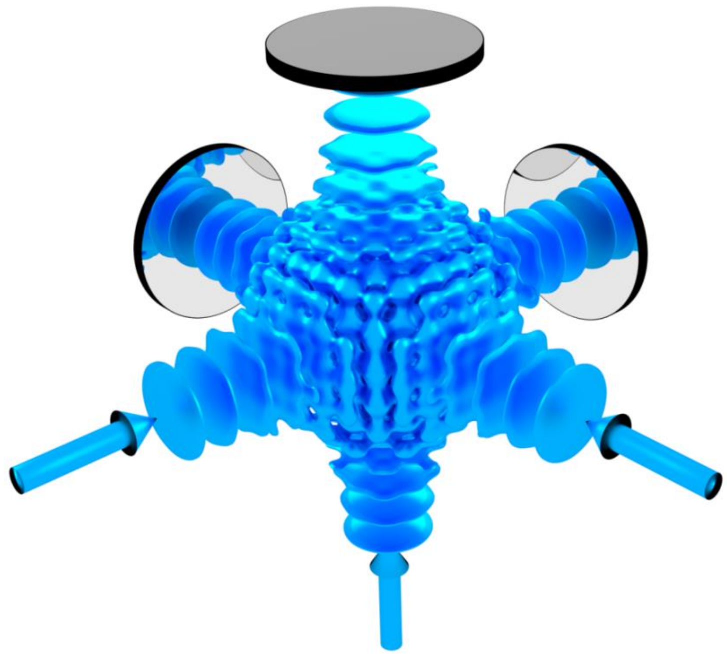
Non-equilibrium dynamics of quantum gases in optical lattices



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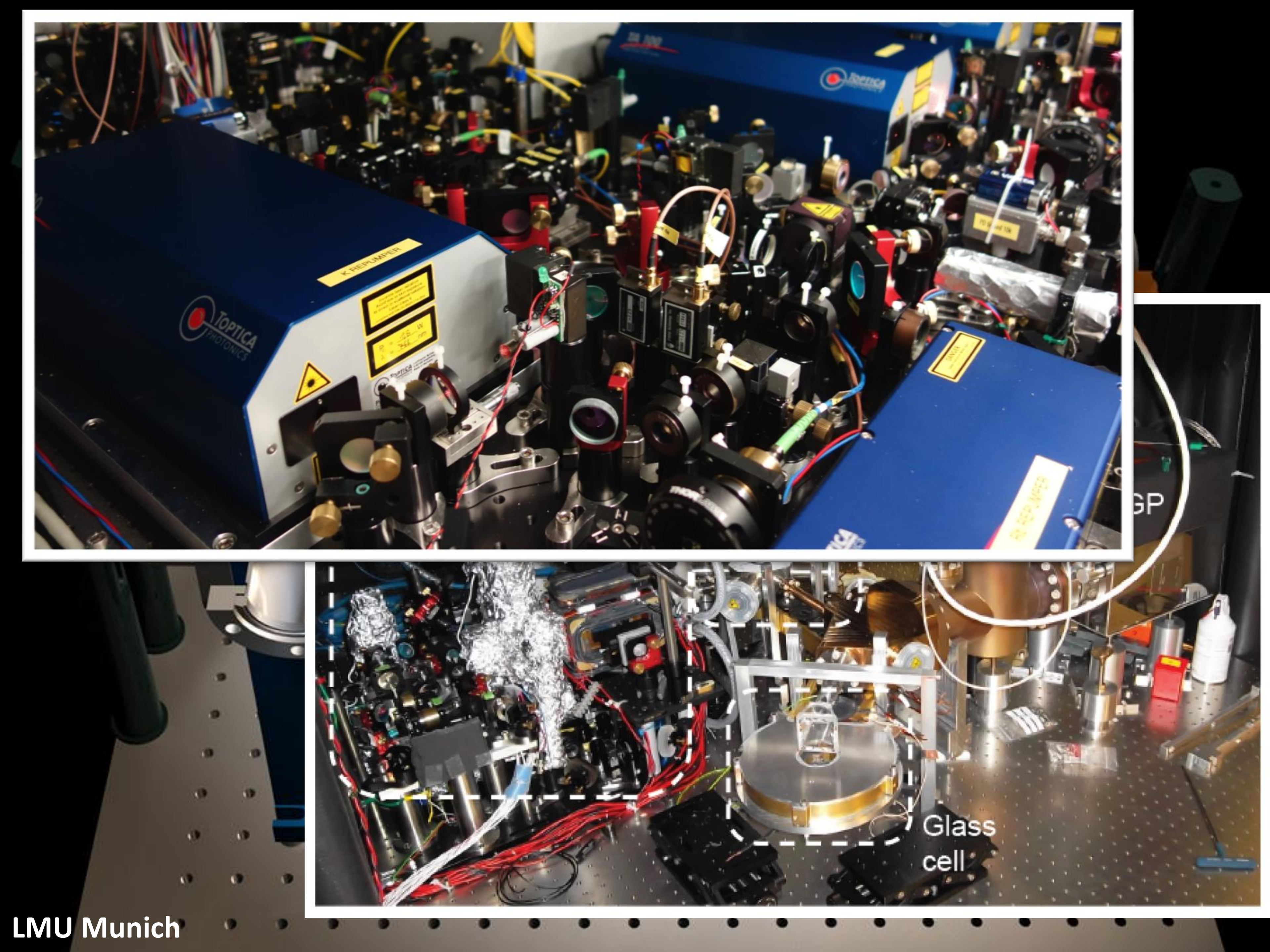
University of Cambridge

UQUAM ultracold quantum matter

AQψS ANALOG QUANTUM SIMULATORS FOR MANY-BODY DYNAMICS

LMU

www.manybody.phy.cam.ac.uk



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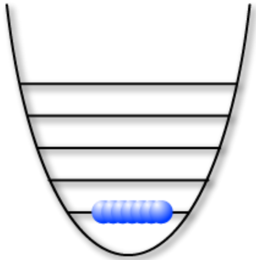
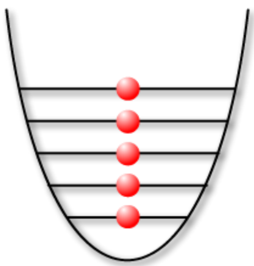
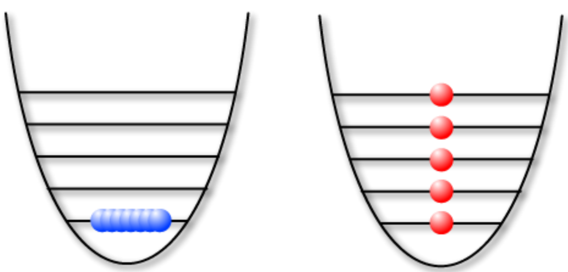


Bose-Einstein condensate of Potassium ^{39}K atoms

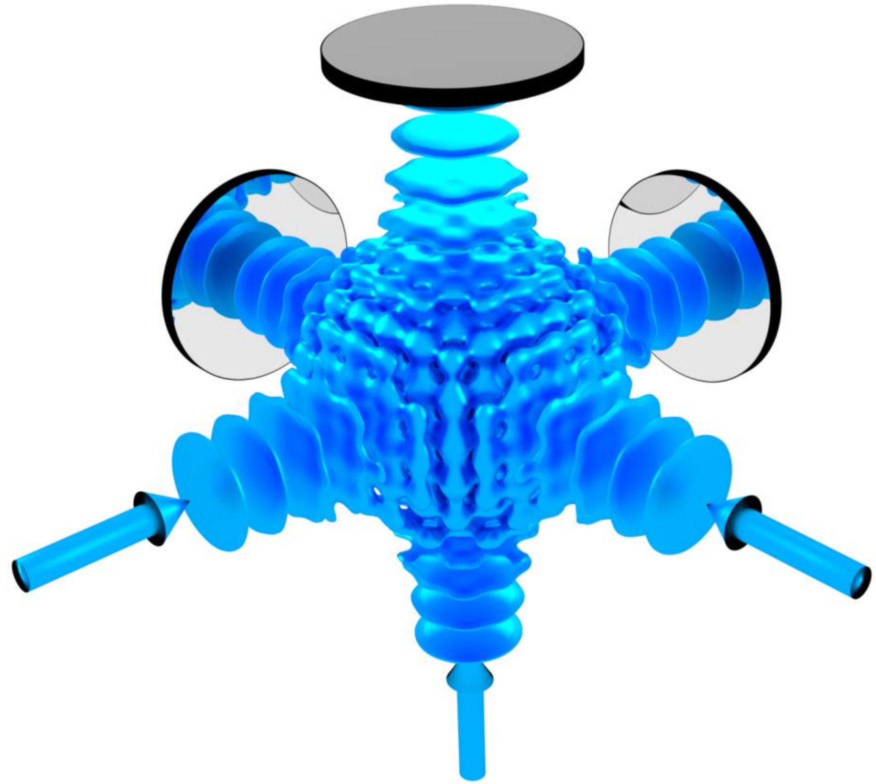


Degenerate Fermi gas ^{40}K



Bose-Fermi mixture $^{87}Rb + ^{40}K$

+

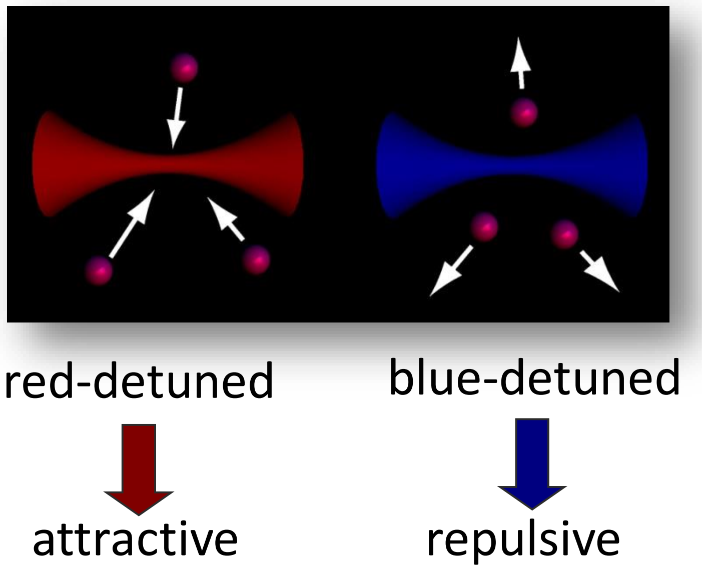


3D optical lattice

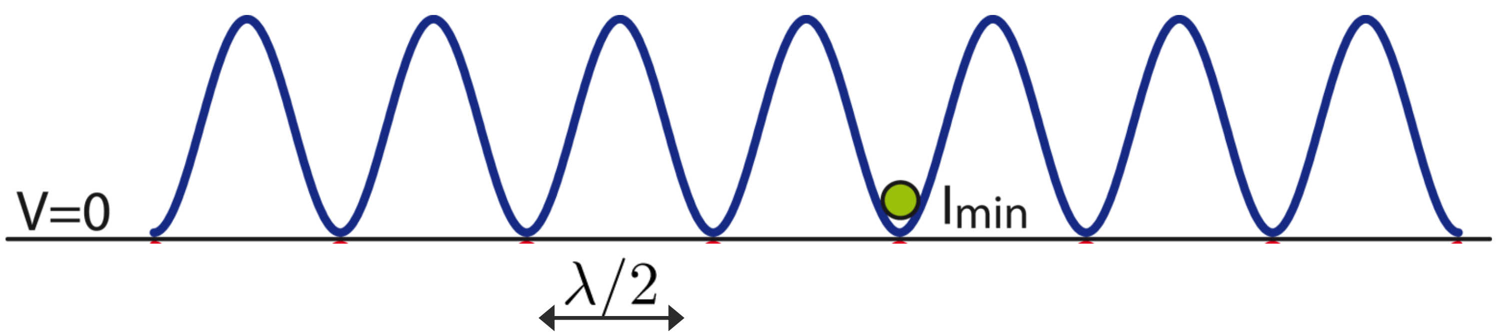
Optical lattices

Dipole potential:
Interaction of atoms with far-detuned light, e.g. optical tweezer, dipole trap

$$\boxed{U_{\mathrm{dip}} \propto \alpha(\omega)\, I(\vec{r})}$$

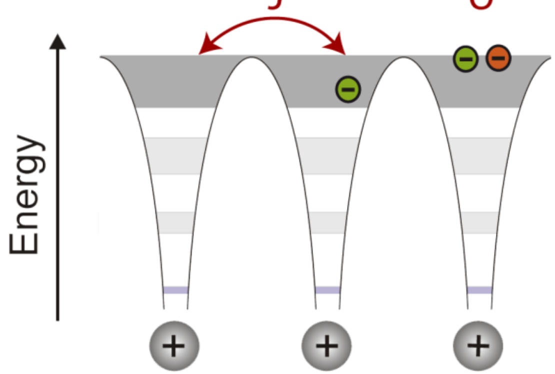


Overlapping two counterpropagating laser beams produce a standing wave:

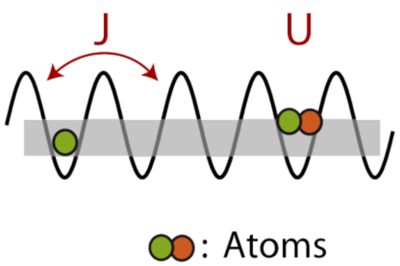


Why lattices?

- Realizes important model Hamiltonians from solid-state physics:
  ➔ e.g. Hubbard models



Potential created by ions

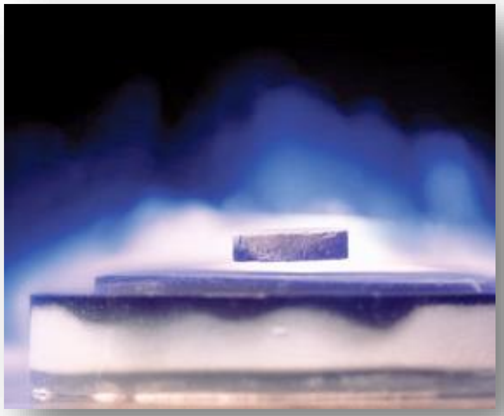


Potential created by standing light wave

Understand and Design Quantum Materials

- High temperature superconductivity
- Quantum Magnetism

Emergent many-body phenomena



YBCO

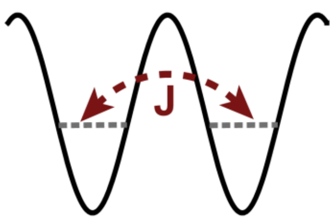
Hubbard Models

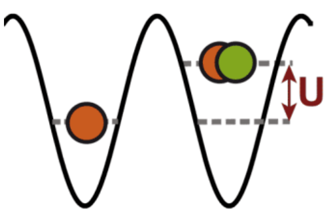
- Bosons:

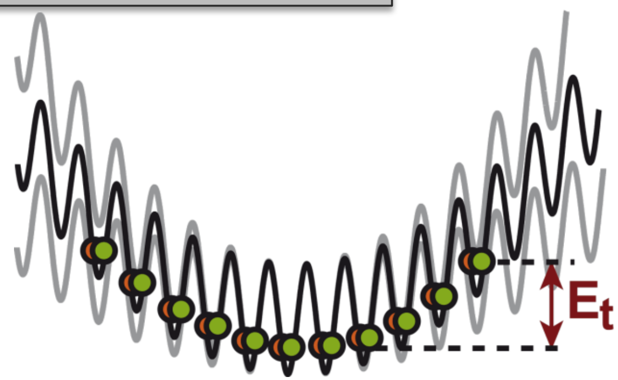
$$H_B = -J \sum_{\langle i,j \rangle} a_i^{+} a_j + \frac{U}{2} \sum_i n_i (n_i - 1)$$

- Fermions:

$$H_F = -J \sum_{\langle i,j \rangle,\sigma} a_{i,\sigma}^{+} a_{j,\sigma} + U \sum_i n_{i,\downarrow} n_{i,\uparrow}$$

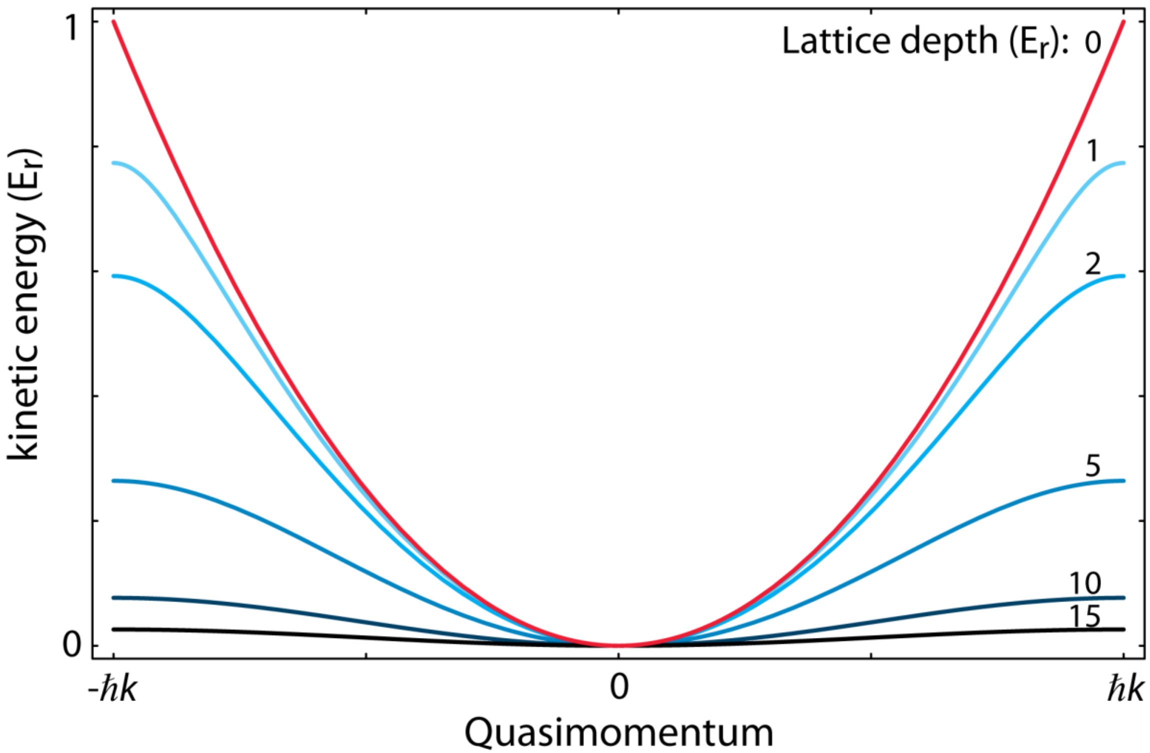


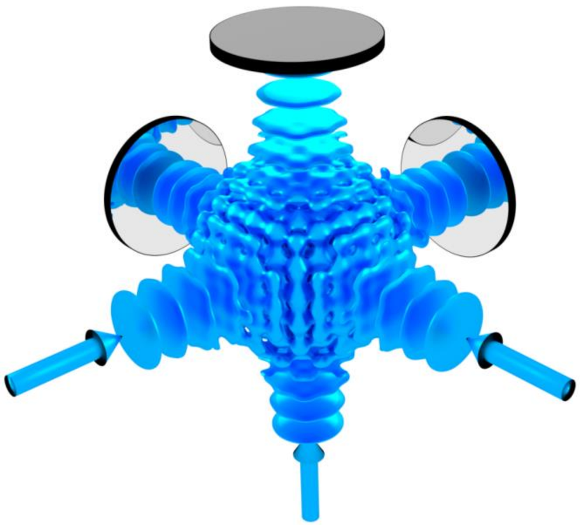




No thermal environment – no external temperature!
(closed quantum system)

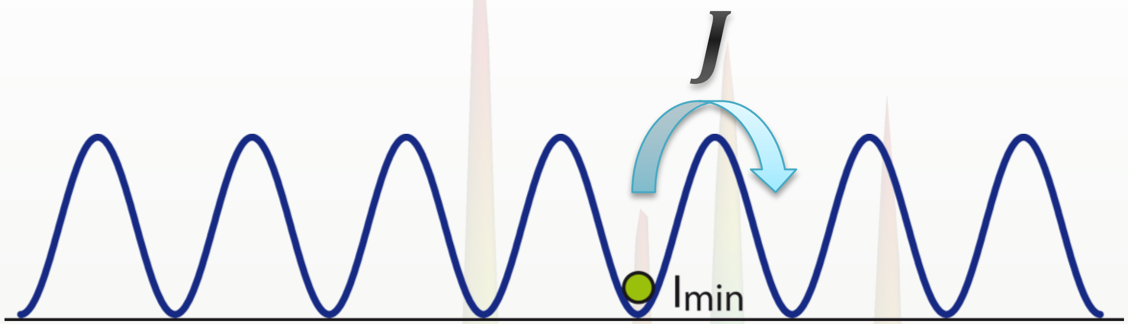
Why lattices ? – Enhances Interactions





- Lattice strongly reduces kinetic energy
  - →enhances interaction effects
  - →strongly interacting phases become available at moderate interactions

Why lattices? - Non-Equilibrium physics

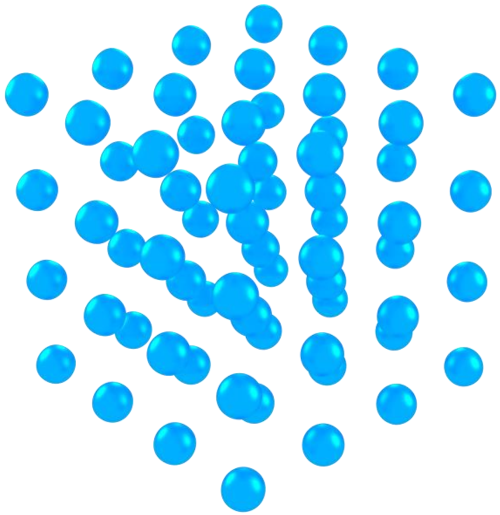


$$J \sim 1 - 1000 \text{ Hz}$$

$$\tau = \frac{\hbar}{J} \sim 0.1\text{ms} - 1\text{s}$$

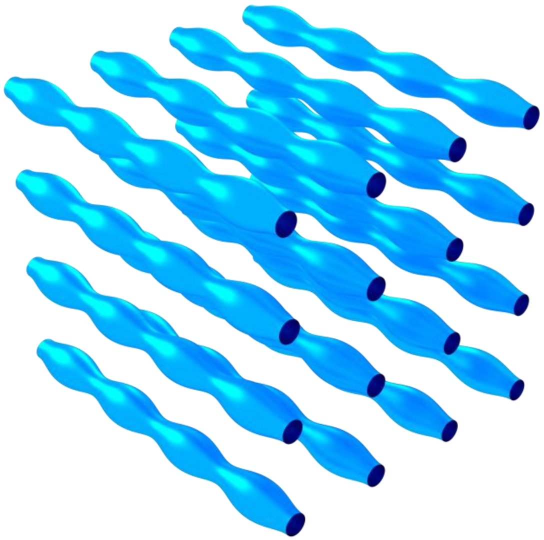
- Can control and observe real-time dynamics

Why lattices? - variable dimensionality



0D

- 3D deep lattice
- isolated wells
- no hopping

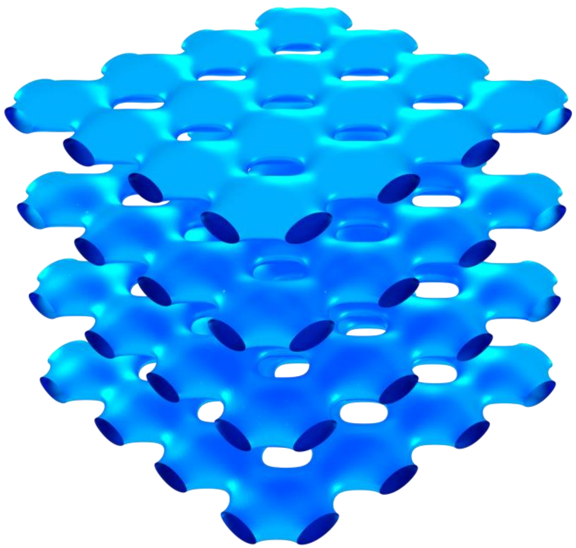


1D

- 2D deep lattice

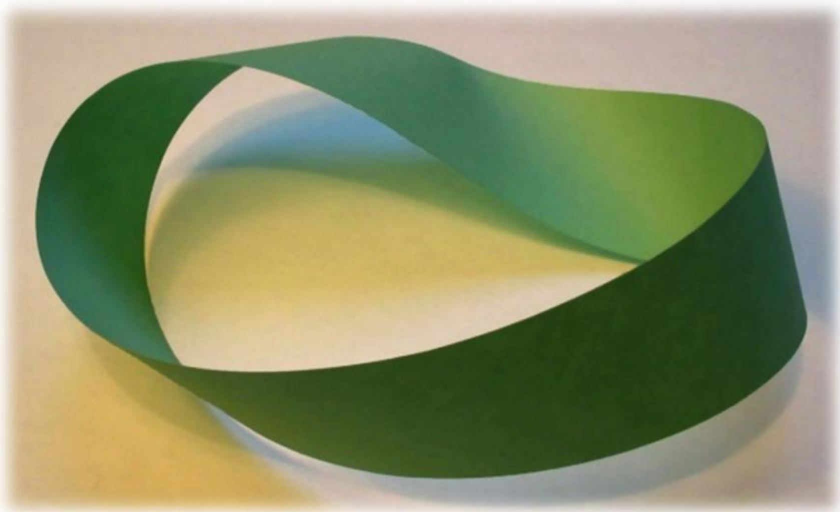
+1D weaker lattice

- 1D hopping



2D

Why lattices? – Topological band strucres



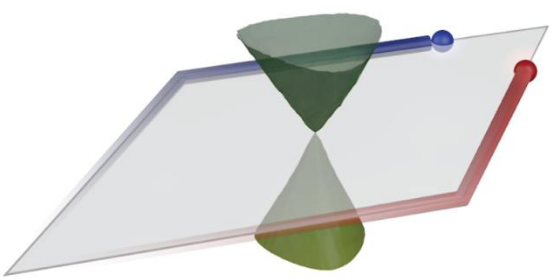
→Topological invariants

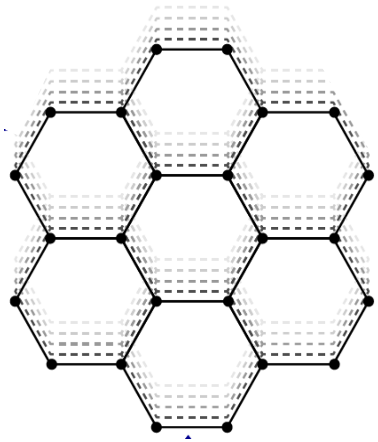
Discrete values, e.g. ± 1, $\mathbb{Z}$

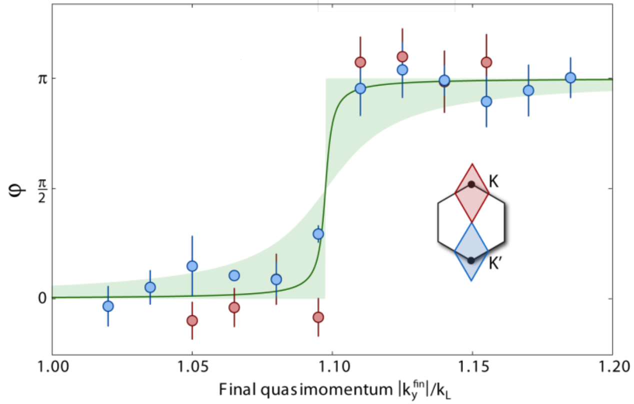
Example: Chern numbers

- Integer Quantum Hall
- Topological insulators

Aharonov-Bohm interferometry & Wilson lines

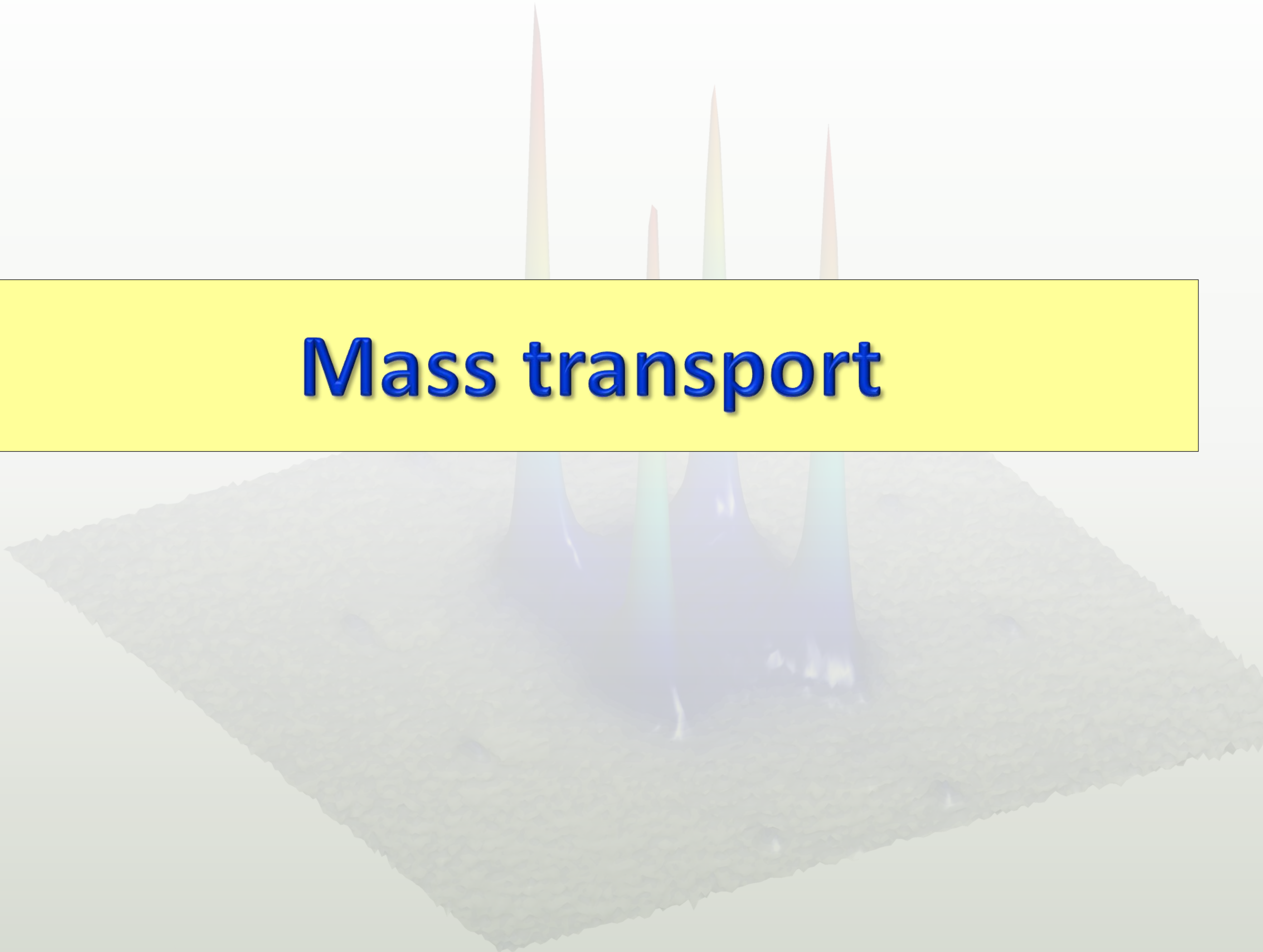






L. Duca *et al.* Science **347**, 288 (2015)
T. Li *et al.* Science **352**, 1094 (2016)

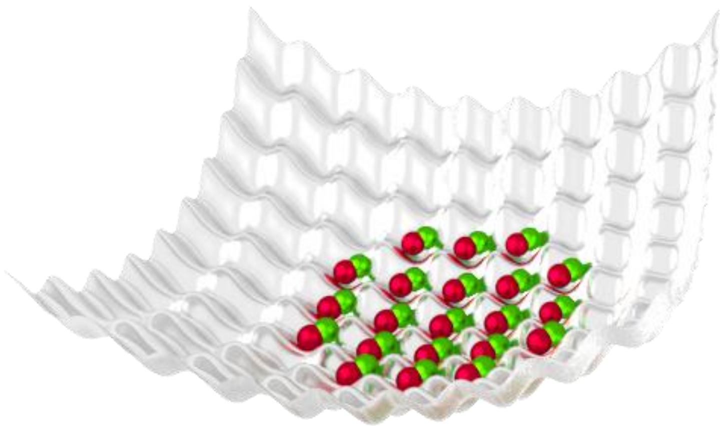
Mass transport



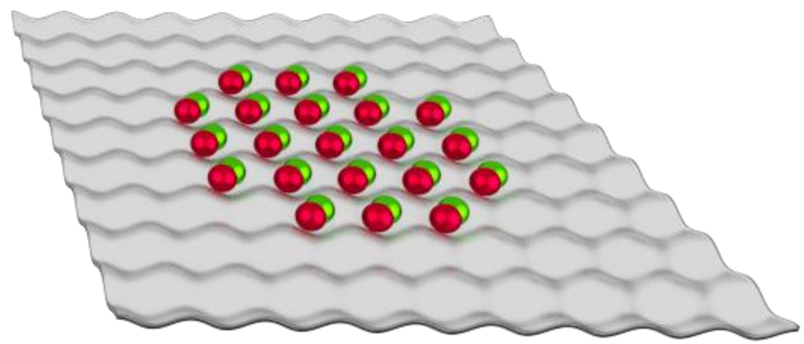
Fermionic Expansion

fermionic ^{40}K

2D



band insulator: two spin states

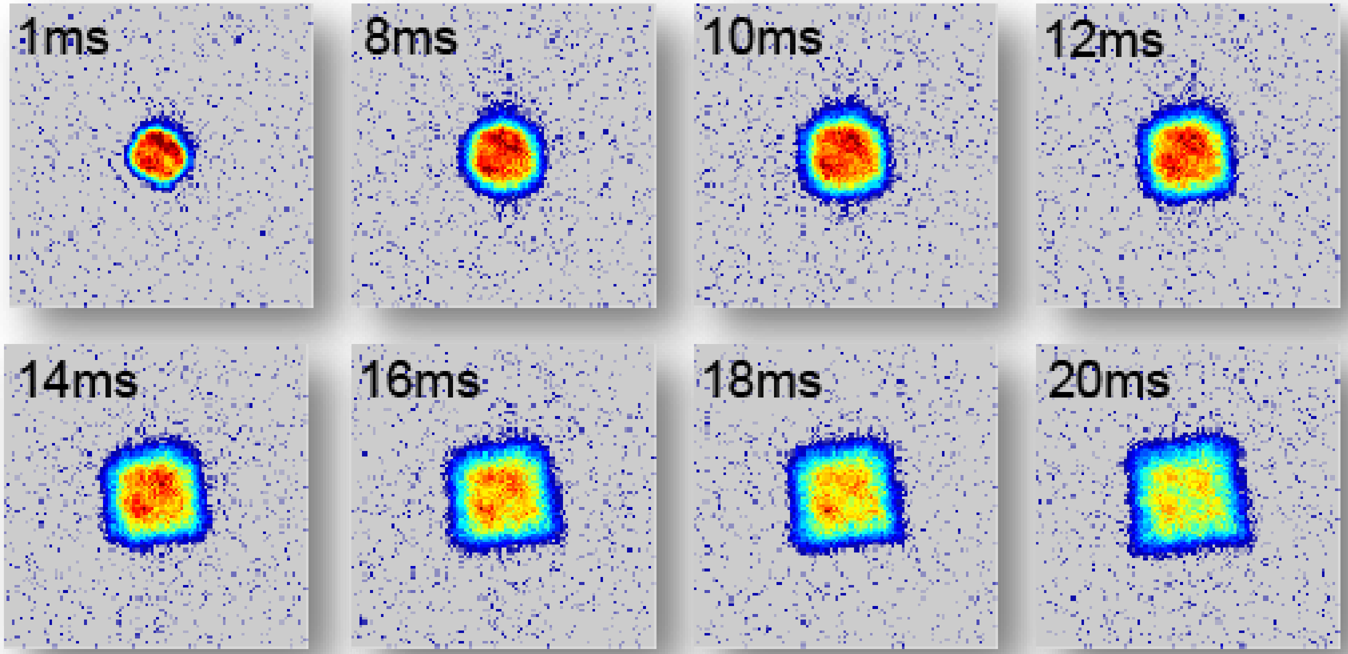


Dynamics within lattice

Sudden Quench

→All energies contribute
→Far from equilibrium

Non-interacting atoms



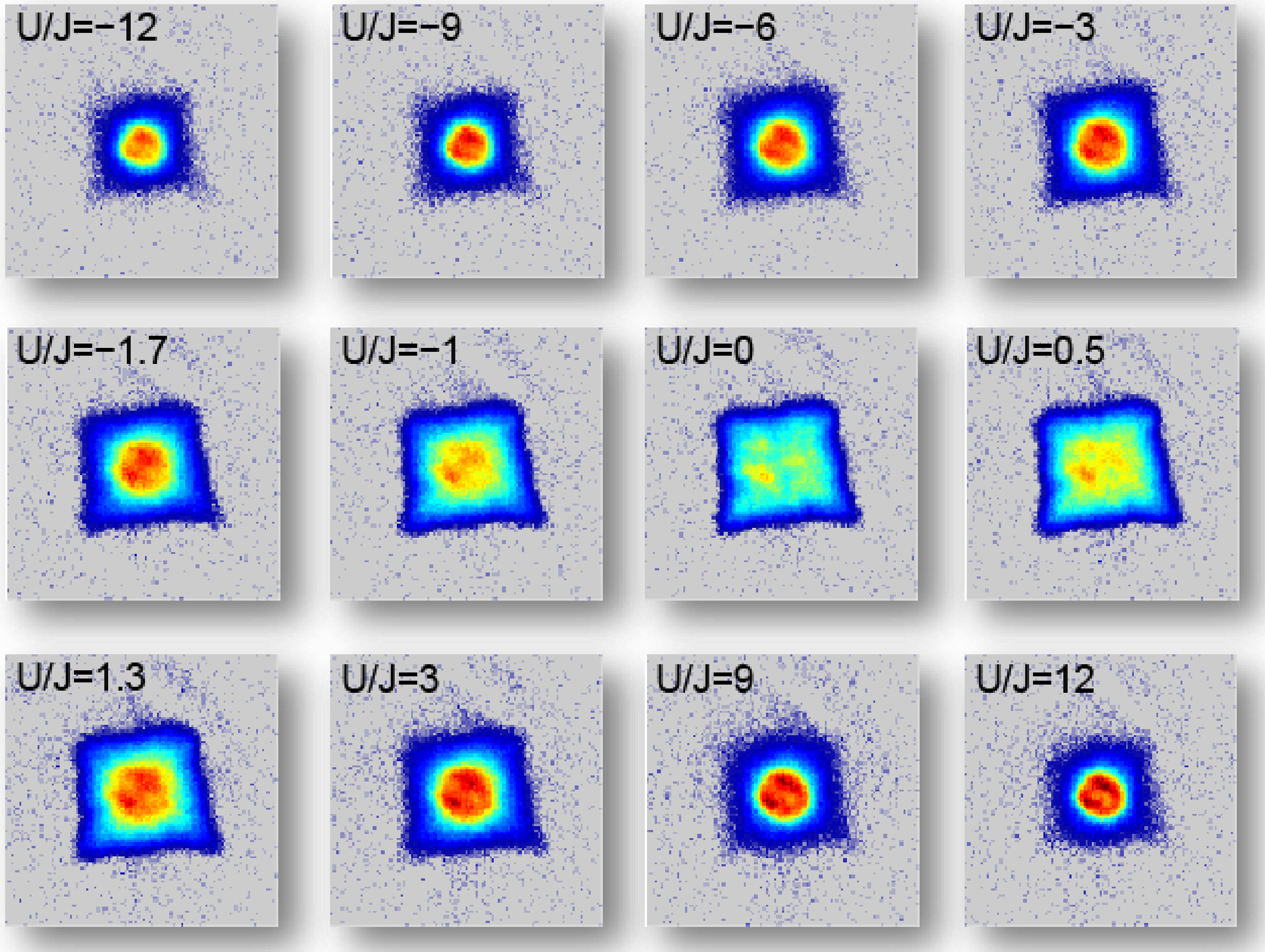
Symmetry of expanding cloud

Symmetry of Brillouin-Zone

Ballistic Expansion

Nat. Phys. 8, 213 (2012)

Interaction



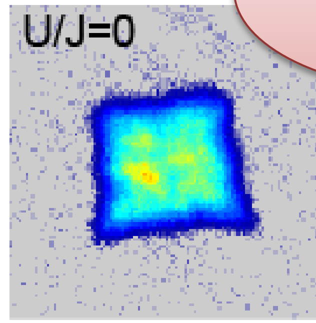
Density distribution after 25ms of expansion

Nat. Phys. 8, 213 (2012)

Theory

1) non-interacting: no collisions

→ **ballistic expansion**



„Flat-top“ distribution

…erplay between **diffusive core** and **ballistic wings**

↓

described by Boltzmann equation

2) interacting:

- low density limit: no collisions

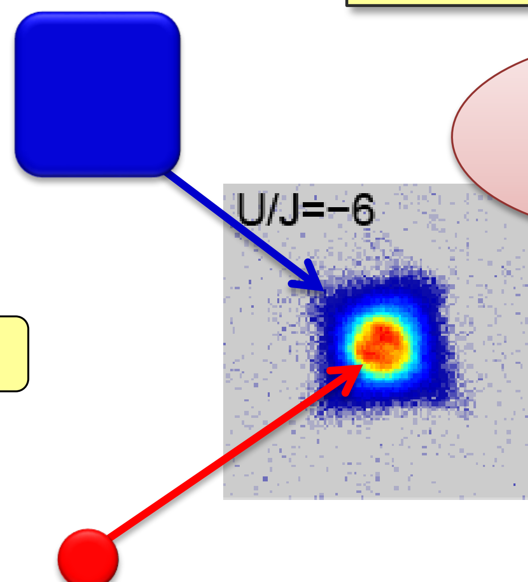
→ **ballistic expansion**

- no thermalisation
- non-isotropic expansion

- higher density: many collisions

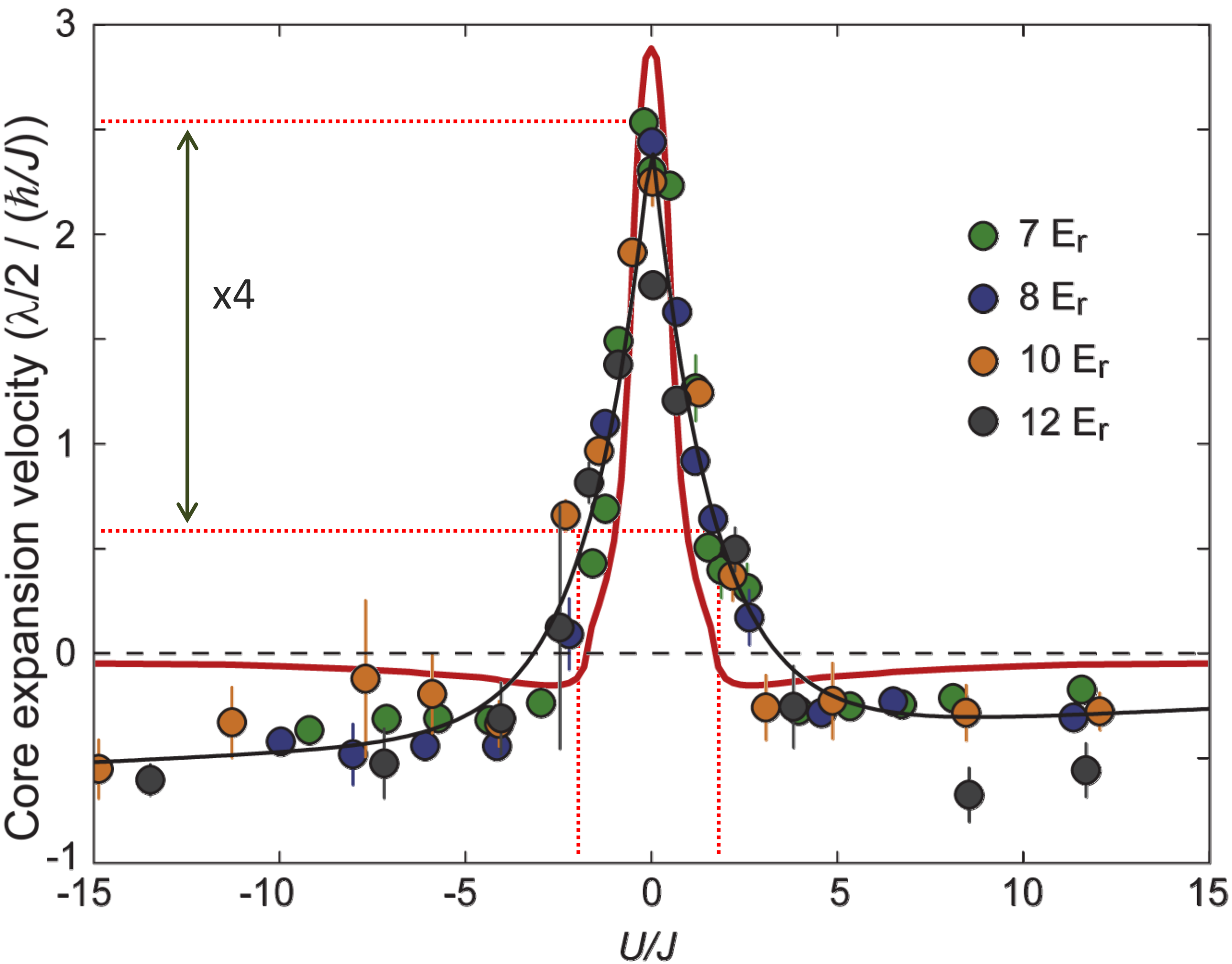
→ **non-linear diffusive expansion**

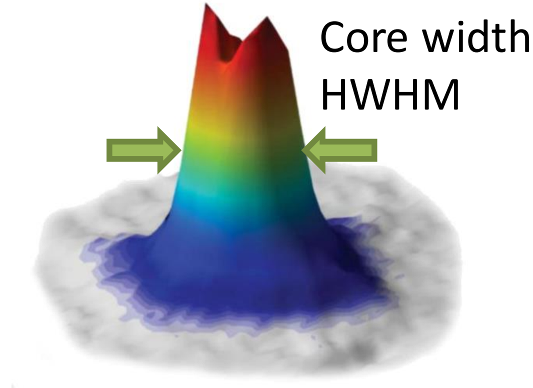
- local thermalisation
- isotropic hydrodynamic expansion



„Bimodal“ distribution

Fermionic Expansion velocity in 2D





Symmetric dynamics:

U ↔ -U

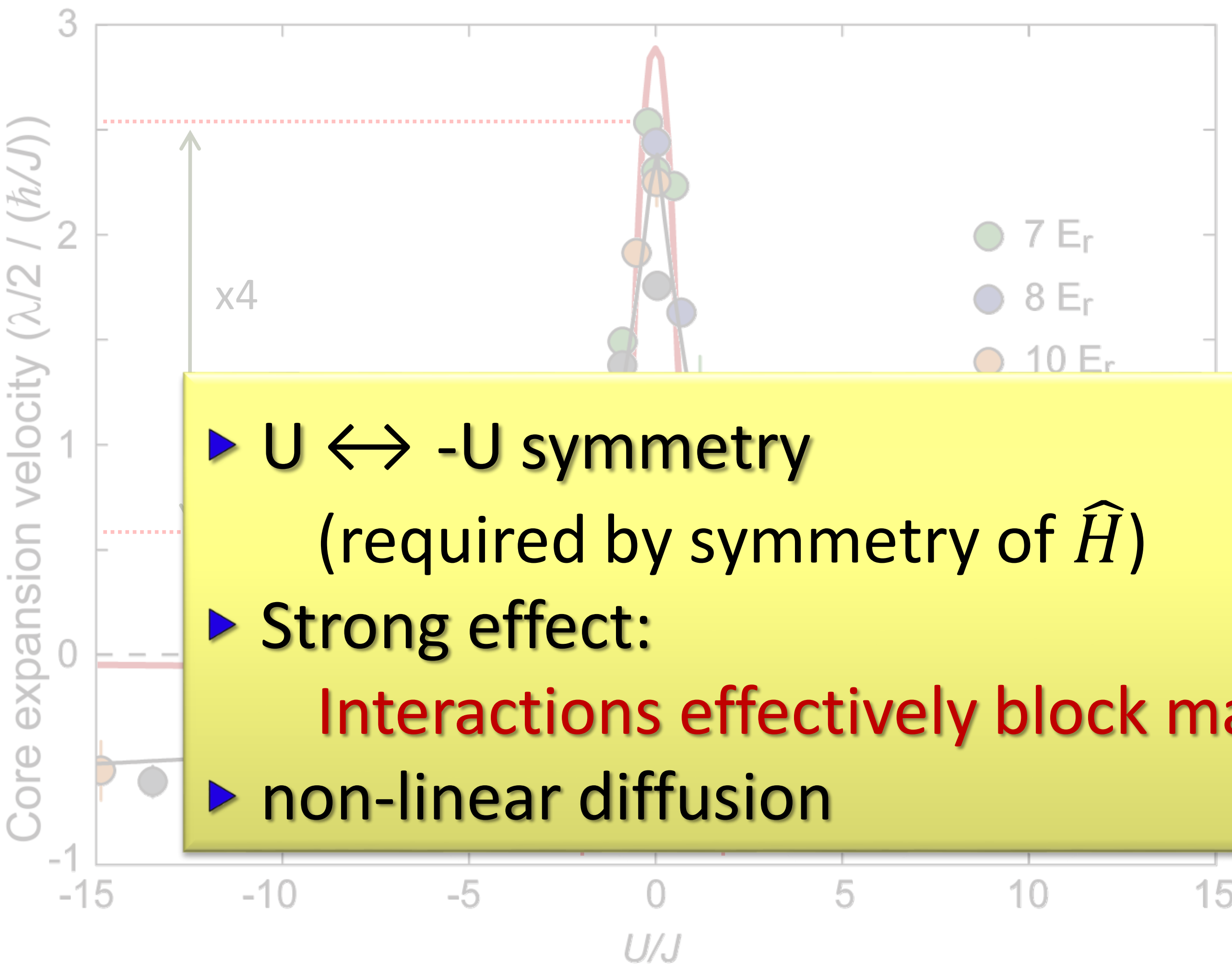
red line:
Boltzmann equation in relaxation time approximation (A. Rosch *et.al.*)

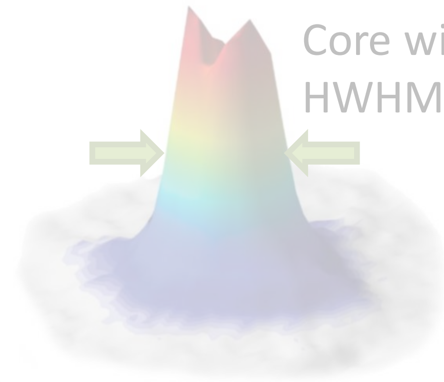
Quantum dynamics is more complex:

- Bound states / correlated tunneling
- Creation of Many-body entanglement
- Quantum distillation (1D) (F. Heidrich-Meisner et.al.)

Nat. Phys. 8, 213 (2012)

Fermionic Expansion velocity in 2D





- U ↔ -U symmetry
  (required by symmetry of $\hat{H}$)
- Strong effect:
  Interactions effectively block mass transport!
- non-linear diffusion

mics:

Boltzmann equation in relaxation time approximation (A. Rosch *et.al.*)

Quantum dynamics is more complex:

- Bound states / correlated tunneling
- Creation of Many-body entanglement
- Quantum distillation (1D) (F. Heidrich-Meisner et.al.)

Nat. Phys. 8, 213 (2012)

The diffusive picture

- Separation between local & global timescales

- frequent scattering

  → fast local relaxation or
  fast local thermalization (in higher dimension)

  → system can be described by $\mu(\vec{r},t), T(\vec{r},t)\ \forall\, t > t_{local}$

- slower global dynamics driven by
  gradients in temperature & chemical potential

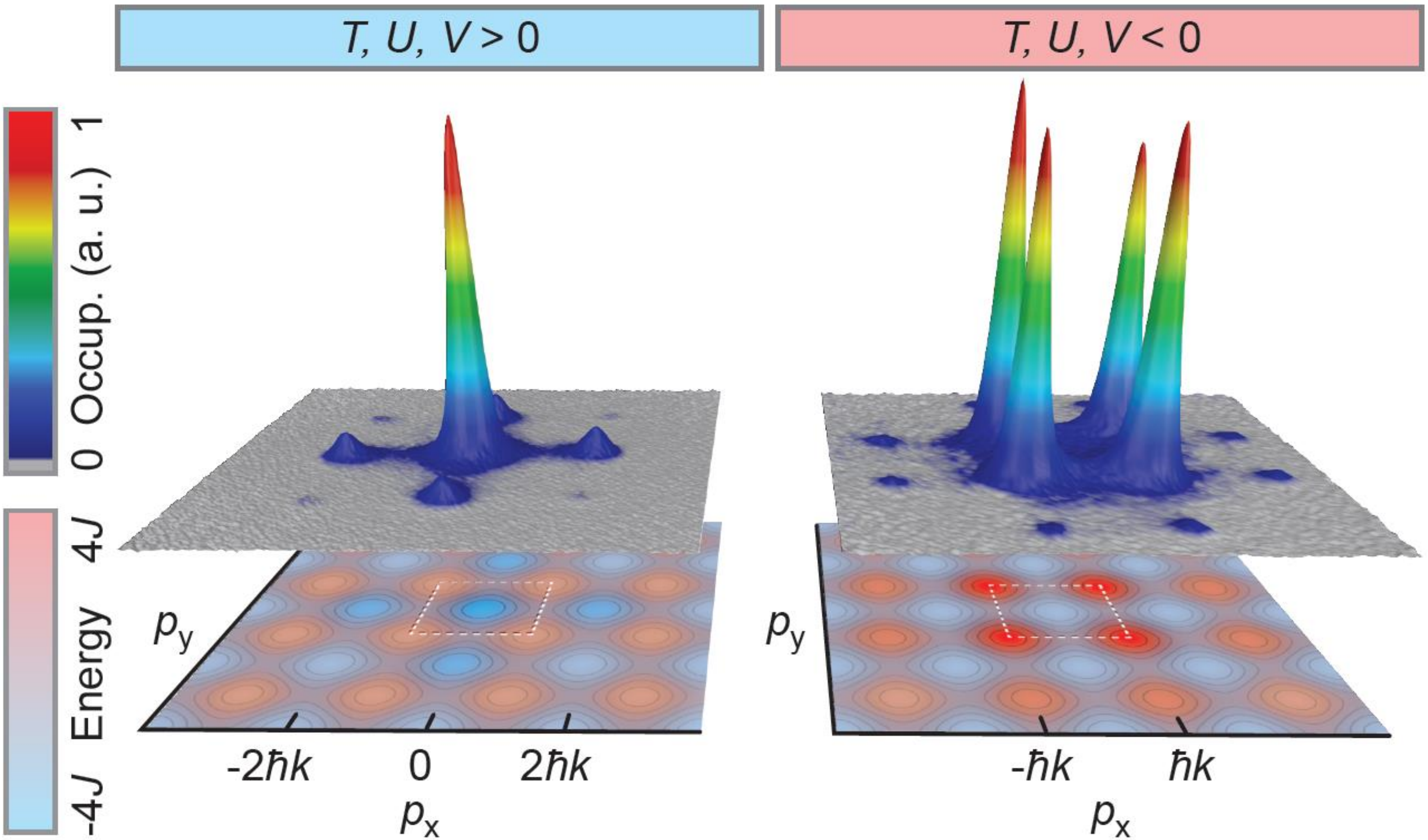
See also: Lux *et al.* PRA, 2014

- $\boldsymbol{U \leftrightarrow -U}$ symmetry requires thermalization to $\boldsymbol{T \leftrightarrow -T}$

Can we see negative Temperatures?

Science 339, 52 (2013)

Bose gas at pos. and negative Temperature

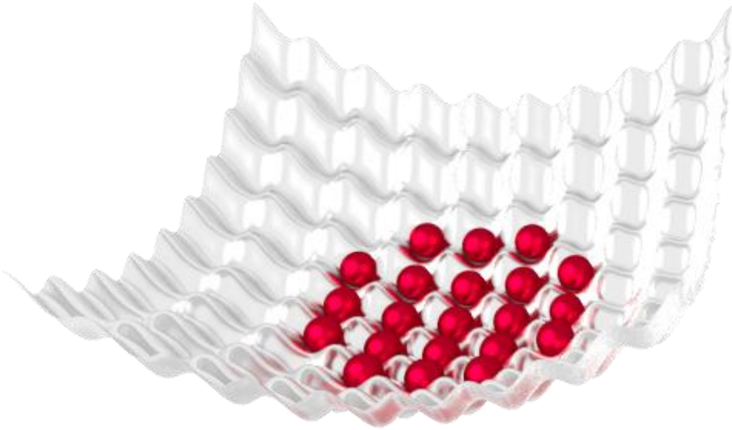


Science 339, 52 (2013)

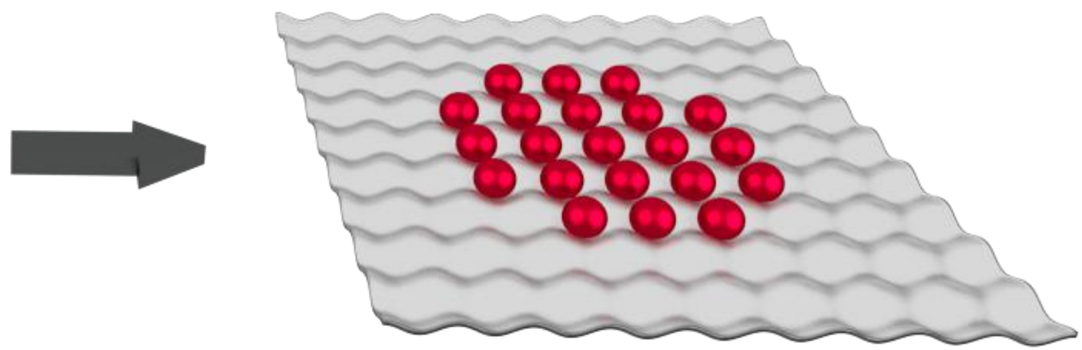
Bosonic Expansion:

bosonic ^{39}K

2D & 1D

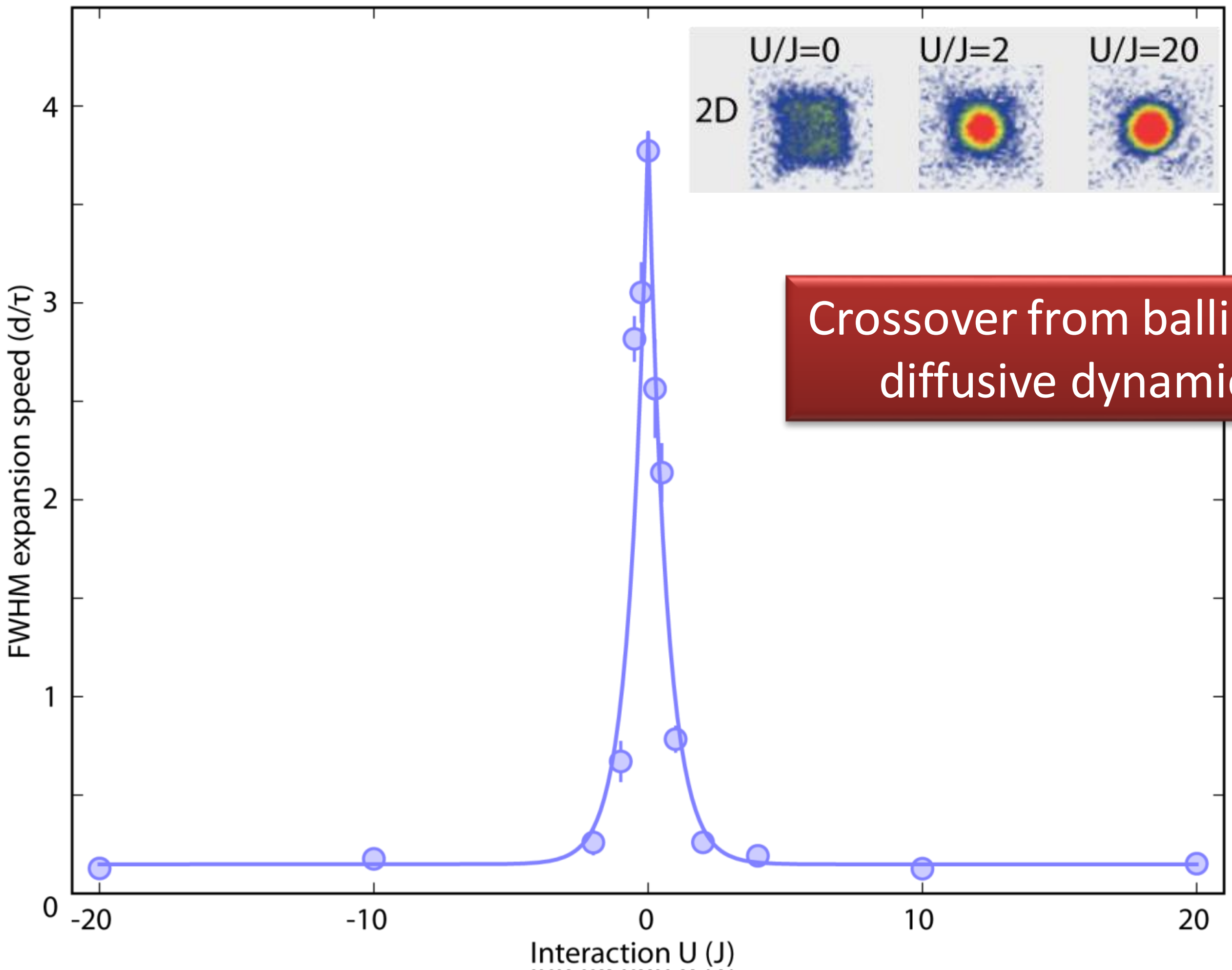


n=1 Mott Insulator



Dynamics within lattice

Bosonic Expansion in 2D



Initial state:
One atom per lattice site
(Product of Fock states)

Crossover from ballistic to diffusive dynamics!

Very similar to Fermionic case!

PRL 110, 205301 (2013)

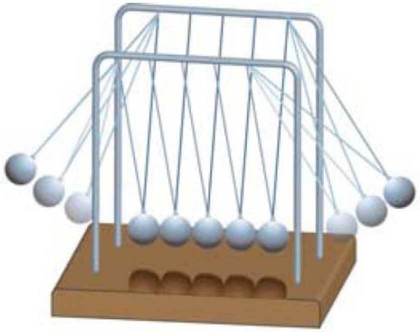
Dynamics in different dimensions

Thermalization?

In low dimensions?

Pictures: WikiMedia Commons

What can be different in 1D?



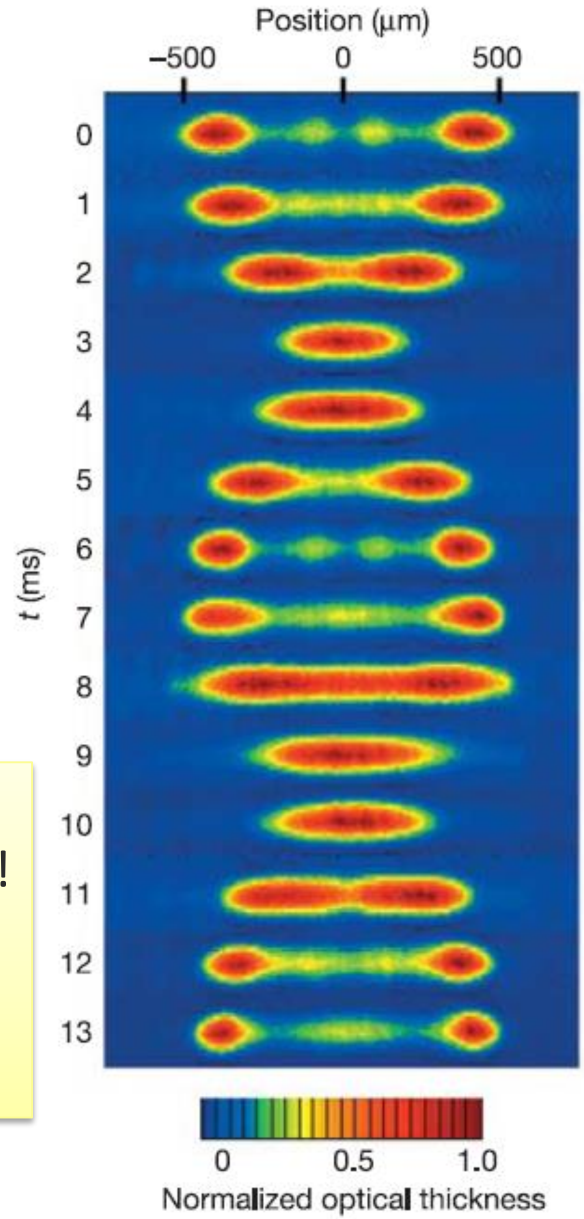
Classically: Two-body collisions can only **exchange** momentum, but ***not redistribute*** it!

$$n(k, t) = const. \text{ w.r.t. } t$$

Repulsive 1D Bosons with point-like interaction without a lattice are **integrable** in homogeneous case!

→**Lieb-Liniger model**

Thermalization constrained by conserved quantities.



T. Kinoshita *et al.*, Nature (2006)

Fineprint: Trap, 3-body collisions, quasi 1D

1D Bosons on a lattice

- additional process *Umklapp scattering* + lattice dispersion
- 1D Bose-Hubbard model is (in general) *not integrable!*

  classically chaotic for intermediate U and intermediate energy

  M. Hiller et al. PRA **79,** 023621 (2009)

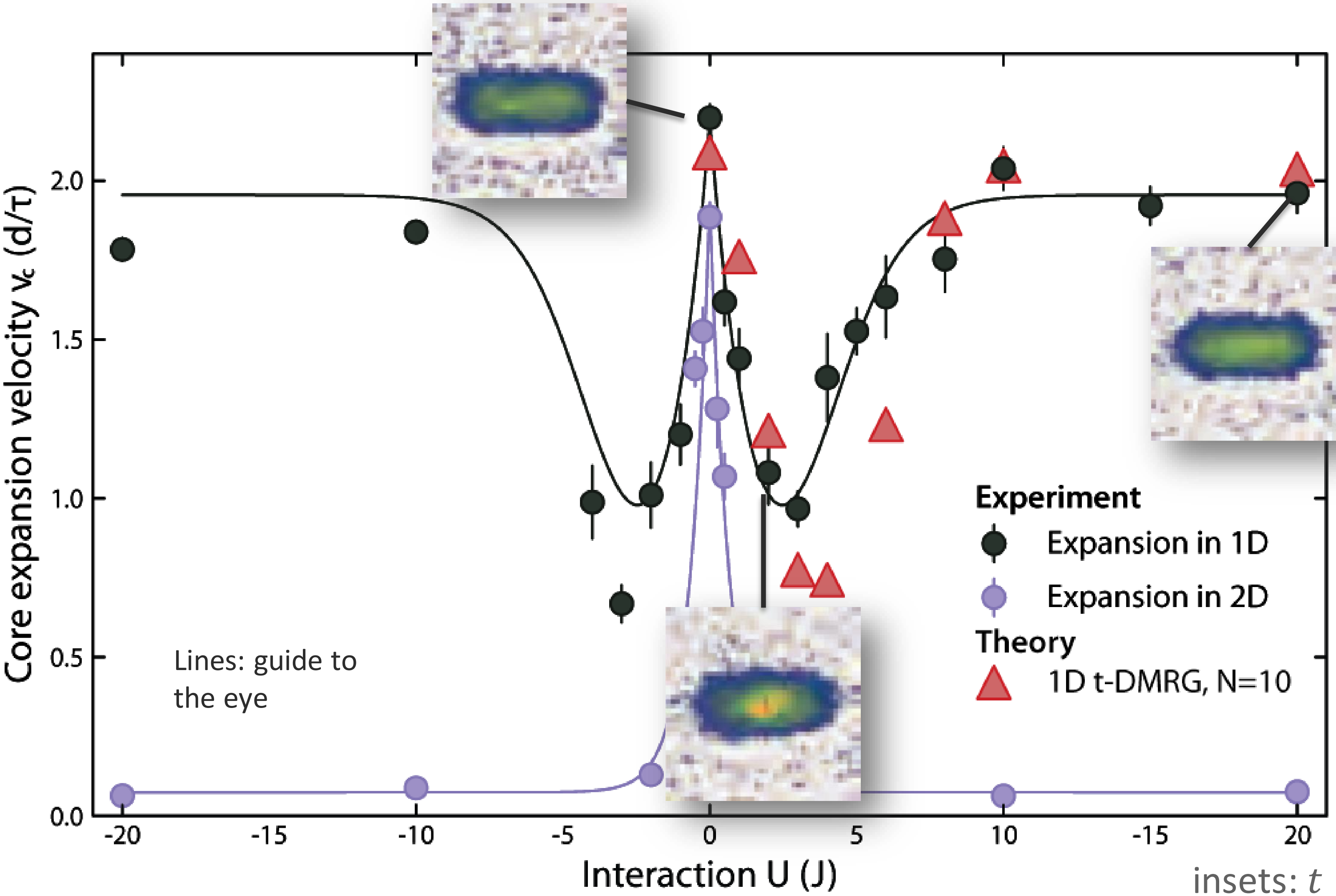
- Integrable limits:
  - Non-interacting
  - Hard-core Bosons: $U \gg J,\ n \in \{0, 1\}$

    i.e. no higher occupancies

  equivalent to non-interacting spinless Fermions

  (Jordan-Wigner transformation)

Bosonic Expansion velocities



insets: $t \approx 36\,\tau$

Onset of diffusive dynamics at intermediate *U*.

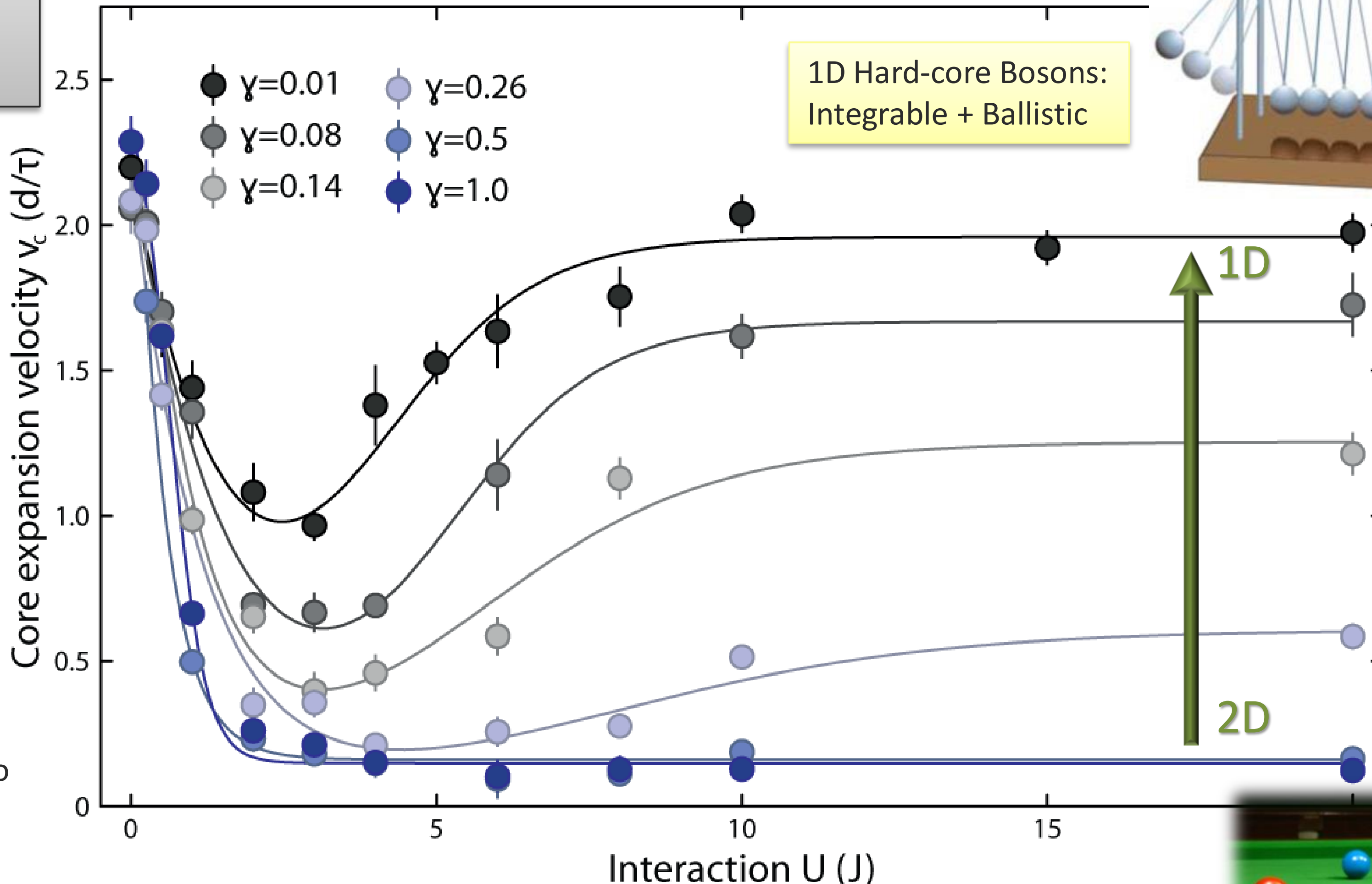
PRL 110, 205301 (2013)

1D-2D Crossover

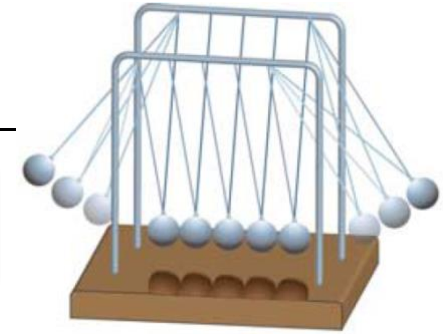
$$\gamma = \frac{J_y}{J_x}$$

1D: $\gamma = 0$

2D: $\gamma = 1$



1D Hard-core Bosons: Integrable + Ballistic



Lines: guide to the eye

- 1D: Perfect agreement with DMRG
- 2D: No exact theory available!

2D Hard-core Bosons: Thermalizing + Diffusive

PRL 110, 205301 (2013)

Alternatives to local thermalization

Many-body localization

Stability of (disorder induced) Anderson localization
in the presence of interactions (and finite energy density)

So what?

Non-ergodic behaviour!

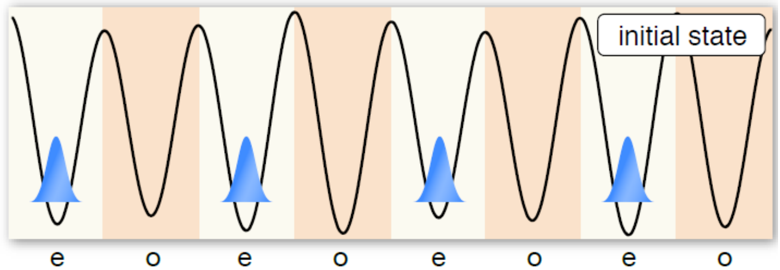
No thermalization, no standard statistical mechanics

➔ *Potential for novel long-time dynamics*

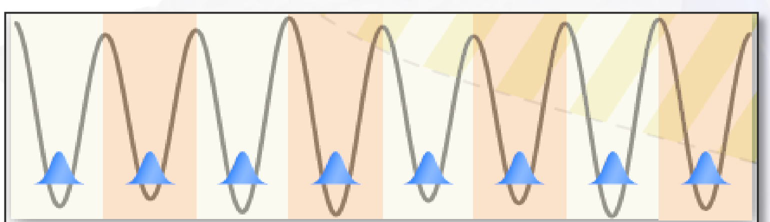
Challenge: MBL requires perfect isolation from environment
(coupling to reservoir → thermalization)

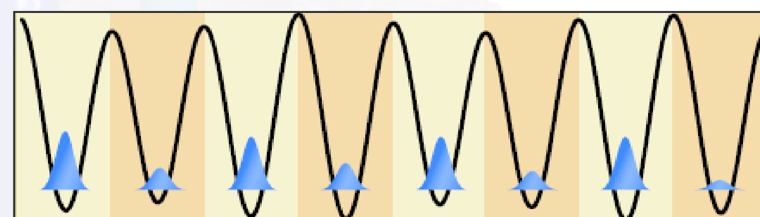
Ergodicity breaking in Many-body localization

Initial state



- fermionic ^{40}K
- free evolution



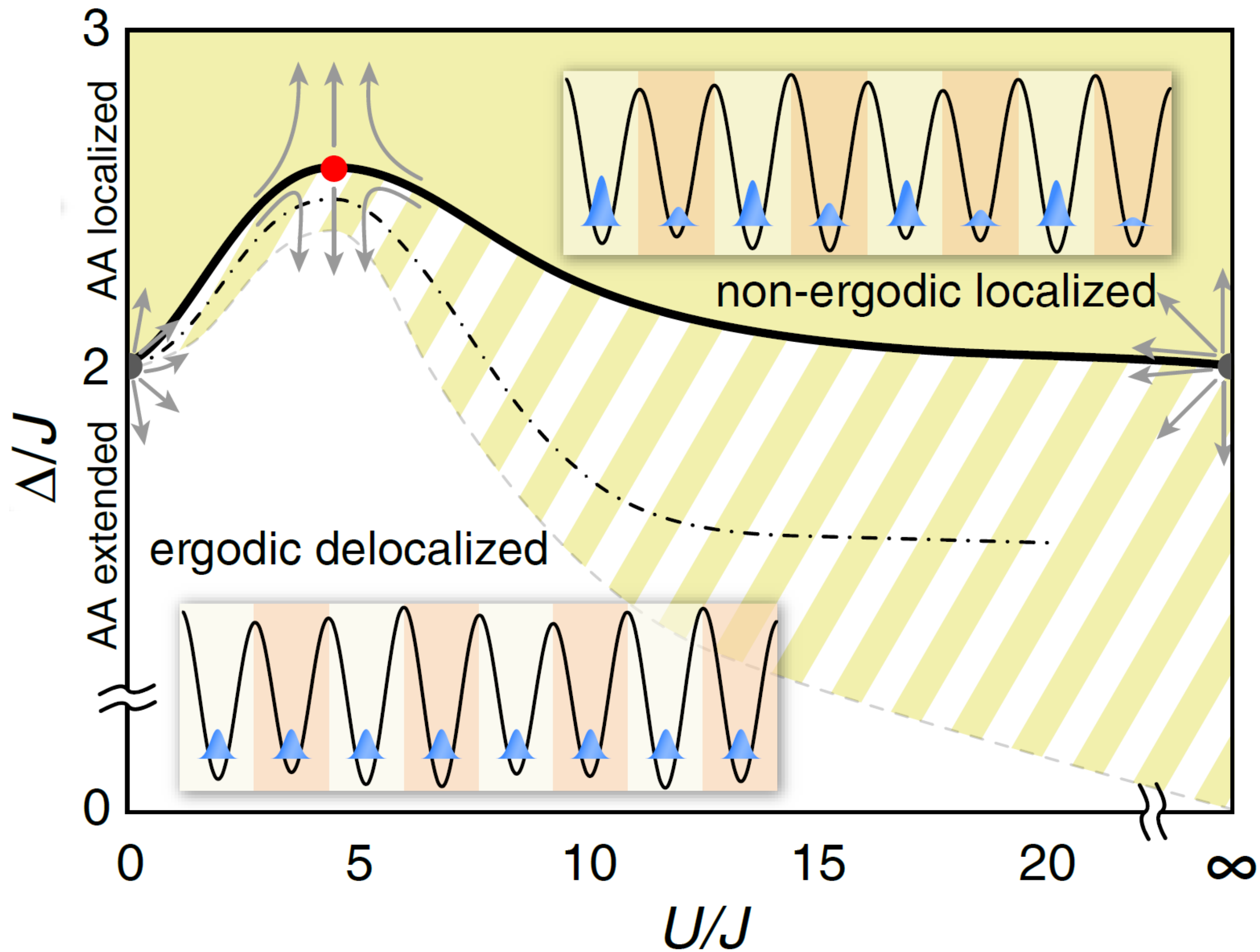


Ergodic time evolution destroys initial CDW

Persistent CDW signals non-ergodic behavior → localization

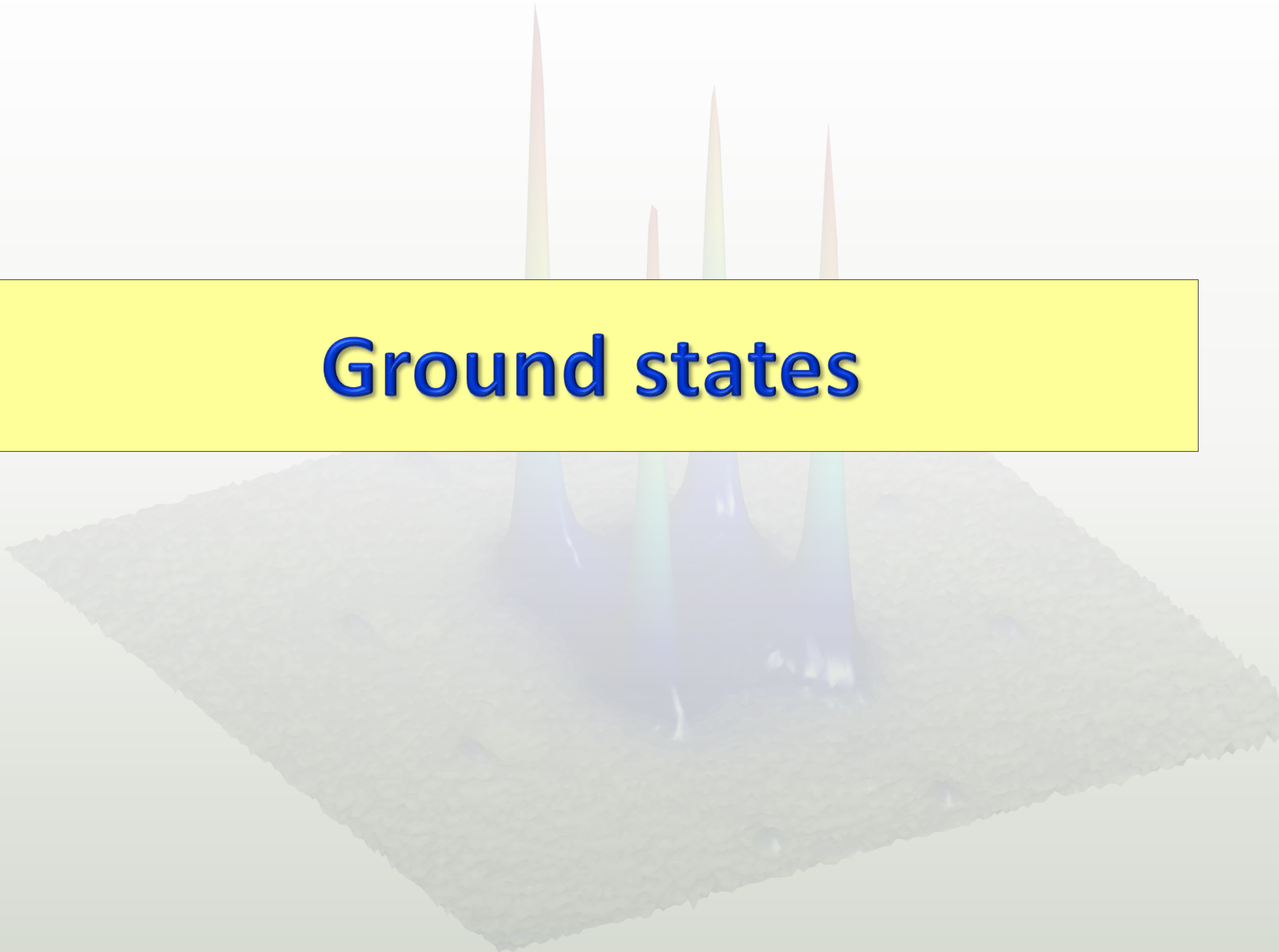
Science **349**,842 (2015)

Phase diagram



Science **349**,842 (2015)

Ground states

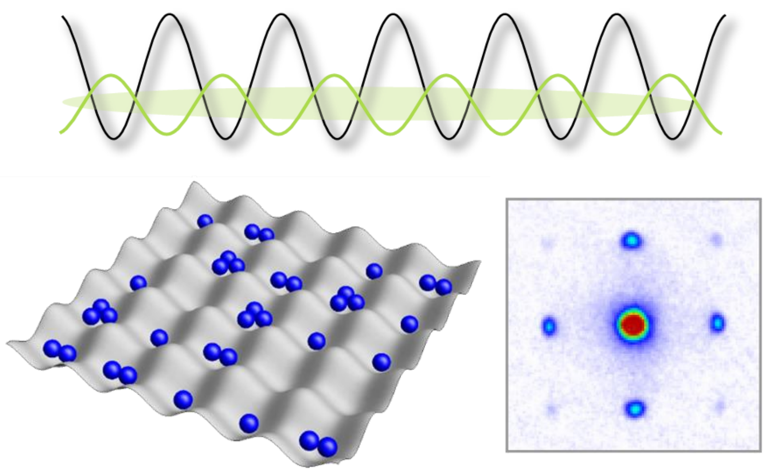


Ground states of Bose-Hubbard model

$$H = -J \sum_{\langle i,j \rangle} a_i^{+} a_j + \frac{U}{2} \sum_i n_i (n_i - 1)$$

$J \gg U$: Superfluid

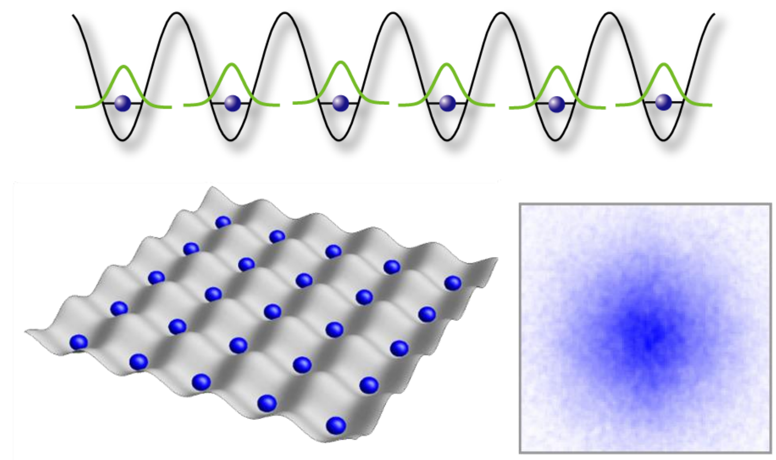
Shallow lattice



- Long-range order
- Phase coherent
- Gapless excitations

$J \ll U$: Mott Insulator

Deep lattice

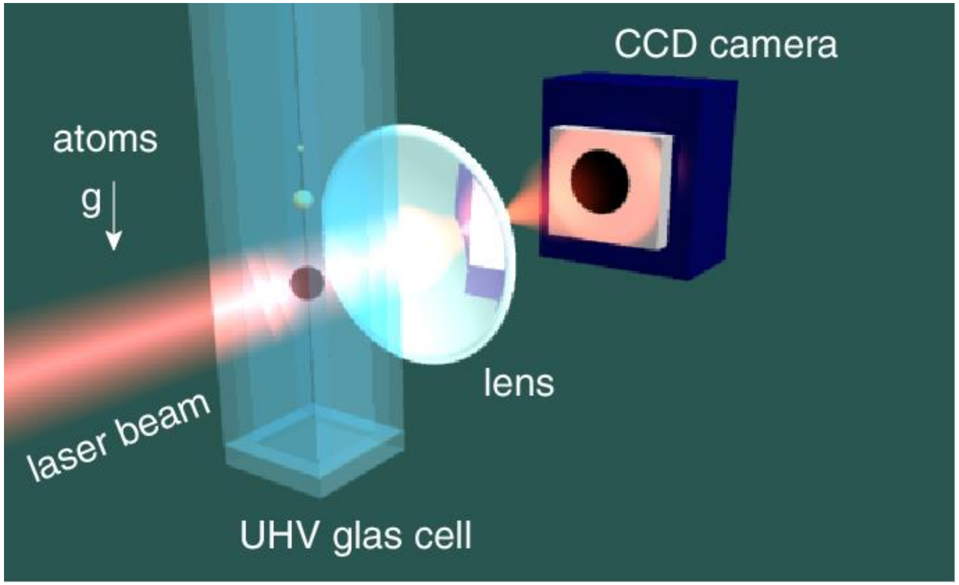


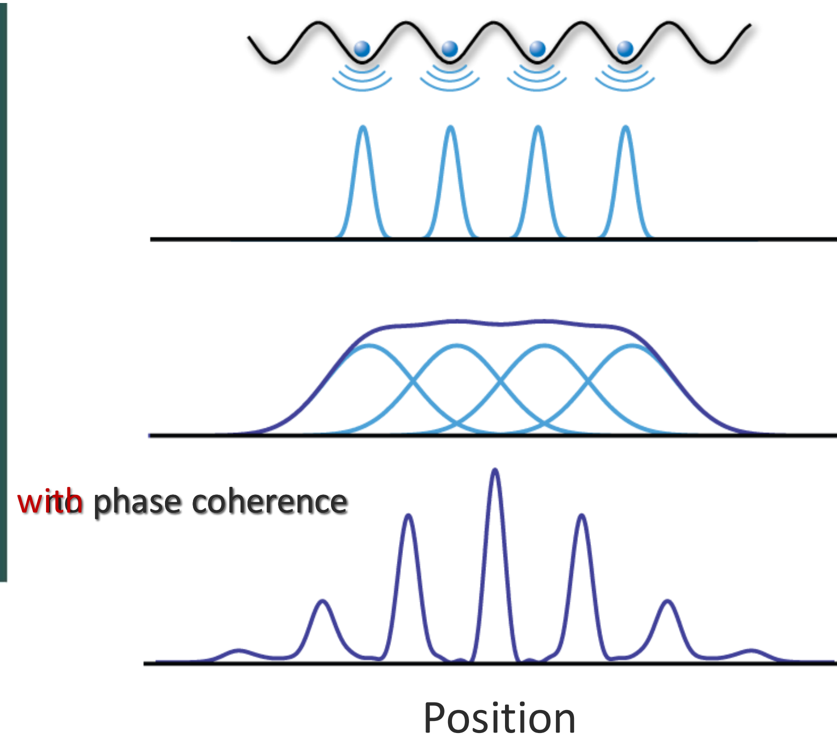
- No Phase coherence
- Gapped

Cubic Lattice:
3D: $U_c \approx 29.3\,J$ at unity filling
1D: $U_c \approx 3.3\,J$ "

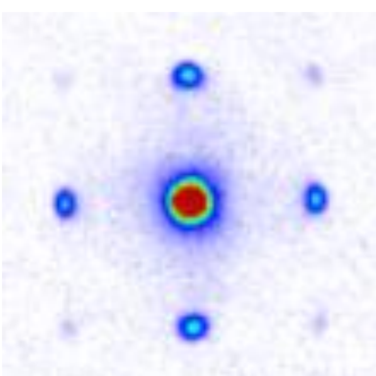
First observation:
Greiner *et al.*
Nature **415**,39 (2002)

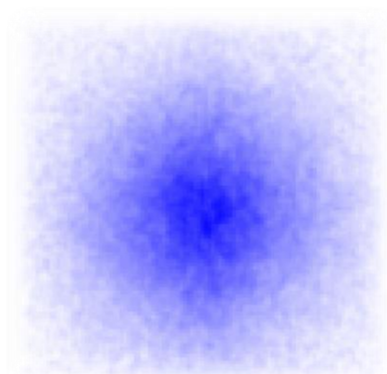
Absorption imaging after time-of-flight





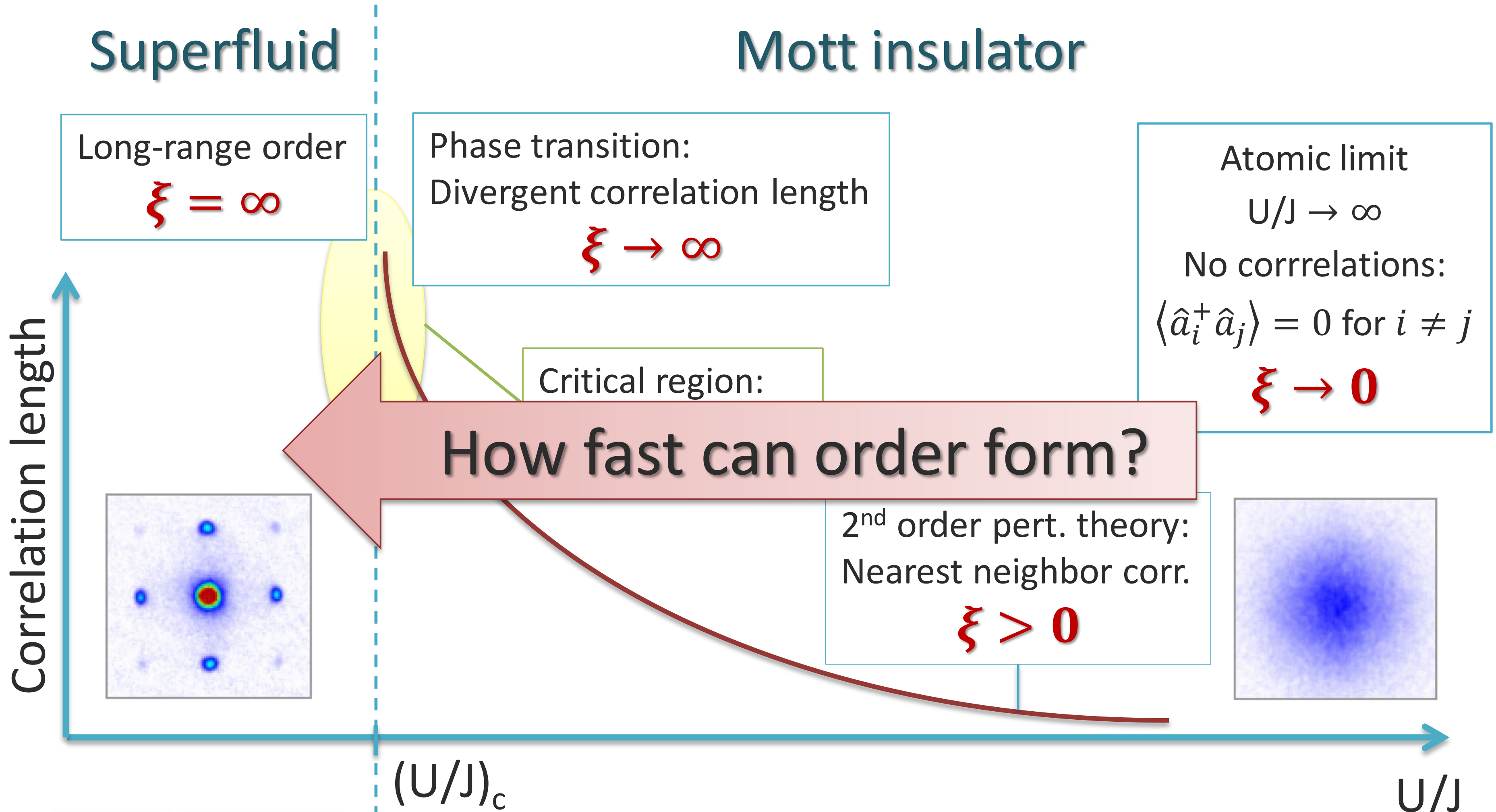
Absorption creates shadow that can be recorded by a CCD camera

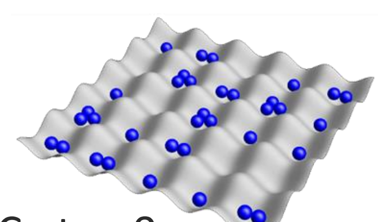




How fast can order form?

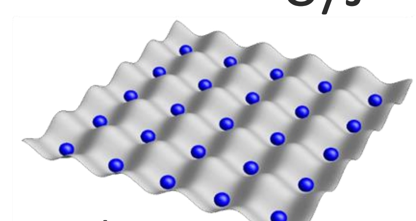
Bose-Hubbard model at n=1





BEC at q=0

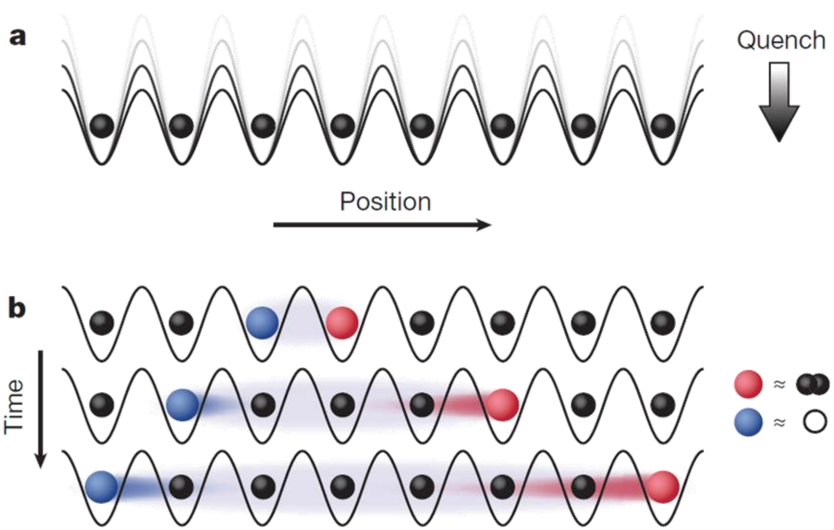
$$H = -J \sum_{\langle i,j \rangle} a_i^+ a_j + \frac{U}{2} \sum_i n_i (n_i - 1)$$

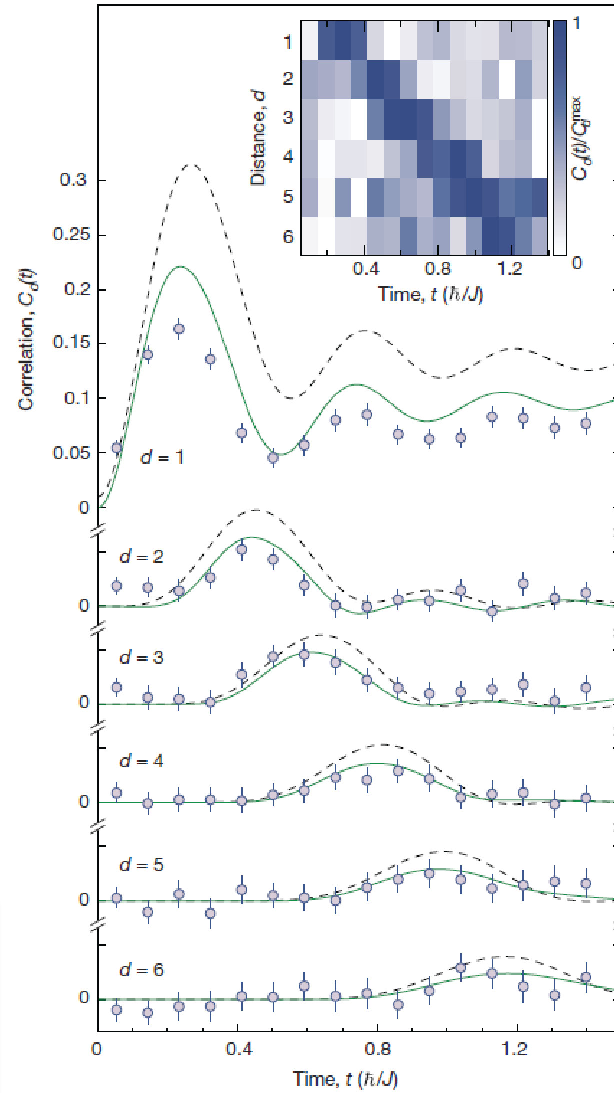


Fock state

Information spread in BHM

- Information spreads via quasiparticles at finite velocity (almost) Lieb-Robinson bound

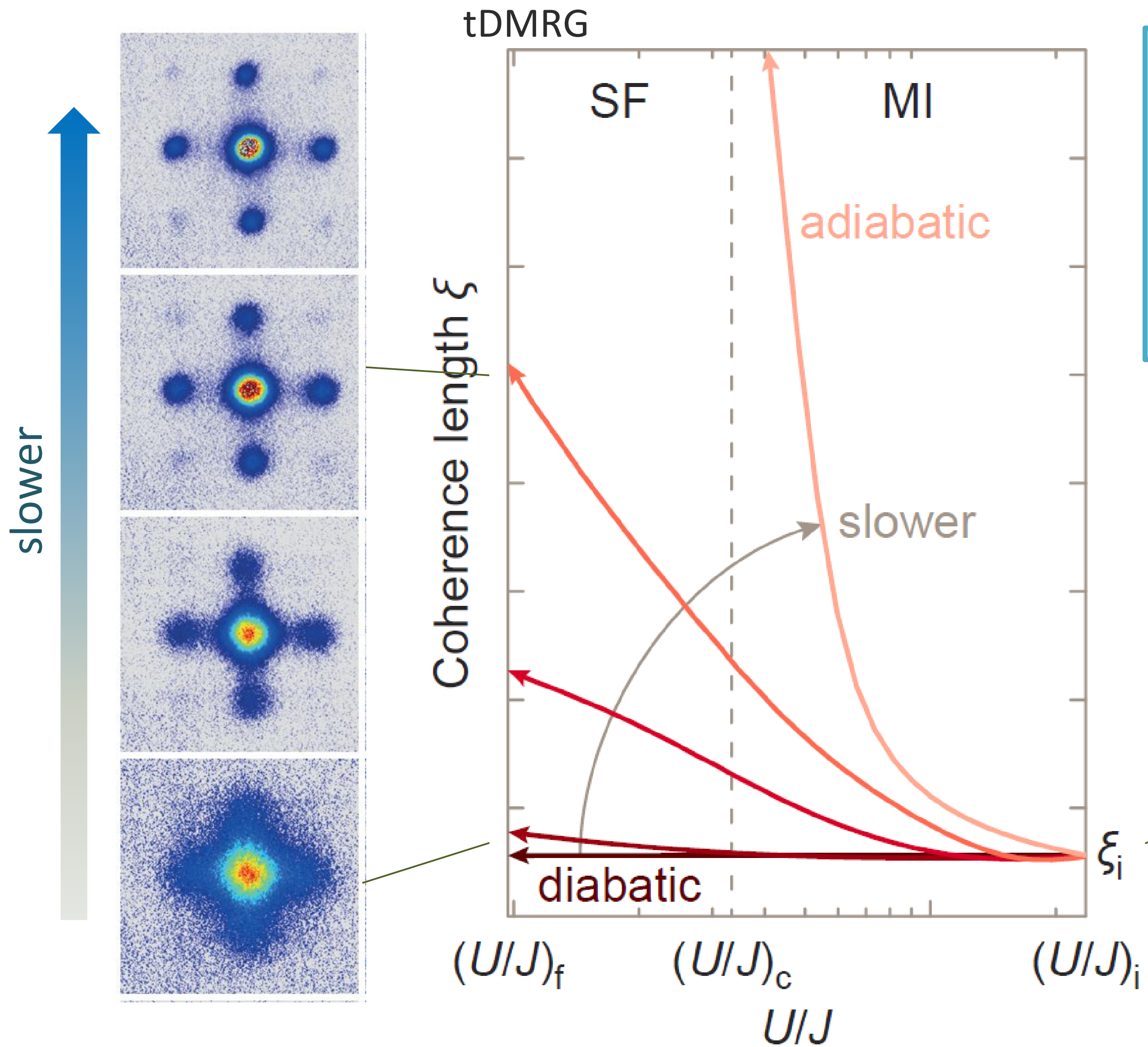




- NOW: Continuous quench
- Quasi-particles are continuously created
- Many quasi-particles → Interactions?

(M. Cheneau et al Nature **481**, 484 (2012))

Mott Insulator to Superfluid transition in BHM

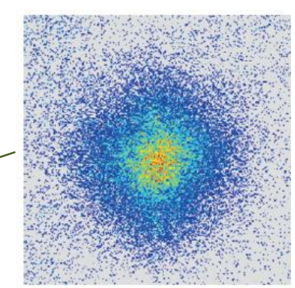


Mott Insulator

lattice ramp

Superfluid regime

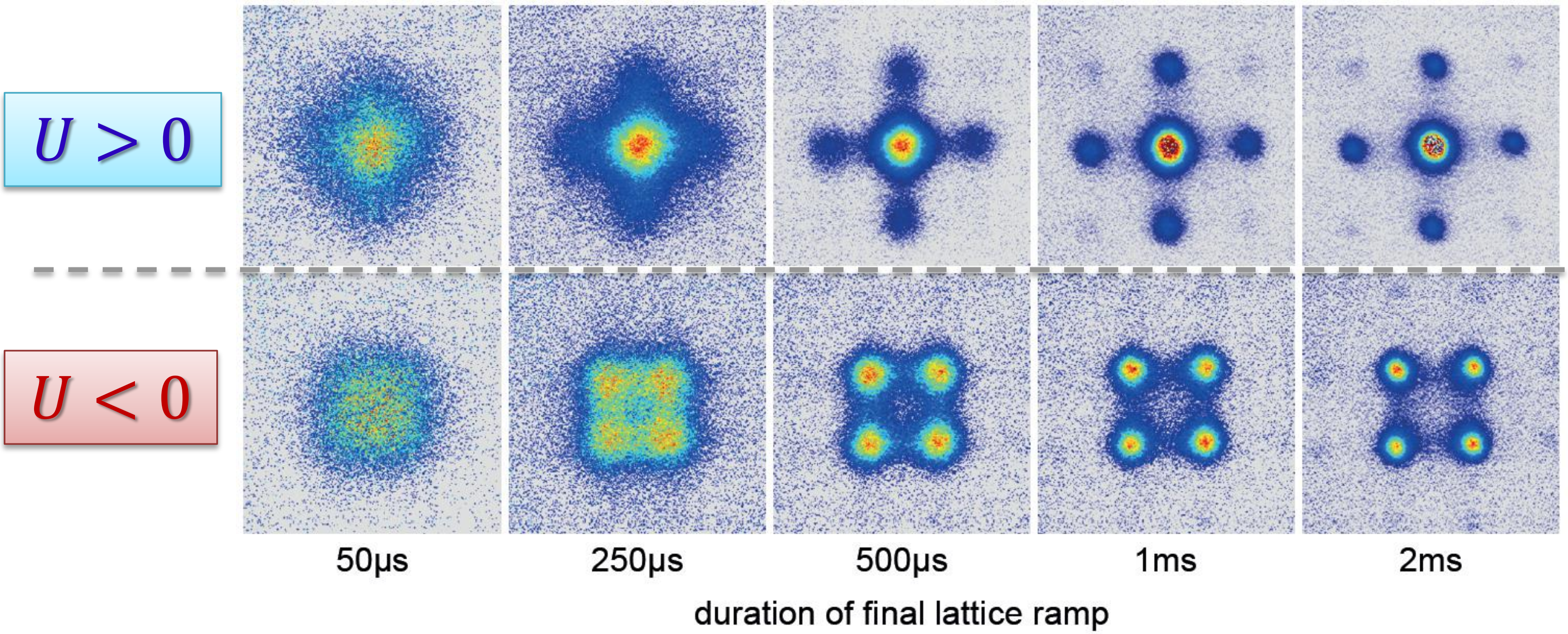
Instantaneous



➔ Establishing coherence requires time

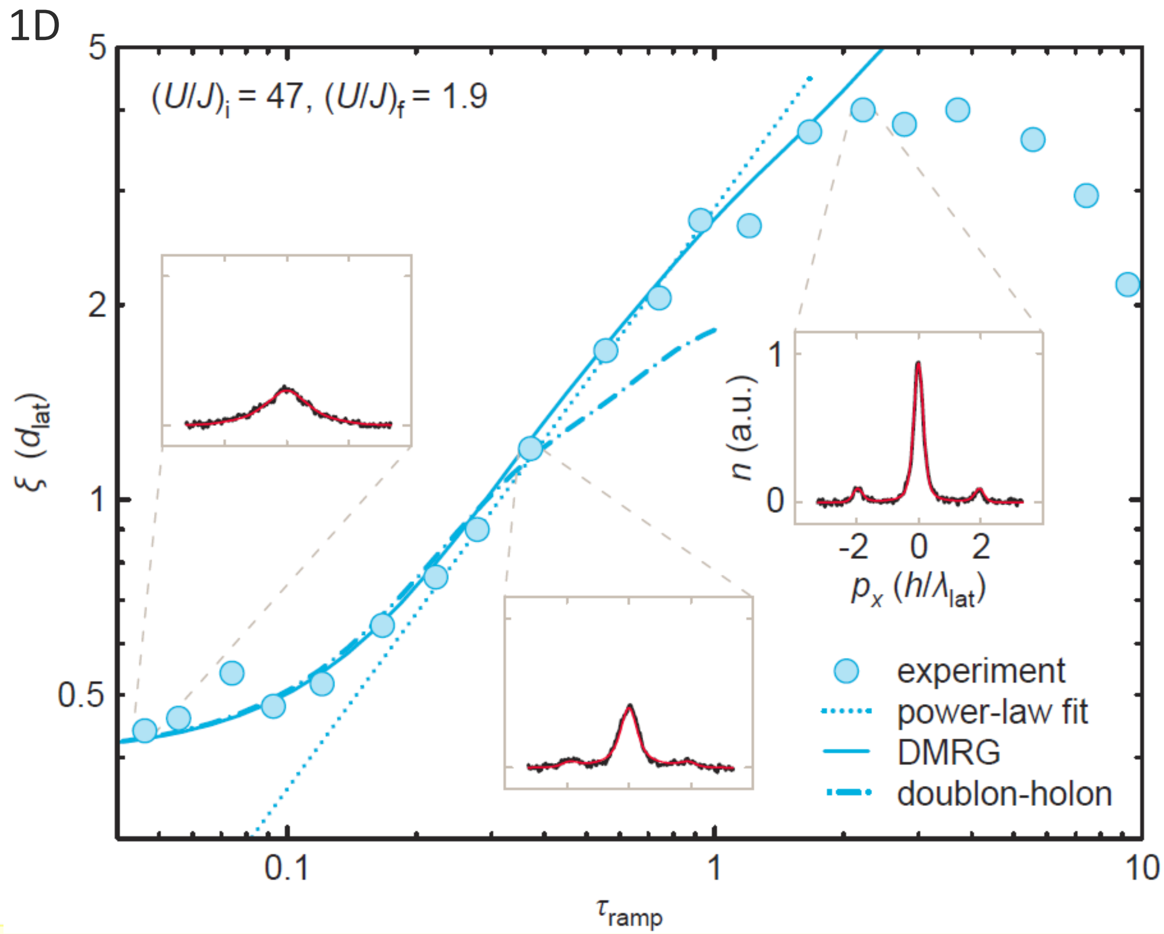
Related Experiments:
Greiner, DeMarco, …

Emergence of Coherence: 2D

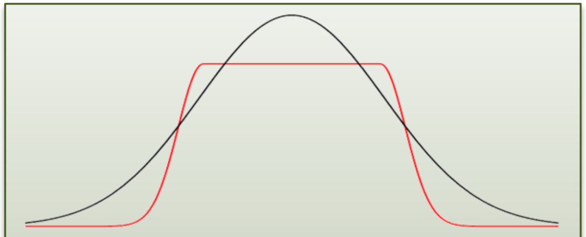


Observe **identical timescales** for U>0 and U<0!
➔ See **generic** behavior of phase transition

Coherence length vs quench time: 1D



Large times:
Dephasing due
to trapping potential



Initial and final ground-state density profile

Trap requires mass redistribution

- Beyond free quasi-particles
- Perfect agreement with DMRG (homogeneous system)
- Power-law behavior over one order of magnitude

S. Braun *et al.*, PNAS **112** 3641 (2015)

Kibble Zurek mechanism

- What can we say about this if we don‘t have a dynamical theory?
- Adiabatic evolution far away from phase transition + dynamics frozen out at phase transition (critical slowing down)
- Kibble-Zurek approximation: adiabatic – impulse – adiabatic (+variants)

Within Kibble-Zurek approximation:

- Only need to know the *transition point* from adiabatic to sudden
- Correlation length at the beginning of impulse stage:

$$\xi \propto \epsilon^{-\nu} \propto \tau_Q^{\frac{\nu}{1+\nu z}}$$

- Scaling governed by critical exponents: ν, z

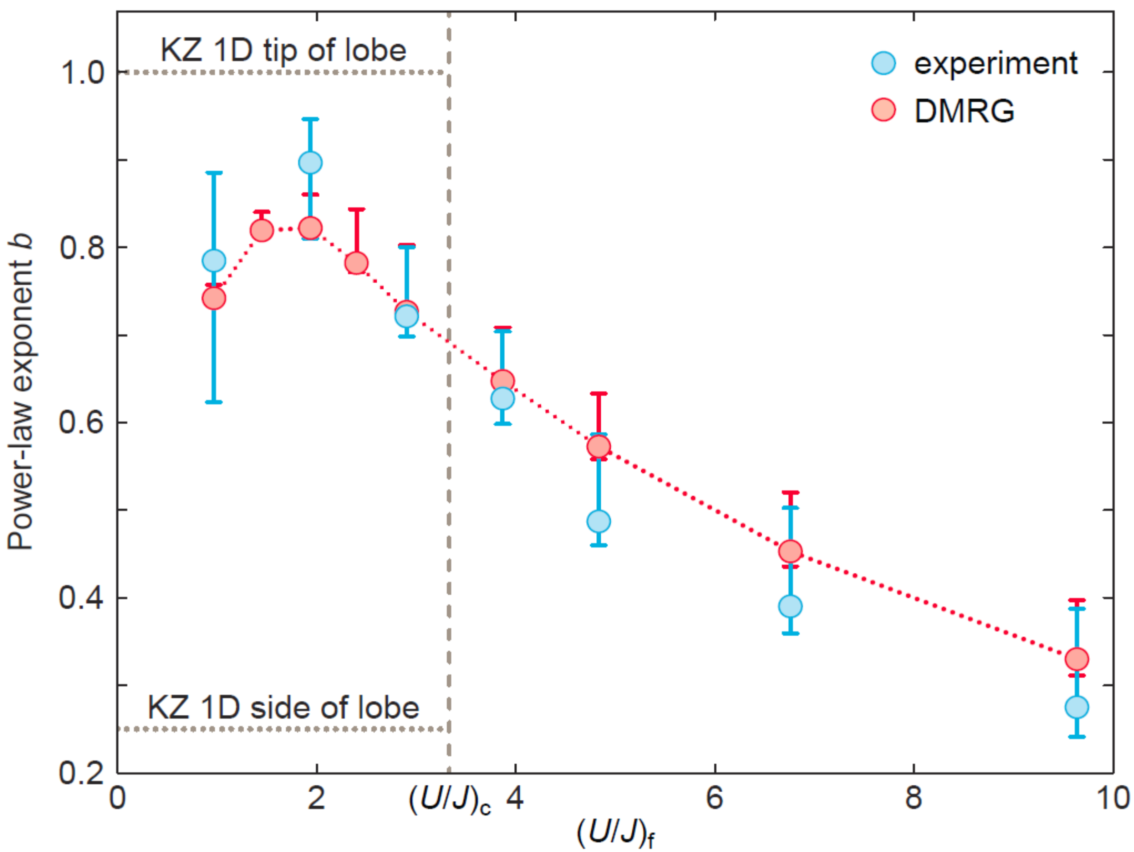
→Assumes that transition points lie in quantum critical region!

1D Exponents

- 1D: Tip of Mott Lobe: $z = 1 \;\; \nu \to \infty$ $\Rightarrow \xi \propto \tau_Q$

Caveat: Kosterlitz-Thouless transition, expect $\nu \to \infty$ only for extremly slow ramps

- 1D: side of Mott Lobe: $z = 2 \;\; \nu = \frac{1}{2}$ $\Rightarrow \xi \propto \tau_Q^{\frac{1}{4}}$

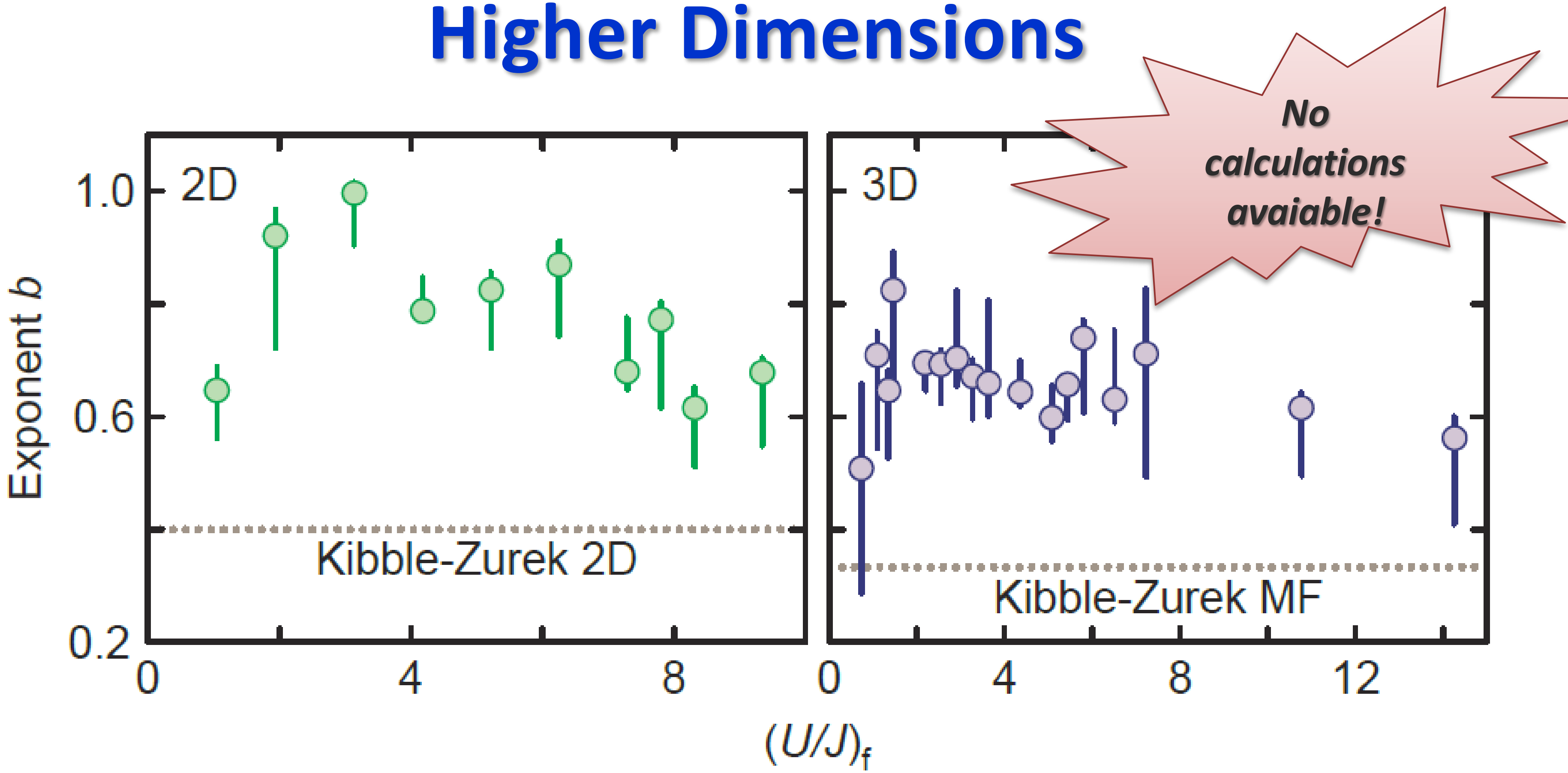


- Exponents depend on initial and final *U/J*
- *Bending down for small interactions*

$\Rightarrow$ KZM not sufficient!

S. Braun *et al.*, PNAS **112** 3641 (2015)

Higher Dimensions



- Same qualitative behaviour – independent of dimension
- No Kosterlitz-Thouless transition

  → KZM predicts pure power law with fixed exponent
- Loose adiabaticity already outside of critical region!

  → Full complexity goes beyond critical behaviour!

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