Hydrodynamic theory of quantum fluctuating superconductivity

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Based on: arXiv/1602.08171 [cond-mat.supr-con]

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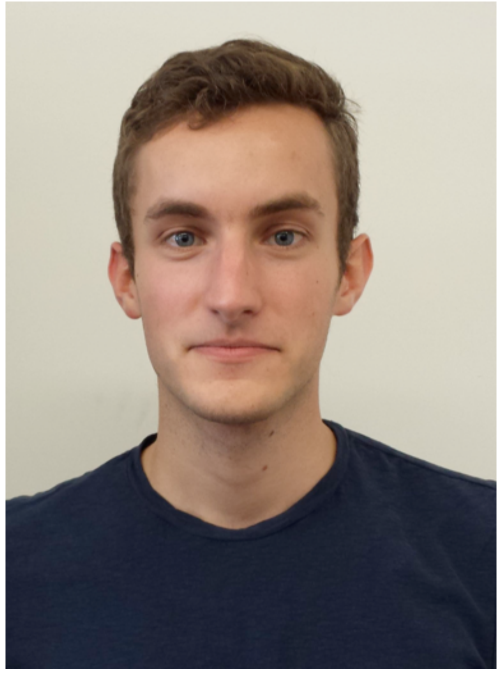

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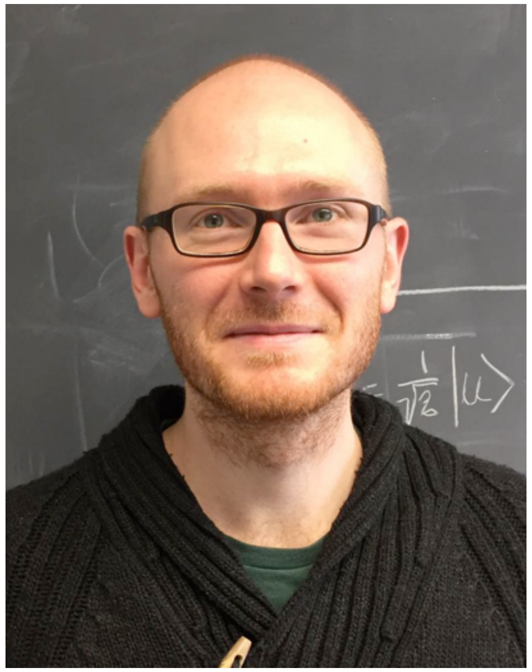

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Hydrodynamic description of conventional metals

- Hydrodynamics:
  → Universal low energy, long wavelength physics.
  → Conserved charges, their currents, Goldstone bosons.

- Conservation law:
$$\frac{\partial \rho}{\partial t} + \nabla \cdot j = 0$$

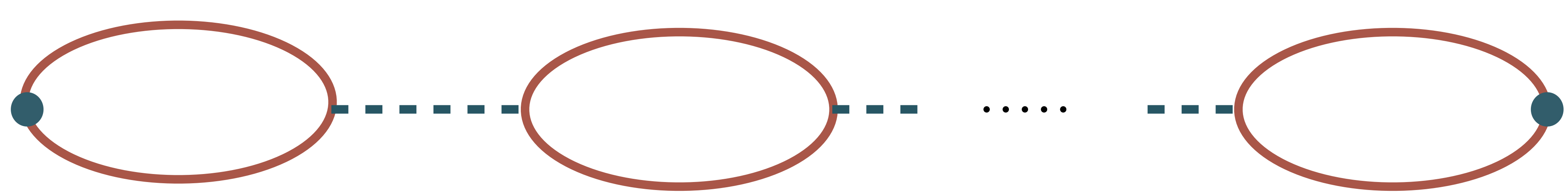
- Constitutive relation (derivative expansion):
$$j = -D\nabla\rho + \cdots$$

- Conductivity (Einstein relation):
$$j = -D\nabla\rho = -D\frac{\partial\rho}{\partial\mu}\nabla\mu = D\chi\, E = \sigma E$$

Comment on screening by Maxwell fields

- Charge in a metal does not diffuse, it decays exponentially.
- This comes from solving Maxwell's equations + Ohm's law.
- The Einstein relation for the conductivity still holds.
- σ measured with respect to total, not external, electric field:

$$j = \sigma E_{\text{tot}} = \sigma \frac{E_{\text{ext}}}{\epsilon(\omega, k)} = \frac{\sigma E_{\text{ext}}}{1 - \frac{1}{k^2}\chi(\omega, k)} = \frac{-i\omega D\chi}{i\omega - D(k^2 + \chi)} E_{\text{ext}}$$



Superfluid hydrodynamics

- Phase ϕ of the order parameter appears in hydrodynamics.
- $u_\phi = \frac{1}{m}\nabla\phi$ is the superfluid velocity.
- 'Josephson relation':

$$m\frac{\partial u_\phi}{\partial t} = \nabla\frac{\partial\phi}{\partial t} = -\nabla\mu + \cdots$$

- Constitutive relation:

$$j = \frac{\rho_s}{m}u_\phi - D\nabla\rho + \cdots$$

- (super-)Conductivity:

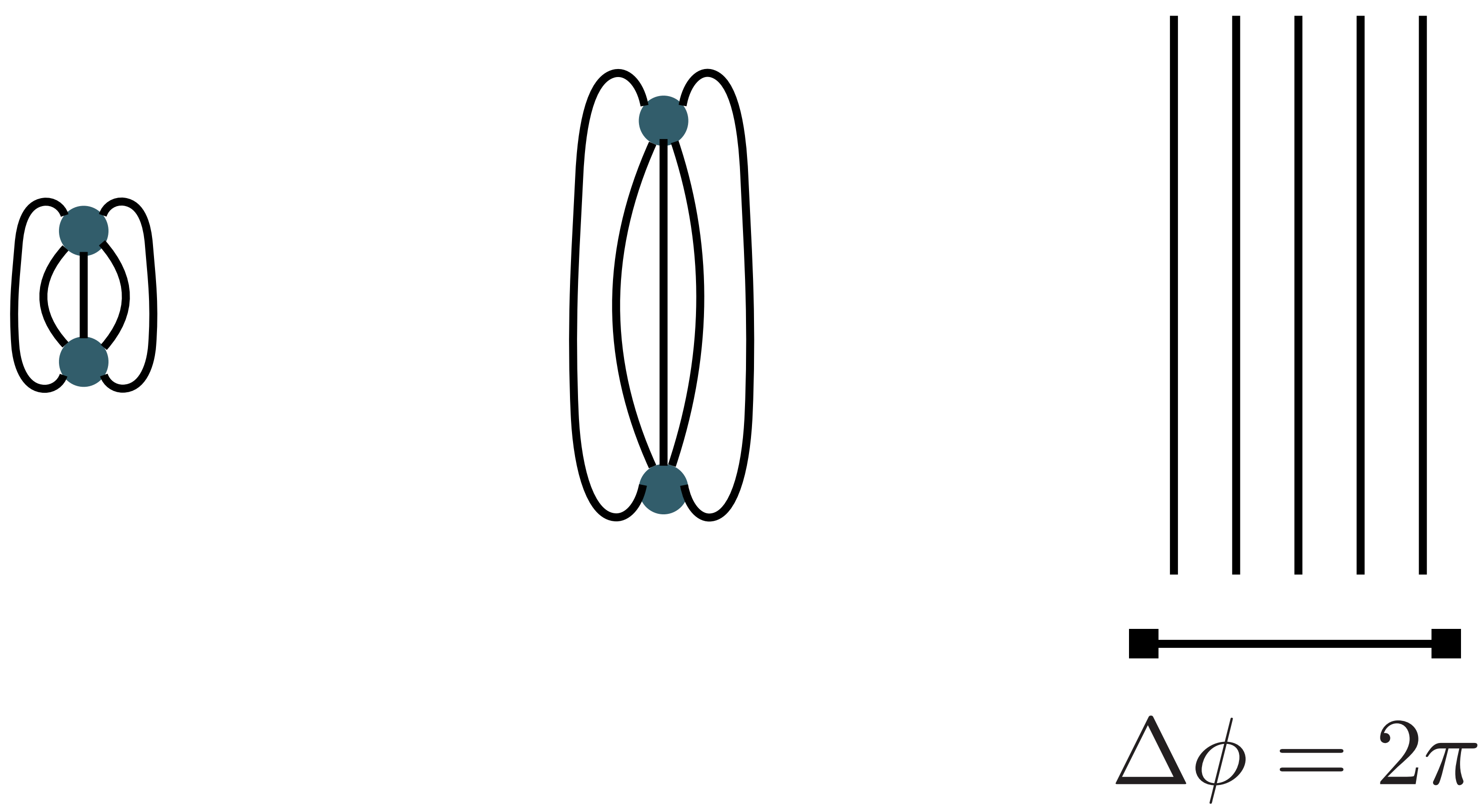
$$j = -\left(\frac{\rho_s}{m^2}\frac{i}{\omega} + D\chi\right)\nabla\mu = \left(\frac{\rho_s}{m^2}\frac{i}{\omega} + D\chi\right)E = \sigma(\omega)E$$

Superconductivity

- ∞ conductivity because: diffusion → second sound mode.
- In a superconductor, the U(1) symmetry is gauged, i.e. coupled to electromagnetism.
- This gaps out the Goldstone/sound mode in the same way the diffusive mode was previously gapped.
- However, the conductivity is, as before, measured with respect to the total electric field. So the unscreened (superfluid) hydrodynamics determines the conductivities.

Vortices and supercurrent relaxation

- In two space dimensions, above picture incomplete.
- Motion of vortices can wind and unwind the supercurrent.



- Expect supercurrent relaxation rate Ω:

$$\sigma(\omega) = \frac{\rho_s}{m^2}\frac{1}{-i\omega + \Omega}$$

Vortices and supercurrent relaxation

- This problem is well understood in some regimes:
  → Thermal BKT proliferation of vortices above T_{BKT}.

- Classical picture: vortices pushed across sample by 'superfluid Magnus force'
  → The core of the vortices is in the normal state.
  → Therefore, motion of vortices creates dissipation.
  → Get

$$\Omega \sim \frac{n_f A_v}{\sigma_{\mathrm{n}}}$$

[Bardeen-Stephen '65]

- Much controversy, however, about whether (quantum) phase-disordered superconductors exist at T = 0.

[review: Phillips-Dalidovich '03]

In the remainder

- Lightening overview of some experiments.
- Develop a fully quantum effective field theoretic formalism for the conductivity of phase-disordered superconductors.
- Illustrate formalism with two examples:

  (i) 'Check': Elegant (re)derivation of Bardeen-Stephen result.

  (ii) Phase disordering by a Chern-Simons interaction ['topologically ordered superfluid vortex liquid'].

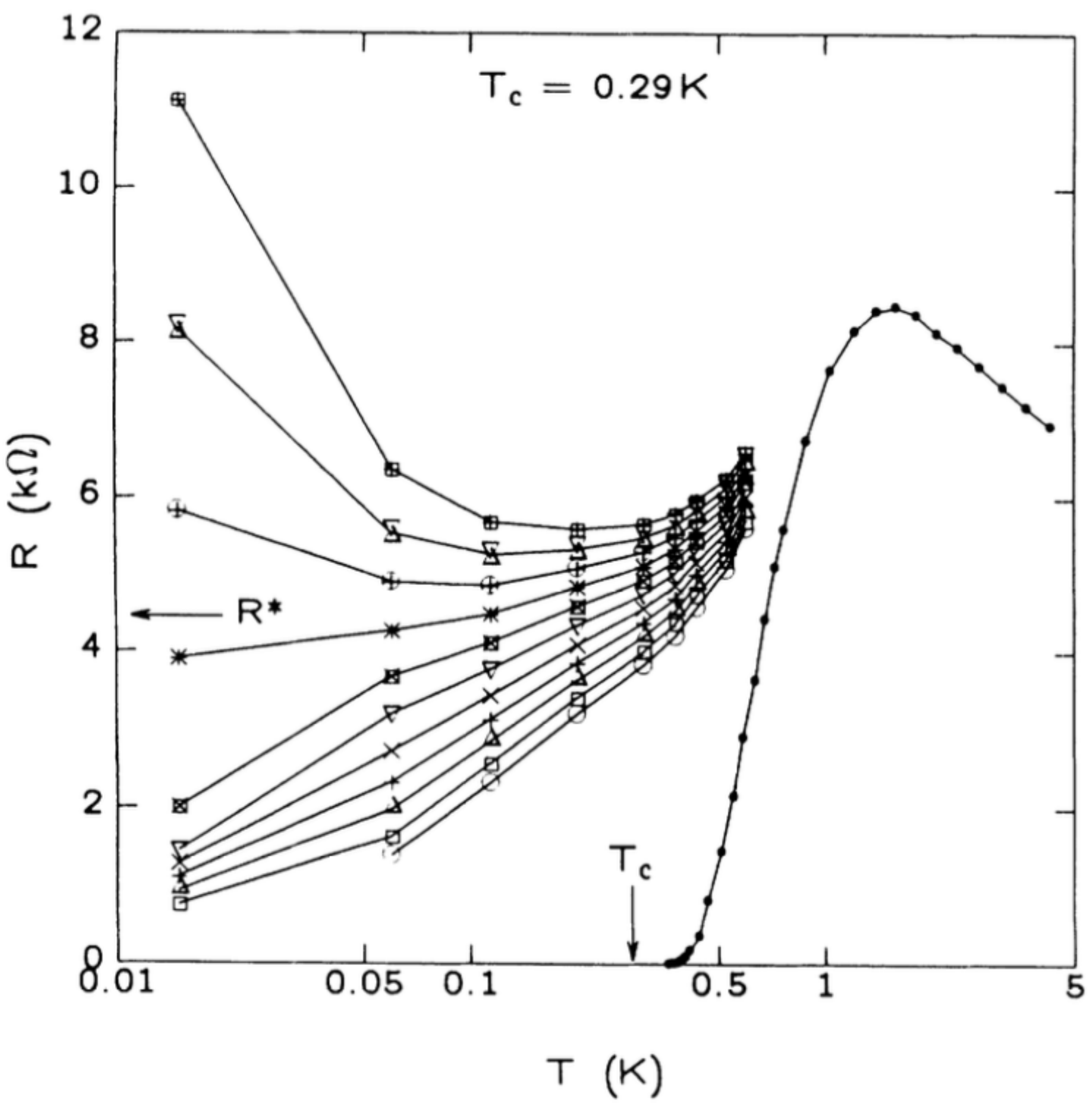
Superfluid-insulator transitions

- In two (spatial) dimensions, conventional theory suggests that as $T \to 0$ electrons will either localize or pair up.
- That is, the phase of matter one expects to find is either an insulator or a superconductor.
- Indeed, early experiments suggested that disordered thin films undergo superconductor-insulator transitions as a function of magnetic field or thickness ($\approx 1/\text{disorder}$).

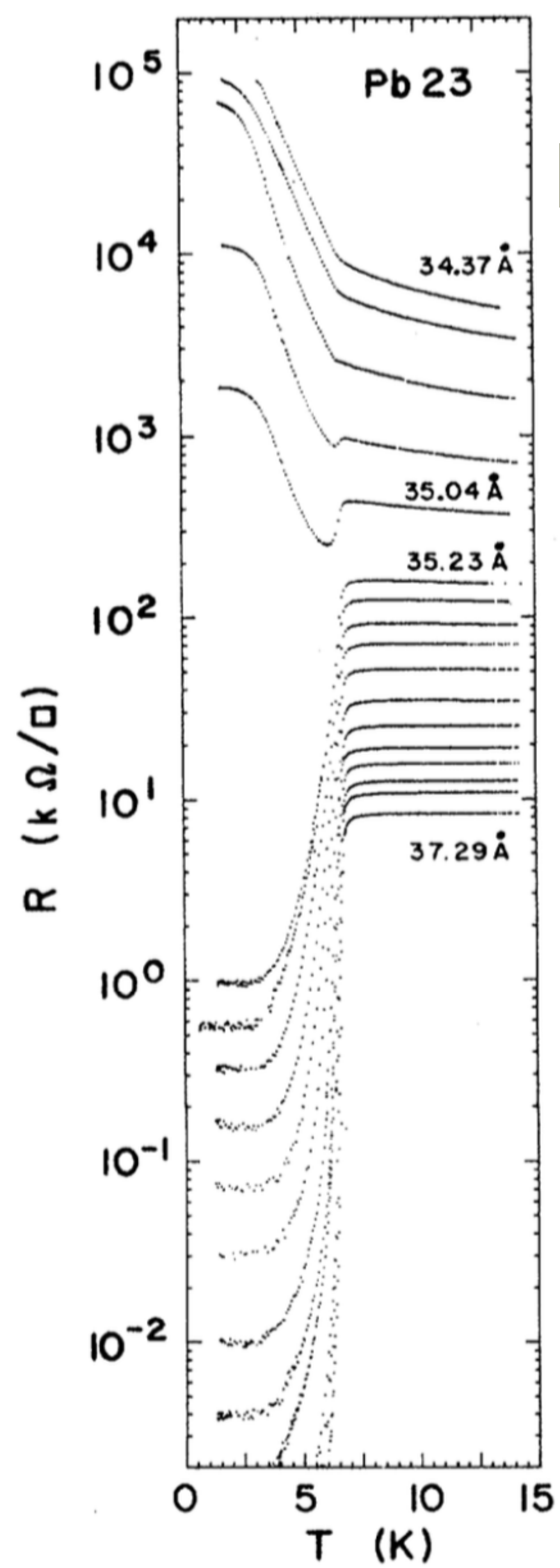
Destroys superconductivity

Favors localization

Superfluid-insulator transitions



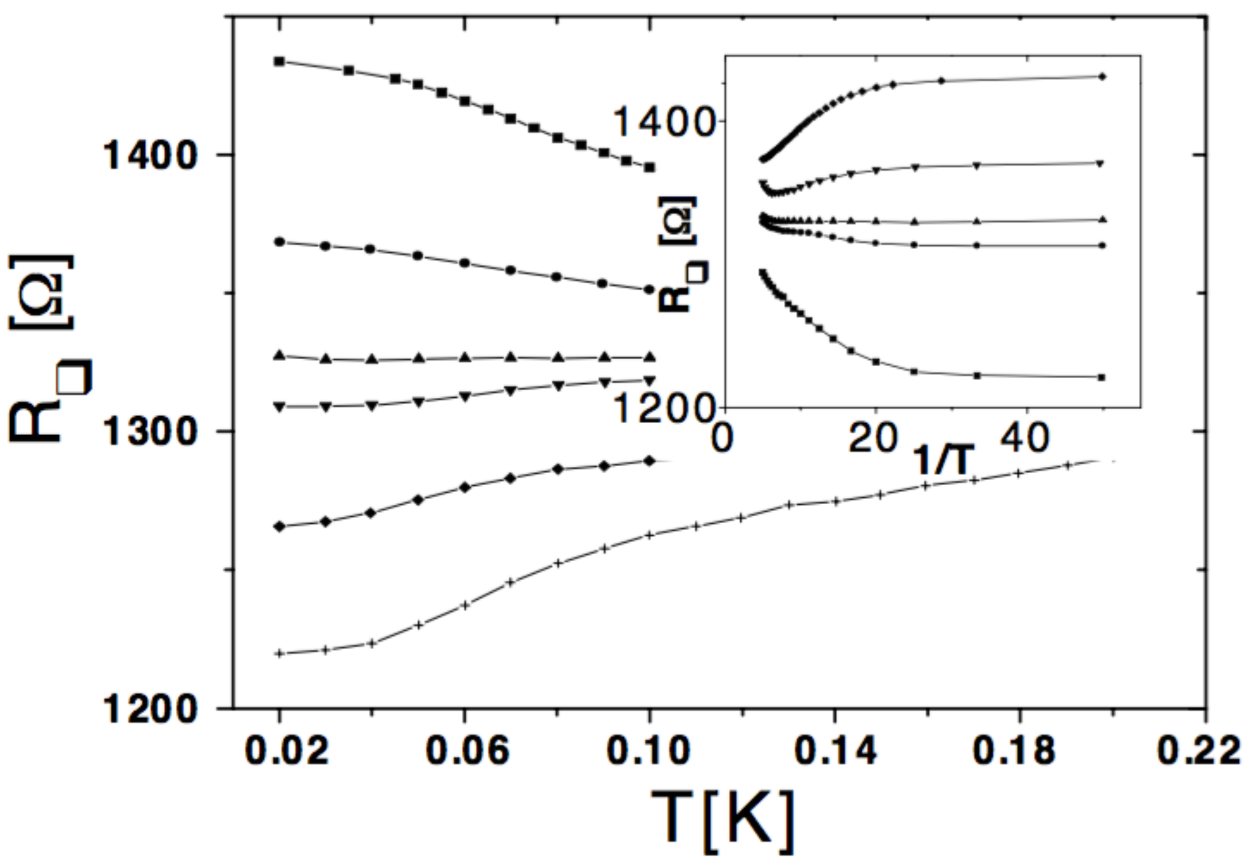
[Hebard and Paalanen ’90, α-InO_x]

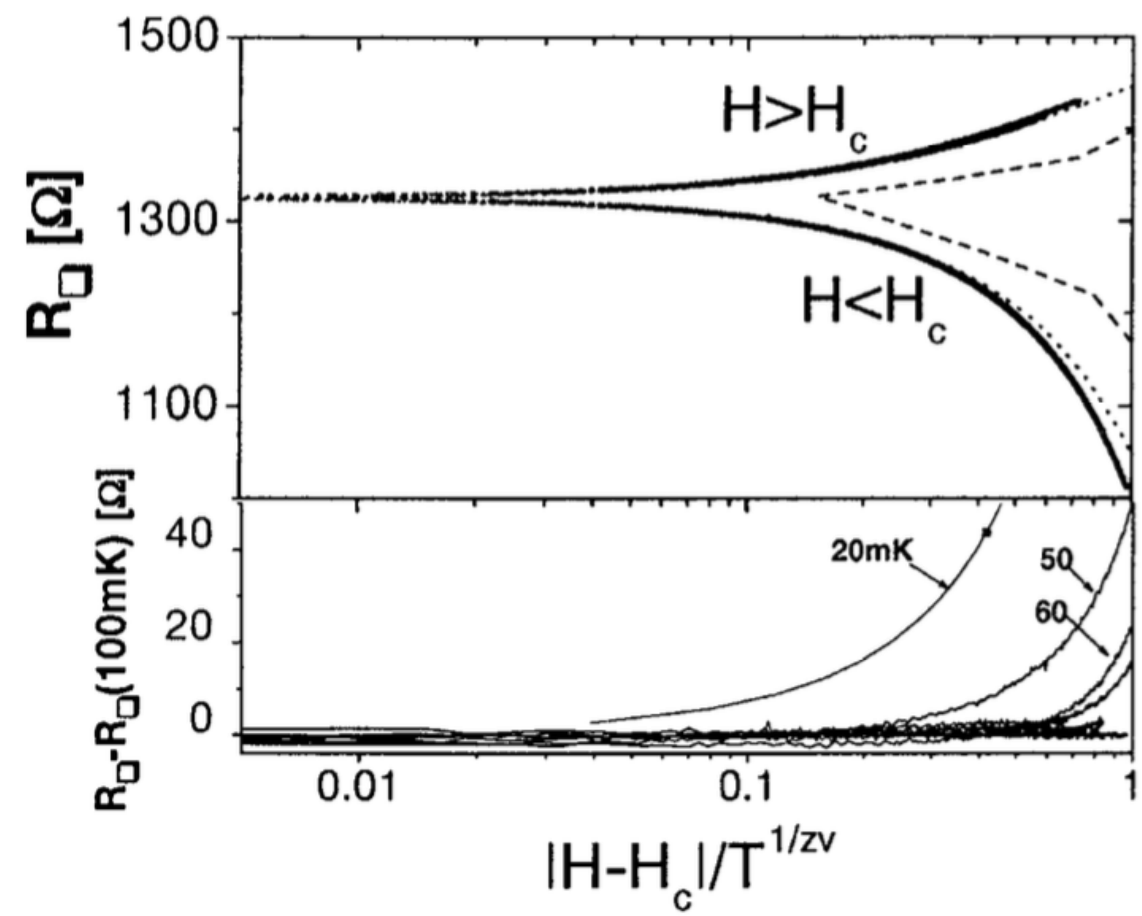


[Jaeger et al. ’89, Pb]

Metallic phases in two dimensions

- Problematically for ‘conventional’ understanding, in weakly disordered films a metallic phase intervenes (at T = 0!) between the superconductor and insulator.





[Mason, Kapitulnik ’99, α-MoGe]

Metallic phases in two dimensions

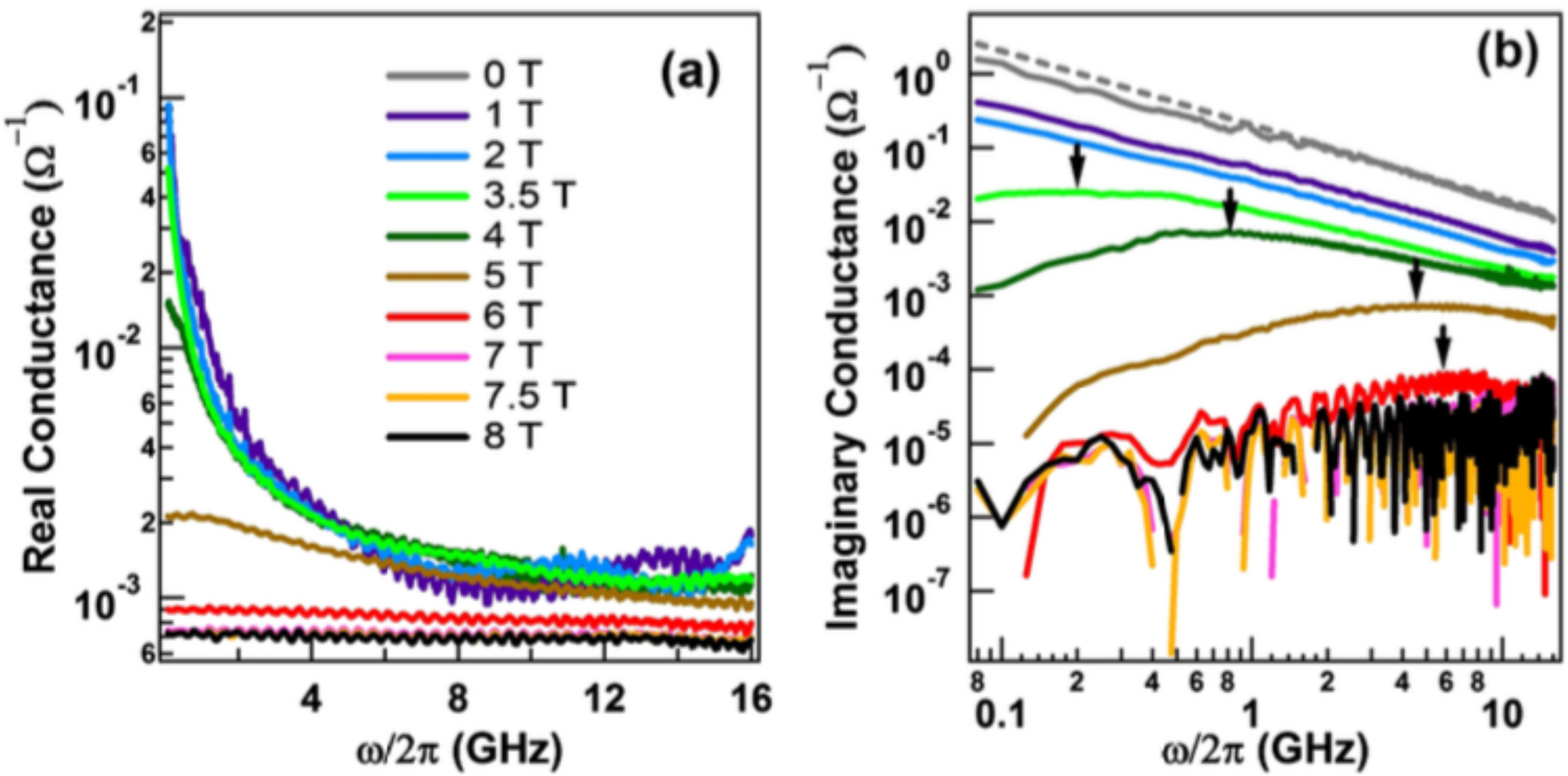
- Often, the residual resistivity of the metallic phase is much smaller than the “normal state” resistivity of the material at temperatures above the “mean field” superconducting temperature.

- Suggests the <u>low energy degrees of freedom of the metallic phases are not the normal state quasiparticles</u>.

- Natural to think of as “failed superconductors” where (quantum!) phase fluctuations have destroyed phase coherence.

Metallic phases in two dimensions

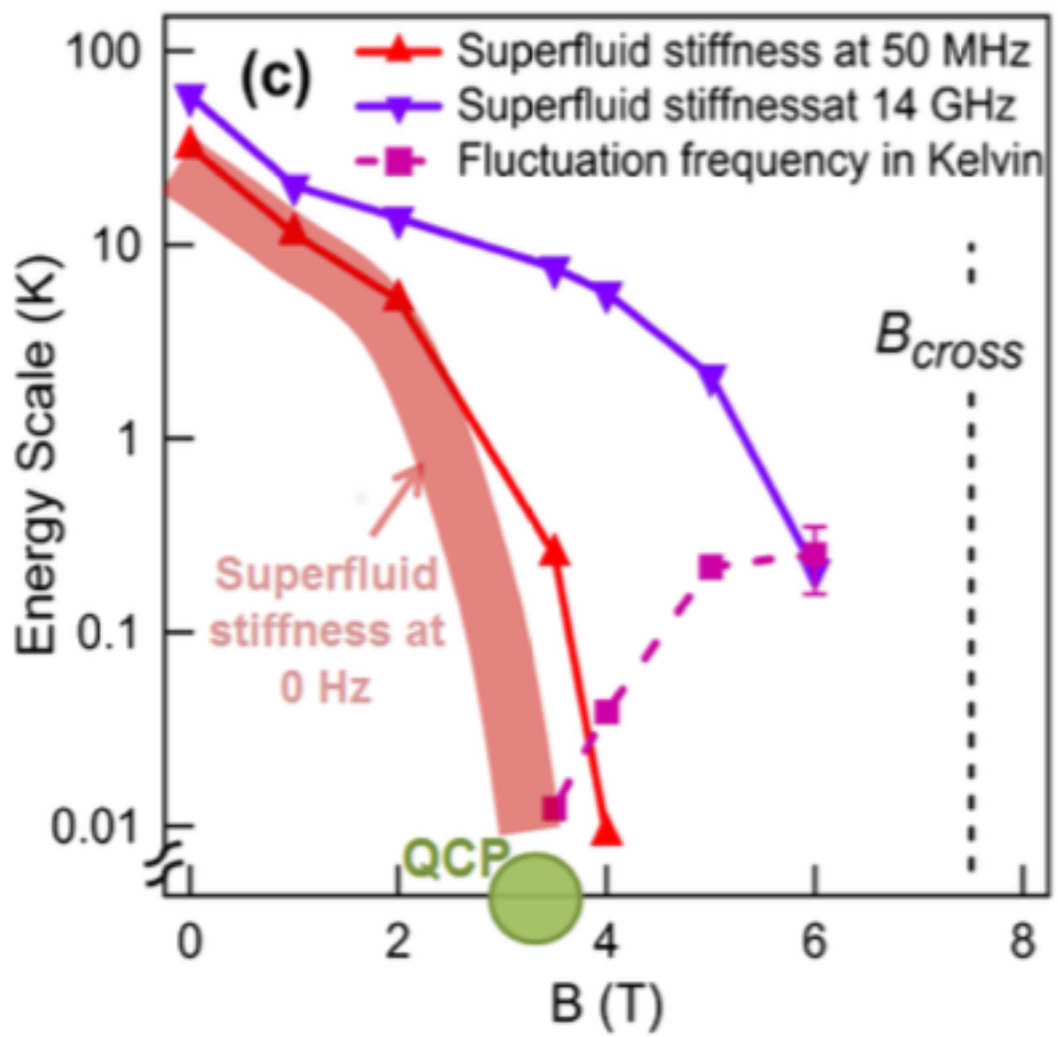
- Direct motivation for our work: observation of a Drude-like peak in the metallic phase of InO_x.



[Liu, Pan, Wen, Kim, Sambandamurthy, Armitage '13]

Metallic phases in two dimensions

- The width of the Drude-like peak goes to zero at the same magnetic field where superconductivity appears.



[Liu, Pan, Wen, Kim, Sambandamurthy, Armitage '13]

What we will and won’t do

- We will not answer the question of whether some given microscopic model remains metallic at T = 0.

- Our point is that universal aspects of this problem can be isolated from the microscopic models.

- Theory of Drude peaks due to fluctuating superconductivity.

- ‘Classification’ of mechanisms of dissipation at T=0.

Memory matrix formalism

- Most discussions of this physics have involved semi-microscopic models with uncontrolled approximations.
- Instead: work in a limit where a hierarchy of timescales allows an effective field theoretic approach.
- Small parameter will be the supercurrent relaxation rate. I.e. want $\Omega \ll T$, etc.
- (Approach inspired by studies in holographic systems over past few years, where slow mode was momentum.)

Memory matrix formalism

[Logic goes back to: Götze and Wölfle '72, Forster '75, …]

- Suppose that $H = H_0 + \varepsilon\, \Delta H$, with $[\Delta H, J_\phi] \neq 0$.
- Then the decay of J_ϕ is slow and dominates σ:

$$\sigma(\omega) = \frac{\chi_{J J_\phi}^2}{\chi_{J_\phi J_\phi}} \frac{1}{-i\omega + \Omega} + \cdots$$

- But now we have a formula for Ω! :

$$\Omega = \epsilon^2 \, \frac{1}{\chi_{J_\phi J_\phi}} \lim_{\omega \to 0} \left. \frac{\mathrm{Im}\, G^R_{i[\Delta H, J_\phi]\, i[\Delta H, J_\phi]}(\omega)}{\omega} \right|_{\epsilon = 0} .$$

Spectral density of states into which J_ϕ can decay. Cf. Fermi Golden rule.

Supercurrent relaxation

- Recap: if an 'almost conserved' operator carries current, rate of the decay determines the conductivity.

- In our case of interest today: $J_\phi = \frac{1}{m}\int d^2x \nabla\phi$

- Need an interaction that doesn't commute with J_Φ.

- Natural building block:

$$\pi_\phi = \frac{\partial f}{\partial \dot{\phi}} = -\frac{\partial f}{\partial \mu} = \rho\,.$$

i.e. charge density is canonically conjugate to the phase:

$$[\phi(x), \rho(y)] = i\delta(x-y)\,.$$

Supercurrent relaxation

- Thus a simple, generic perturbation of the superfluid state is the short range Coulombic interaction:

$$\Delta H = \frac{\lambda}{2} \int d^2x \, \rho(x)^2 \, .$$

- At first glance looks like commutator is trivial total derivative:

$$i[\Delta H, J_\phi] = -\frac{\lambda}{m} \int d^2x \nabla \rho(x)$$

- However, the phase appearing in J_ϕ is only defined outside of vortex cores! Above integral is then also only over the outside of vortex cores. Integral over all space vanishes:

  → integral over vortex cores.

Supercurrent relaxation

- The memory matrix formula for Ω becomes an integral of the two point function of ρ over the vortex core.

- Using the diffusive behavior of ρ in normal state, the Bardeen-Stephen formula drops out exactly.

$$\Omega \sim \frac{n_f A_v}{\sigma_{\rm n}}$$

- So we discover the quantum origin of this formula. Charge interactions enhance phase fluctuations:

$$\Delta\rho \, \Delta\phi \gtrsim \hbar$$

Beyond Bardeen-Stephen (in progress)

- Real life vortices are not infinitely large. The diffusive form of the charge density correlator is therefore not exact. For small vortices, it will not even be approximately correct.

- Work in progress: generalize Bardeen-Stephen formula allowing for non-diffusive dynamics of the charge density.

- Part of the controversy around T=0 metallic phases is 'where does the dissipation occur'? From our approach it is manifest that if the phase-relaxing interacting is local, dissipation must be due to vortex cores.

Supercurrent relaxation without parity

- With parity and time-reversal broken, a second very natural ΔH exists.

- Suppose the low energy effective theory is coupled to an emergent Chern-Simons gauge field:

$$\mathcal{L} = \mathcal{L}_{\mathrm{matter}} + j_\mu \left(A^\mu + a^\mu\right) - \frac{1}{2\lambda'}\epsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho$$

- Integrating out the gauge field generates

$$\mathcal{L}' = \frac{\lambda'}{2} j_\mu \frac{\epsilon^{\mu\nu\rho}\partial_\rho}{\partial_\sigma \partial^\sigma} j_\nu \quad \Rightarrow \quad \Delta H = \frac{\lambda'}{2}\int \frac{d^2k}{(2\pi)^2}\frac{\rho_{-k}(\nabla\times j)^z_k}{k^2} + \mathrm{h.c.}$$

Supercurrent relaxation without parity

- Non-locality of induced interaction leads to a nonzero time dependence of J_ϕ everywhere. In fact:

$$i[\Delta H, J_\phi^i] = -\frac{\lambda'}{m}\epsilon^{ij} J^j \ .$$

- Rough physical picture:
  Current = Flow of charge
  → Flow of emergent magnetic flux (CS term)
  → Flow of vortices
  → Relaxation of supercurrent in transverse direction!

- Ω depends on charge flow in normal component.

Supercurrent relaxation without parity

- Result for conductivities:

$$\sigma_{xx} = -\frac{m^2}{\lambda'^2 \rho_s} \frac{\omega(\omega\Omega + i(\Omega^2 + \Omega_H^2))}{(-i\omega + \Omega)^2 + \Omega_H^2}\,,$$

$$\sigma_{xy} = -\frac{1}{\lambda'} - \frac{m^2}{\lambda'^2 \rho_s} \frac{\omega^2 \Omega_H}{(-i\omega + \Omega)^2 + \Omega_H^2}\,,$$

- Feature: ‘supercyclotron resonance’ at

$$\omega_\star = \pm\Omega^H - i\Omega = \frac{\lambda' \rho_s}{m^2} \frac{1}{\pm 1 - \lambda'(\pm\sigma_0^H - i\sigma_0)}\,.$$

Conductivities of the normal component of superfluid.

Chern-Simons superfluid hydrodynamics

- The expressions for the conductivities can be alternatively derived directly from superfluid hydrodynamics coupled to a Chern-Simons gauge field.

- Dissipation in this case is not due to vortex cores, but to the normal component of the superfluid.

- If the normal component only has a Hall conductivity (e.g. a superfluid coupled to a quantum Hall state), obtain nontrivial dissipationless frequency dependent dynamics.

Recap [see arXiv/1602.08171]

- Superfluid relaxation occurs if perturbations of effective Hamiltonian do not commute with the supercurrent.
- Starting with perturbations of superfluid hydrodynamics gives controlled entry point. This works even if the underlying microscopic dynamics is strongly correlated.
- Gave two examples, with and without parity:
  (1) With parity: recovered Bardeen-Stephen.
  (2) Without parity: ‘supercyclotron resonance’ determined by conductivities of normal component.