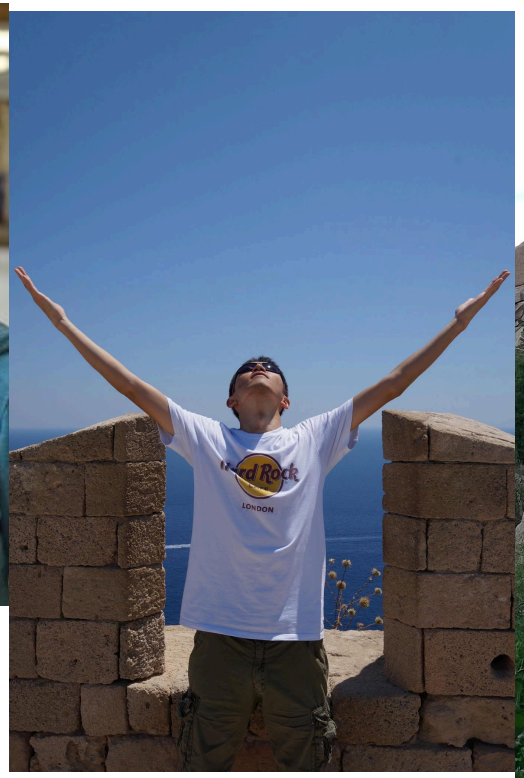
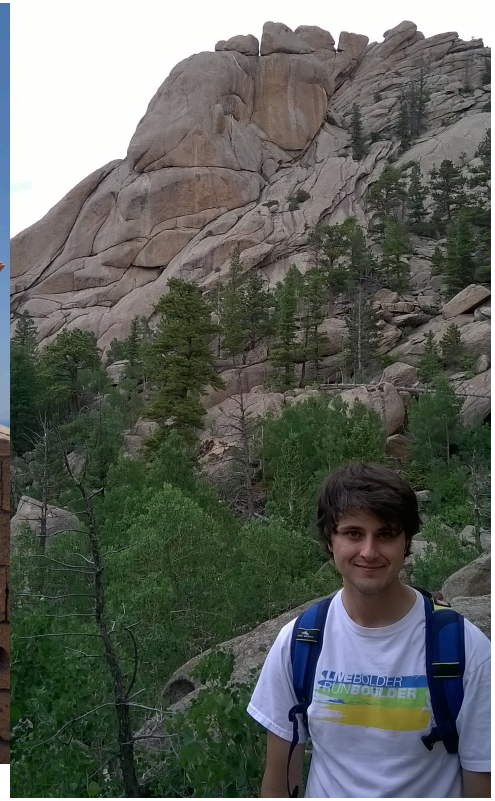
Holographic Predictions for Novel Collective Modes in Cold Atom Experiments

Paul Romatschke

CU Boulder & CTQM

...in collaboration with...



J. Brewer & PR, 1508.01199 (PRL)

H. Bantilan, J. Brewer, T. Ishii, W. Lewis & PR, 1605.00014

String Theory Based Predictions for Novel Collective Modes in Strongly Interacting Fermi Gases

H. Bantilan[1,2], J.T. Brewer[3], T. Ishii[4,5], W.E. Lewis[4], and P. Romatschke[4,5]

Report of the Referee -- AT11474/Bantilan

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references are unfortunately missing. In fact, I find both the introduction and motivation and also the summary of the paper rather superficial and hardly understandable for an audience interested in ultracold gases which the authors eventually want to adress.

Experimental summary

Fluid	T[K]	η [Pa · s]	η/n [h]	η/s [h/k]
Water	370	2.9×10^{-4}	85	8.2
Helium-4	2	1.2×10^{-6}	0.5	1.9
Lithium-6 (unitarity)	23×10^{-6}	$\leq 1.7\times10^{-15}$	≤ 1	≤ 0.5
QGP	2×10^{12}	$\leq 5\times10^{11}$	-	≤ 0.4

[From E.~Taylor; originally adapted from Teaney and Schafer, 2009]

Strongly Interacting Quantum Fluids (SIQFs)

- Good fluid is when mean free path $\lambda \ll L$ (system size)
- Mean free path is related to viscosity η:

$$\lambda \sim \eta / P$$

- For relativistic systems $P \sim s\,T$ so

$$\lambda \sim \eta / s / T$$

and for weak coupling $g \ll 1$

$$\eta / s \sim 1 / g^4$$

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- For relativistic systems $P \sim s\,T$ so

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For weak coupling $g \ll 1$: $\eta/s \sim 1/g^4$

For strong coupling $g \gg 1$: $\eta/s \to \text{const}$

Strongly Interacting Quantum Fluids (SIQFs)

- Good fluid if mean free path λ is small
- λ is small if η/s is small
- η/s is small if coupling is strong

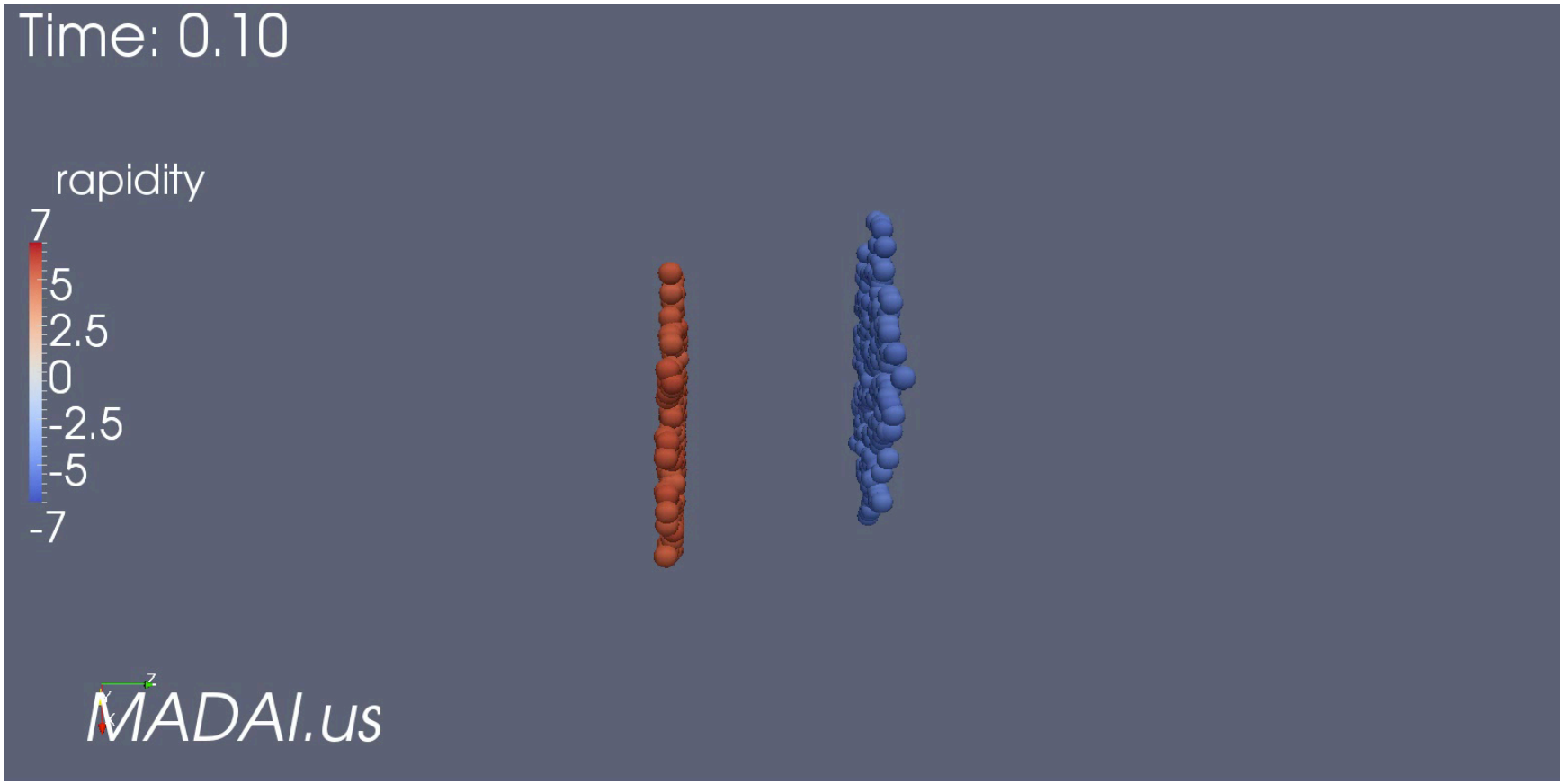
The lower η/s is for a fluid, the stronger it is coupled!

SIQFs: Black Holes

- Holographic dual to field theories with $g \rightarrow \infty$
- Damping of horizon fluctuations measures shear viscosity
- For a large class of (string-theory inspired) black holes $\eta/s = 1/(4\pi) \sim 0.08$

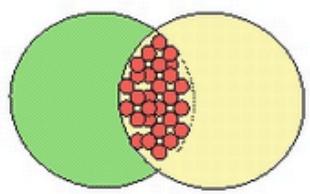
[Policastro, Son, Starinets, PRL 2001; "KSS", PRL2005]

Heavy-Ion Collisions

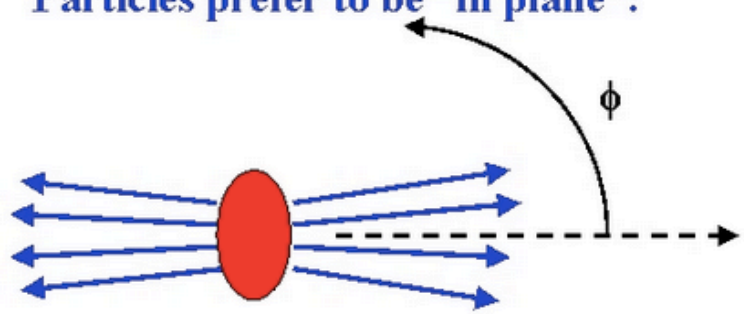


Heavy-Ion Collisions and Elliptic flow V_2

Beam's eye view of a non-central collision:

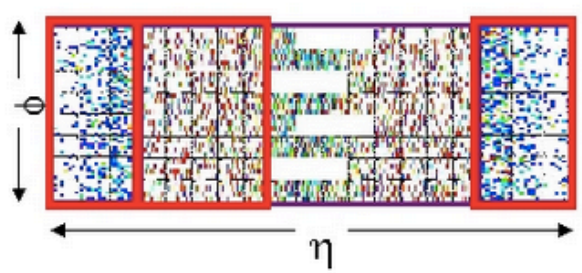


Particles prefer to be "in plane":



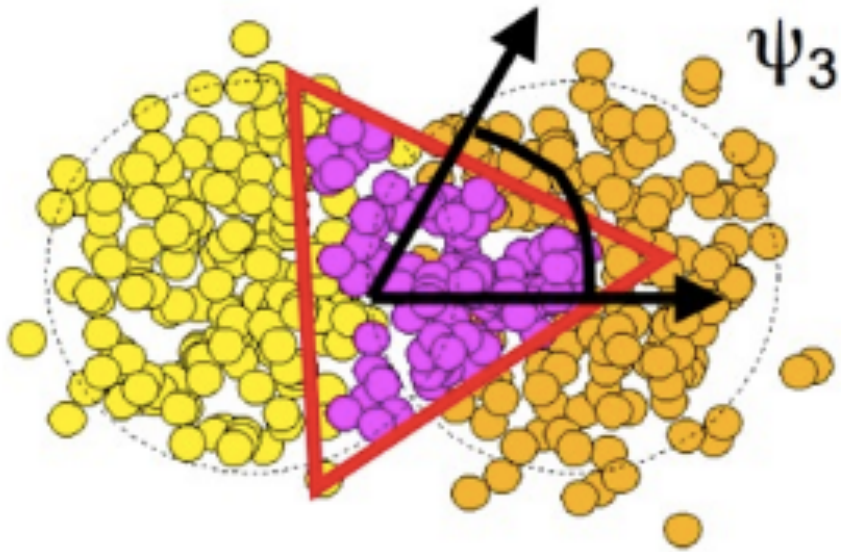
$$dN/d(\phi - \Psi_R) = N_0 (1 + 2V_1 \cos(\phi - \Psi_R) + 2V_2 \cos(2(\phi - \Psi_R)) + \ldots)$$

Elliptic flow



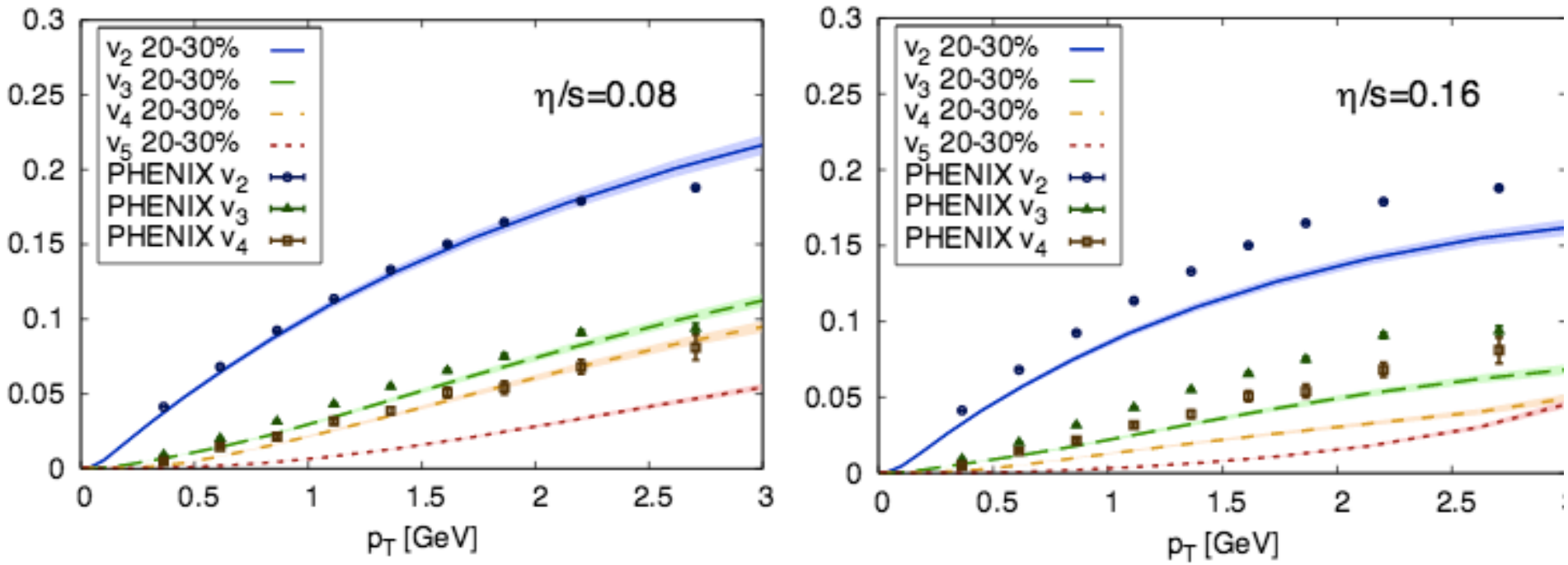
Mark D. Baker

Heavy-Ion Collisions and Triangular flow V_3



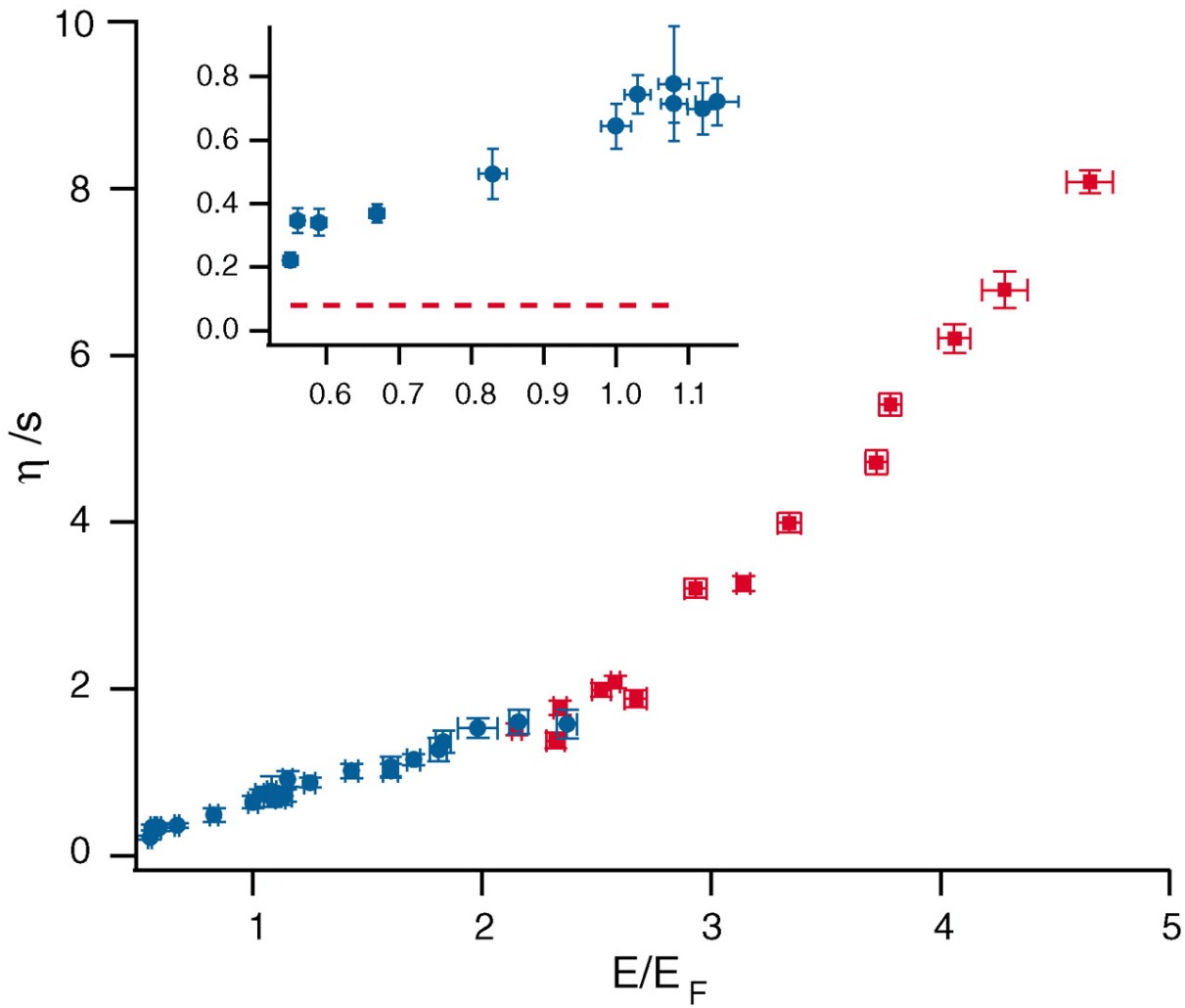
$dN/d\phi \sim V_3 \cos(3\phi)$

SIQF: Quark-Gluon Plasma



[Schenke et al. PRC 2012]

SIQF: Cold Unitary Fermi Gas



[Cao et al, Science 2011]

Universality in Strongly Interacting Quantum Fluids (SIQFs)

- Strongly coupled N=4 SYM: $\eta/s \approx 0.08$
- QCD @ T~200 MeV: $\eta/s \approx 0.1\text{-}0.3$
- Cold ^{6}Li close to unitarity: $\eta/s \approx 0.3$
- $Bi_2Sr_2CaCu_2O_{8+\delta}$ @ T~100K: $\eta/s \approx 0.1\text{-}0.2$

Very different systems, very similar hydrodynamic transport properties

Universality beyond hydro

Hydrodynamic Transport in SIQFs is similar

What if this similarity extends beyond Hydrodynamics?

Let's look for similarities at non-zero frequency/transient timescales (non-equilibrium)

Example: QNMs

- Sound mode excitations in N=4 SYM: Linear response function (correlator) has poles:

$$\omega_h = \pm c_0|\mathbf{k}| - i\frac{\eta \mathbf{k}^2}{4P_0}\frac{2}{3}\,, \quad \omega_{nh,1} \simeq 2\pi T(\pm 1.73 - 1.34i)\,,$$

- These are QN modes of black hole in AdS_5
- Real time energy density response:

$$\delta\rho = \int dk\, e^{ikx-i\omega t}\, G = \int dk\, \Sigma_i\, e^{ikx-i\omega t}/(\omega-\omega_i)$$

Example: QNMs

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- These are QN modes of black hole in AdS_5
- Real time energy density response:

$$\delta\rho_{BH}(t,\mathbf{x}) \propto e^{\pm i c_0 |\mathbf{k}| + i\mathbf{k}\cdot\mathbf{x} - \Gamma_h t \mathbf{k}^2} + \sum_{n=1}^{\infty} a_{nh,n} e^{\pm i \mathrm{Re}\omega_n t - \Gamma_{nh,n} t},$$

So what?

- QNMs tell us about far-from-equililibrium thermalization
- First non-vanishing QNM gives convergence radius of hydrodynamic (gradient) expansion
- Strong coupling: QNMs; weak coupling: branch cuts; finding QNMs (maybe) tells us about presence of quasiparticles?

Universality beyond hydro

- If similarity between SIQFs extends beyond hydro, there should be something like QNMs in all SIQFs
- Generically, I call these ‘non-hydro modes’ (but I should think of a better name)
- If universality argument holds, knowing non-hydro modes in one SIQFs could teach you about transport in another…

Wouldn’t it be nice to have experimental data on non-hydro modes?

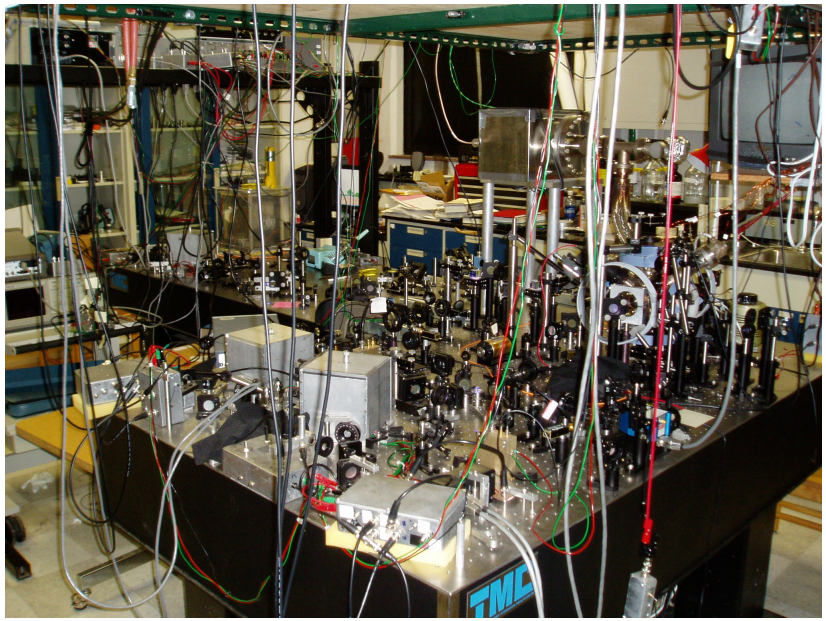
Hunt for “experimental” non-hydro mode signatures in SIQFs

- Heavy-ions: hopeless (no real-time info)
- Lattice QCD: very hard (sign problem)
- High-Tc-superconductors: ???
- Ultracold Quantum Gases: ☺
- Others????

Hunting for Quasi-Normal Modes in Cold Atoms

Ultracold Fermi gases

- Cloud of cold atoms in a trapping potential
- Interactions tunable via magnetic field
- Experimentally realized in D=2,3
- For strong interactions, “unitary Fermi gas”



Predictions for strongly interacting Fermi gases

- Want: strong-coupling, real-time model
- Available: gauge/gravity duality
- No known exact gravity dual for Fermi gases
- Have to do with what's available
- Need to get basic symmetries right

Short version

- We use Lifshitz BHs at zero density
- Right symmetry (non-relativistic)
- Right equation of state

Long version: Lifshitz Black Holes

- "Original" gauge/gravity duality for relativistic systems (invariance under (t,x)->λ(t,x))
- Lifshitz scaling: (t,x)->(λ^zt, λ t) with different z
- Gravity dual with Lifshitz scaling

$$\mathrm{d}s^2 = \frac{\ell^2}{r^2}\mathrm{d}r^2 - \frac{r^{2z}}{\ell^{2z}}\mathrm{d}t^2 + \frac{r^2}{\ell^2}\mathrm{d}\vec{x}_{d-1}^2$$

[Kachru, Liu, Mulligan; Tarrio, Vandoren]

- Known black hole/black brane solutions

e.g. [Danielsson, Thorlacius;Mann; Amado, Faedo;Tarrio, Vandoren]

- Field content: gravity, scalar+gauge fields

Boundary theory for simple Lifshitz BHs

- Finite temperature T
- Zero chemical potential μ
- Known shear viscosity over entropy density
- (Analytically) known quasinormal modes for probe scalar in arbitrary D & z

[Symbesma, Vandoren; Bantilan et al;]

Lifshitz BHs	Fermi Gases
• Scale invariance $(t,x) \to (\lambda^z t, \lambda t)$	• Non-relativistic $(t,x) \to (\lambda^2 t, \lambda t)$
• Boundary theory R^{D+1}	• D=2 and D=3
• Black brane with T	• Temperature T
• Zero density	• Non-zero density
• Equation of state $\varepsilon z = P D$	• Equation of state $\varepsilon = P D/2$

Need to choose z=2

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Quasi-normal modes for Lifshitz BHs

- For probe scalar with dimension Δ
- For scaling exponent z=2
- For D=z, analytic: $w_n = -i\,\Gamma_n$

$$\omega_n^{(d=2)} = 0\,, \quad \Gamma_n^{(d=2)} = \left(n - 1 + \frac{\Delta}{2z}\right) \times 4\pi k_B T/\hbar\,, \quad n \geq 1\,.$$

- For D=z+1, numerical

	$\Delta = 3$		$\Delta = 4$	
n	$\omega_n\hbar/(4\pi k_B T)$	$\Gamma_n\hbar/(4\pi k_B T)$	$\omega_n\hbar/(4\pi k_B T)$	$\Gamma_n\hbar/(4\pi k_B T)$
1	0.2812	0.5282	0.3560	0.7540
2	0.5776	1.437	0.6507	1.663
3	0.8714	2.342	0.9446	2.568
4	1.165	3.246	1.239	3.472

Lifshitz BH – Fermi Gas Conversion

- Lifshitz Black Hole: scalar operator with dimension Δ
- Fermi gas: density perturbations
- Number density operator: fermionic bilinear with operator dimension $\Delta=D$
- Or maybe energy density operator? fermionic bilinear with derivative, $\Delta=D+1$?
- Don't know, so let's take $\Delta=D\pm 1/2$

Lifshitz BH – Fermi Gas Conversion

- Quasinormal modes

$$\omega_n^{(d=2)} = 0\,, \quad \Gamma_n^{(d=2)} = \left(n - 1 + \frac{\Delta}{2z}\right) \times 4\pi k_B T/\hbar\,, \quad n \geq 1\,.$$

- Γ proportional to 4π T
- Same units as s T/η
- What about

$$4\pi k_B T/\hbar \to (\epsilon + P)/\eta$$

- With ε = P D/2 all reference to temperature gone

Lifshitz BH – Fermi Gas Conversion

- Proposition: $4\pi k_B T/\hbar \to (\epsilon + P)/\eta$

- Consequence:

$$\Gamma_n^{d=2} = 2\,(n-1+\Delta/2z)\,P/\eta$$

- E.g. $n=1$, $\Delta=2$, $z=2$

$$\Gamma_1^{d=2} = P/\eta$$

- Generalization of Lifshitz BH result to arbitary (small) η
- Same scaling in Kinetic Theory (Boltzmann)

Fermi Gas – Lifshitz BHs

Our Proposition is $4\pi k_B T/\hbar \to (\epsilon + P)/\eta$

and thus

$$\Gamma^{d=2} = 2\,(n-1+\Delta/2z)\,P/\eta$$

(similar for $\Gamma^{d=3}$) for strong coupling.

Should be testable for BHs with $1/\lambda$ corrections and/or finite density calculation.

Fermi Gas – Lifshitz BHs

Our Proposition is $$4\pi k_B T/\hbar \to (\epsilon + P)/\eta$$

and thus

$$\Gamma^{d=2} = 2\,(n-1+\Delta/2z)\,P/\eta$$

(similar for $\Gamma^{d=3}$) for strong coupling.

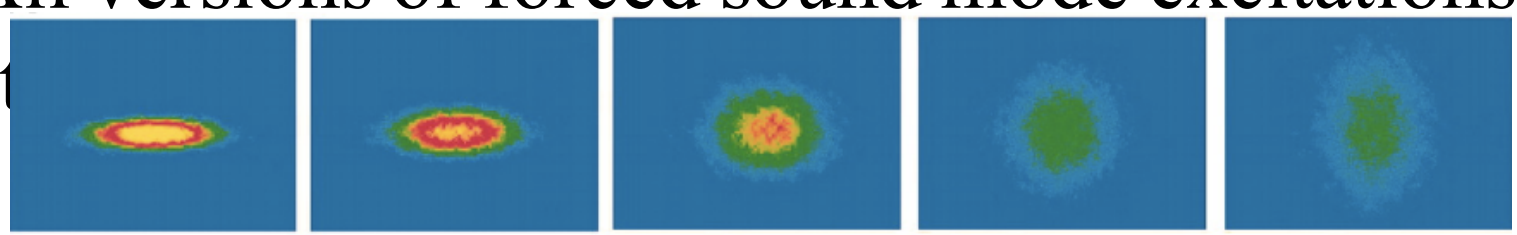
- Damping rate independent of trapping frequency
- Can use exp' hydro damping rate $\Gamma_H \sim \eta/P$ to express Γ_{NH} in terms of exp' quantities

Non-hydro modes for Lifshitz BHs have be calculated

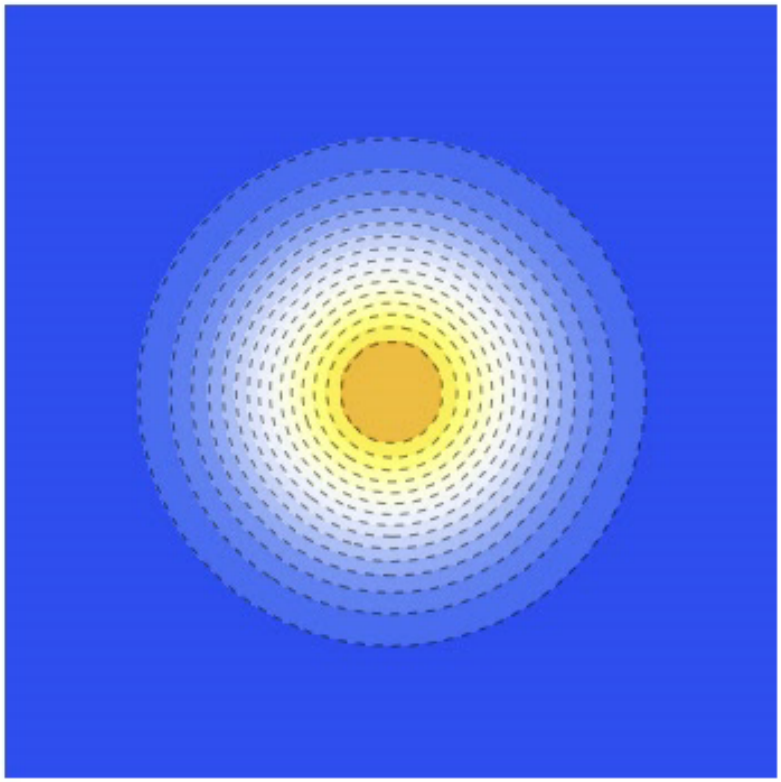
How to connect calculations to real Fermi gas experiments?

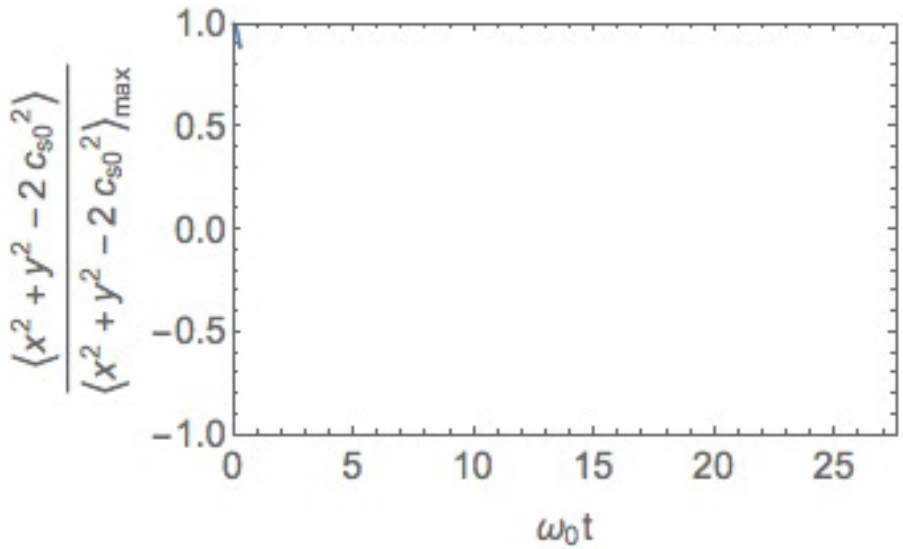
Collective modes in trapped Fermi gases

- Trapping potential (e.g. $\omega^2_0 \; x^2$) determines hydrostatic density profile of atoms
- By suddenly changing the potential (e.g. from asymmetric to symmetric) one can excite oscillations in the whole atom cloud
- Several "modes" possible: sloshing, breathing, quadrupole, hexapole, etc.
- All versions of forced sound mode excitations at



Breathing (Monopole) Mode

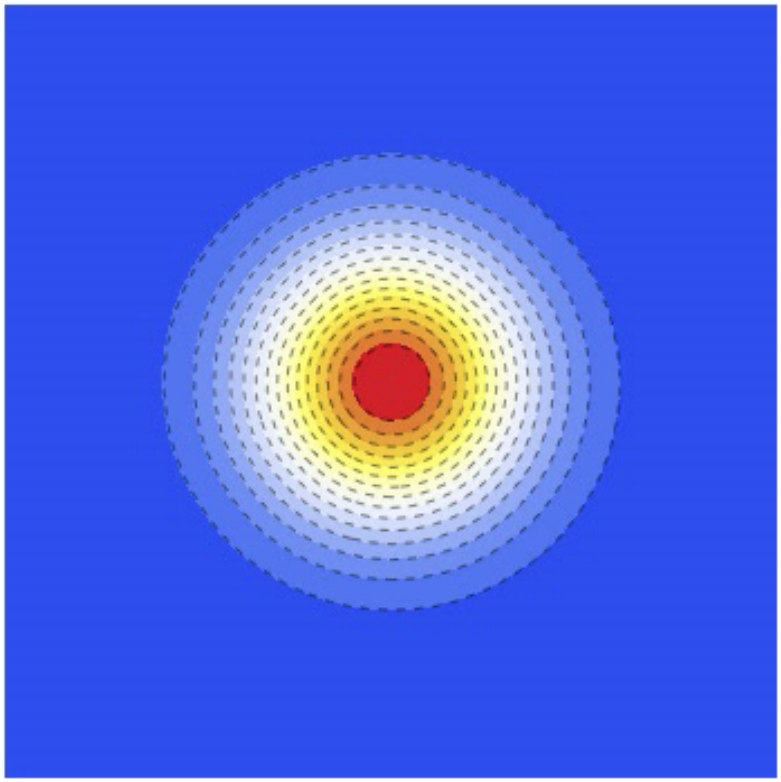


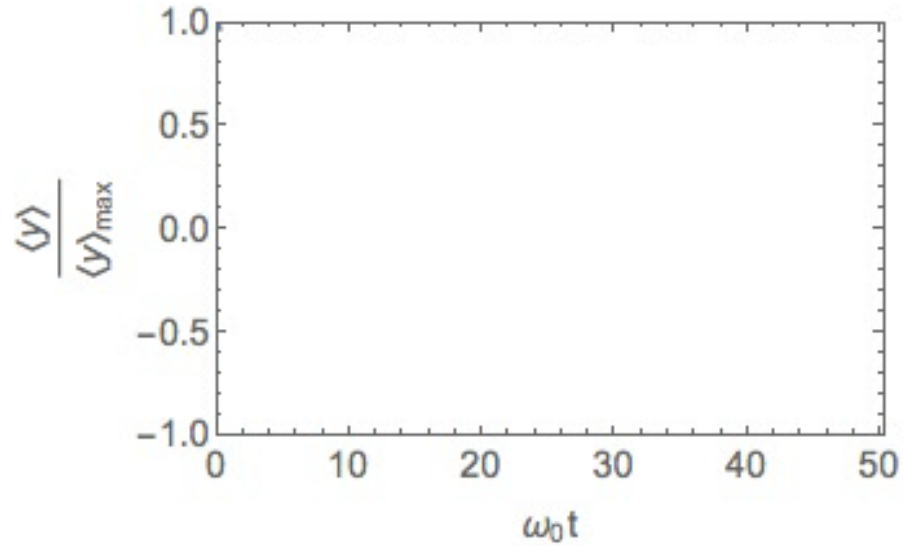


$$\omega_B = \sqrt{\frac{10}{3}}\,\omega_0 \quad \Gamma_B = \frac{\eta}{3P}\,\omega_0^2$$

[Will Lewis & PR, in prep]

Sloshing (Dipole) Mode

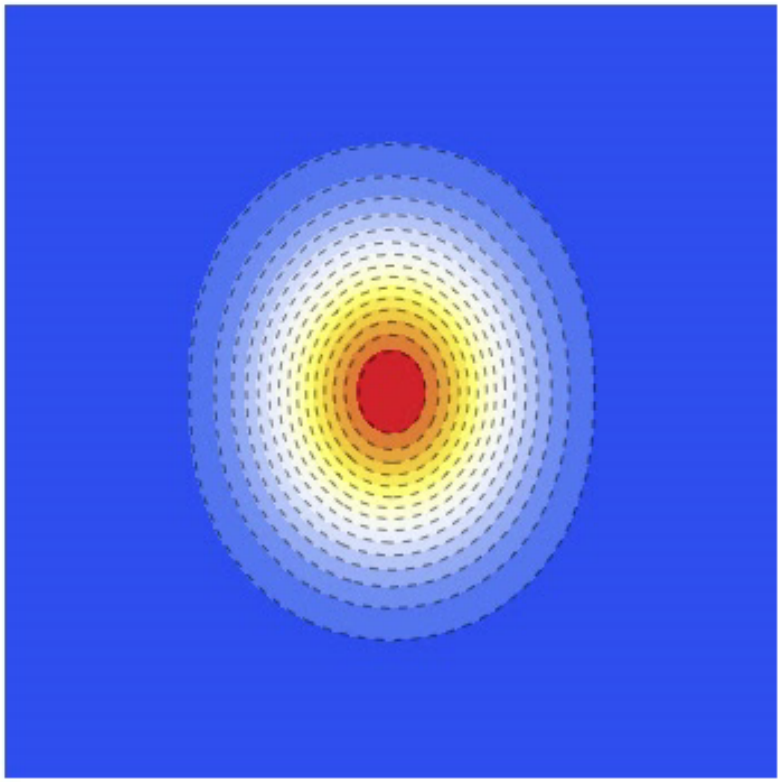


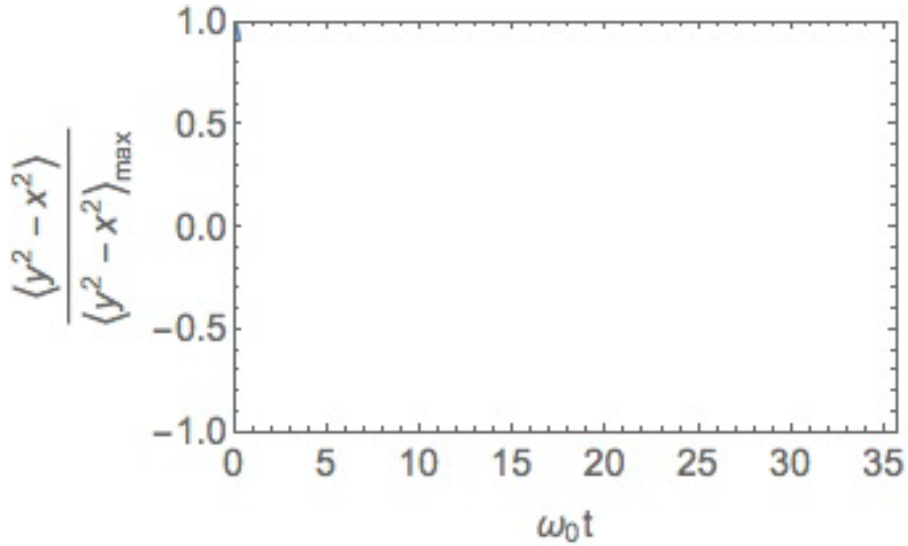


$\omega_S = \omega_0$ $\Gamma_S = 0$

[Will Lewis & PR, in prep]

Quadrupole Mode

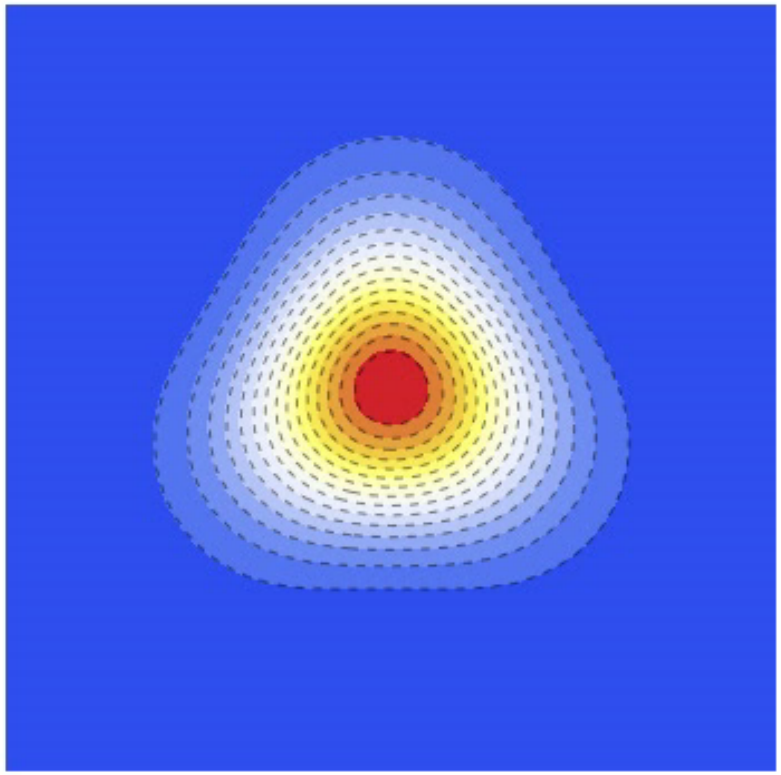


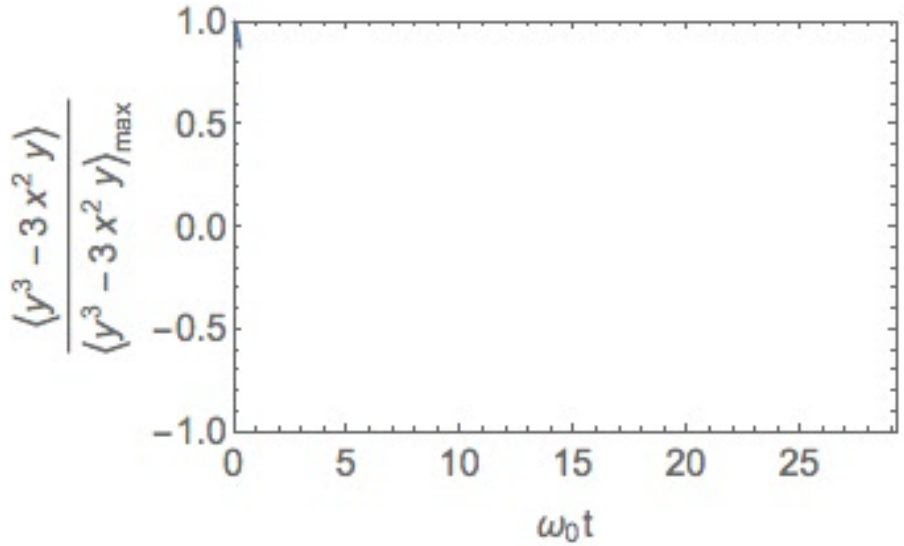


$$\omega_Q = \sqrt{2}\,\omega_0 \qquad \Gamma_Q = \frac{\eta}{P}\,\omega_0^2$$

[Will Lewis & PR, in prep]

Hexapole Mode





$$\omega_H = \sqrt{3}\,\omega_0 \qquad \Gamma_H = \frac{2\eta}{P}\,\omega_0^2$$

[Will Lewis & PR, in prep]

Fermi Gas Collective Modes

- Fit time-evolution
- Hydro-inspired fitting function

$$F(t) = a_h \cos(w_h t) e^{-\Gamma_h t}$$

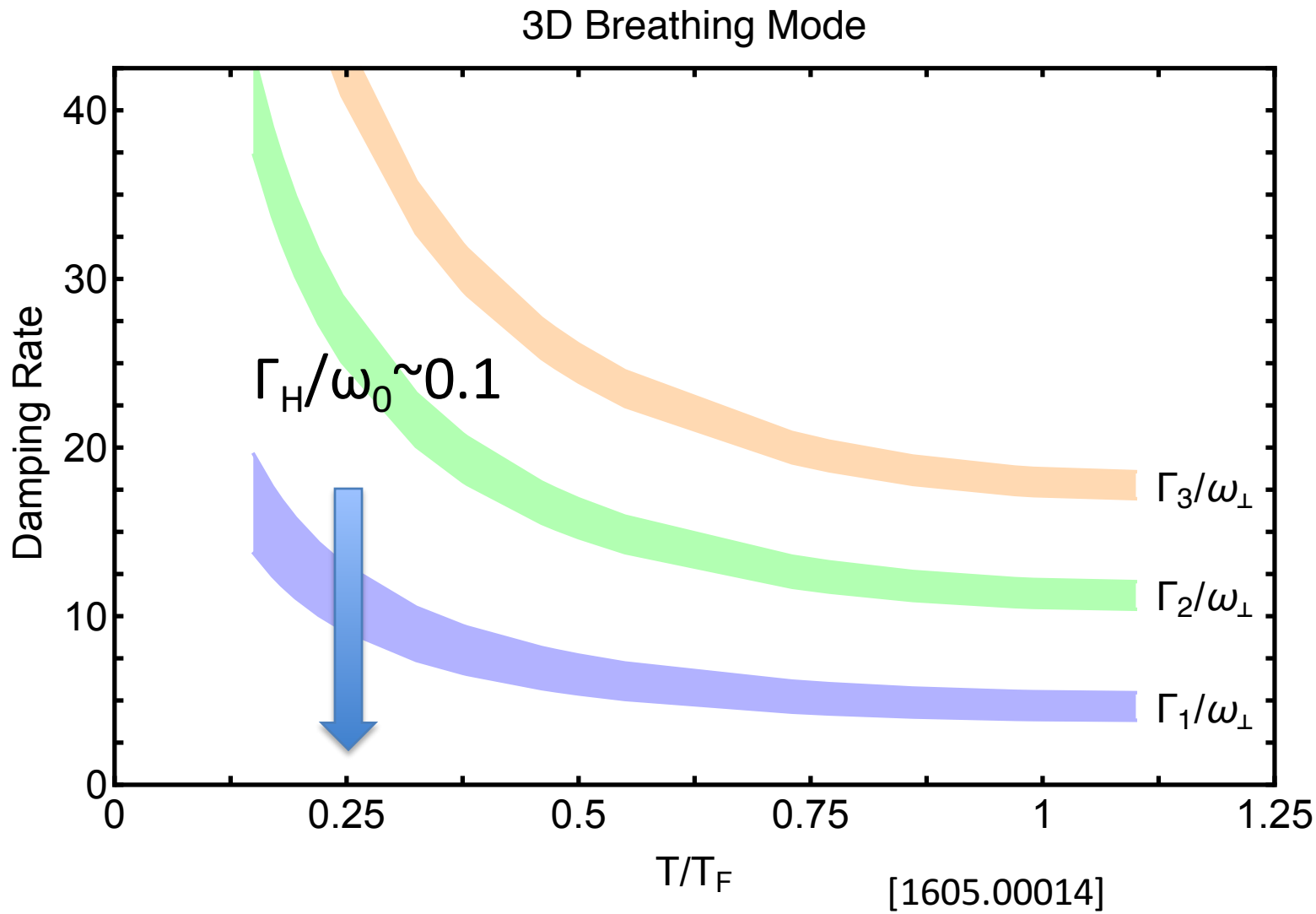
- Frequencies ω and damping rates Γ of hydrodynamic modes routinely extracted in experiment

Quasinormal modes

- Infinite number of Lifshitz black hole quasinormal modes
- Expect real-time response of the form

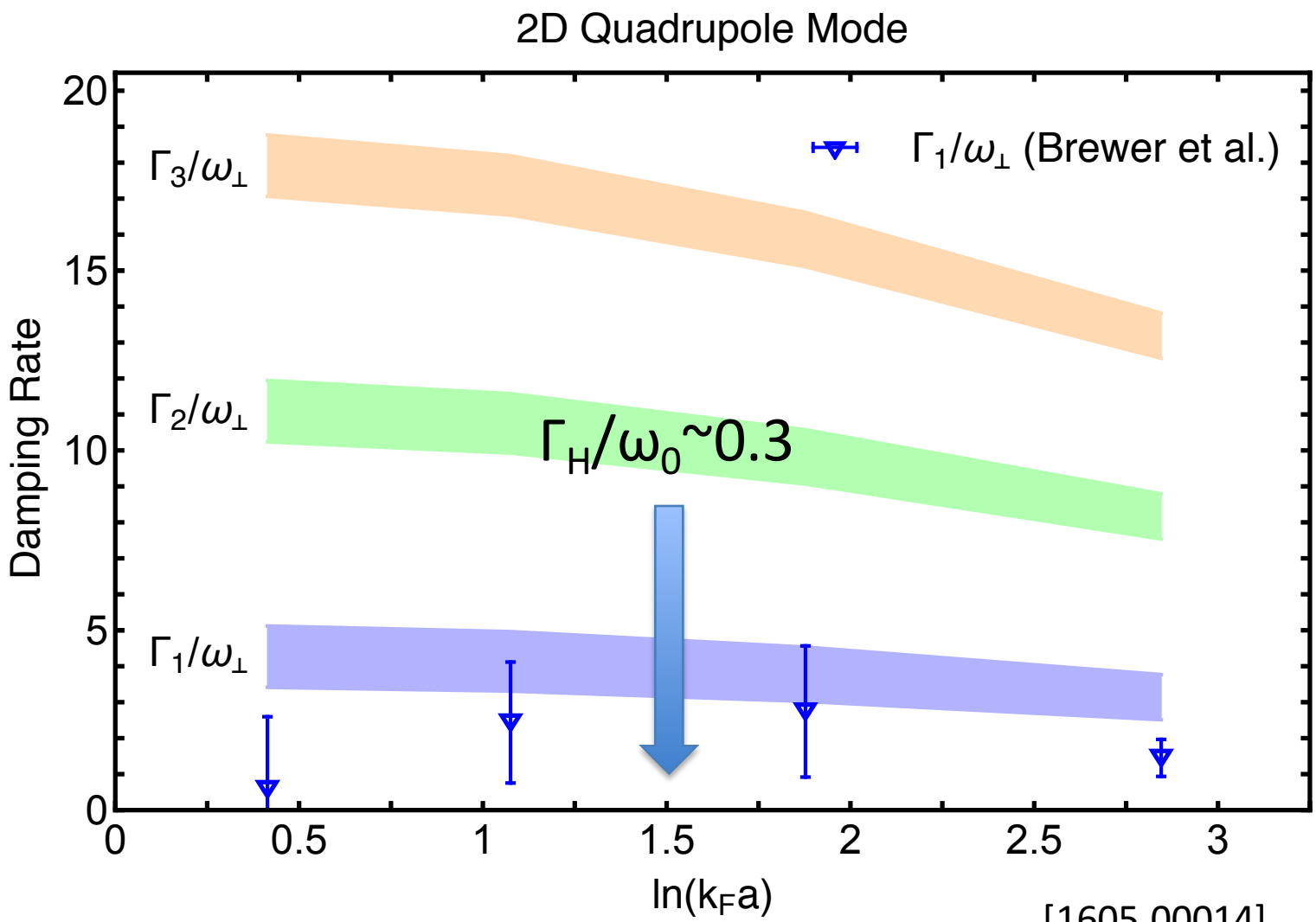
$$F(t) = a_h \cos(w_h t) e^{-\Gamma_h t} + a_{nh} \cos(w_{nh} t) e^{-\Gamma_{nh} t}$$

Lifshitz BH Preditions for Fermi Gases



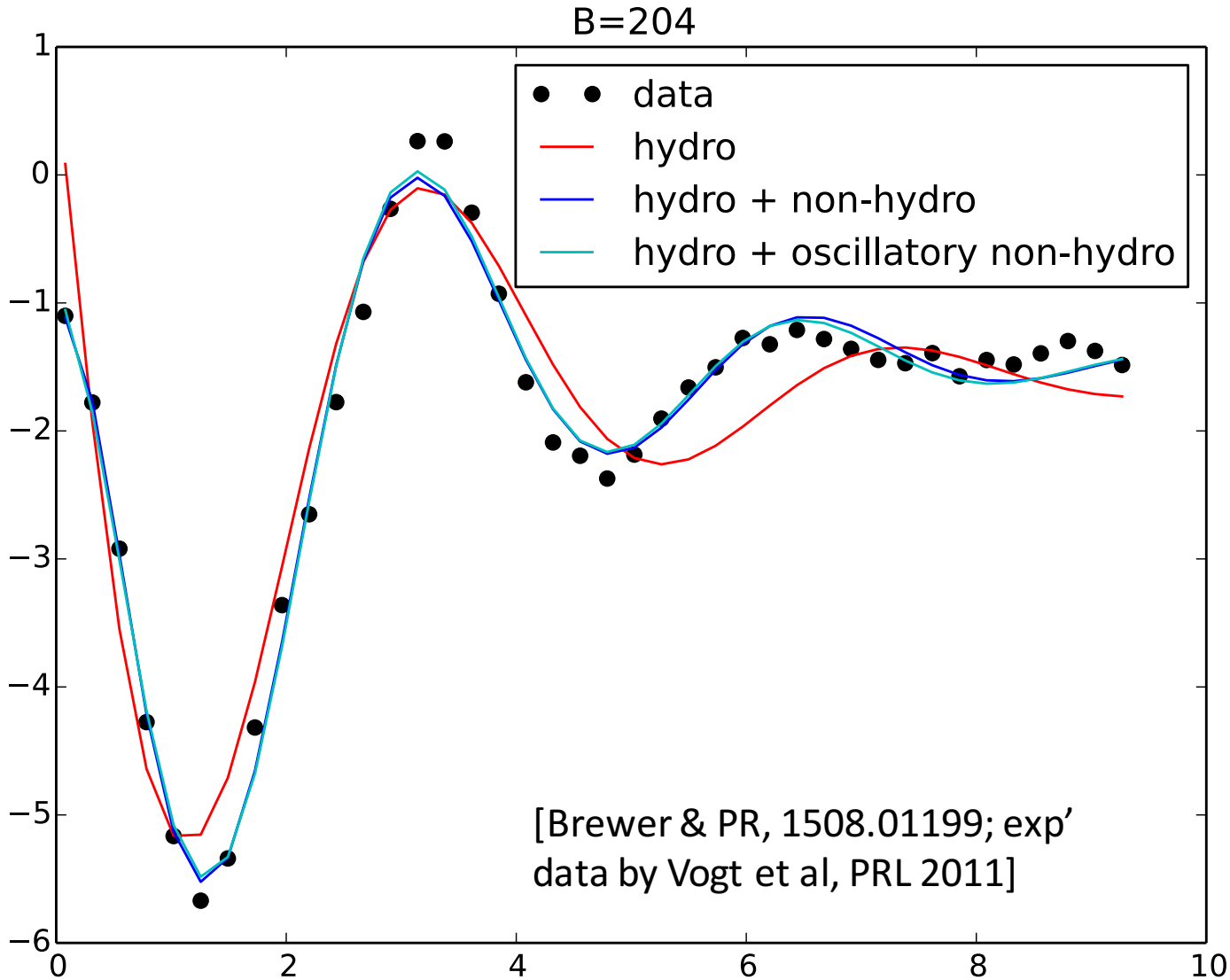
[1605.00014]

Lifshitz BH Preditions for Fermi Gases



[1605.00014]

D=2, quadrupole mode



We need better data!

(bending the ear of experimentalists)

Amplitudes

- Prediction of novel, non-hydro collective modes
- Novel modes have high damping rates
- Can estimate (first) non-hydro mode amplitude compared to hydro amplitude:

$$(\omega_H + \Gamma_H)/(\omega_1 + \Gamma_1) \sim \omega_H/\Gamma_1$$

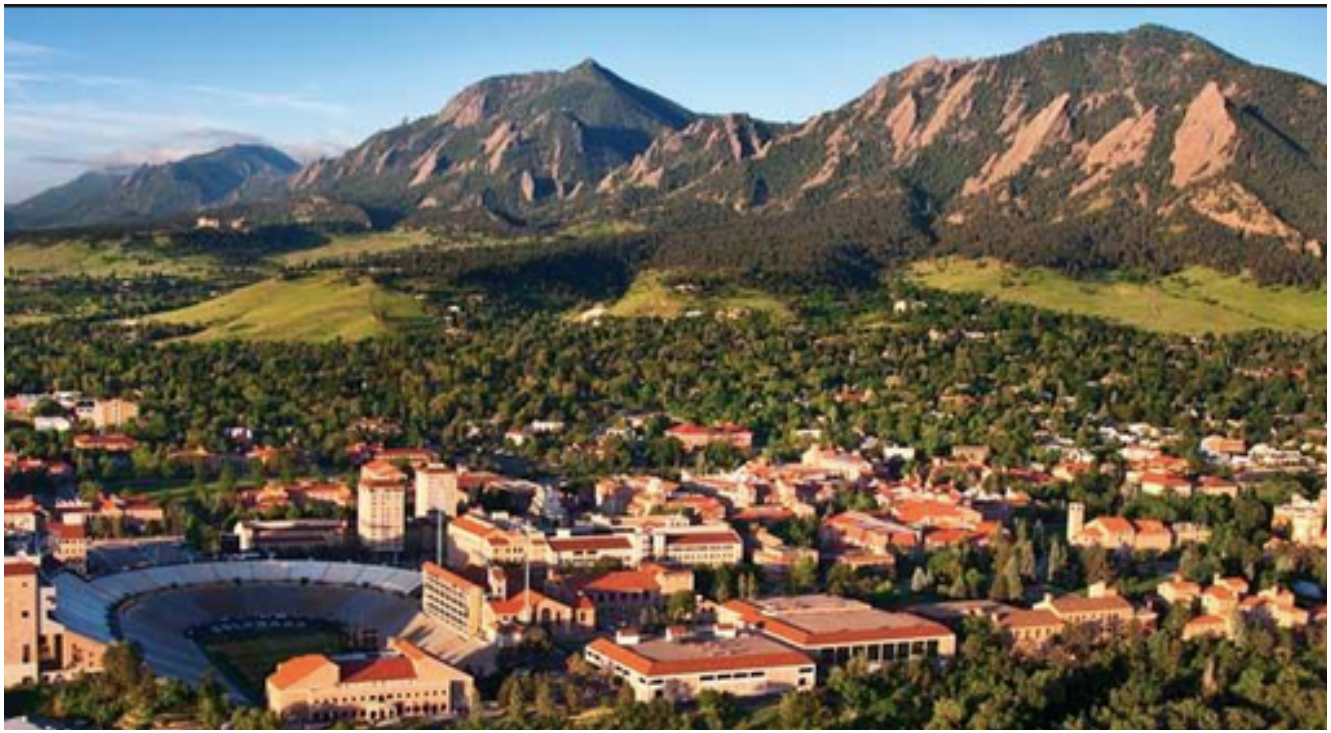
- For 3D case, predict non-hydro mode amplitude to be 20 percent of hydro mode

Summary

- Prediction of novel type of non-hydro collective modes from Lifshitz BHs
- Key assumption: $4\pi k_B T/\hbar \to (\epsilon + P)/\eta$
- Predict frequencies, damping rates of non-hydro mode
- Predict amplitude of first non-hydro mode
- Predictions for (first) non-hydro mode testable by state-of-the-art experiments with in-situ cloud measurements & good time resolution

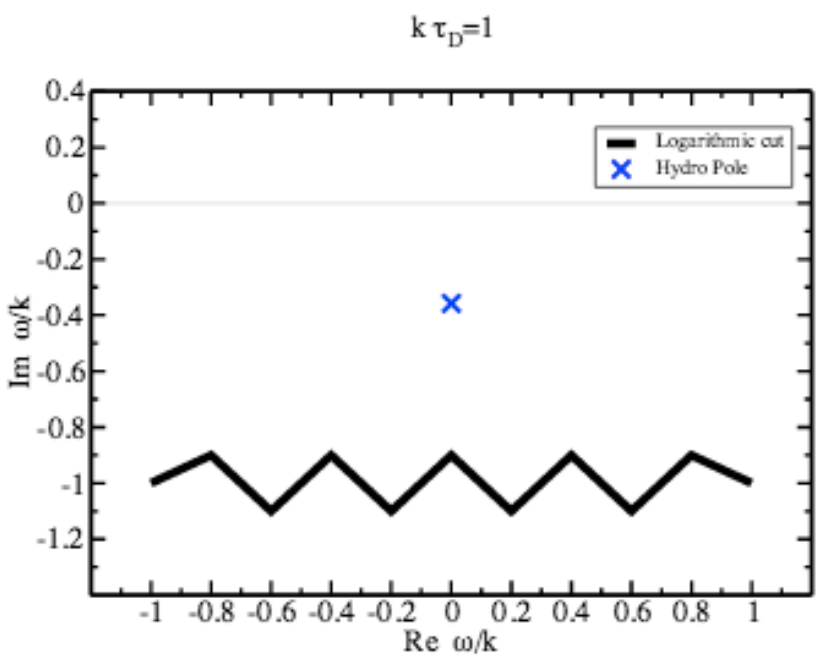
Postdoc Applications Welcome!

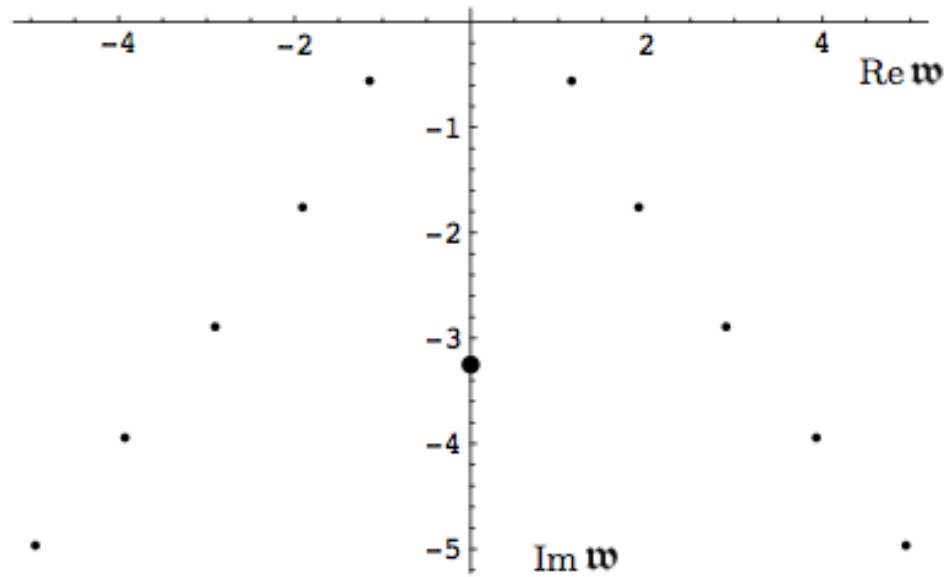
- CU ‘Nuclear’ group expects to open postdoc position for Fall’17
- Applications welcome (also after Jan 7!)



Bonus Material

Weak vs Strong Coupling

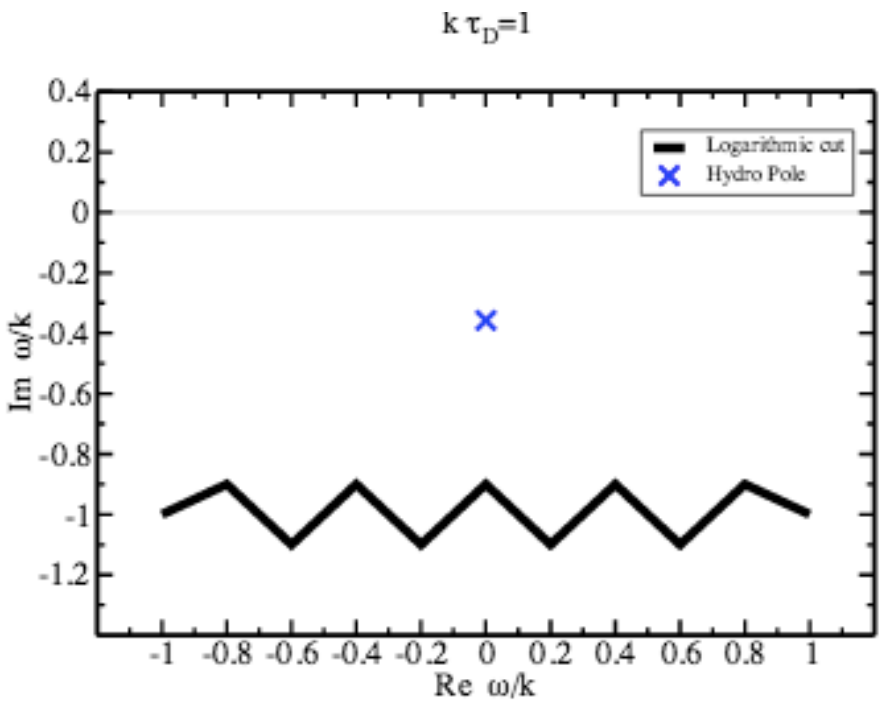


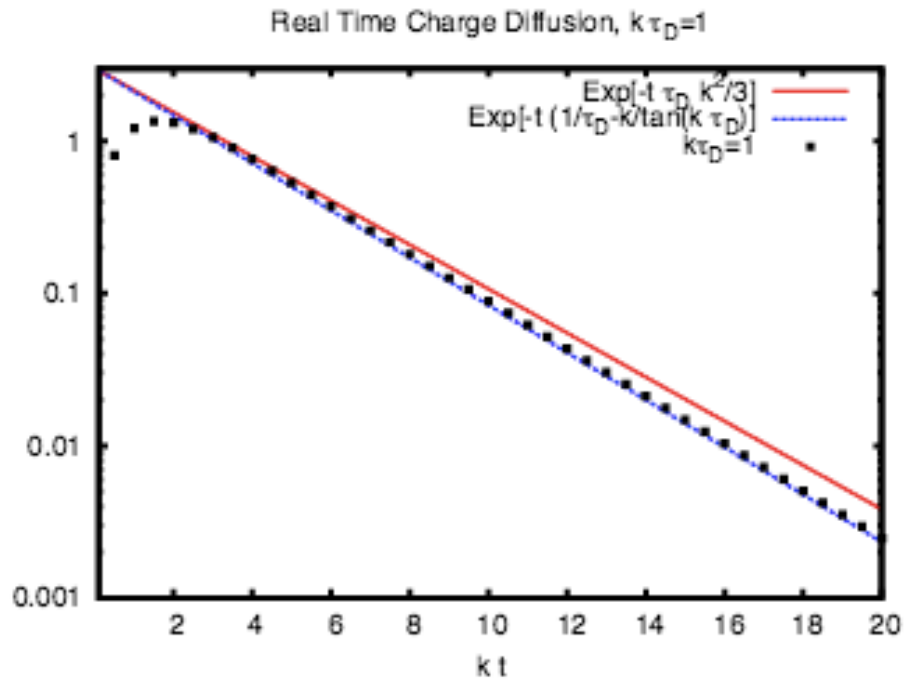


Weak coupling
[PR, 1512.02641]

Strong Coupling
[Kovtun, Starinets, hep-th/0506184]

Weak coupling

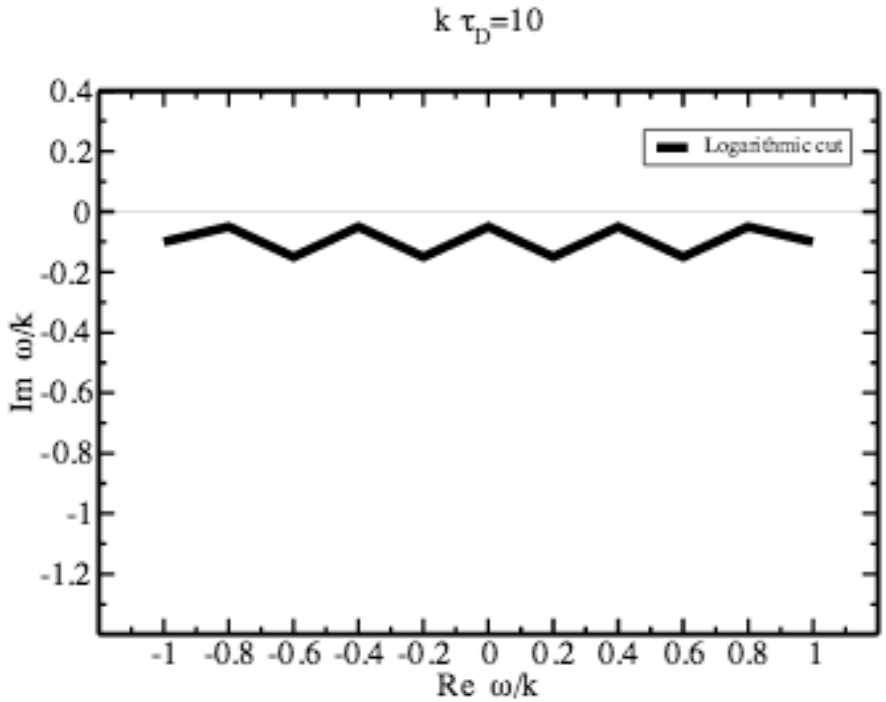


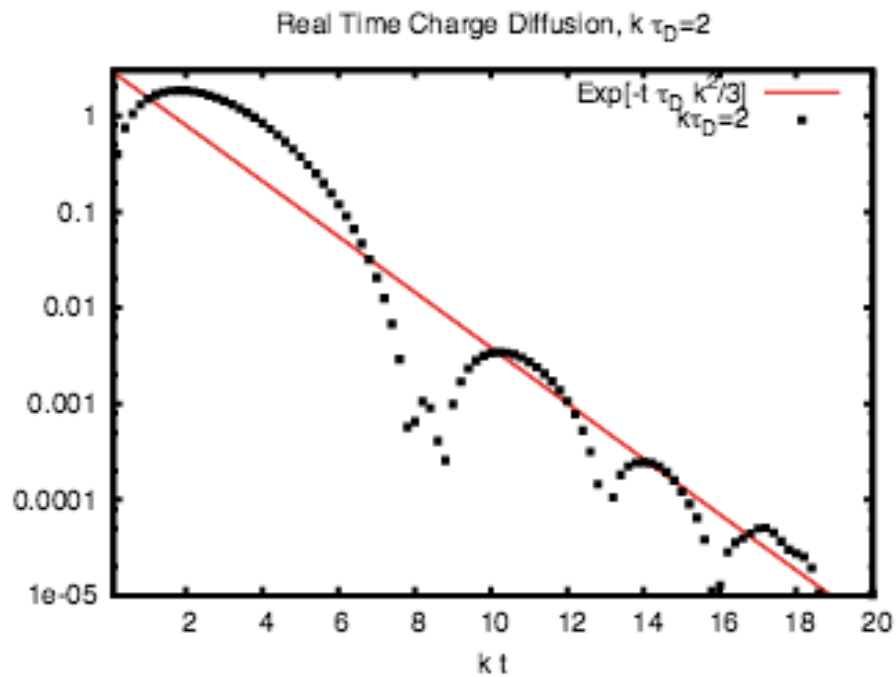


[PR, 1512.02641]

"H"-phase: hydro pole exists

Weak coupling





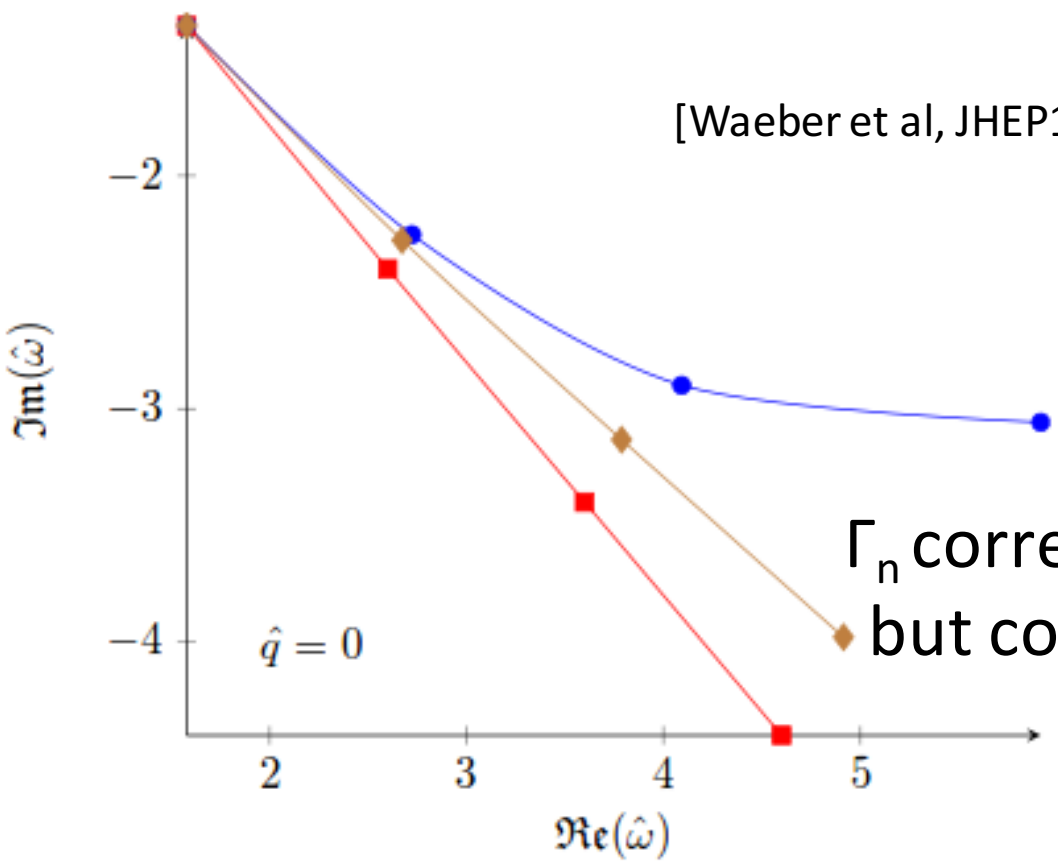
[PR, 1512.02641]

"G"-phase: hydro pole does not exist

Aside: Hydrodynamic onset Transition

- Kinetic theory (in RTA) predicts onset of hydrodynamic pole at $k\,\tau_D<\pi/2$
- For at $k\,\tau_D> \pi/2$ no hydrodynamic pole ("G" phase)
- Within the regime of applicability of kinetic theory if τ_D is large
- Same qualitative conclusion from strong coupling side ($k> \pi\, T/|\lambda_{GB}|$) [1605.02173]

AdS BH with finite λ corrections



Γ_n corrections linear in γ, but coefficient doesn't match?

- $\eta/s=1/(4\pi)(1+120\,\gamma)$; $\gamma=1/8\zeta(3)\lambda^{-3/2}$

Fermi Gas – Kinetic Theory

- Weak Coupling Dynamics: Boltzmann equation with relaxation time τ_R
- Can calculate collective mode dispersion relations for trapped Fermi gas
- Find

$$i\omega_Q(w_Q^2 - 4\omega_\perp^2)\tau_R + (2\omega_\perp^2 - \omega^2) = 0\,, \quad D = 2,3$$

$$i\omega_B(w_B^2 - 4\omega_\perp^2)\tau_R + \left(\frac{10}{3}\omega_\perp^2 - \omega_B^2\right) = 0\,, \quad D = 3\,, \quad (9)$$

Fermi Gas – Kinetic Theory

- Weak Coupling Dynamics: Boltzmann equation with relaxation time τ_R
- Non-hydro modes!

$$\frac{\omega_{B,nh}}{\omega_\perp} = -\frac{5i}{6\tau_R\omega_\perp}\,(D=2)\,, \quad \frac{\omega_{Q,nh}}{\omega_\perp} = -\frac{i}{2\tau_R\omega_\perp}\,,$$

- Relation to viscosity in kinetic theory:

$$\tau_R = \eta/P$$

- Boltzmann: $\Gamma \sim P/\eta$

Fermi Gas – Kinetic Theory

In kinetic theory, can rigorously derive

$$\Gamma \sim P/\eta$$

for weak coupling.

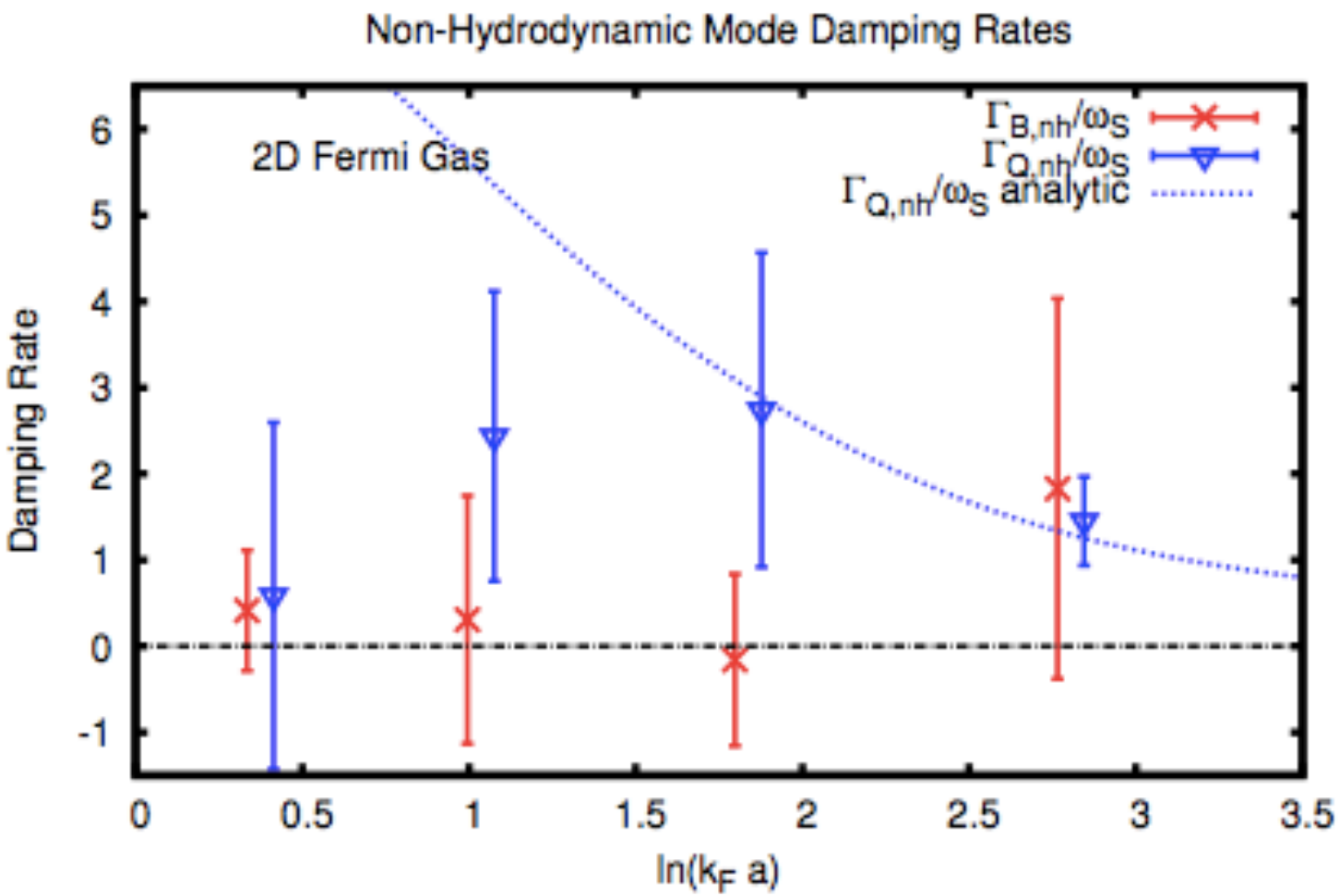
Fermi Gas – Kinetic Theory

- Hydro limit of Boltzmann equation: small mean free time (τ_R->0)
- Hydro Modes
- Non-hydro modes: $\omega=-i/\tau_R$
- Relation to viscosity in kinetic theory:

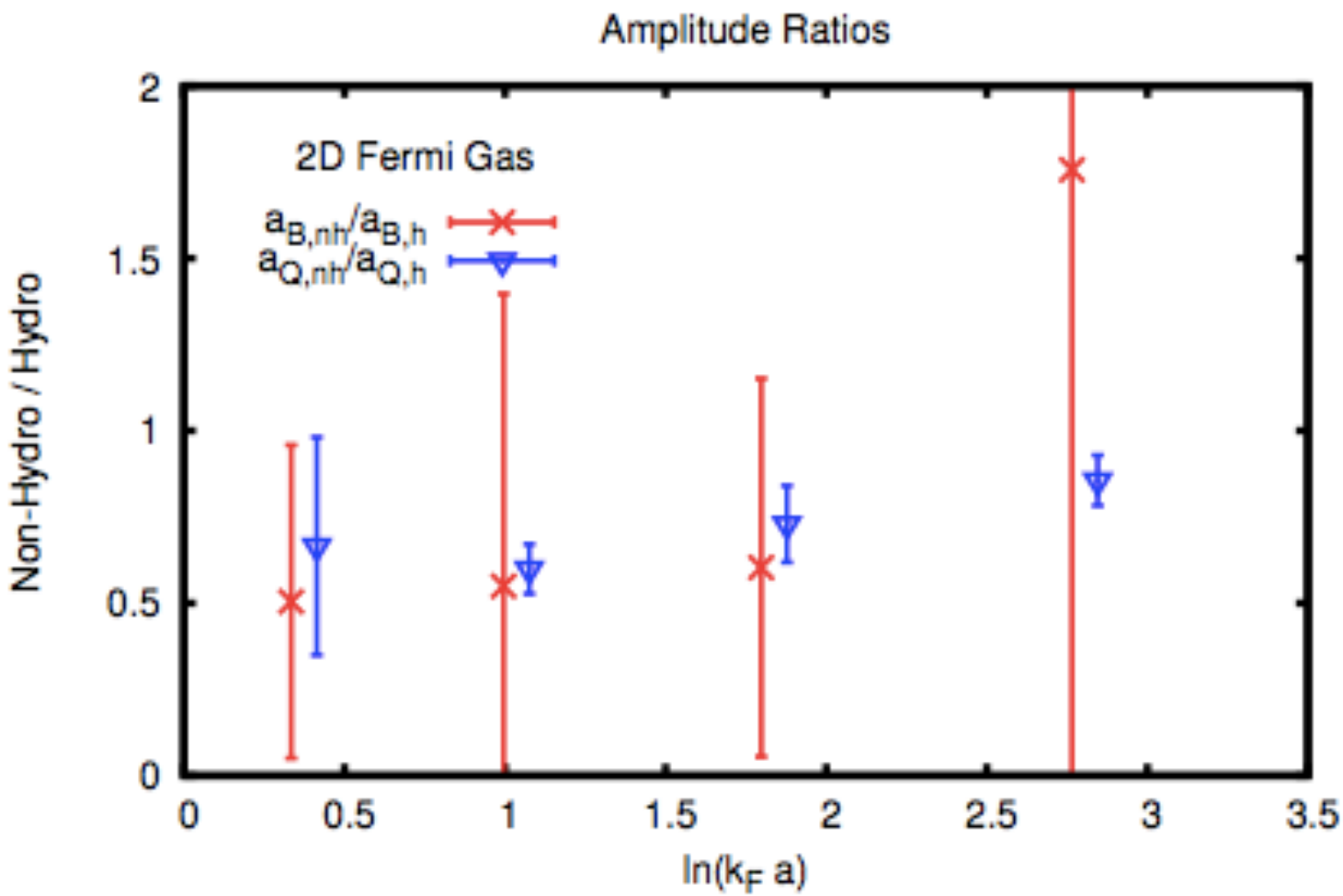
$$\tau_R=\eta/P$$

- Boltzmann: $\Gamma=P/\eta$

Hints for non-hydro modes D=2



Hints for non-hydro modes D=2

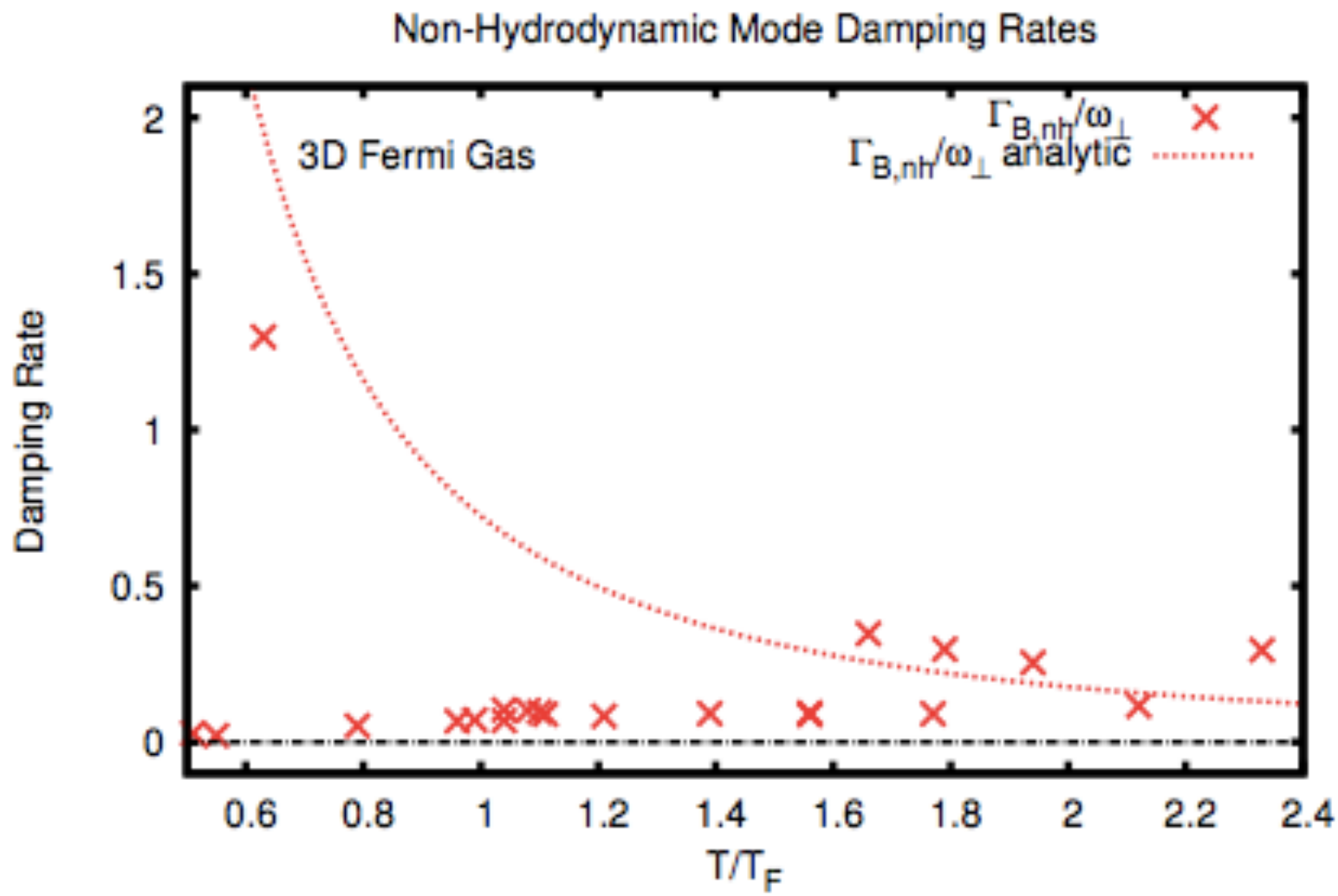


Hints for non-hydro in D=2

- No hint for a non-hydro mode in D=2 breathing mode
- Some hints for a non-hydro mode in D=2 quadrupole mode
- Damping rate for non-hydro quadrupole mode consistent with kinetic theory analytic estimate; uses $\omega_\perp \tau_R = 0.12\left(1 + \frac{4}{\pi^2}\ln^2(k_F a)\right)$

[1507.05975]

Hints for non-hydro modes D=3



Hints for non-hydro in D=3

- Some hints for a non-hydro mode in D=3 breathing mode
- (Some) extracted damping rates for non-hydro breathing mode consistent with kinetic theory analytic estimate; uses

$$\omega_\perp \tau_R = \frac{45\pi}{4} \frac{\omega_\perp}{T_F} \frac{T^2}{T_F^2}$$

[0809.1814]