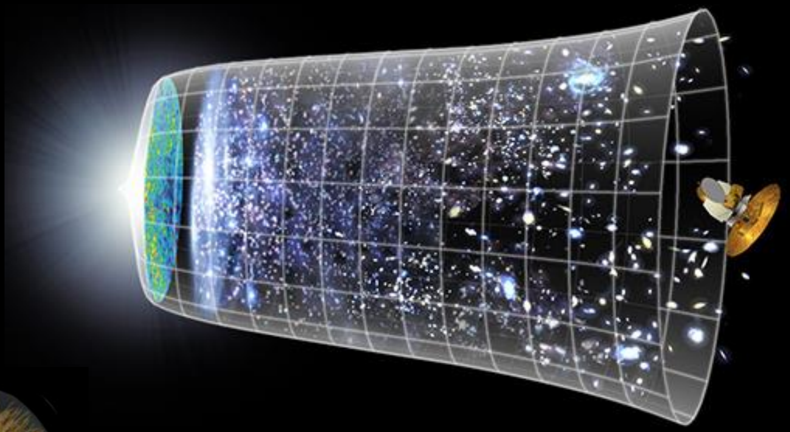
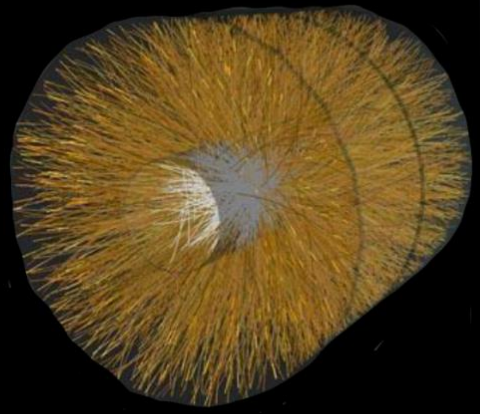
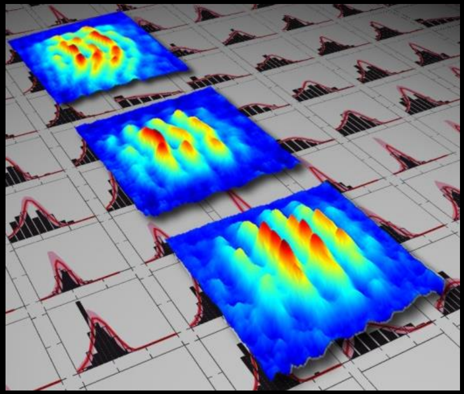
Universality far from equilibrium



WMAP Science Team



ALICE/CERN



TU Vienna

J. Berges

Heidelberg University

ISOQUANT SFB1225

Workshop on non-equilibrium physics and holography, July 2016

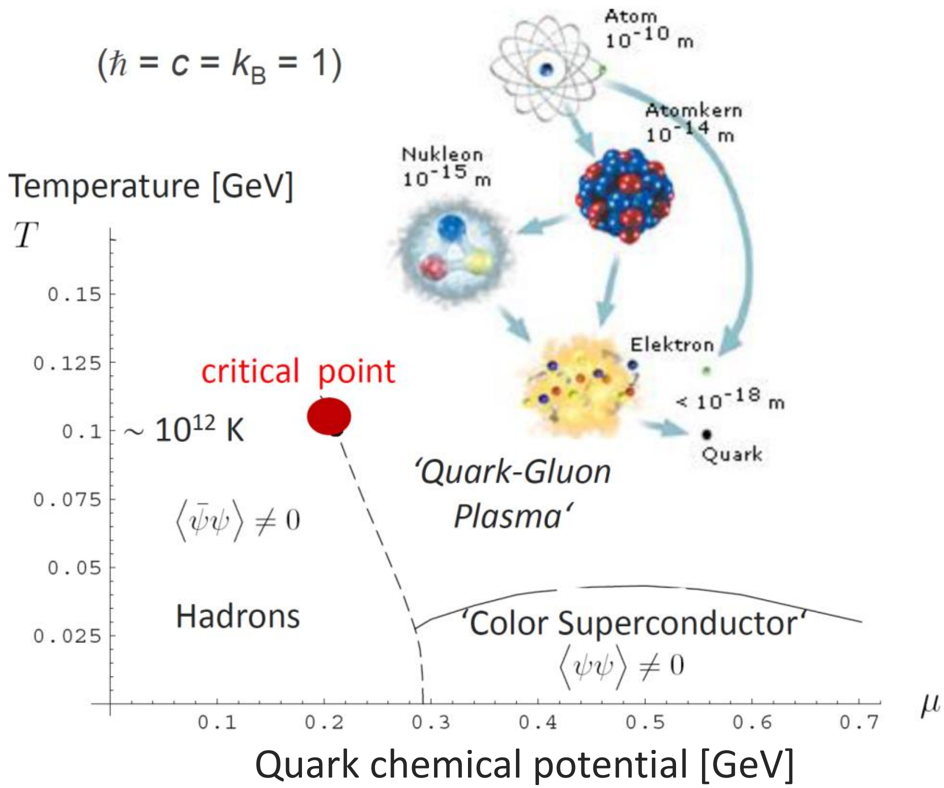
Content

- Thermalization dynamics in isolated quantum systems
- Universality far from equilibrium: Nonthermal fixed points in early-universe cosmology, relativistic heavy-ion collisions, and ultracold quantum gases
- Infrared cascade and dynamics of Bose condensation
- Some puzzles and challenges

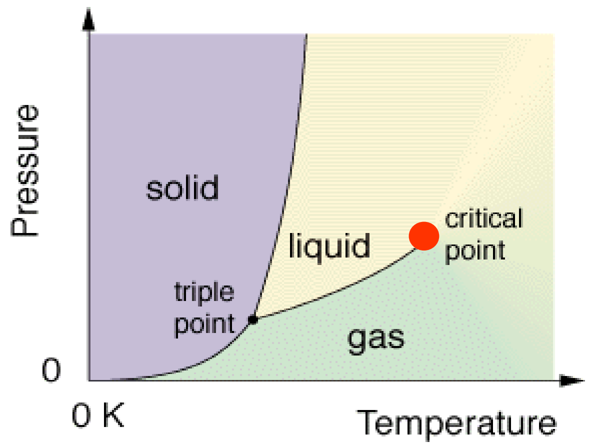
Universality in thermal equilibrium

Example: Schematic phase diagram of

- **strong interaction matter (QCD)**



- **typical liquid-gas system**



Universal critical exponents: e.g.

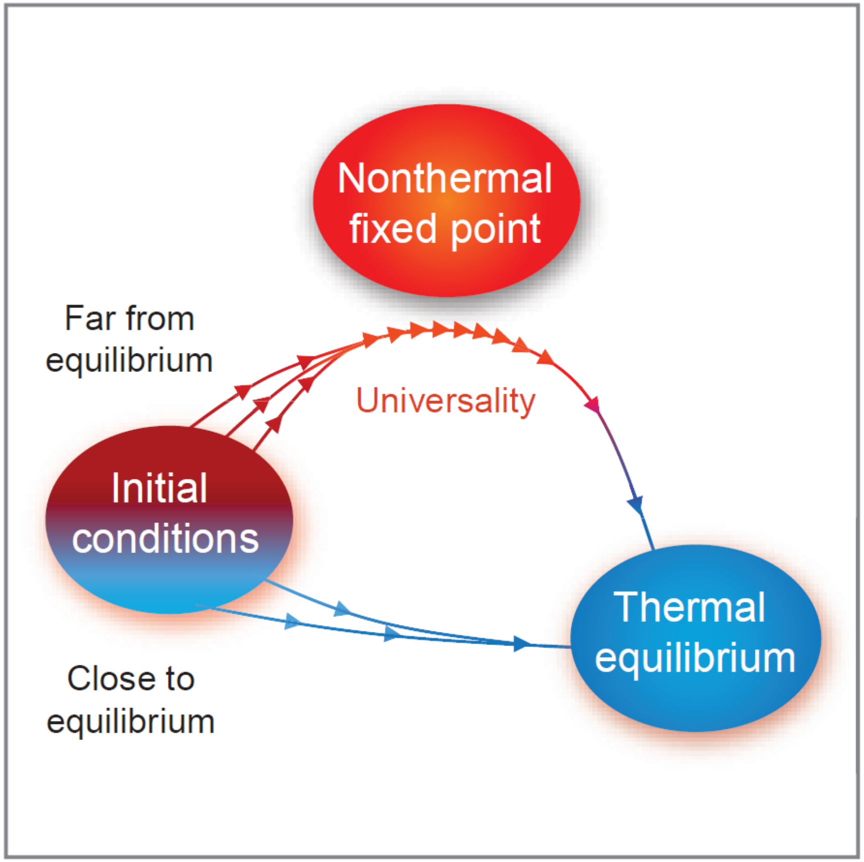
$$\boxed{\text{order parameter} \sim (T_c - T)^{\beta}}$$

Ising universality class (d=3)

→ **Fine-tuning of critical parameter(s) to observe universal behavior (relevant operators for renormalization group fixed points)**

Universality far from equilibrium

Example: Schematic thermalization for isolated quantum systems



→ No fine-tuning to observe far-from-equilibrium universal behavior (irrelevant operators for nonthermal renormalization group fixed points)

Extreme conditions

Dimensionless combination of

coupling strength

field^2 expectation value (vacuum/ thermal equilibrium/nonequilibrium)

$$\frac{\lambda \cdot \langle \varphi^2 \rangle}{Q^2} \sim 1$$

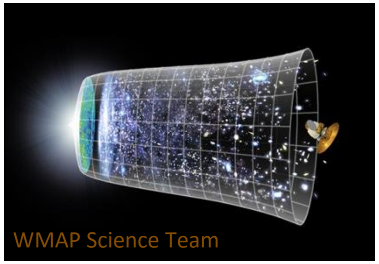
characteristic energy/momentum^2

$\rightarrow$ Extreme conditions can enhance the loss of details about microscopic properties (coupling strengths, initial conditions, …)

Isolated quantum systems in extreme conditions

Early-universe inflaton dynamics

Preheating after inflation (~10^{16} GeV)



Relativistic heavy-ion collision experiments

Quark-Gluon Plasma (~100 MeV ~10^{12} K)

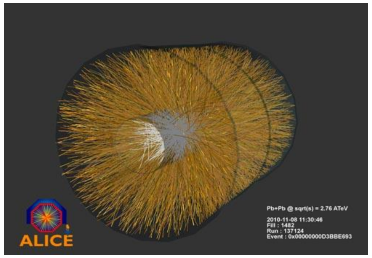
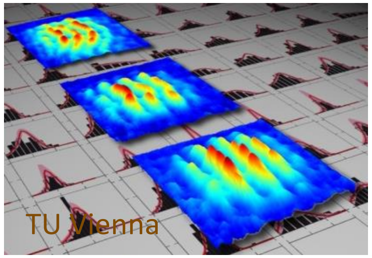
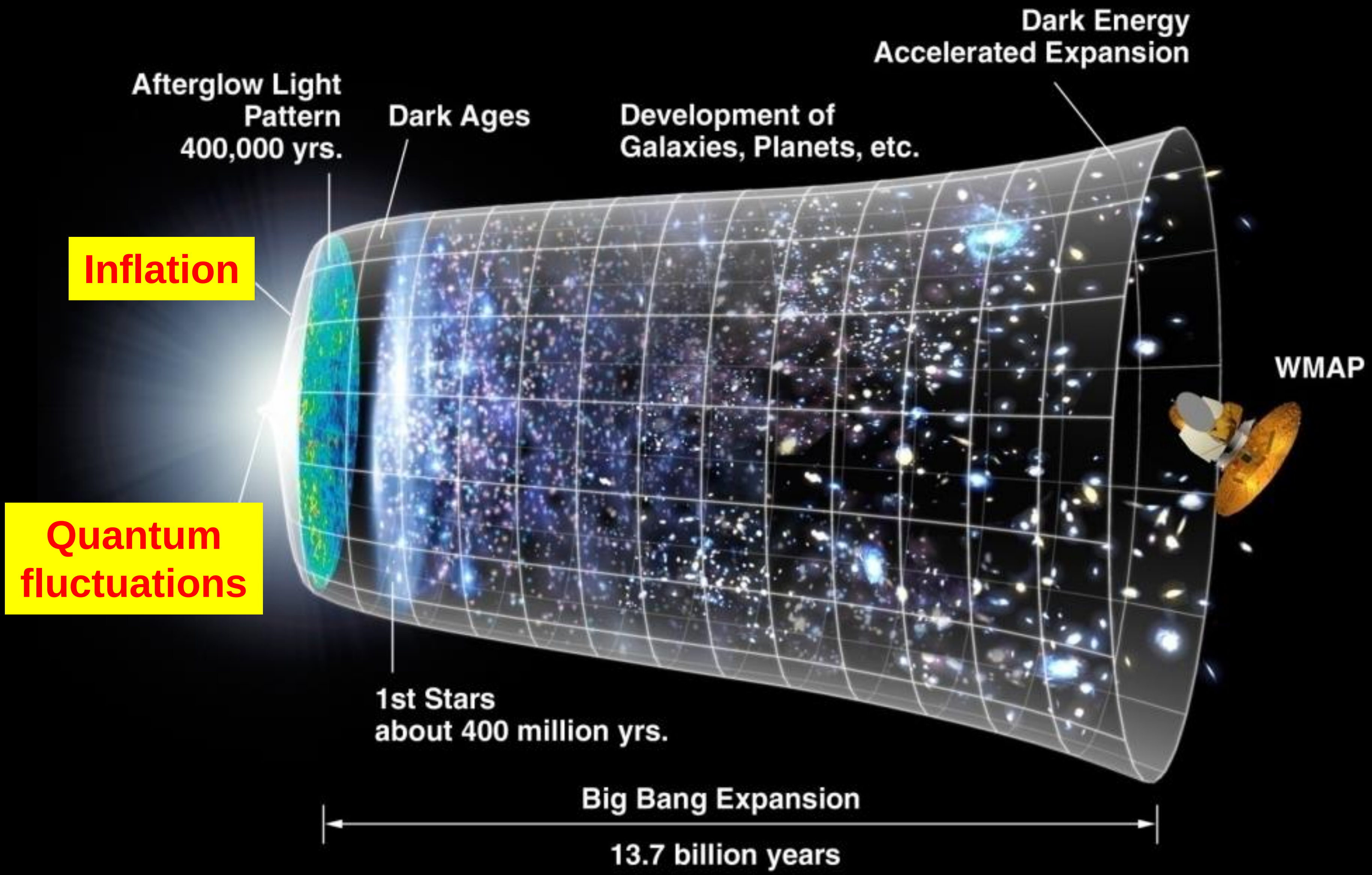


Table-top experiments with ultracold atoms

Strong quenches at nanokelvins



Evolution of the Universe



WMAP Science Team

`Preheating´ from parametric resonance instability

- All particles/radiation diluted away after inflation $\rightarrow$ `vacuum-like´

- Particle production after inflation *Kofman, Linde, Starobinsky, PRL 73 (1994) 3195*

$$V(\phi,\chi) = \frac{1}{2}m^2\phi^2 + \frac{1}{2}\mu^2\chi^2 + \frac{\lambda_\phi}{4!}\phi^4 + \frac{g}{2}\phi^2\chi^2$$

$$\boxed{\phi \gg m/\sqrt{\lambda}} \quad , \quad \phi \sim 0.3 M_P \quad , \quad \lambda_\phi \lesssim 10^{-12} \quad , \quad g^2 \lesssim \lambda$$

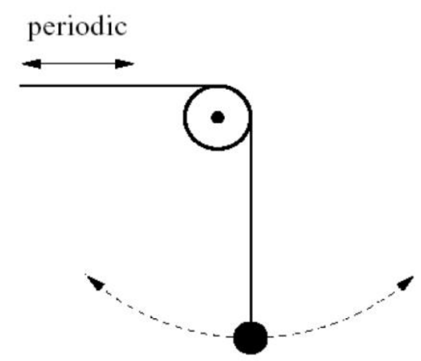
massless reheating: $m = \mu = 0$ $\qquad M_P \equiv (8\pi G)^{-1/2} \simeq 2.435 \cdot 10^{18}\mathrm{GeV}$

$$\frac{d^2\chi_k}{dt^2} + (k^2 + g\phi^2(t))\chi_k = 0 \quad \Rightarrow \quad \boxed{\chi_{k\simeq Q}(t) \sim e^{\gamma_Q t/2}}$$

nonequilibrium instability

Classical oscillator analogue:

$$\left.\begin{array}{l} \omega(t) \leftrightarrow \phi(t), \\ x(t) \leftrightarrow \chi_{k=0}(t) \end{array}\right\} \qquad \ddot{x} + \omega^2(t)x = 0$$



Macroscopic fields, condensates and fluctuations

- In a quantum theory the field amplitude corresponds to the expectation value of a (here relativistic, real) Heisenberg field operator $\Phi(t, \mathbf{x})$:

$$\phi(t) = \langle \Phi(t, \mathbf{x}) \rangle \equiv \mathrm{Tr}\, \{ \varrho_0 \, \Phi(t, \mathbf{x}) \}$$

time-dependent expectation value

density operator at some `initial´ time t = 0

- Fluctuations derive from correlation functions, e.g. spatially homogeneous:

`quantum-half´

volume: $(2\pi)^3\delta(\mathbf{0}) \to V$

occupation number distribution

$$\frac{f(t, \mathbf{p}) + 1/2}{\omega(t, \mathbf{p})} + (2\pi)^3 \delta(\mathbf{p}) \phi_0^2(t)$$

dispersion

condensate²

$$\equiv \frac{1}{2} \int \mathrm{d}^3 x \, e^{-i\mathbf{p}\mathbf{x}} \, \langle \Phi(t, \mathbf{x}) \Phi(t, 0) + \Phi(t, 0) \Phi(t, \mathbf{x}) \rangle$$

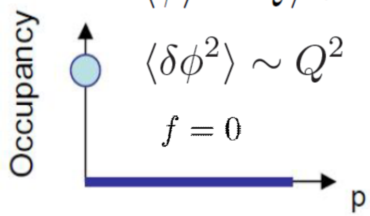
Preheating: Insensitivity to initial condition details

Example: `*Inflaton*´ $\lambda\phi^4$ theory ($\lambda \ll 1$), $\phi = \phi_0 + \delta\phi$

1. Large initial field:

$\langle\phi\rangle \sim Q/\sqrt{\lambda}$

$\langle\delta\phi^2\rangle \sim Q^2$

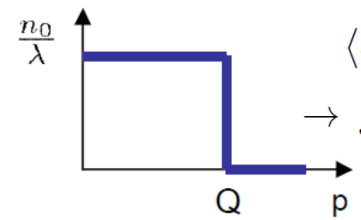


$\boxed{f(Q) \sim e^{\gamma_Q \Delta t}}$

$\Delta t \sim Q^{-1} \log \lambda^{-1}$

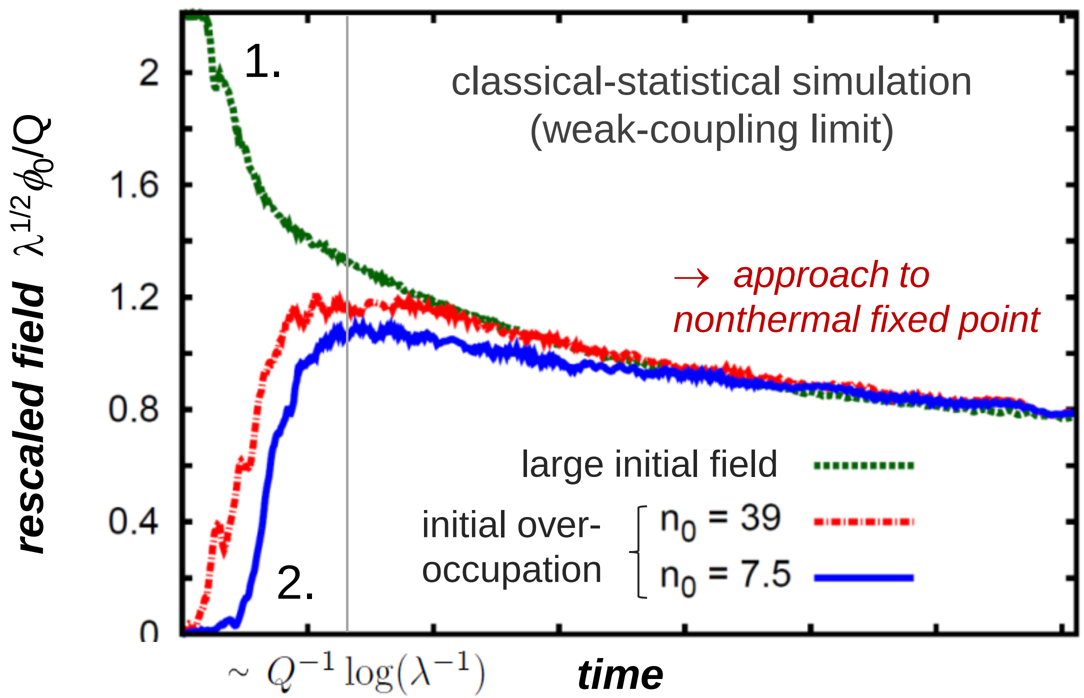
$\longrightarrow$ *instability*

2. High occupancy:



$\langle\delta\phi^2\rangle \sim Q^2/\lambda$

$\to f(Q) \sim 1/\lambda \gg 1$



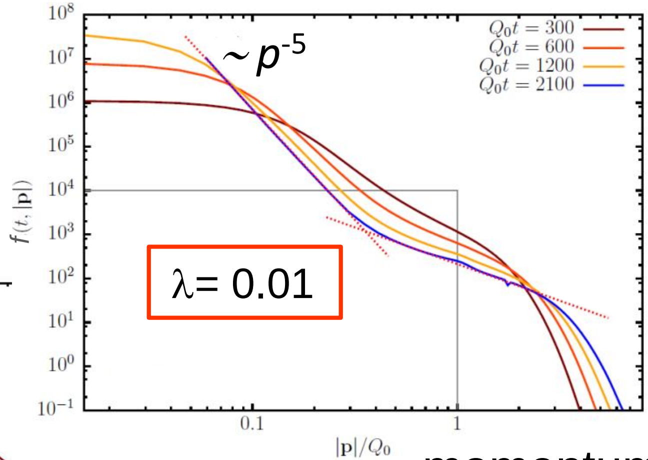
Berges, Boguslavski, Schlichting, Venugopalan, JHEP 1405 (2014) 054

Nonthermal fixed point: Insensitivity to coupling strength

E.g. scalar *N*-component $\lambda\phi^4$ *quantum* theory (1/*N* to NLO 2PI)**:**

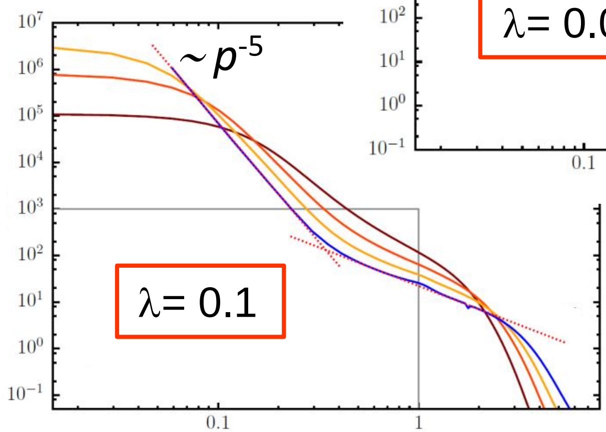
Occupation number distribution:

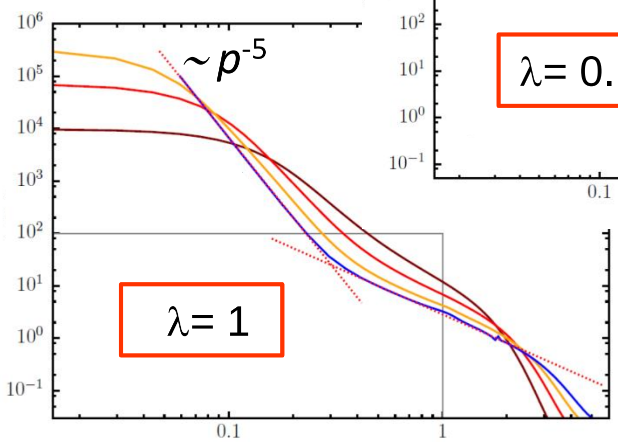
Berges, Wallisch, arXiv:1607.02160



momentum

N = 4

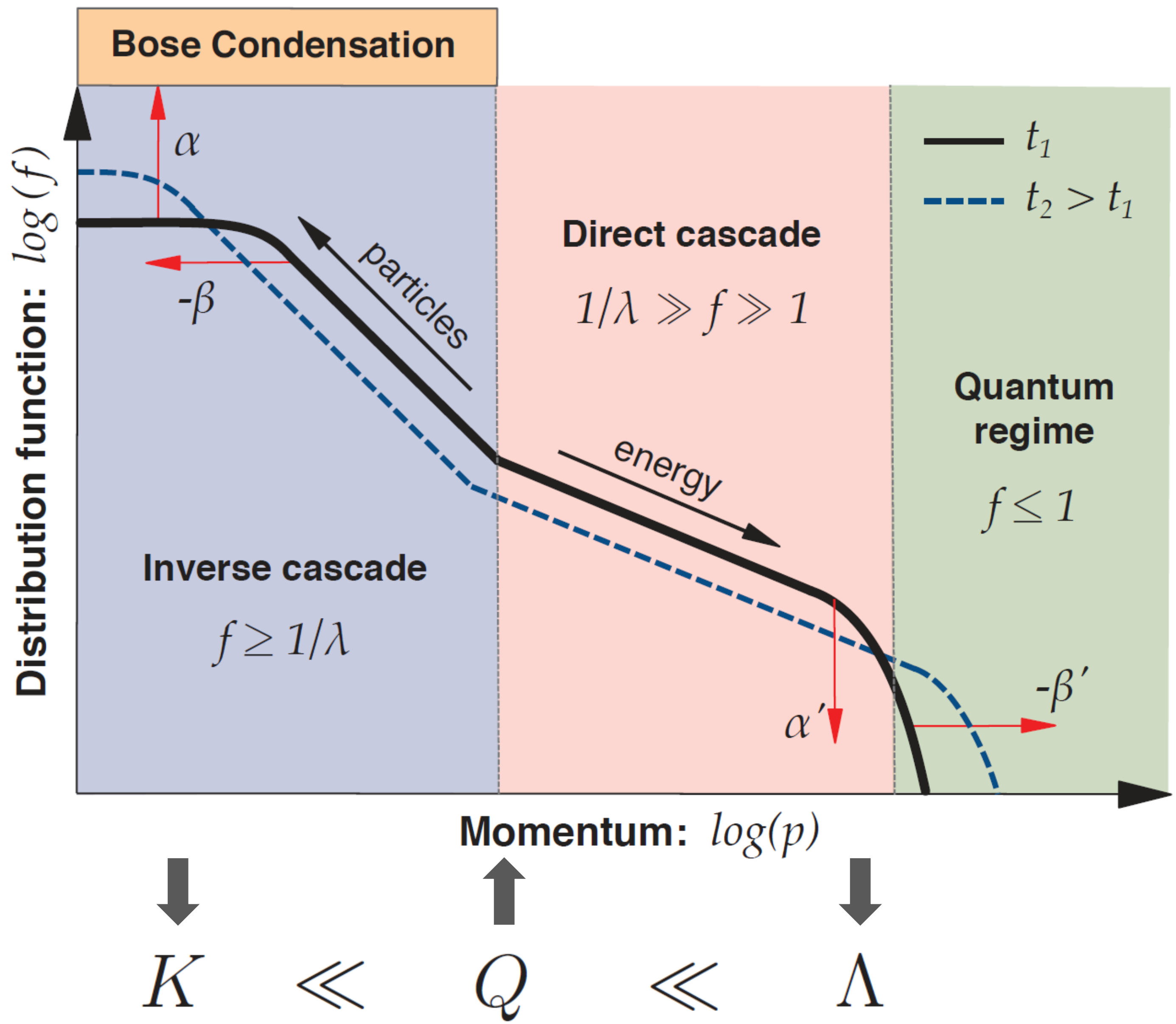




(similar results for strong-field initial conditions)

Universal scaling behavior for wide range of couplings!

Schematic behavior near nonthermal fixed point: dual cascade



Particle versus energy transport

$$K \ll Q \ll \Lambda$$

number conservation:

$$\dot{n}_Q = \dot{n}_K + \dot{n}_\Lambda$$

energy conservation:

$$Q\dot{n}_Q = K\dot{n}_K + \Lambda\dot{n}_\Lambda$$

$$\dot{n}_K = \frac{\Lambda - Q}{\Lambda - K}\dot{n}_Q \simeq \dot{n}_Q$$

$$\dot{n}_\Lambda = \frac{Q - K}{\Lambda - K}\dot{n}_Q \simeq \frac{Q}{\Lambda}\dot{n}_Q$$

$$\Rightarrow \quad \Lambda\dot{n}_\Lambda \simeq Q\dot{n}_Q$$

Particles are transported towards lower scales, energy towards higher scales

Self-similarity

$$f(t,\mathbf{p}) = s^{\alpha/\beta} f(s^{-1/\beta} t, s\mathbf{p})$$

$$s^{-1/\beta} t = 1$$
$$\Rightarrow$$

$$\boxed{f(t,\mathbf{p}) = t^{\alpha}\, f_S(t^{\beta}\mathbf{p})}$$

Time-independent scaling function: $f_S(t^{\beta}\mathbf{p}) \;\equiv\; f(1, t^{\beta}\mathbf{p})$

Scaling exponents α and β determine rate and direction of transport:

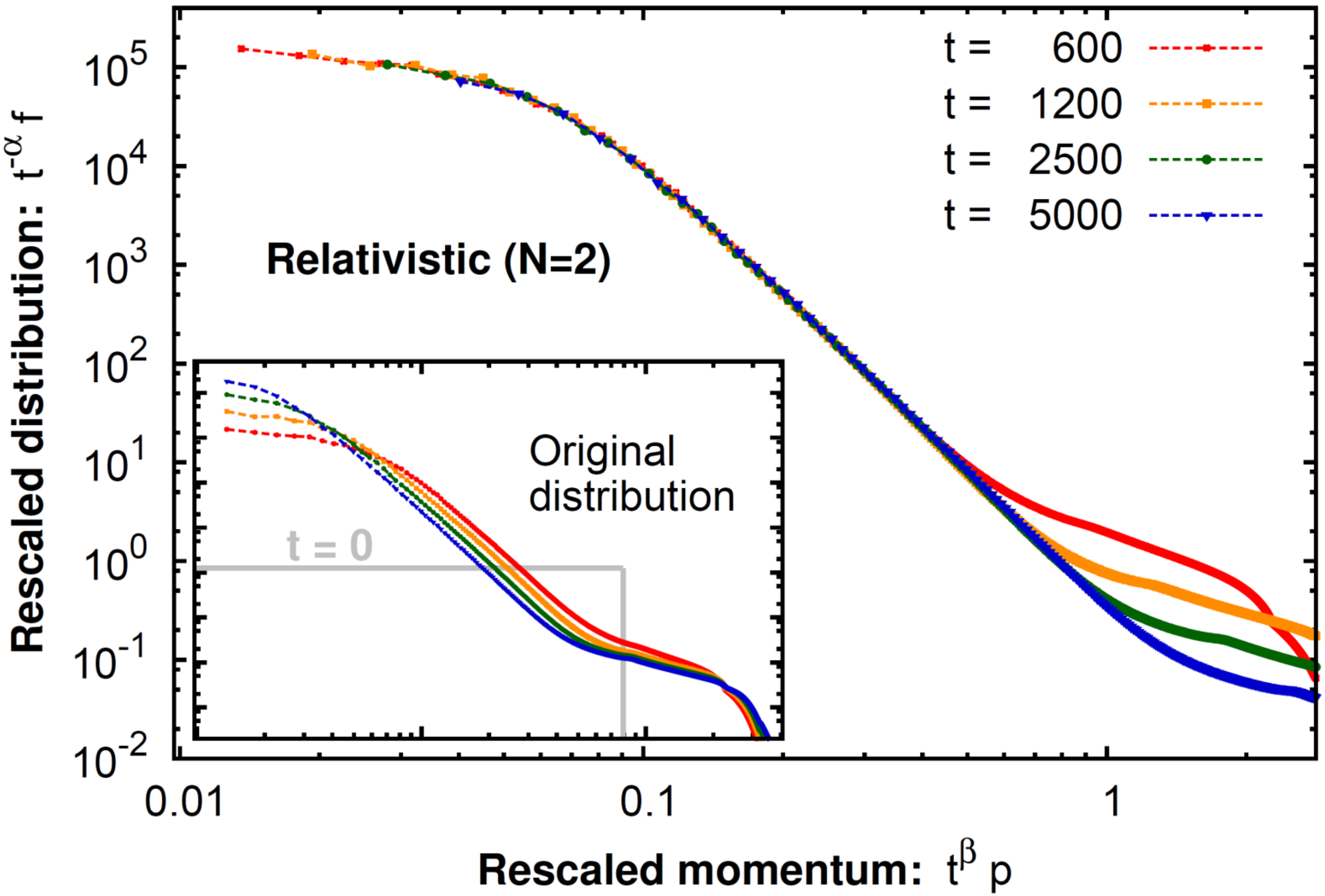
$$K(t_1) = K_1 \qquad \Rightarrow \qquad K(t) = K_1 (t/t_1)^{-\beta}$$

$$f(t, K(t)) \sim t^{\alpha}$$

e.g. $\alpha > 0$, $\beta > 0$: particle transport towards lower momentum scales

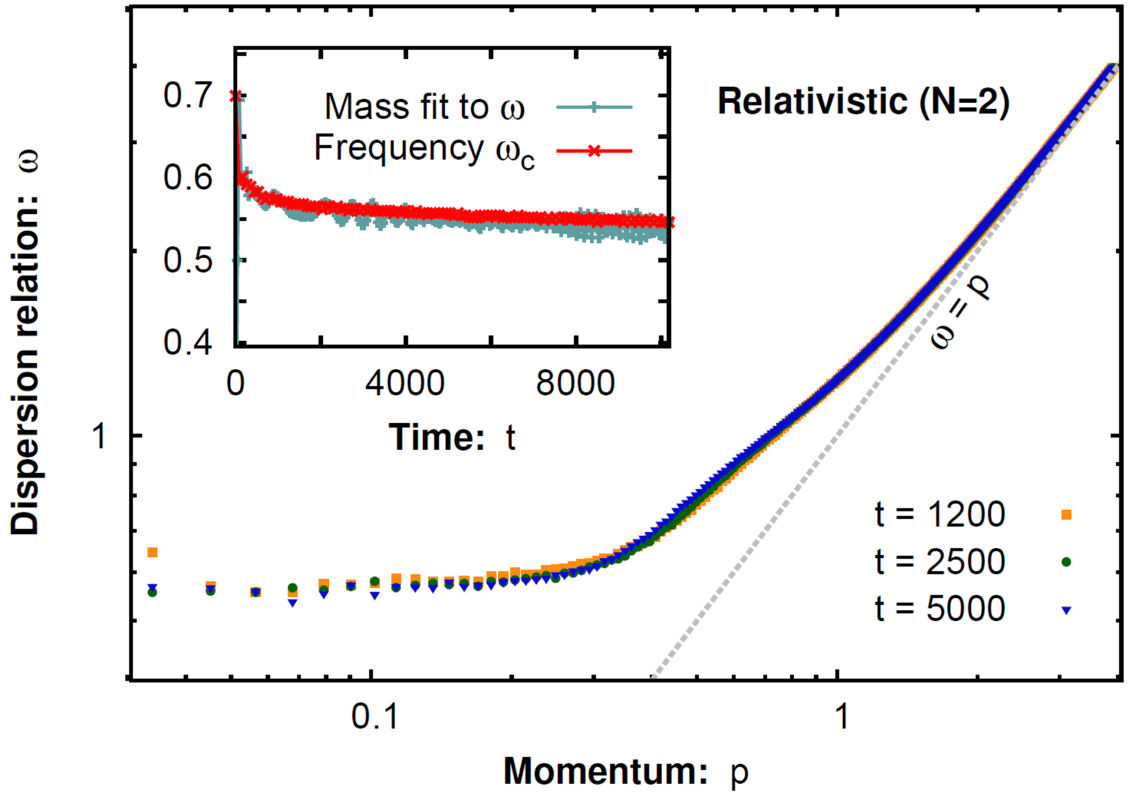
Self-similar dynamics: infrared scaling

$$f(t,\mathbf{p}) = t^{\alpha}\, f_S(t^{\beta}\mathbf{p}) \;, \quad \alpha = 1.51 \pm 0.13\,, \qquad \beta = 0.51 \pm 0.04$$



Piñeiro Orioli, Boguslavski, Berges, PRD 92 (2015) 025041

Mass scale separating non-relativistic infrared regime



- ***non-relativistic infrared dynamics*** expected because of the generation of a ***mass gap*** (condensate + medium)

→ relativistic & non-relativstic field theories have same infrared scaling

Piñeiro Orioli, Boguslavski, Berges, PRD 92 (2015) 025041

Non-relativistic system: Dilute quantum gases

- Interacting bosons with s-wave scattering length a, interatomic distance $\sim n^{-1/3}$

`diluteness' parameter:

$$\boxed{\zeta = \sqrt{na^3}}$$

`inverse coherence length':

$$\boxed{Q = \sqrt{16\pi a n}}$$

- E.g. Gross-Pitaevskii equation for dilute Bose gas:

$$i\partial_t \psi(t,\mathbf{x}) = \left(-\frac{\nabla^2}{2m} + g|\psi(t,\mathbf{x})|^2\right)\psi(t,\mathbf{x})$$

$$g = 4\pi a/m \qquad N_{\text{total}} = \int d^3x |\psi(t,\mathbf{x})|^2$$

Dilute bose gas in extreme conditions

- E.g. in the mean-field approximation for a spatially homogeneous system without condensate the interaction term leads to a constant energy shift:

$$\Delta E = 2g\langle|\psi|^2\rangle = 2gn = 2g\int\frac{d^3p}{(2\pi)^3}f_{\mathrm{nr}}(|\mathbf{p}|)$$

occupation number

- For the overoccupied Bose gas with

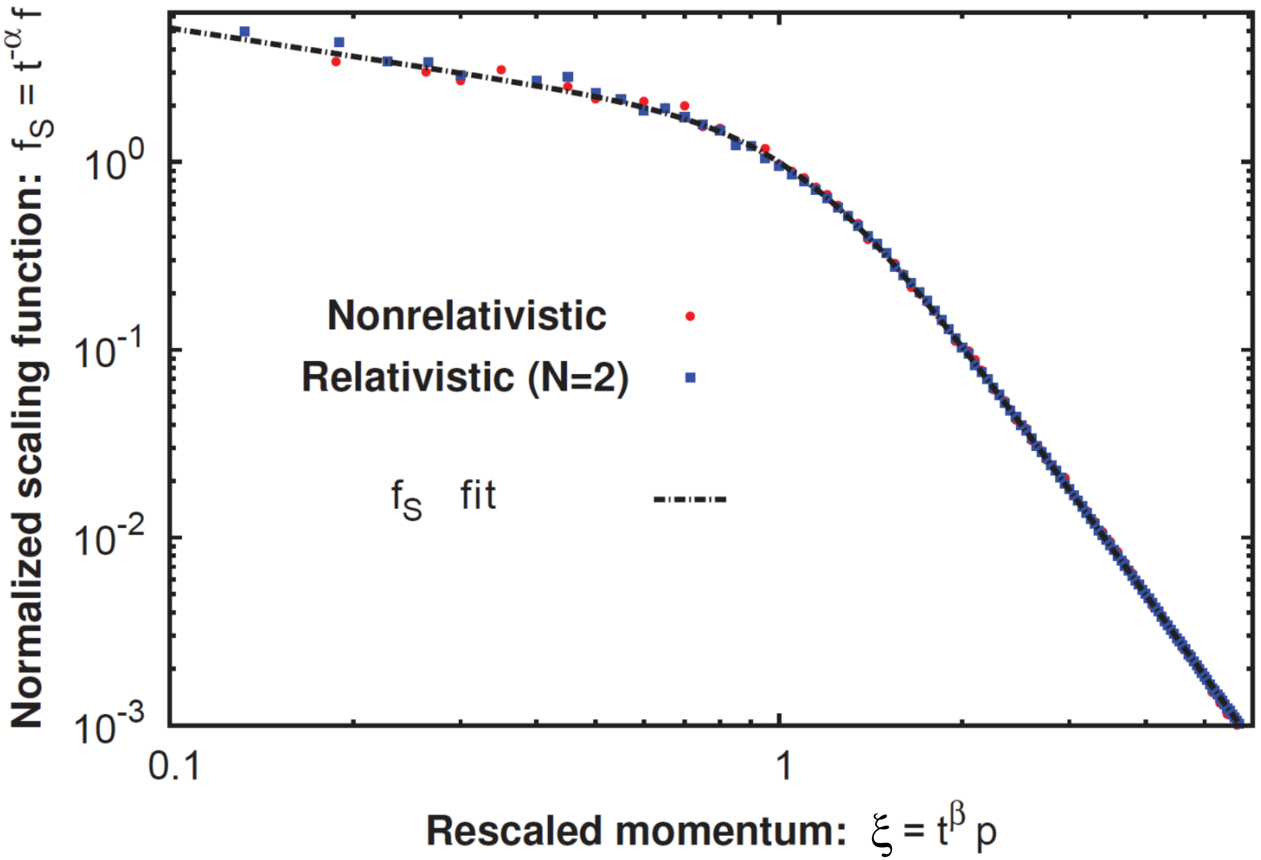
$$f_{\mathrm{nr}}(Q) \sim \frac{1}{\zeta}$$

the mean-field shift in energy is of the same order as the relevant kinetic energy $Q^2/2m$ *irrespective of the coupling g*, since

$$g\int \mathrm{d}^3p\, f_{\mathrm{nr}}(\mathbf{p}) \sim g\,Q^3 f_{\mathrm{nr}}(Q) \sim g\frac{Q^3}{\zeta} \sim g\frac{Q^3}{mgQ} \sim \frac{Q^2}{m}$$

$$\left.\begin{aligned} a &= mg/(4\pi) \\ Q &= \sqrt{16\pi a n}\end{aligned}\right\} \quad Q = 2\sqrt{mgn} \xrightarrow{\zeta = \sqrt{na^3}} \zeta = mgQ/(16\pi^{3/2})$$

Universal scaling form of the distribution function



Piñeiro Orioli, Boguslavski, Berges, PRD 92 (2015) 025041

$$f_S(\xi) \simeq \frac{A(\kappa_> - \kappa_<)}{(\kappa_> - 2)(\xi/B)^{\kappa_<} + (2 - \kappa_<)(\xi/B)^{\kappa_>}} \quad , \; \kappa_< \simeq 0.5 \;\; , \;\; \kappa_> \simeq 4.5$$

$$f_S(\xi = B) = A \quad , \quad df_S(\xi = B)/d\xi = -2A/B$$

Estimating scaling properties

$$\frac{\partial f(t,\mathbf{p})}{\partial t} = C\,[f]\,(t,\mathbf{p})$$ *`collision integral´*

$$C[f](t,\mathbf{p}) \underset{\uparrow}{=} s^{-\mu}\,C[f](s^{-1/\beta}t, s\mathbf{p}) \underset{\uparrow}{=} t^{-\beta\mu}\,C[f_S](1, t^{\beta}\mathbf{p})$$

$$f(t,\mathbf{p}) = s^{\alpha/\beta} f(s^{-1/\beta}t, s\mathbf{p}) \qquad\qquad s^{-1/\beta}t = 1$$

Use $$\frac{\partial}{\partial t}\left[t^{\alpha}\, f_S(t^{\beta}\mathbf{p})\right] = t^{\alpha-1}\left[\alpha + \beta\,\mathbf{q}\cdot\nabla_{\mathbf{q}}\right] f_S(\mathbf{q})\big|_{\mathbf{q}=t^{\beta}\mathbf{p}}$$

Time-independent *fixed point equation:*

$$\boxed{\left[\alpha + \beta\,\mathbf{p}\cdot\nabla_{\mathbf{p}}\right] f_S(\mathbf{p}) = C[f_S](1,\mathbf{p})}$$

+ scaling relation: $$\boxed{\alpha - 1 = -\beta\mu}$$

Micha, Tkachev, PRD 70 (2004) 043538

Conservation laws

$$n = \int \frac{d^d p}{(2\pi)^d} f(t,\mathbf{p}) = t^{\alpha-\beta d} \int \frac{d^d q}{(2\pi)^d} f_S(\mathbf{q})$$

$$\Rightarrow \quad \boxed{\textit{particle conservation:} \quad \alpha = \beta d}$$

$$\epsilon = \int \frac{d^d p}{(2\pi)^d} \omega(\mathbf{p}) f(t,\mathbf{p}) = t^{\alpha-\beta(d+z)} \int \frac{d^d q}{(2\pi)^d} \omega(\mathbf{q}) f_S(\mathbf{q})$$

$$\Rightarrow \quad \boxed{\textit{energy conservation:} \quad \alpha = \beta(d+z)}$$

$$\omega(\mathbf{p}) = s^{-z} \omega(s\mathbf{p})$$

Perturbative estimate (Gross-Pitaevskii)

$$C^{2\leftrightarrow 2}[f](t,\mathbf{p}) = \int \mathrm{d}\Omega^{2\leftrightarrow 2}(\mathbf{p},\mathbf{l},\mathbf{q},\mathbf{r})$$
$$\times\left[(f_{\mathbf{p}}+1)(f_{\mathbf{l}}+1)f_{\mathbf{q}}f_{\mathbf{r}} - f_{\mathbf{p}}f_{\mathbf{l}}(f_{\mathbf{q}}+1)(f_{\mathbf{r}}+1)\right]$$
$$\overset{1\,\ll\, f_{\mathbf{p}}}{\simeq} \int \mathrm{d}\Omega^{2\leftrightarrow 2}(\mathbf{p},\mathbf{l},\mathbf{q},\mathbf{r})\;\left[(f_{\mathbf{p}}+f_{\mathbf{l}})f_{\mathbf{q}}f_{\mathbf{r}} - f_{\mathbf{p}}f_{\mathbf{l}}(f_{\mathbf{q}}+f_{\mathbf{r}})\right]$$

Using:
$$\int \mathrm{d}\Omega^{2\leftrightarrow 2}(\mathbf{p},\mathbf{l},\mathbf{q},\mathbf{r}) \sim g^2 \int \frac{d^d l}{(2\pi)^d}\frac{d^d q}{(2\pi)^d}\frac{d^d r}{(2\pi)^d}$$
$$\times(2\pi)^{d+1}\,\delta^{(d)}(\mathbf{p}+\mathbf{l}-\mathbf{q}-\mathbf{r})\,\delta(\omega_{\mathbf{p}}+\omega_{\mathbf{l}}-\omega_{\mathbf{q}}-\omega_{\mathbf{r}})$$

gives scaling relation: $\alpha - 1 = -\beta\,[3d-(d+2)-3\alpha/\beta]$

$$\overset{\alpha\,=\,\beta d}{\Rightarrow}\quad \textit{nonrel. particle transport:}\;\; \alpha = -\frac{d}{2},\;\; \beta = -\frac{1}{2}$$

Negative perturbative exponents do not account for inverse particle cascade!

Beyond perturbation theory: large-*N* expansion to NLO

$$\int \mathrm{d}\Omega^{2\leftrightarrow 2}(\mathbf{p},\mathbf{l},\mathbf{q},\mathbf{r}) \longrightarrow \int d\Omega^{\mathrm{NLO}}[f](t,\mathbf{p},\mathbf{l},\mathbf{q},\mathbf{r}) \sim \int \frac{d^d l}{(2\pi)^d}\frac{d^d q}{(2\pi)^d}\frac{d^d r}{(2\pi)^d}$$
$$\times (2\pi)^{d+1}\,\delta^{(d)}(\mathbf{p}+\mathbf{l}-\mathbf{q}-\mathbf{r})\,\delta(\omega_{\mathbf{p}}+\omega_{\mathbf{l}}-\omega_{\mathbf{q}}-\omega_{\mathbf{r}})$$
$$\times\, g^2_{\mathrm{eff}}[f](t,\mathbf{p},\mathbf{q})\,,$$

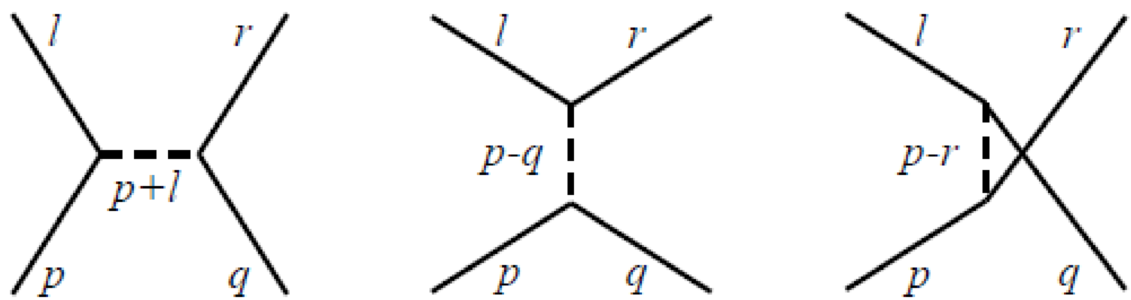


FIG. 11. Illustration of different scattering channels. The vertex correction at NLO may be viewed as an effective interaction, which involves the exchange of an intermediate particle.

based on Berges Nucl. Phys. A 699 (2002) 847; Aarts et al. Phys. Rev. D 66 (2002) 045008

Vertex correction (NLO 1/*N*)

$$g_{\mathrm{eff}}^2(t,\mathbf{p},\mathbf{q}) \equiv \frac{g^2}{|1+\Pi_{\mathrm{nr}}^R(t,\omega_{\mathbf{p}}-\omega_{\mathbf{q}},\mathbf{p}-\mathbf{q})|^2}$$

one-loop retarded self-energy:

$$\Pi_{\mathrm{nr}}^R(t,\omega,\mathbf{p}) = g\int\frac{\mathrm{d}^d q}{(2\pi)^d}\, f(t,\mathbf{p}-\mathbf{q})$$
$$\times\left[\frac{1}{\omega_{\mathbf{q}}-\omega_{\mathbf{p}-\mathbf{q}}-\omega-i\epsilon}+\frac{1}{\omega_{\mathbf{q}}-\omega_{\mathbf{p}-\mathbf{q}}+\omega+i\epsilon}\right]$$

scaling behavior:

$$\Pi_{\mathrm{nr}}^R(t,\omega_{\mathbf{p}},\mathbf{p}) = s^{\alpha/\beta-d+2}\,\Pi_{\mathrm{nr}}^R(s^{-1/\beta}t,\omega_{s\mathbf{p}},s\mathbf{p})$$

$\alpha/\beta \geq d$, $\Pi_{\mathrm{nr}}^R(t,\omega_{\mathbf{p}},\mathbf{p}) \gg 1$ in the infrared:

$$\Rightarrow \qquad g_{\mathrm{eff}}^2(t,\mathbf{p},\mathbf{q},\mathbf{r}) = s^{-2(\alpha/\beta-d+2)}\,g_{\mathrm{eff}}^2(s^{-1/\beta}t,s\mathbf{p},s\mathbf{q},s\mathbf{r})$$

Scaling solution at NLO 1/*N*

$$C_{\mathrm{nr}}^{\mathrm{NLO}}[f](t,\mathbf{p}) = s^{-(2-\alpha/\beta)}\, C_{\mathrm{nr}}^{\mathrm{NLO}}[f](s^{-1/\beta}t, s\mathbf{p})$$
$$= t^{\alpha-2\beta}\, C_{\mathrm{nr}}^{\mathrm{NLO}}[f_S](1, t^{\beta}\mathbf{p})$$

gives scaling relation: $\alpha - 1 = \alpha - 2\beta$

$$\Rightarrow \boxed{\textit{nonrel. transport: } \beta = \frac{1}{2} \;\; \textit{of} \begin{cases} \textit{particles: } & \alpha = d/2 \\ \textit{energy: } & \alpha = (d+2)/2 \end{cases}}$$

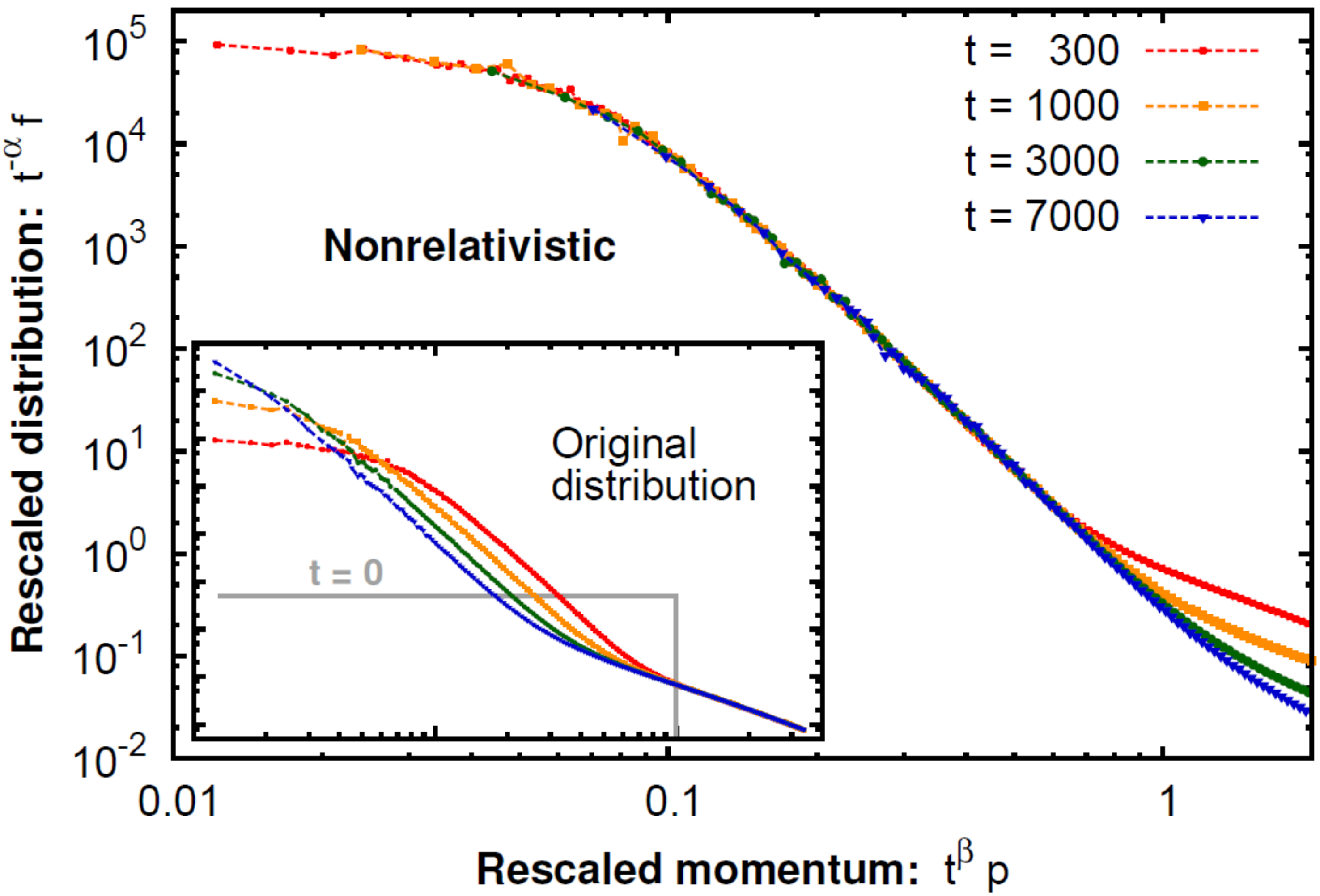
Positive nonperturbative exponents can describe inverse cascade!

NLO result in good agreement with full numerical simulation

Piñeiro Orioli, Boguslavski, Berges, PRD 92 (2015) 025041

Self-similar dynamics from classical-statistical simulations

$$f(t,\mathbf{p}) = t^{\alpha}\, f_S(t^{\beta}\mathbf{p})\ , \quad \alpha = 1.66 \pm 0.12\,, \qquad \beta = 0.55 \pm 0.03$$



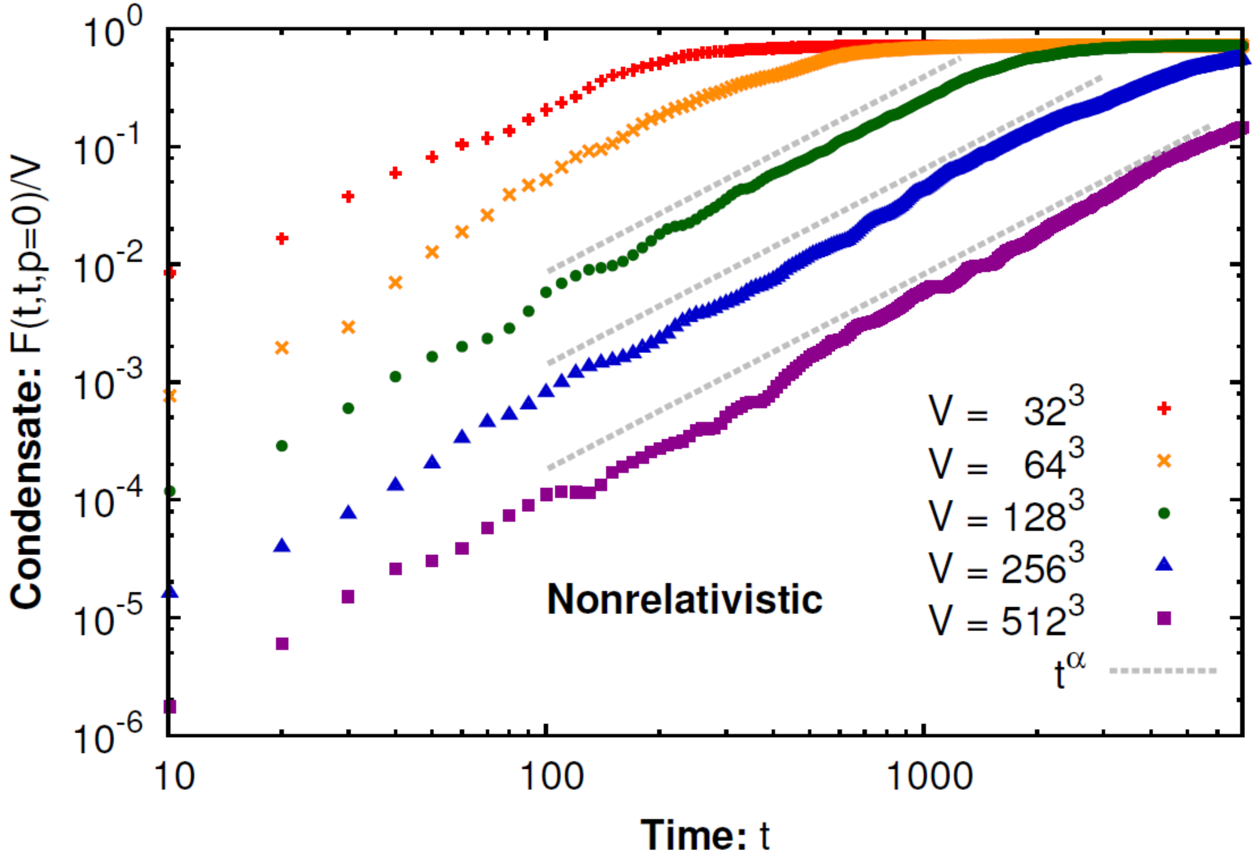
Piñeiro Orioli, Boguslavski, Berges, PRD 92 (2015) 025041

Condensation far from equilibrium

$$F(t,t',\mathbf{x}-\mathbf{x}') = \frac{1}{2}\langle \psi(t,\mathbf{x})\psi^*(t',\mathbf{x}') + \psi(t',\mathbf{x}')\psi^*(t,\mathbf{x})\rangle$$

$$f(t,\mathbf{p}) + (2\pi)^3\delta^{(3)}(\mathbf{p})|\psi_0|^2(t) \equiv \int d^3x\, e^{-i\mathbf{px}}\, F(t,t,\mathbf{x})$$

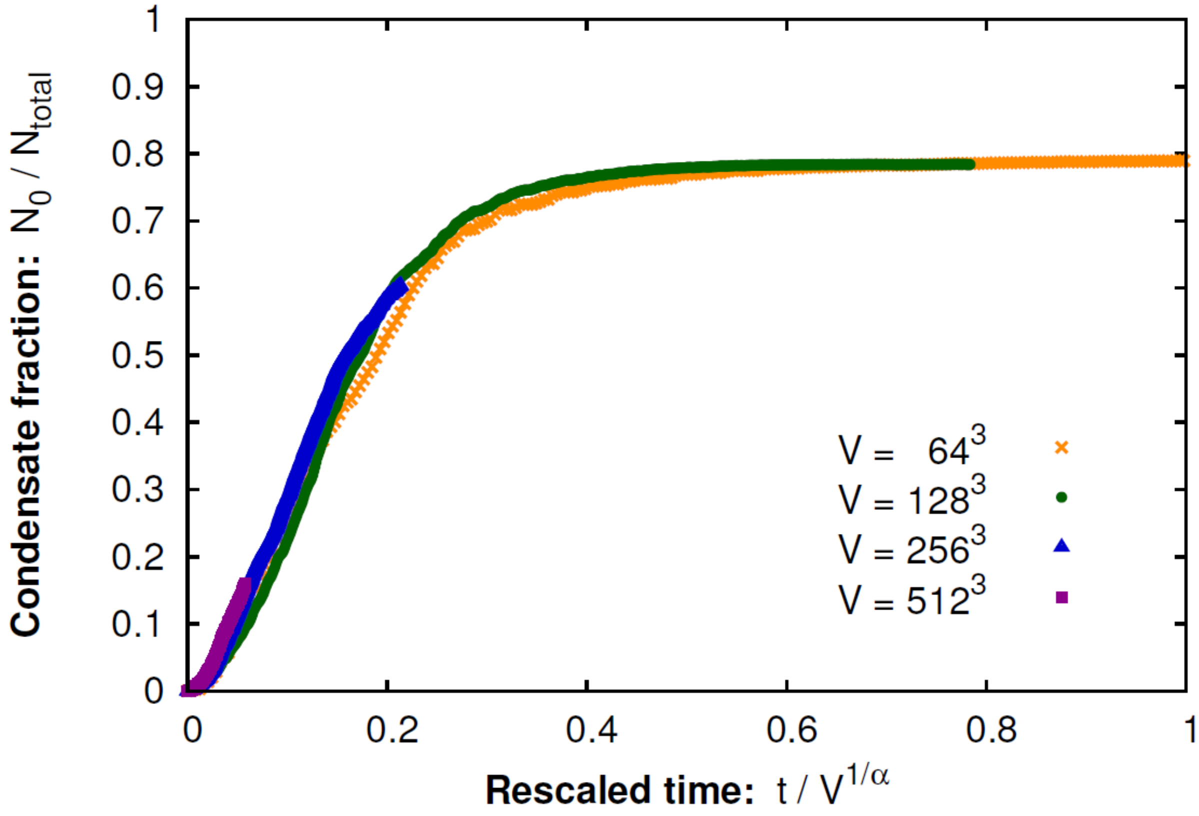
volume: $(2\pi)^3\delta(\mathbf{0}) \to V$



Condensation time

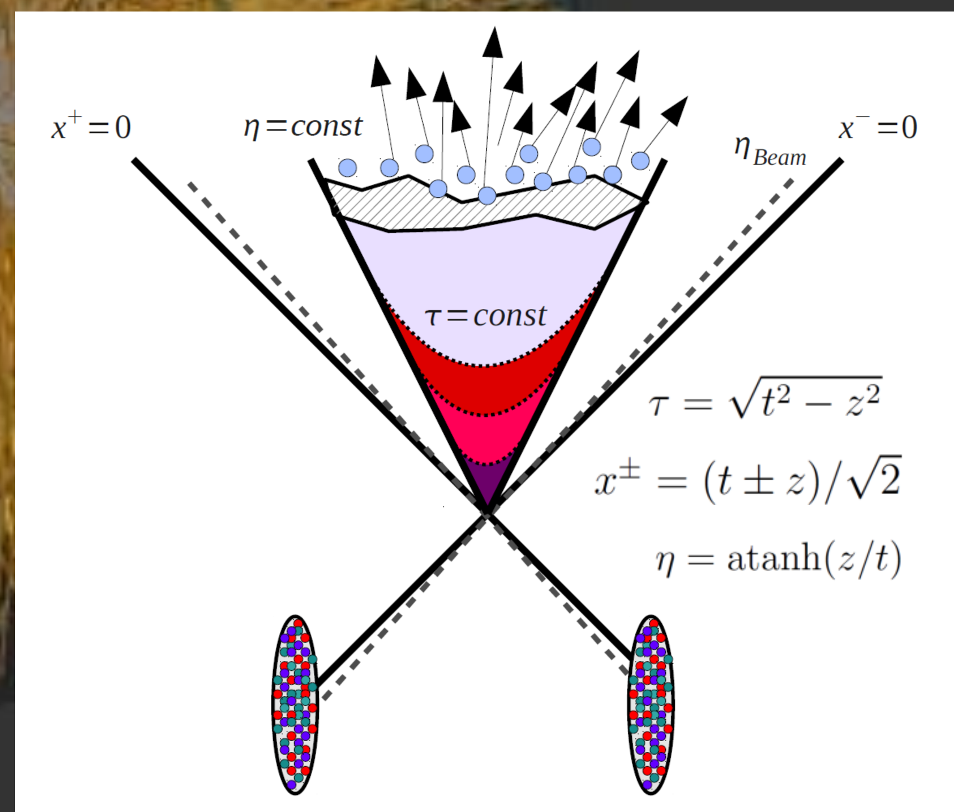
$$\frac{N_0(t)}{N_{\text{total}}} = \frac{|\psi_0|^2(t)}{\int d^3p/(2\pi)^3 f(t,\mathbf{p}) + |\psi_0|^2(t)} \quad , \quad V^{-1}F(t,t,\mathbf{p}=0) \sim t^{\alpha}$$

$$\Rightarrow \qquad \boxed{t_f \simeq t_0 \left(\frac{|\psi_0|^2(t_f)}{f(t_0,0)} \right)^{1/\alpha} V^{1/\alpha}}$$



Analytic estimates agree well with simulations!

Thermalization dynamics in relativistic heavy-ion collisions



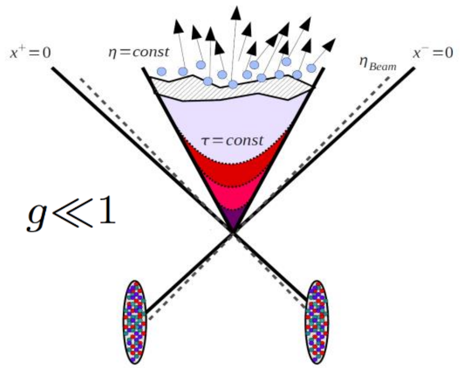
Heavy-ion collisions in the high-energy limit

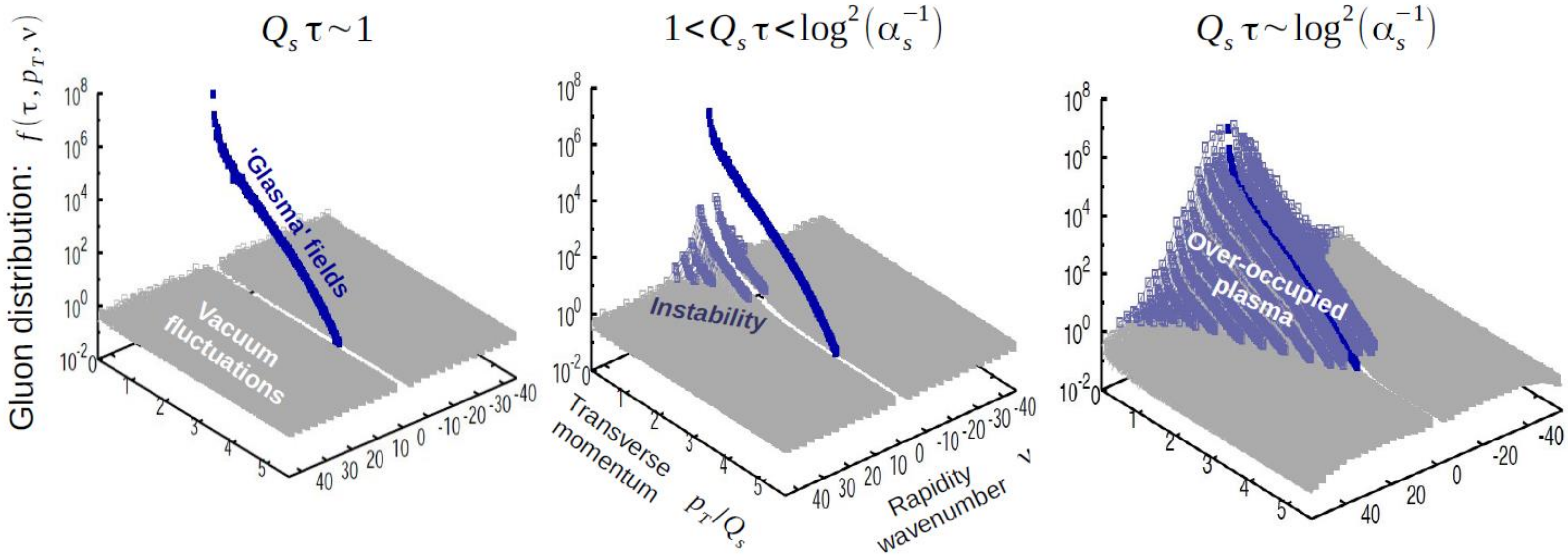
Large initial gauge fields: $\langle A \rangle \sim Q_s/g$

CGC: Lappi, McLerran, Dusling, Gelis, Venugopalan, Epelbaum…

Small initial (vacuum) fluctuations: $\langle \delta A^2 \rangle \sim Q_s^2$

$\rightarrow$ ***plasma instabilities***





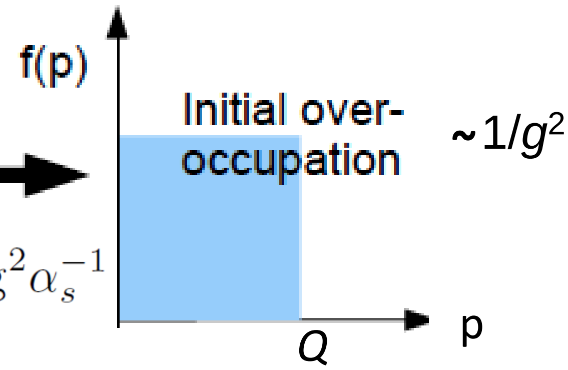
JB, Schenke, Schlichting, Venugopalan, NPA931 (2014) 348 for initial spectrum from *Epelbaum, Gelis, PRD88 (2013) 085015*. ***Plasma instabilities from wide range of initial conditions:***

Mrowczynski; Rebhan, Romatschke, Strickland; Arnold, Moore, Yaffe; Bödecker; Attems, … Romatschke, Venugopalan; J.B., Scheffler, Schlichting, Sexty; Fukushima, Gelis …

Overoccupied non-Abelian plasma

Glasma + vacuum fluctuations → Plasma Instabilities →

$\tau_0 \sim Q_s^{-1} \log^2 \alpha_s^{-1}$



- To discuss attractor: Initial overoccupation described by family of distributions at τ_0 (read-out in Coulomb gauge)

occupancy parameter

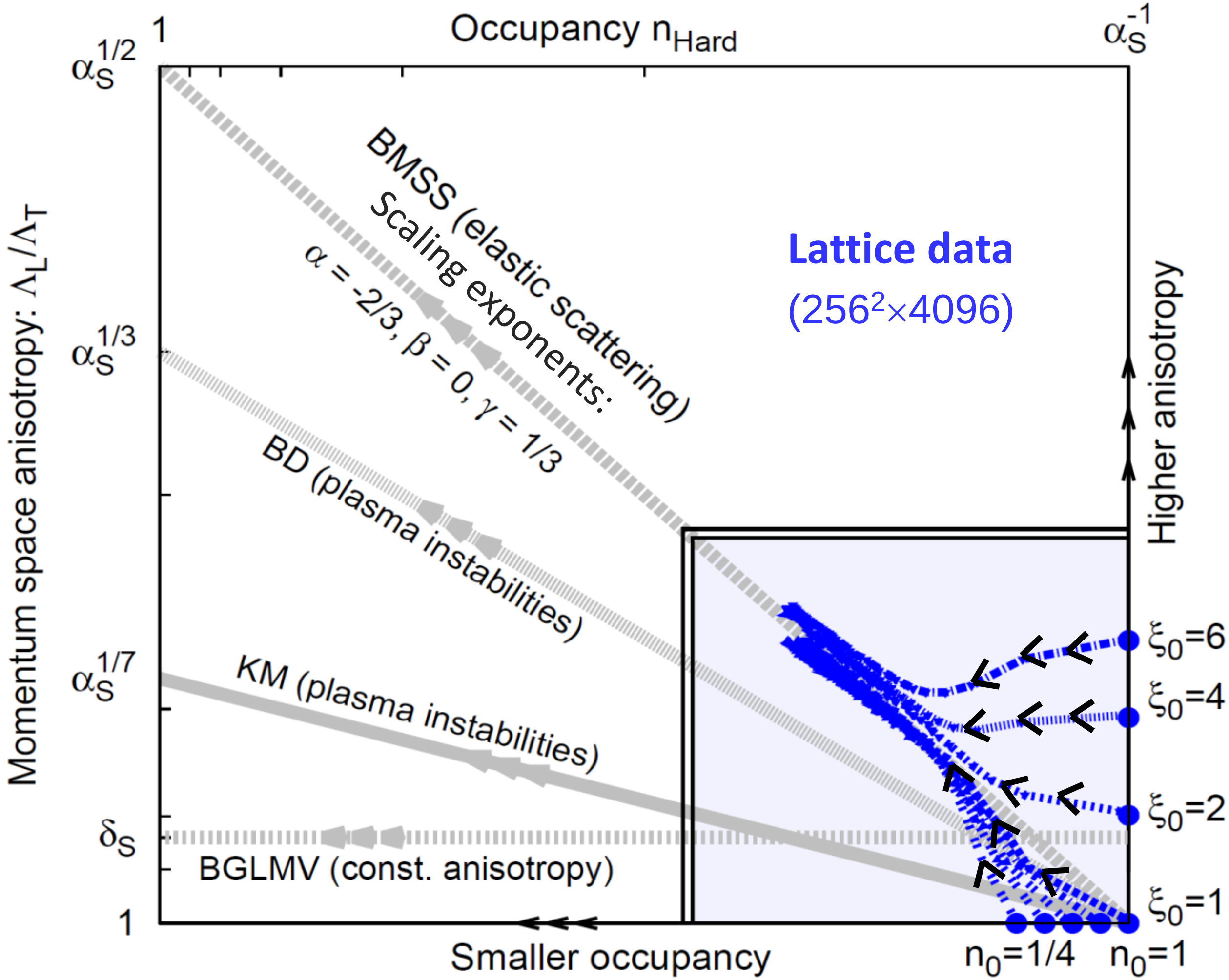
$$f(\mathrm{p_T}, \mathrm{p_z}, \tau_0) = \frac{n_0}{2g^2}\,\Theta\left(Q_s - \sqrt{\mathrm{p_T^2} + (\xi_0 \mathrm{p_z})^2}\right)$$

anisotropy parameter
(controls “prolateness” or “oblateness”
of initial momentum distribution)

J.B., Boguslavski, Schlichting, Venugopalan, PRD89 (2014) 074011; 114007; PRL114 (2015) 061601

Nonthermal fixed point

Evolution in the `anisotropy-occupancy plane´

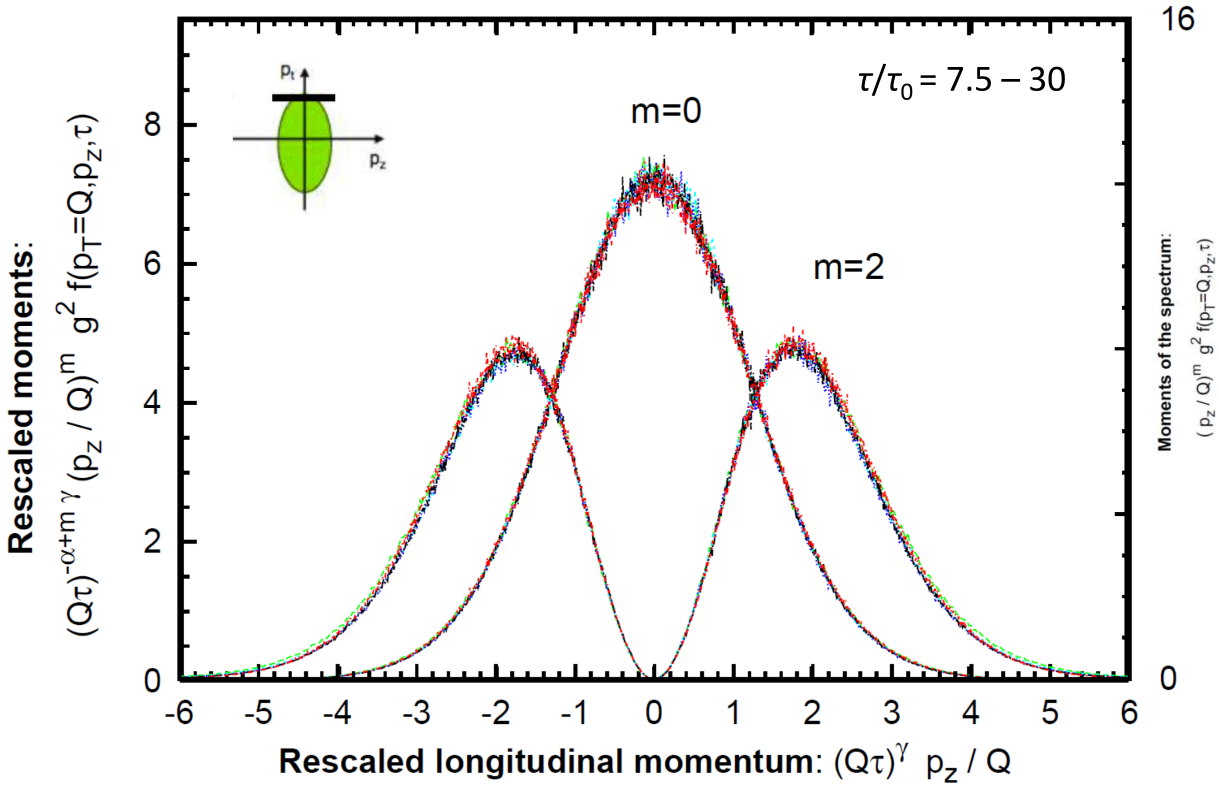


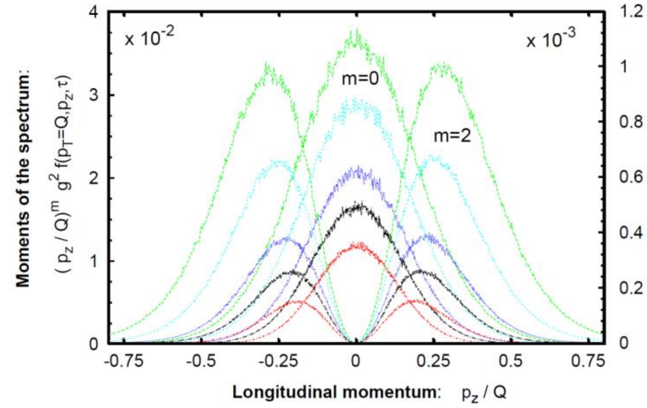
J.B., Boguslavski, Schlichting, Venugopalan, PRD 89 (2014) 074011; ibid. 114007

`Bottom-up´* scaling emerges as a consequence of the fixed point!

*Baier et al, PLB 502 (2001) 51

Self-similar evolution





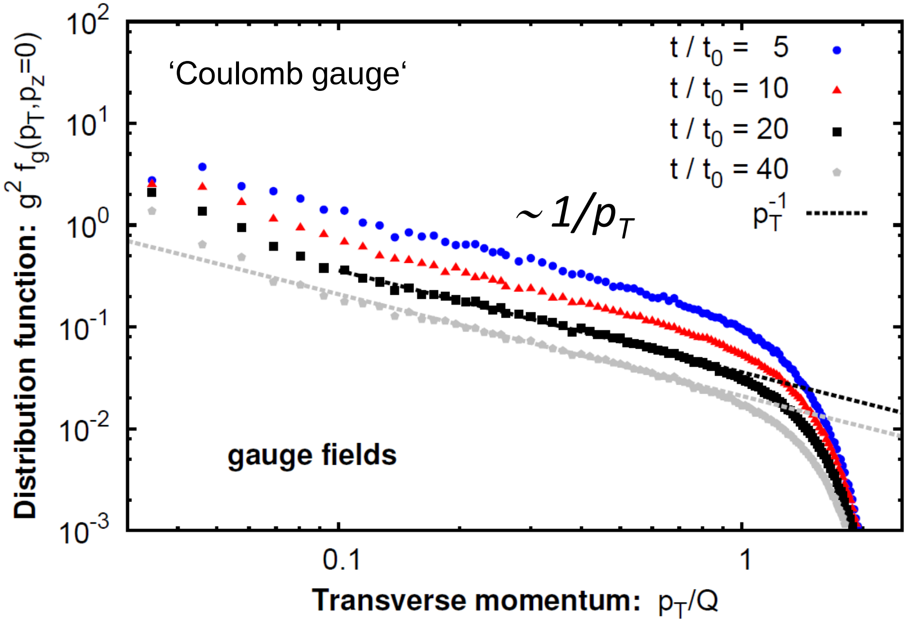
Scaling exponents: $\alpha = -2/3\,, \qquad \beta = 0\,, \qquad \gamma = 1/3$

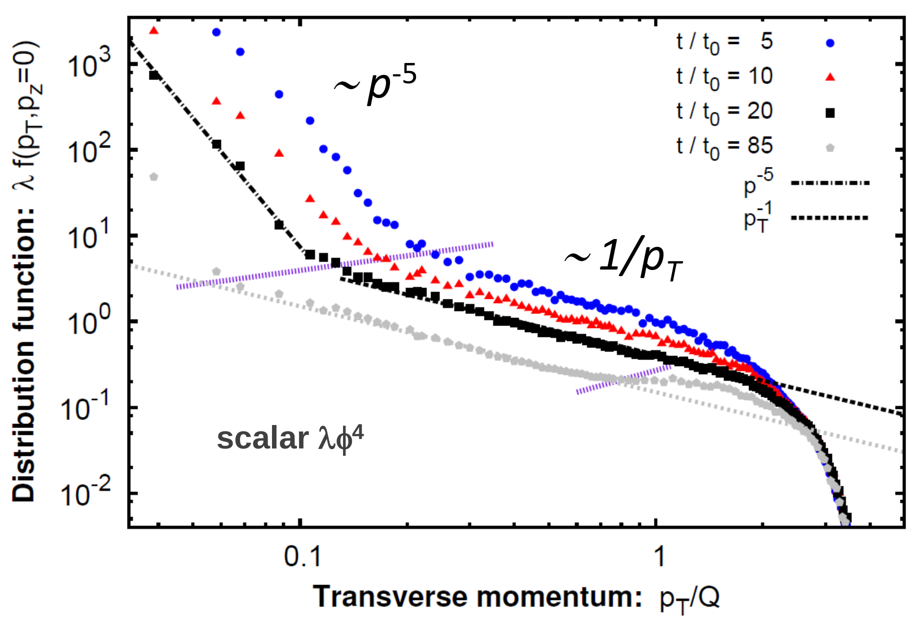
and scaling distribution function f_S:

$$\boxed{f(\mathrm{p_T}, \mathrm{p_z}, \tau) = (Q\tau)^{\alpha} f_S\Big(\underbrace{(Q\tau)^{\beta}\mathrm{p_T}, (Q\tau)^{\gamma}\mathrm{p_z}}_{\text{stationary fixed-point distribution}}\Big)}$$

Comparing gauge and scalar field theories

with longitudinal expansion





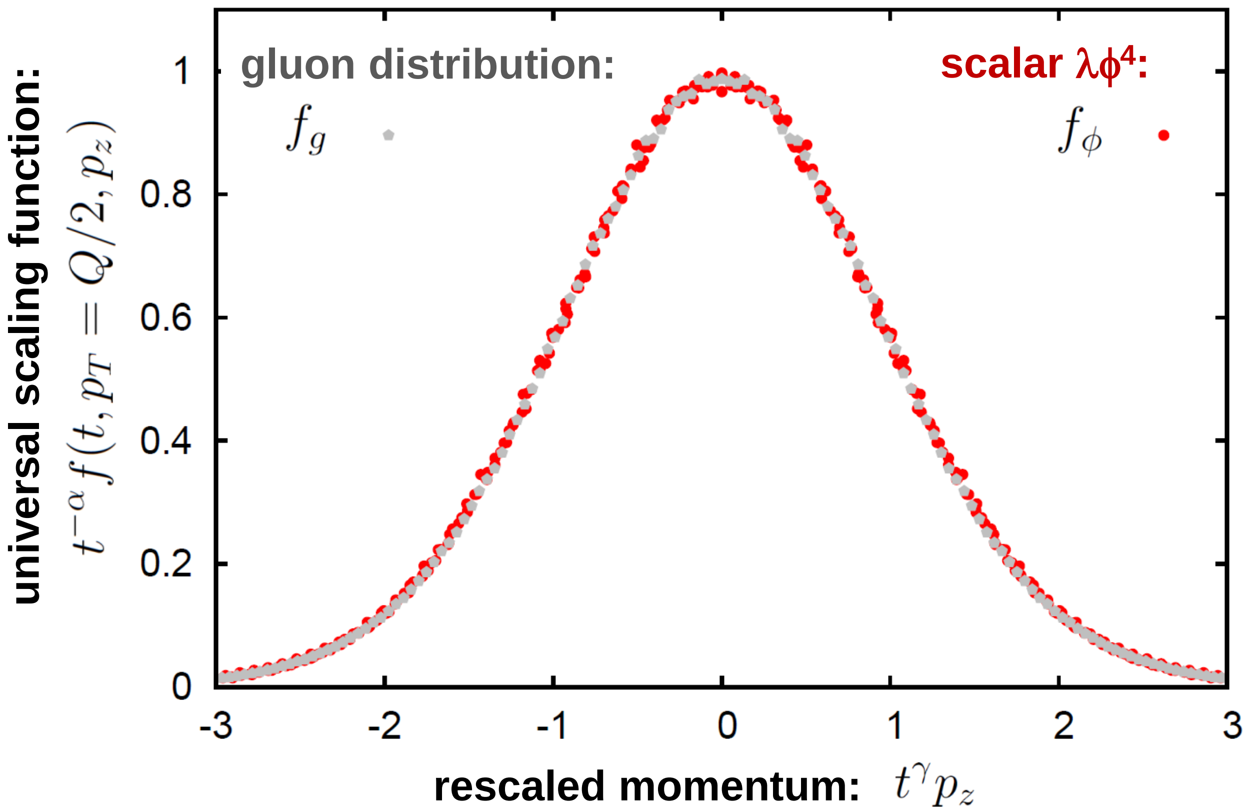
- For gauge & scalar fields: Inertial range of ***thermal-like transverse spectrum ~1/p_T*** even as longitudinal distribution is being \`squeezed´
- Strongly enhanced infrared regime for scalars: ***inverse particle cascade leading to Bose condensation***, $\sim 1/p^5$ as in isotropic ***superfluid turbulence***
- At latest available times for scalars a flat distribution for $p_T \gtrsim Q$ emerges

J.B., Boguslavski, Schlichting, Venugopalan, Phys. Rev. Lett. 114 (2015) 6, 061601

Universality far from equilibrium

- Same ***universal exponents and scaling function*** in $1/p_T$ inertial range

$$\alpha = -2/3 \quad , \quad \beta = 0 \quad , \quad \gamma = 1/3$$



$\rightarrow$ **Remarkably large universality class far from equilibrium!**

Some puzzles and challenges

- The scaling solution seems well understood for the gauge theory (BMSS) – but the corresponding kinetic theory arguments fail for the scalar theory

Consider the Boltzmann equation

$$[\partial_\tau - \frac{p_Z}{\tau}\partial_{p_z}]f(p_T, p_Z, \tau) = C[f](p_T, p_Z, \tau)$$

with a self-similar evolution

$$f(p_T, p_Z, \tau) = (Q\tau)^\alpha f_S((Q\tau)^\beta p_T, (Q\tau)^\gamma p_Z)$$

→ ***Non-thermal fixed point solution*** $(f \gg 1)$

$$[\alpha + \beta p_T \partial_{p_T} + (\gamma - 1) p_Z \partial_{p_z}] f_S(p_T, p_Z) = Q^{-1} C[f_S](p_T, p_Z)$$

→ ***Scaling exponents determined by scaling relations for***

- Small angle elastic scattering $(2\alpha - 2\beta + \gamma = -1)$ ⟵ for gauge theory
- Energy conservation $(\alpha - 3\beta - \gamma = -1)$
- Particle number conservation $(\alpha - 2\beta - \gamma = -1)$

→ $\alpha = -2/3, \beta = 0, \gamma = 1/3$ ***in excellent agreement with lattice data!***

However: No dominance for small angle scattering in scalar theory!
Much more general principle underlying nonthermal fixed point?

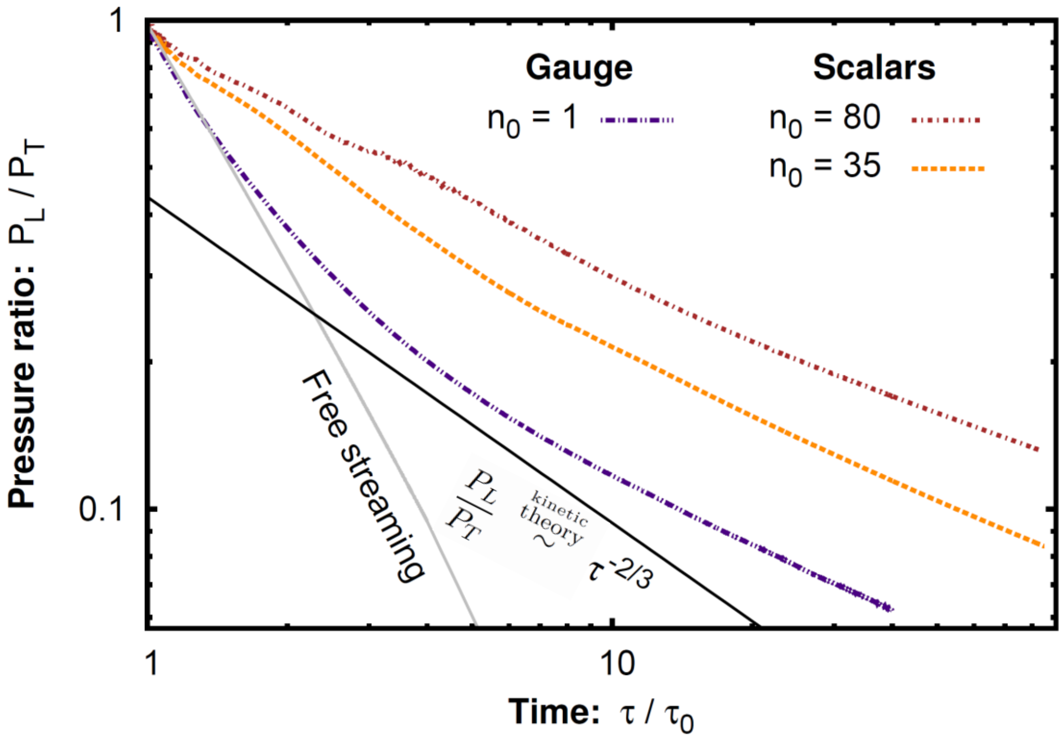
- In the weak-coupling limit, kinetic theory is expected to have an overlapping range of validity with classical-statistical simulations

$f(p) \sim 1/g^2$	$1/g^2 > f(p) > 1$	$f(p) < 1$
('overoccupied')	(classical particle)	(quantum)

$\underbrace{\text{('overoccupied') ... (classical particle)}}$ ***classical-statistical lattice gauge theory***

$\underbrace{\text{(classical particle) ... (quantum)}}$ ***kinetic theory***

However, kinetic theory cannot reproduce important quantities such as P_L / P_T characterizing isotropization of the longitudinally expanding plasma:



- Scaling behavior of P_L / P_T the same for scalar and gauge field simulations!
- In scalar theory behavior of P_L / P_T known to arise from infrared contributions (Bose condensation)

→ Nonperturbative despite (weak) coupling parameter

- Perturbative estimates extrapolated beyond the weak-coupling regime suggest the absence of transient universal scaling behavior

  Gauge: Kurkela, Zhu, PRL 115 (2015) 182301; Scalar: Epelbaum, Gelis, Jeon, Moore, Wu, JHEP 09 (2015) 117

  However, no such indications (yet no expansion) from nonperturbative estimates in scalar quantum field theory *JB, Wallisch, arXiv:1607.02160*
  (and holographic superfluids?) *Ewerz et al., JHEP 18 (2015) 1505 (cf. also Adams, Chesler, Liu, Science 341 (2013) 368)*

- By now, detailed understanding of the dynamics of Bose condensation in scalar quantum field theory (NLO-1/N)/vertex-resummed kinetic equation

  However, no such understanding in gauge theories – despite indications for infrared contributions to gauge invariant quantities (P_L / P_T)

$\rightarrow$ *Holography / functional renormalization group / solving QFTs by ultracold quantum gas measurements / …*

Universality far from equilibrium