 \pm 22 \text{ MeV}, \chi^2/\text{dof} = 4/2 \ (12\%) \]
- All fits combined
  \[ m_W = 80387 \pm 19 \text{ MeV}, \chi^2/\text{dof} = 6.6/5 \ (25\%) \]
Combined W Mass Result, Error Scaling
### $p_T(\nu)$ Fit Systematic Uncertainties

<table>
<thead>
<tr>
<th>Systematic (MeV/c²)</th>
<th>Electrons</th>
<th>Muons</th>
<th>Common</th>
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<tbody>
<tr>
<td>Lepton Energy Scale</td>
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<tr>
<td>Lepton Energy Resolution</td>
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<tr>
<td>$u_{</td>
<td></td>
<td>}$ efficiency</td>
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<tr>
<td>Lepton Removal</td>
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<tr>
<td>$p_T(W)$ model</td>
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<td>Parton Distributions</td>
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<tr>
<td><strong>Total</strong></td>
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## Combined Fit Systematic Uncertainties

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### $p_T(\ell)$ Fit Systematic Uncertainties

<table>
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<td><strong>19</strong></td>
<td><strong>18</strong></td>
<td><strong>16</strong></td>
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</table>
QCD Background Estimation in Muon Channel
Decay-in-Flight Background Estimation in Muon Channel
Motivation II

- Separate fits for $M_H$ using only leptonic and only hadronic measurements of asymmetries: marginal difference in preferred Higgs mass (from M. Chanowitz, February 2007 Seminar, Fermilab)

Possible explanations:
- Statistical fluctuation
- Systematic experimental bias

New physics contributions:

To raise $M_H$ prediction of leptonic asymmetries:
- Minimal SuperSymmetric Standard Model (Altarelli et. al.)
- $4^{th}$ family of fermions (Okun et. al.)
- Opaque branes (Carena et. al.)

New physics in $b$-quark asymmetry requires large modification to $Zbb$ vertex
Parameters of Electro-Weak Interactions

At tree level, all of the observables can be expressed in terms of three parameters of the SM Lagrangian: $v$, $g$, $g'$ or, equivalently, $v$, $e$, $s \equiv \sin \theta_W$ (also $c \equiv \cos \theta_W$)

$$
\alpha = \frac{e^2}{4\pi}, \quad G_F = \frac{1}{2\sqrt{2}v^2}, \quad m_Z = \frac{ev}{\sqrt{2}sc}, \quad m_W = \frac{ev}{\sqrt{2}s}, \quad s_{\text{eff}}^2 = s^2,
$$

Radiative corrections to the relations between physical observables and Lagrangian params:

$$
\begin{align*}
    m_Z^2 &= \frac{e^2v^2}{2s^2c^2} + \Pi_{ZZ}(m_Z^2) \\
    m_W^2 &= \frac{e^2v^2}{2s^2} + \Pi_{WW}(m_W^2)
\end{align*}
$$

$$
G_F = \frac{1}{2\sqrt{2}v^2} \left[ 1 - \frac{\Pi_{WW}(0)}{m_W^2} + \delta_{VB} \right]
$$
Radiative Corrections to W Boson Mass

All these corrections can be combined into relations among physical observables, e.g.:

\[
m_W^2 = m_Z^2 \left[ \frac{1}{2} + \frac{1}{2} \sqrt{1 - \frac{2\sqrt{2}\pi \alpha}{G_F m_Z^2} (1 + \Delta r)} \right]
\]

\(\Delta r\) can be parametrized in terms of two universal corrections and a remainder:

\[
\Delta r = \Delta \alpha(m_Z) - \frac{c^2}{s^2} \Delta \rho + \Delta r_{\text{rem}}
\]

The leading corrections depend quadratically on \(m_t\) but only logarithmically on \(m_H\):

\[
\Delta \rho = \frac{\Pi_{ZZ}(0)}{m_Z^2} - \frac{\Pi_{WW}(0)}{m_W^2} \approx \frac{3 \alpha}{16\pi c^2} \left( \frac{m_t^2}{s^2 m_Z^2} + \log \frac{m_H^2}{m_W^2} + \ldots \right)
\]

\[
\frac{\delta m_W^2}{m_W^2} \approx \frac{c^2}{c^2 - s^2} \Delta \rho , \quad \delta \sin^2 \theta_{\text{eff}} \approx -\frac{c^2 s^2}{c^2 - s^2} \Delta \rho
\]
Radiative Corrections to Electromagnetic Coupling

\[ \alpha = \frac{e^2}{4\pi} \left[ 1 + \lim_{q^2 \to 0} \frac{\Pi_{\gamma\gamma}(q^2)}{q^2} \right] \]

\[ \begin{align*}
e^- & \xrightarrow{\gamma} e^+ \\
e^+ & \xrightarrow{\gamma} e^- + e^+ \\
\end{align*} \]

this one is tricky: the hadronic contribution to \(\Pi'_{\gamma\gamma}(0)\) cannot be computed perturbatively

We can however trade it for another experimental observable:

\[ R_{\text{had}}(q^2) = \frac{\sigma_{\text{had}}(q^2)}{\sigma_{\ell^+\ell^-}(q^2)} \]

\[ \alpha(m_Z) = \frac{e^2}{4\pi} \left[ 1 + \frac{\Pi_{\gamma\gamma}(m_Z)}{m_Z} \right] = \frac{\alpha}{1 - \Delta\alpha(m_Z)} \]

\[ \Delta\alpha(m_Z) = \Delta\alpha_{\ell}(m_Z) + \Delta\alpha_{\text{top}}(m_Z) + \Delta\alpha_{\text{had}}^{(5)}(m_Z) \]

\[ \Delta\alpha_{\text{had}}^{(5)}(m_Z) = -\frac{m_Z^2}{3\pi} \int_{4m^2_{\pi}}^{\infty} \frac{R_{\text{had}}(q^2) dq^2}{q^2 (q^2 - m^2_Z)} = 0.02758 \pm 0.00035 \]

(This hadronic contribution is one of the biggest sources of uncertainty in EW studies)
## Systematic Uncertainties in QED Radiative Corrections

<table>
<thead>
<tr>
<th>Effects/Uncertainties</th>
<th>CDF0</th>
<th>CDF1a</th>
<th>CDF1b</th>
<th>CDFII 200pb(^{-1})</th>
<th>CDFII 2.3fb(^{-1})</th>
<th>D(\Phi) 1fb(^{-1})</th>
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</table>
Consistency check of COT alignment procedure

- Fit separate helices to cosmic ray tracks on each side
- Compare track parameters (e.g., Curvature, shown below) of the two tracks: a measure of track parameter bias

CDFII preliminary

False curvature smaller than 0.1% for 40 GeV track, over the length of the COT
Previous $M_W$ vs $M_{\text{top}}$
Updated $M_W$ vs $M_{top}$