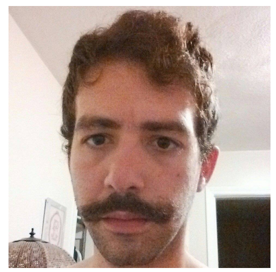
Black Hole Collapse in Large-C CFT

Julian Sonner

Oxford
13 July 2016



Tarek Anous (MIT)

Thomas Hartman (Cornell)

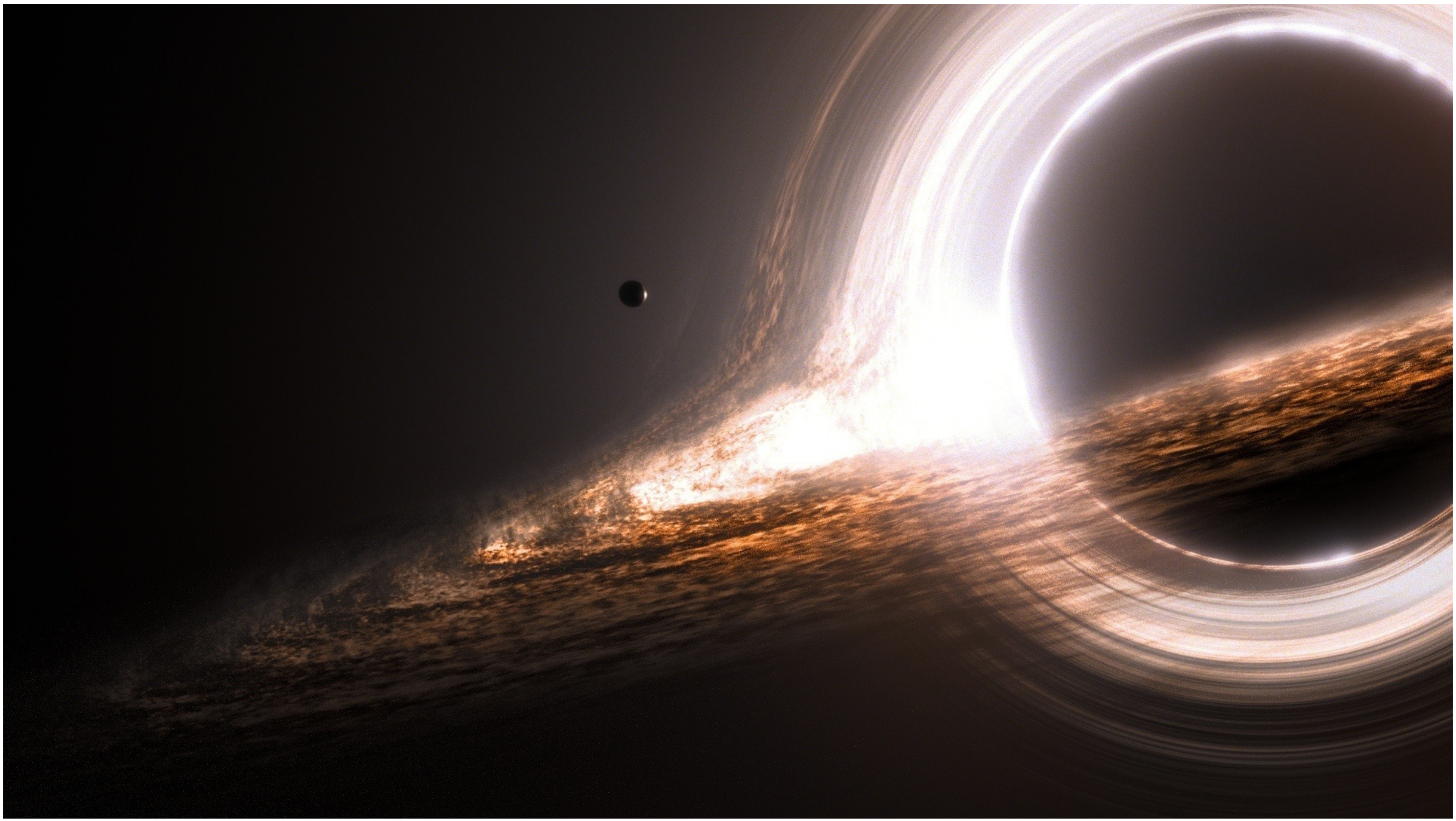
Antonin Rovai (Geneva)

SwissMAP
The Mathematics of Physics
National Centre of Competence in Research

FNSNF
Fonds national suisse
Schweizerischer Nationalfonds
Fondo nazionale svizzero
Swiss National Science Foundation

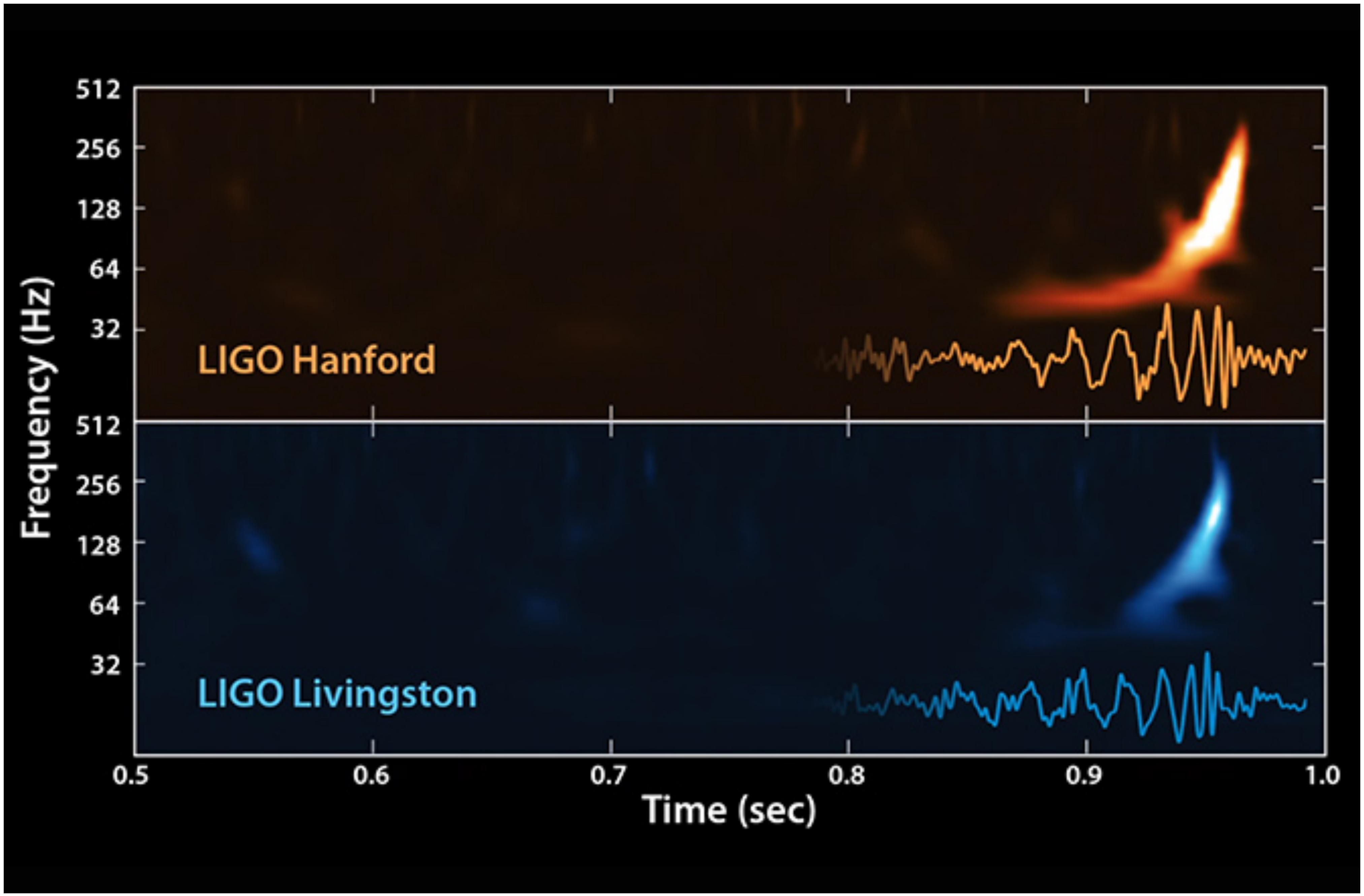
Introduction

The black hole as movie star: Gargantua



‘Gargantua’, C. Nolan & K. Thorne

The black hole as news sensation: GW150914



‘Ligo Black Hole Binary’ R. Drever, R. Weiss, K. Thorne et mult. al.

$$S = \frac{k_B c^3 A}{4 G_N \hbar}$$

black holes are thermodynamic systems

their entropy is proportional to the area of the event horizon

information loss paradox:

a BH formed from a pure state will evolve into a mixed state (of Hawking radiation)

holography:

a theory of quantum gravity should have information ~ area

General Plan

AdS/CFT relates gravity (often in AdS) to unitary field theory (often CFT)

Lots of progress gravity ➛ CFT (my favorite: AdS/CMT)

Less known about CFT ➛ (quantum) gravity

➙ despite developments in CFT, CMT:

- time evolution and spread of entanglement
- thermalization of closed quantum systems (e.g. via eigenstates)
- non-perturbative methods (e.g. bootstrap)

Thermalization ➛ BH formation (& evaporation)

Outline

1. Reminder about black holes

2. CFT implosions

3. Thermalization @ large c

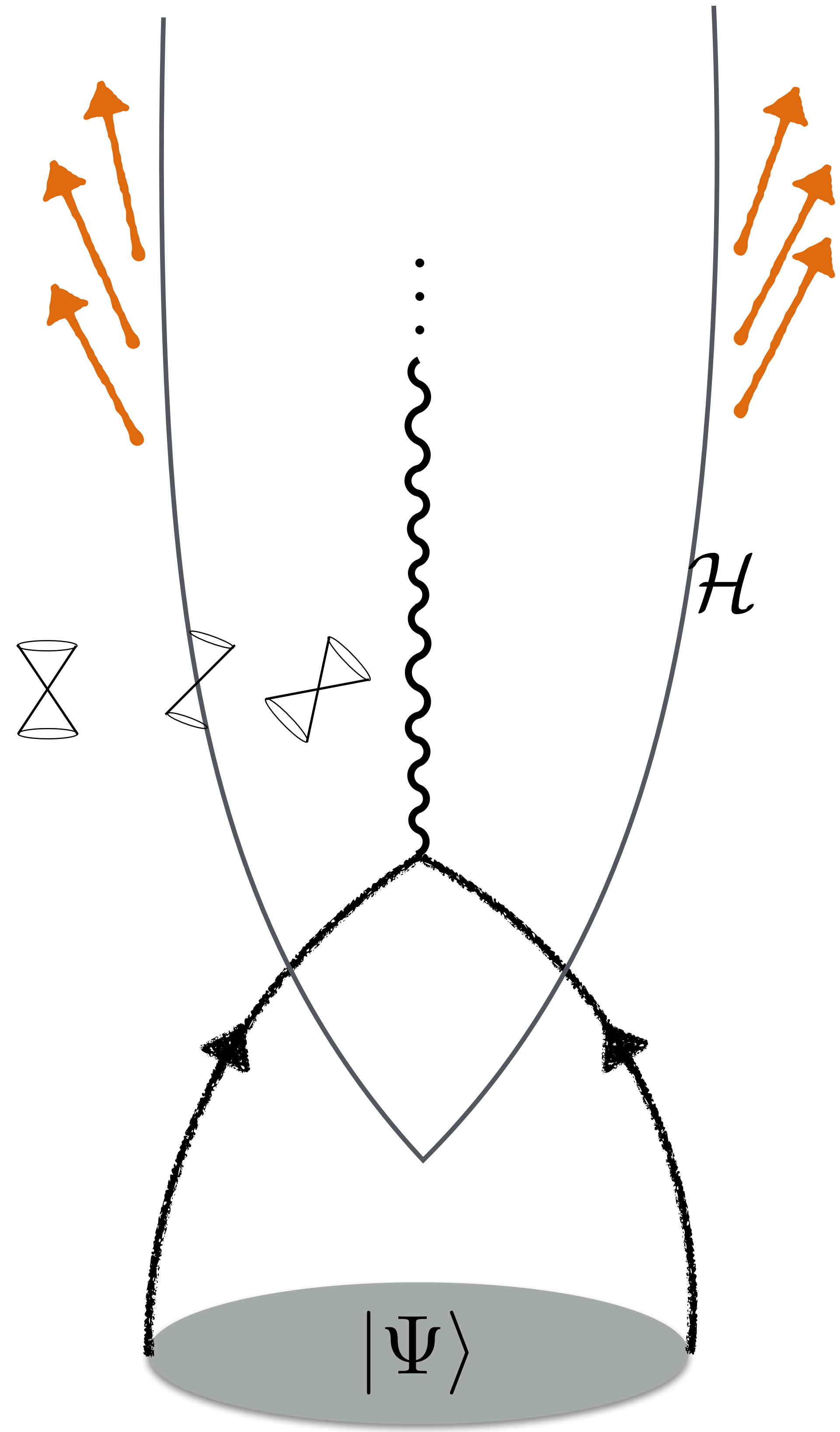
4. Conclusions

“the trouble with black holes”

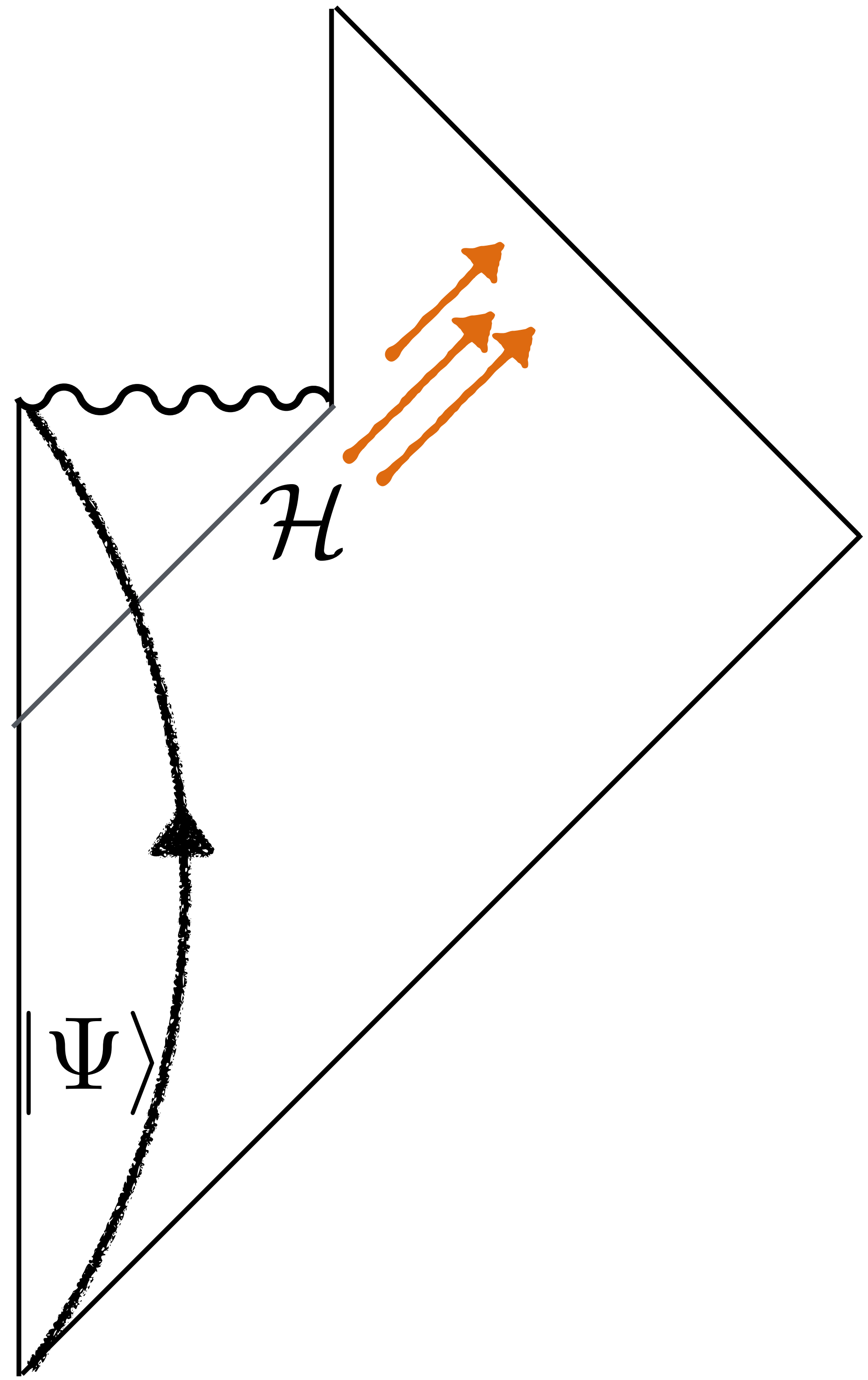
- outgoing **Hawking radiation** is thermal $\rho_{\mathrm{Gibbs}}(T_H)$

- a **horizon** $\mathcal{H}$ cloaks the singularity

- initial pure state $|\Psi\rangle$ of matter collapses inwards



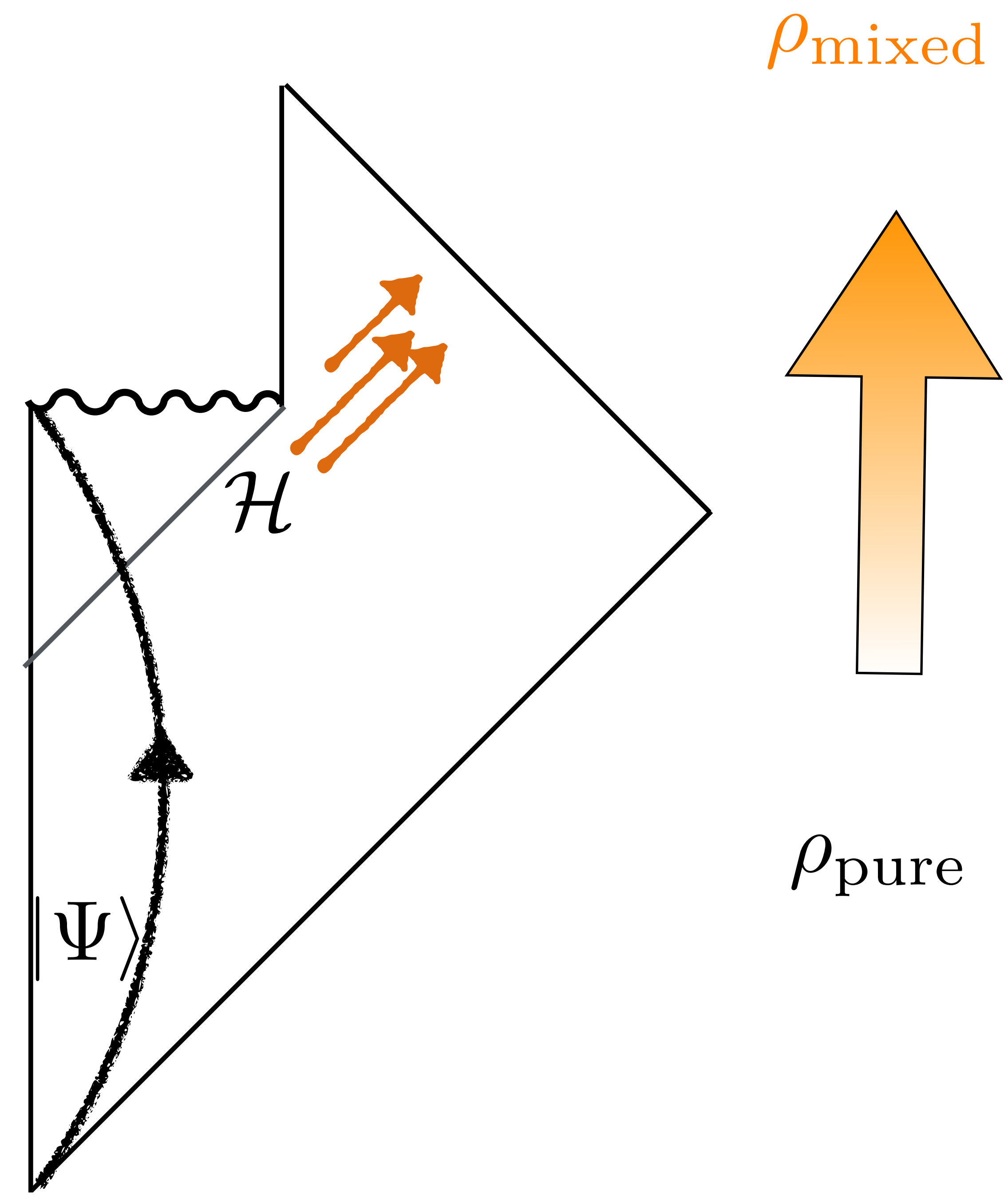
black holes evaporate



The Paradox

- gravity as an EFT implies pure to mixed evolution
- fundamentally incompatible with a unitary S-matrix

1. quantum gravity is non-unitary
2. gravity EFT makes no sense
3. (subtle) corrections to Hawking result



Holography:

quantum gravity $=$ quantum field theory

hence AdS/CFT only allows for options 2 & 3.

the anti-information loss paradox:

how does an obviously unitary theory lose information?

The Plan

(of a first-principles calculation in holographic CFT)

1. define an initial state in CFT which forms a black hole
   ➢ “quantum quench”

2. understand time evolution in strong-coupling regime
   ➢ non-equilibrium CFT (monodromy method / CFT_2)

3. analyze suitable observables (e.g. correlations & EE)
   ➢ strongly-coupled CFT thermalization

4. diagnose signs of information loss & recovery
   ➢ unitarity constraints on correlations

Unitarity vs Thermalization

(constraints on long-time correlations from unitarity)

Correlations in a closed quantum system, e.g.

$$G(t) = \mathrm{tr}\rho\mathcal{O}(t)\mathcal{O}(0)$$

Time average over a large time T cannot vanish by unitarity

$$\lim_{T\to\infty} \overline{|G(t)|^2} \neq 0$$

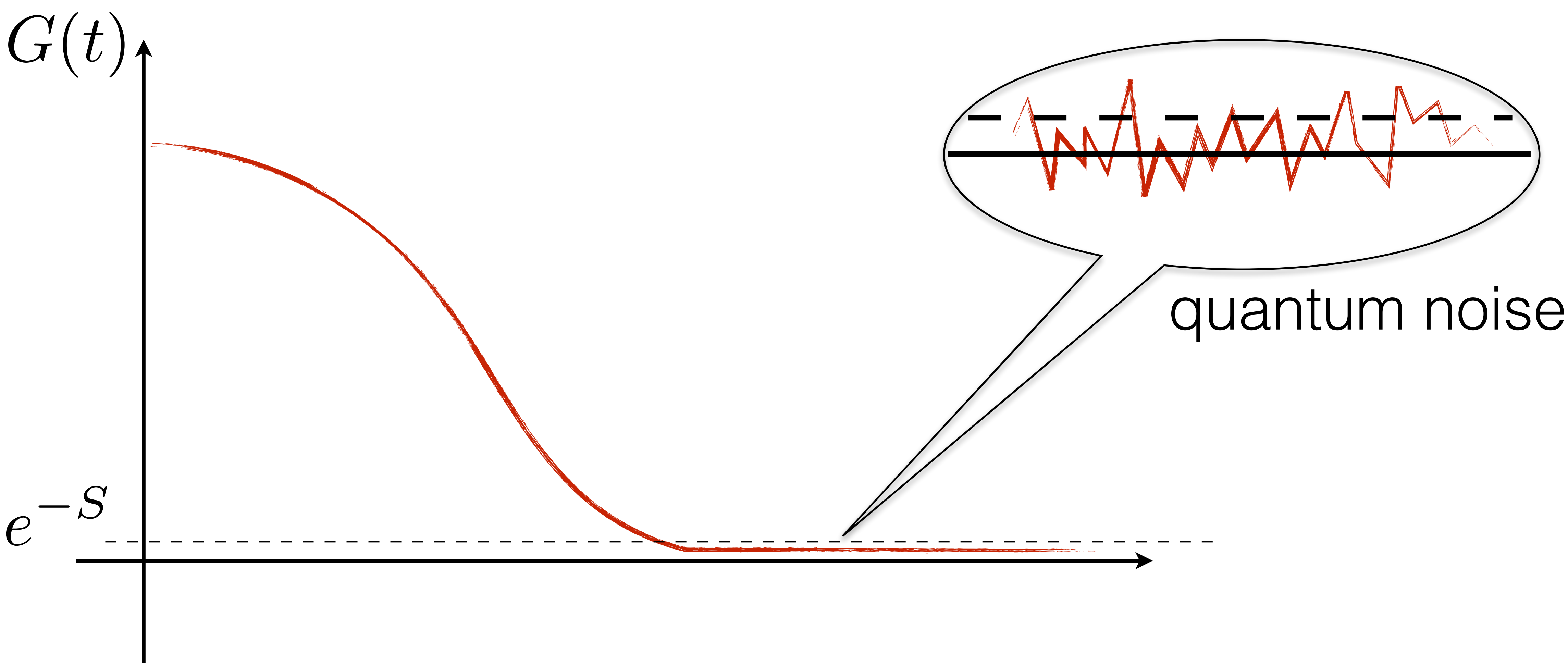
Need to assume spectrum is generic (no specific ordering principle)

➢ can appeal e.g. to ETH so estimate $G(t) \sim e^{-S}$ at late times

Unitarity vs Thermalization

(constraints on long-time correlations from unitarity)

$$\rho = e^{-\beta H}$$



CFT implosions

Looking for the Right Place

0D matrix models(IOP,…): connection to geometry?
2D black hole (CGHS): solvable but very different

3D story shares salient features of 4D (and higher)
in fact central to micro-state counting success (D1-D5)

the trouble: no local degrees of freedom (Achucarro & Townsend):

$$S_{3\mathrm{D}} = S_{\mathrm{CS}}[A] - S_{\mathrm{CS}}[\bar{A}]$$

other side of the coin: CFT_2 puts powerful tools at our disposal

3D Gravity + Matter

➙ add matter: get local dof. BUT need new tools

focus on a universal sector, by defining a 1/c expansion:

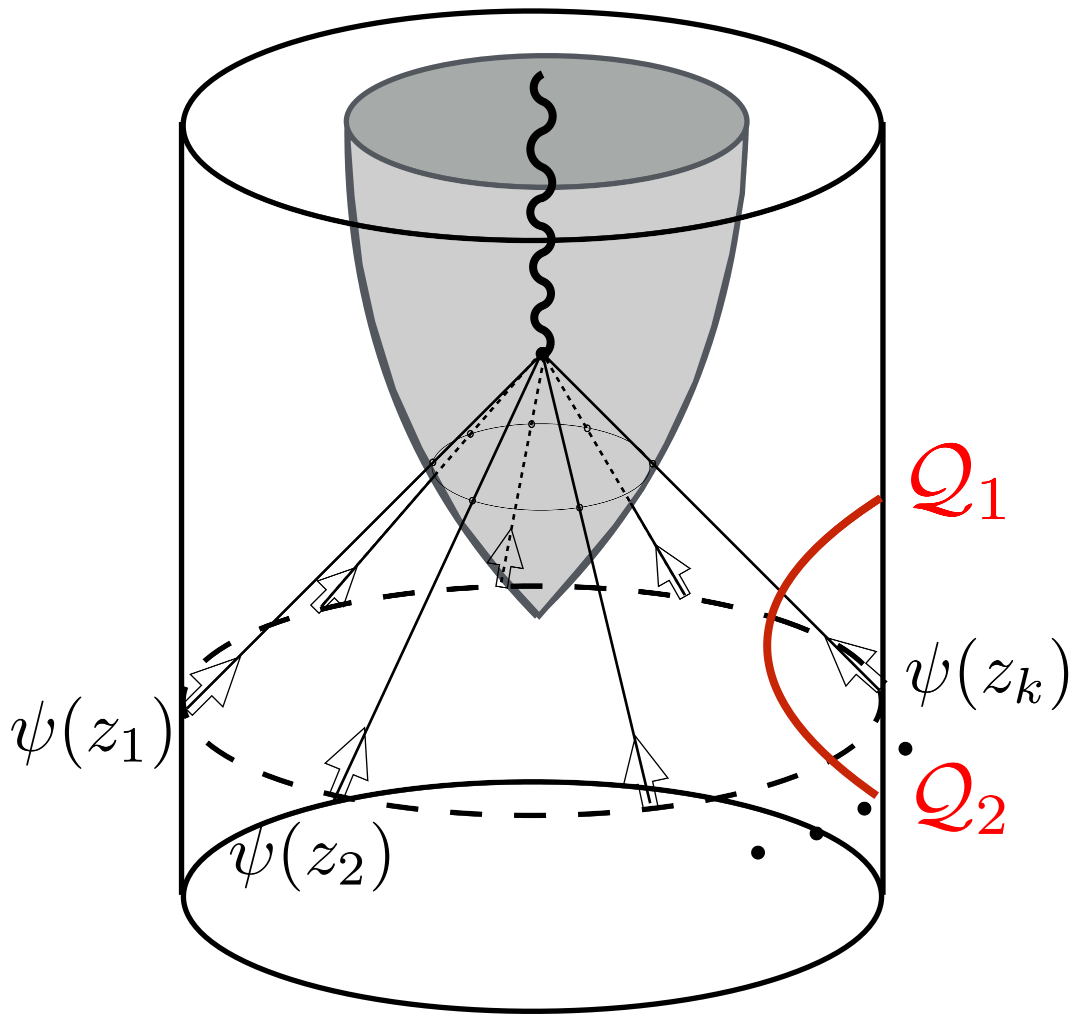
➙ any microscopic theory in this class defines some 3D quantum gravity theory (sparse spectrum)

3D gravity + matter non-trivial, but solvable
➙ ideal place to study BH puzzles!

From bulk point of view this is G_N expansion

CFT_2 gives a non-perturbative definition of quantum gravity.

The Black Hole in the Tin Can



global AdS_{d+1}

Throw in a shell of n dust particles

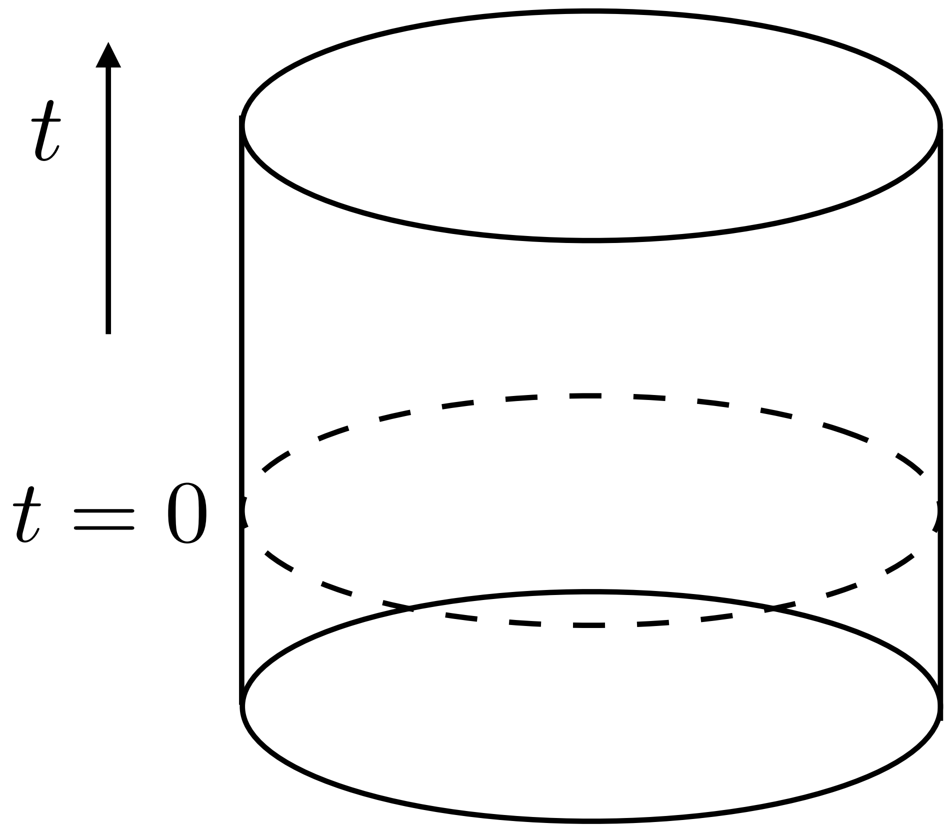
smooth limit: $n \to \infty$

BH collapse: Vaidya metric

Use light operators $\mathcal{Q}$ to probe geometry as function of t

remark: certain quantities such as entanglement entropy are sensitive to behind horizon physics (away from equilibrium)

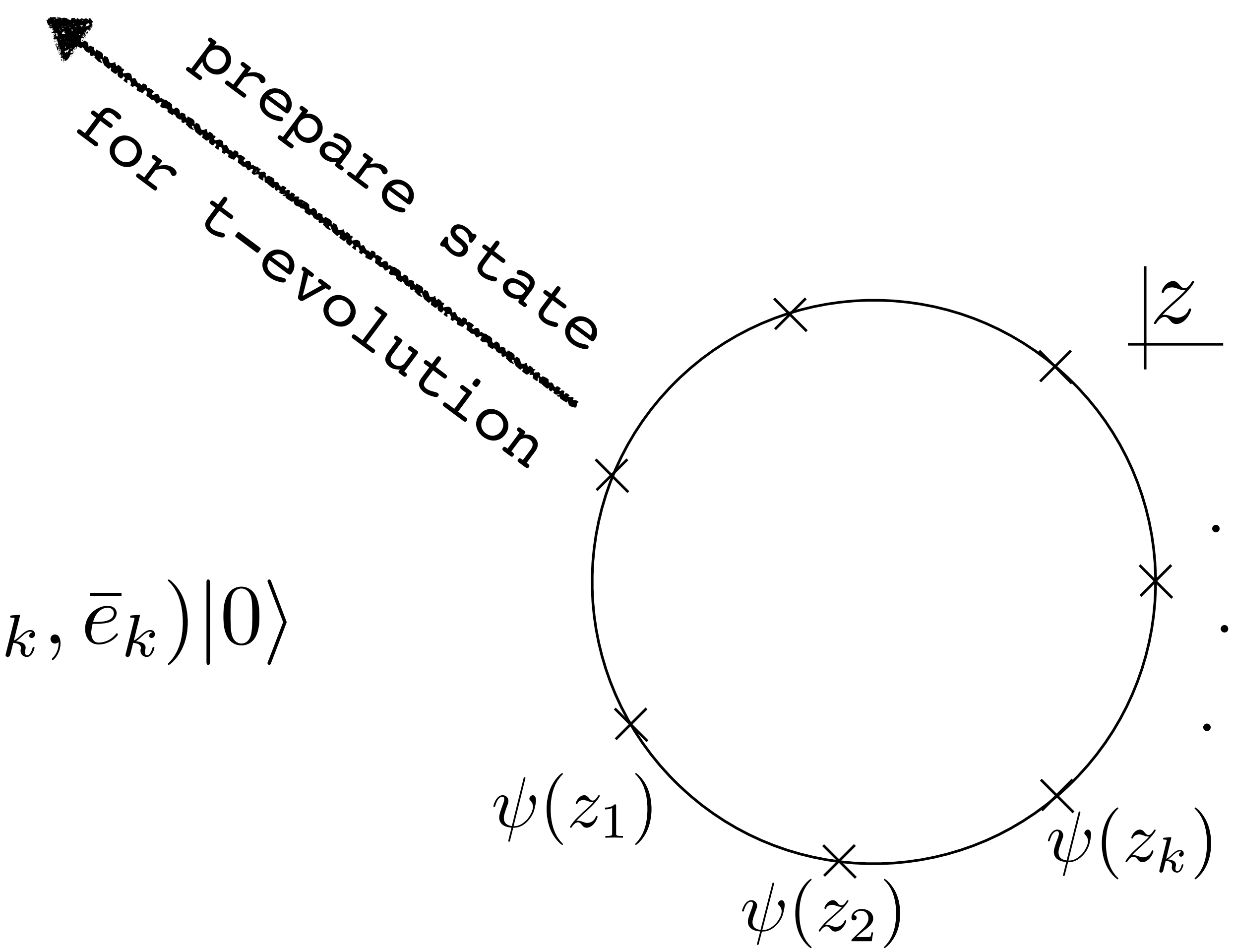
Translating to the CFT



start in excited state at t=0:

prepare by Euclidean path integral
➛ regulator σ

$$|\mathcal{V}\rangle = \frac{1}{\mathcal{N}} \prod_{k=1}^{n} \psi(e_k, \bar{e}_k)|0\rangle$$



Interrogating the CFT

Start probing the physics via 2n + p correlations

$$G(1,2,\dots p) = \langle \mathcal{V} | \mathcal{Q}_1 \, , \dots \, \mathcal{Q}_p | \mathcal{V} \rangle$$

we want to approach smooth, semi-classical gravity

$$c \to \infty$$

$$n \to \infty$$

$$\sigma \to 0$$

$$E \sim n h_\psi / \sigma \to \mathcal{O}(c)$$

infinite-point correlations in strongly-coupled CFT!

Benefits of 2D CFT

in the semi-classical limit (large c), get sum of exponentials

$$G(1,2,\ldots p) = \sum_{\text{blocks}} a_k e^{-\frac{c}{6} f_k^{(n)}(1,2,\ldots p)}$$

correlator approximated by largest term, the identity block

“it from **id**”

the dominant contribution comes from the identity Virasoro block, that is the unit operator **id** *and all its descendants T, ∂T, T^2 $T\partial T$…, (multi-graviton exchange in bulk)*

subleading corrections exponentially suppressed in $e^{-c} \sim e^{-1/G}$

still need to calculate the semi-classical block:

CONFORMAL SCALAR FIELD ON THE HYPERELLIPTIC CURVE AND CRITICAL ASHKIN-TELLER MULTIPOINT CORRELATION FUNCTIONS

Al.B. ZAMOLODCHIKOV

Scientific Council of "Cybernetics", Academy of Sciences, USSR

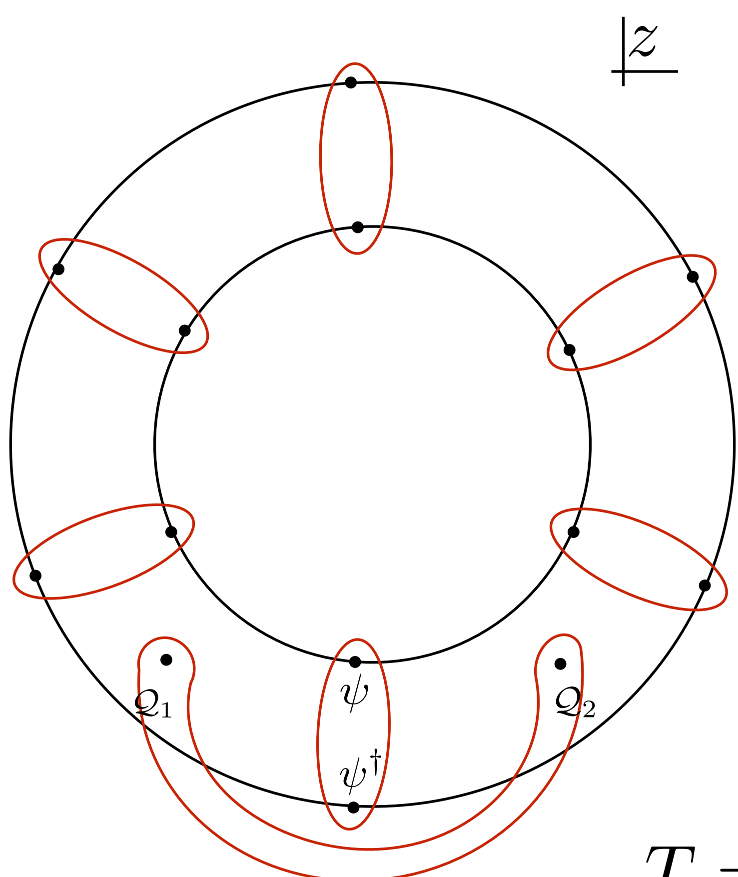
Received 3 December 1986

A multipoint conformal block of Ramond states of the two-dimensional free scalar field is calculated. This function is related to the free energy of the scalar field on the hyperelliptic Riemann surface under a particular choice of boundary conditions. Being compactified on the

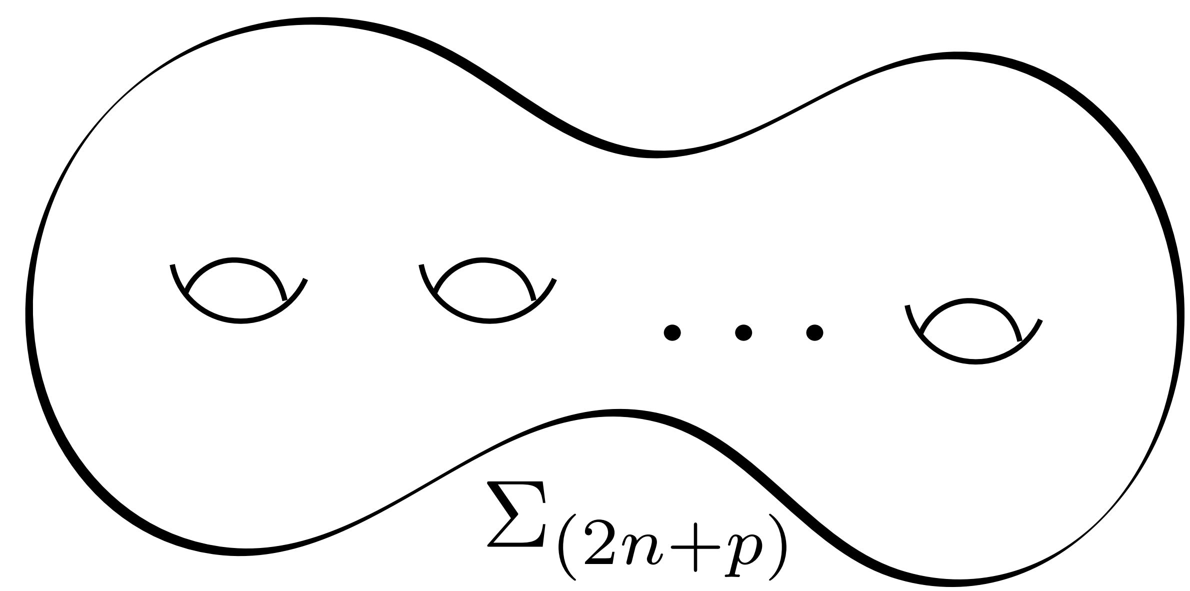
USSR: fighting hyper-intelligent Robot overlords with CFT?

The Monodromy Method

each contraction of operators in the plane defines a cycle



defines

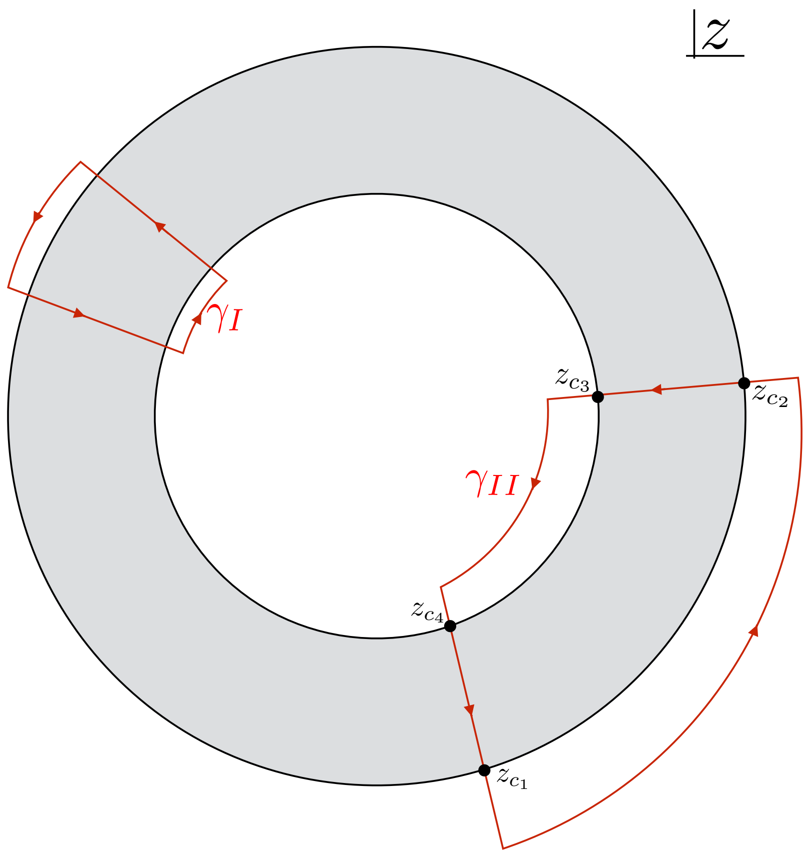


$$T = \sum_{k=1}^{2n+p} \left[\frac{6h/c}{(z-z_k)^2} + \frac{c_k}{z-z_k} \right]$$

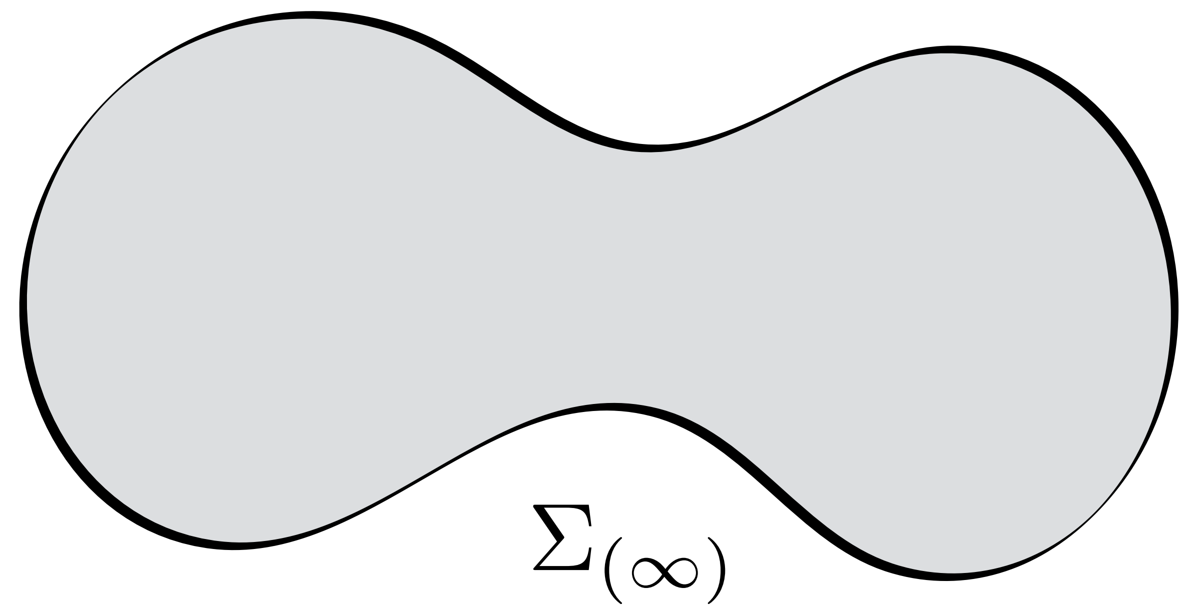
fix monodromies of $y''(z) + Ty(z) = 0 \xrightarrow{c_k} f_k^{(n)}(1,2,\ldots p)$

Taking the smooth Limit

generally a hard problem, big simplification occurs for n $\longrightarrow \infty$



defines



stress tensor $\longmapsto$ distribution on $\Sigma_{(\infty)}$

continuum monodromy method

$$f_0^\infty(1, 2, \ldots, p)$$

3D semi-classical gravity

$$\mathcal{L}_{\text{geo}}(1, 2, \ldots p)$$

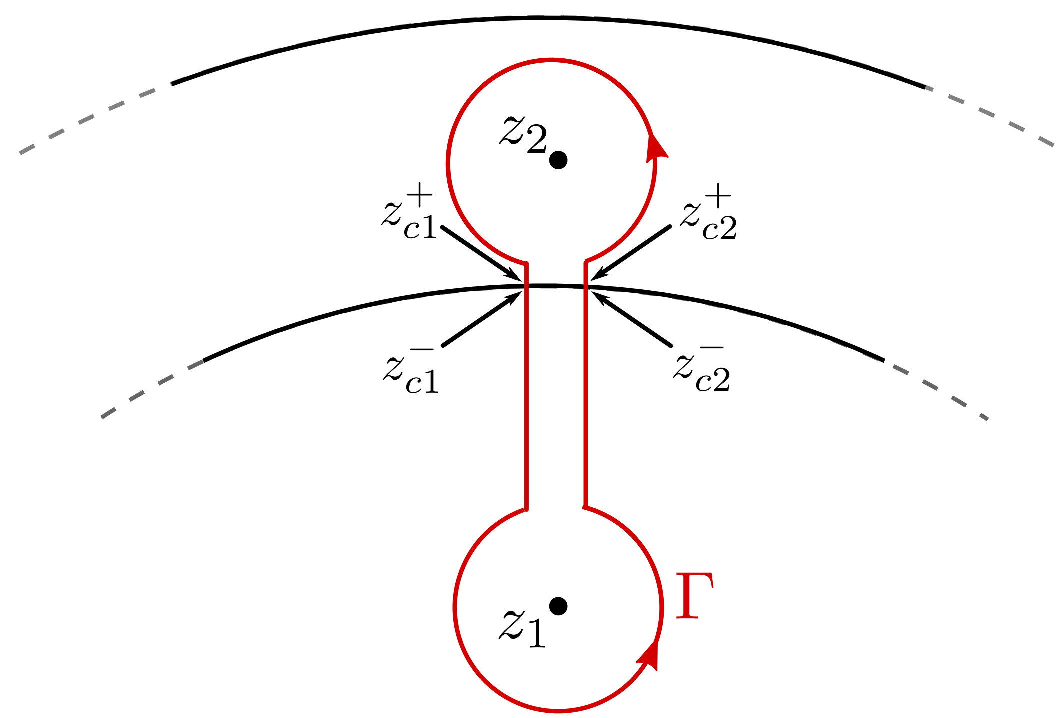
Two-point Autocorrelation

let us now return to the black hole and compute

$$G(t) = \mathrm{tr}\rho\mathcal{O}(t)\mathcal{O}(0)$$

in the collapse state $|\mathcal{V}\rangle$

$$G(1,2) =$$



can be done analytically:

$$\boxed{G(t_1,t_2) = \left(\frac{1}{\pi T}\cos\left(\frac{t_1}{2}\right)\sinh\left(\pi T t_2\right) - 2\sin\left(\frac{t_1}{2}\right)\cosh\left(\pi T t_2\right)\right)^{-2\Delta^{\mathcal{Q}}}}$$

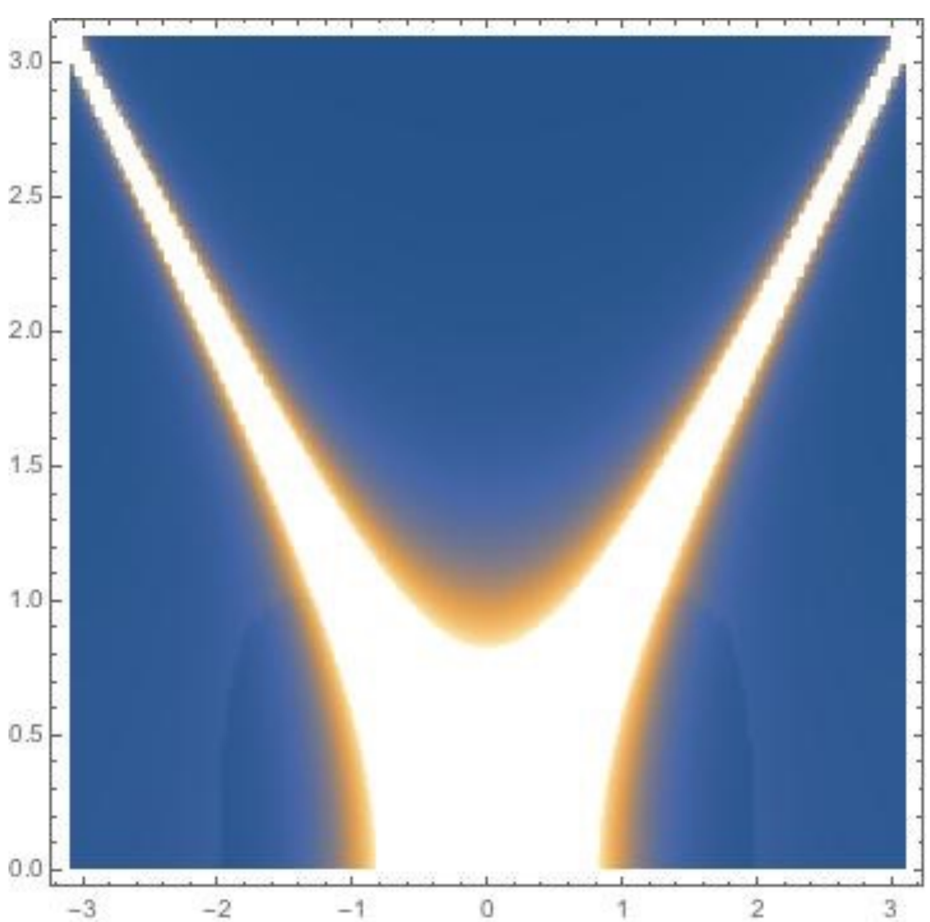
General two-point function

we are also interested in the general case

$$G(t,x) = \mathrm{tr}\rho\mathcal{O}(t,x)\mathcal{O}(0)$$

some illustrative results:

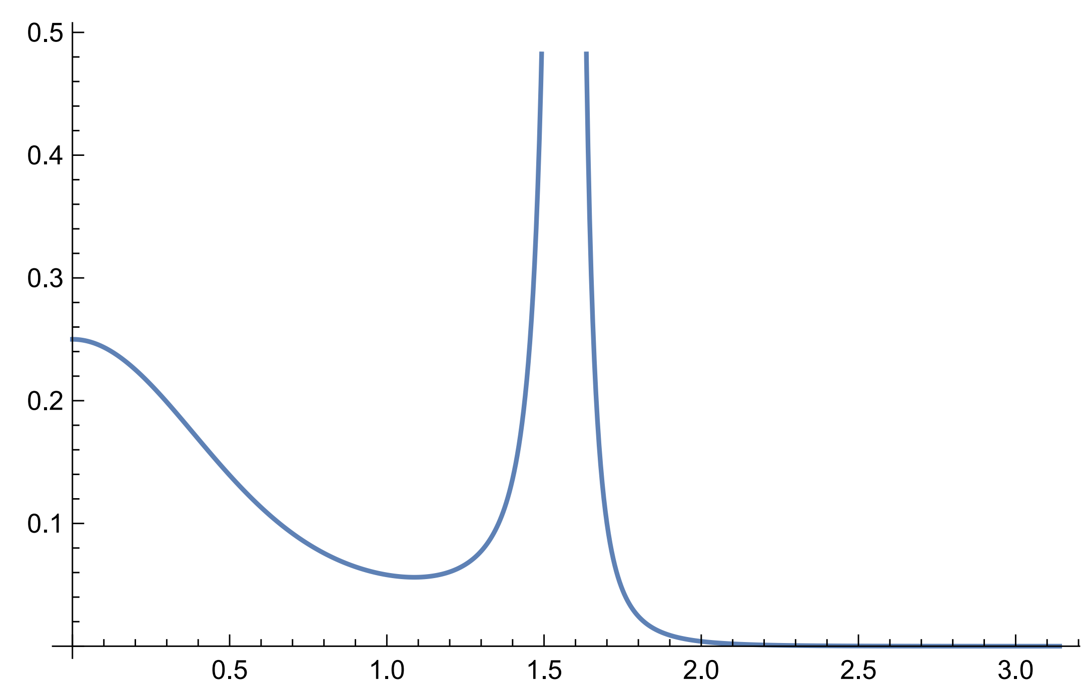
t



θ

[on S^1]

$\mathrm{Re}G(\pi/2,\theta)$



θ

$$G(t,x) \sim e^{-2\pi T \Delta^{\mathcal{Q}} x}$$ [on line]

Physical Consequences

not (yet) known from gravity (but matches known limits)

$\Longrightarrow$ CFT prediction for 3D gravity

The correlation function decays without bound at large time

$$\boxed{G(t_1, t_2) \sim \exp(-\frac{2\pi\Delta^{\mathcal{Q}}t}{\beta})}$$

Manifestly in conflict with unitarity: **CFT loses information!**

Can also compute entanglement entropy of interval A

$$\boxed{S(A) \to S_{\mathrm{Gibbs}}(A;T)} \longrightarrow \rho(A) = \rho_{\mathrm{Gibbs}}(A;T)$$

Restoring Unitarity

This is the anti-information paradox: what happened to unitarity?

$$|G(t)| = \left| \sum_{n,k} e^{i(E_n - E_k)t} \Psi_n^*(\mathcal{V}) \langle n|\mathcal{Q}|k\rangle \langle k|\mathcal{Q}|\mathcal{V}\rangle \right| \neq 0$$

➙ correlations cannot become arbitrarily small in $|\mathcal{V}\rangle$

Neglected contributions exponentially suppressed at t=0 (must be present due to crossing symmetry, e.g.)

$$\sum_{k \neq \mathrm{vac}} a_k e^{-\frac{c}{6} f_k^{\infty}(1,2,\ldots p)} \sim e^{-S}$$

restore unitary at large time ➙ non-perturbative effects in $1/G_N$

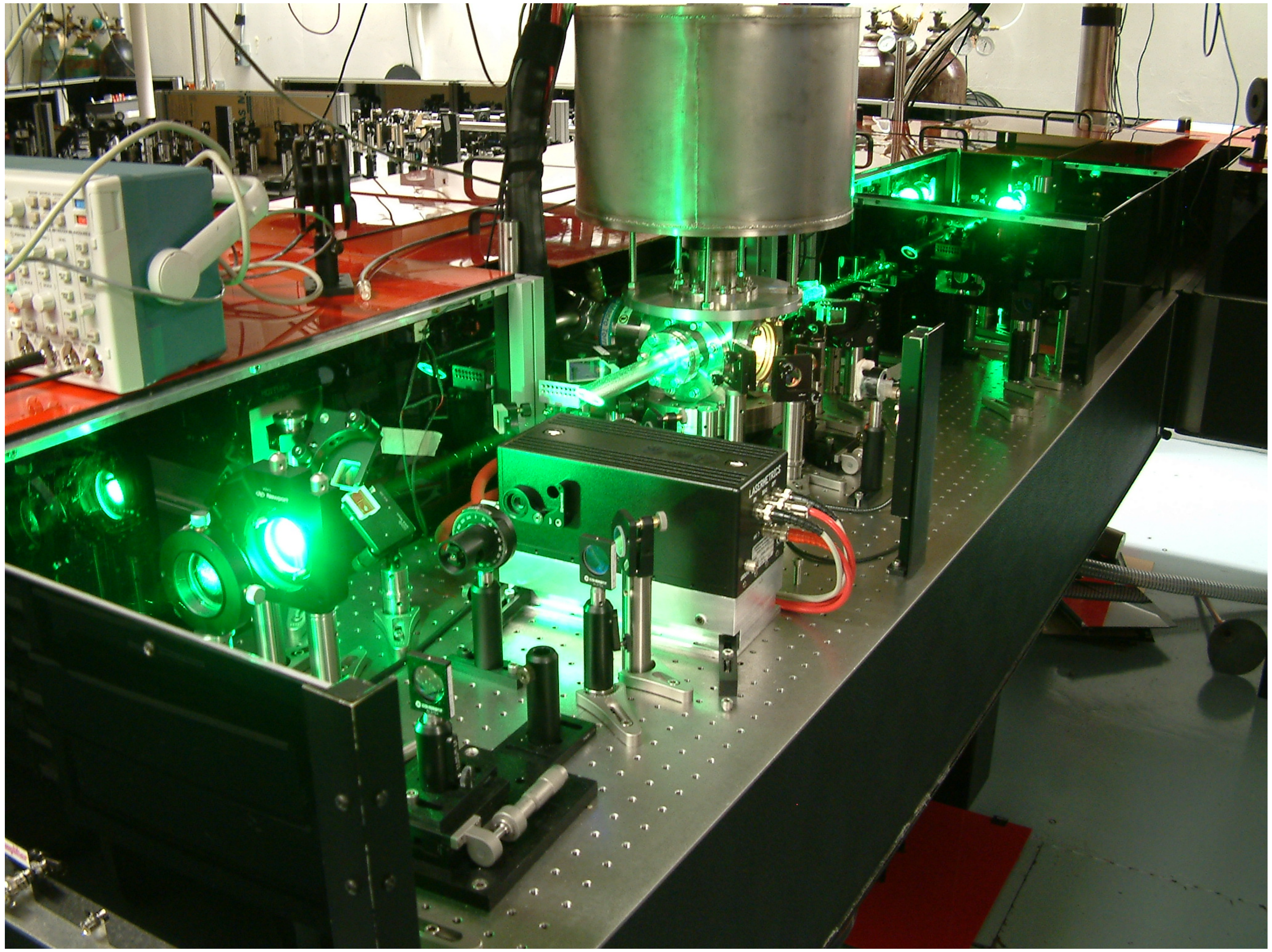
Conclusions

time-dependent 3D quantum gravity with matter in 1/c expansion
'it from id' ➙ ideal arena to think about quantum BHs

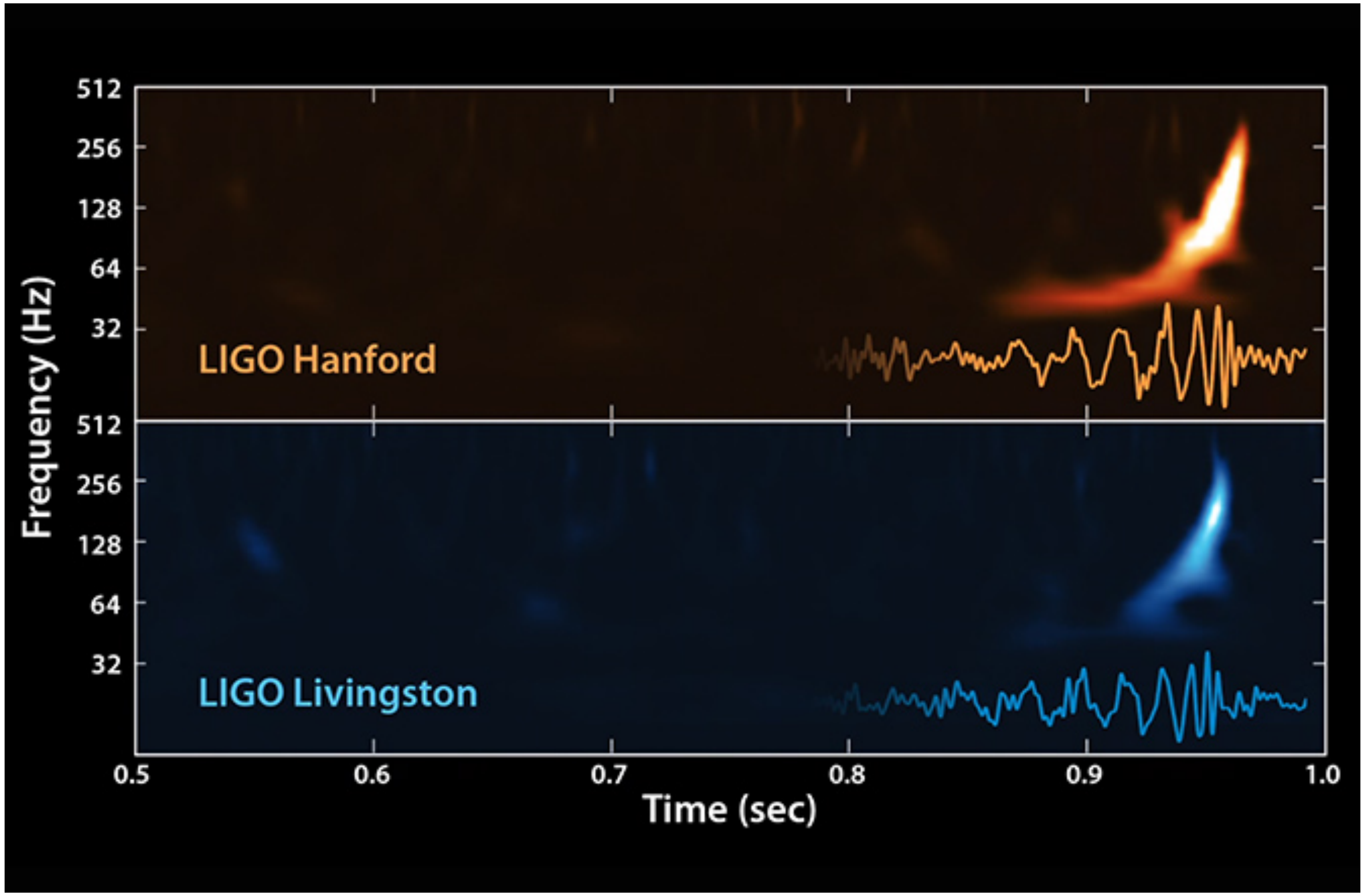
translates to detailed questions about thermalization in stongly-coupled CFT. New approach using monodromy (conformal blocks…)

correlation functions seemingly violate unitarity (naïve).
non-perturbative corrections in c restore unitarity

on gravity side these correspond to non-perturbative effects in G_{N}.
geometric interpretation? bulk interpretation?



AdS/CMT: quantum gravity in the lab



~~AdS/CMT: quantum gravity in the lab~~

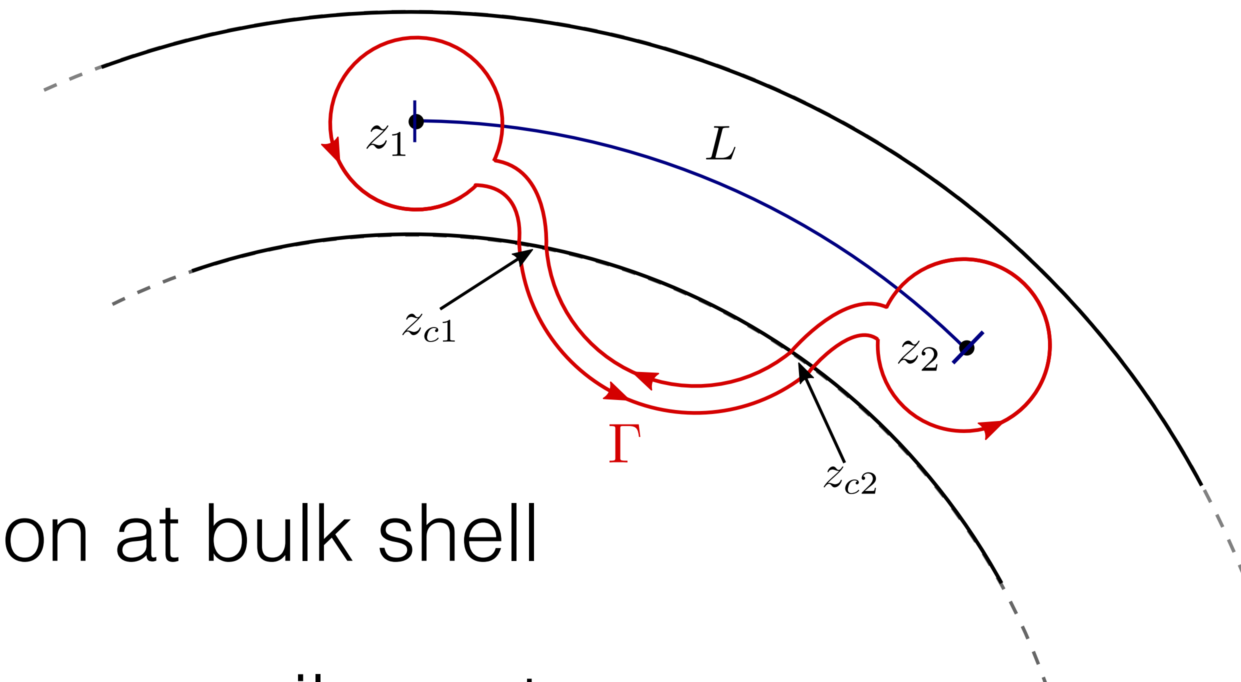
AdS/CMT: condensed matter in the Universe!

thank you!

entanglement entropy

Q-type operators ➙ twist insertions: $G_q(t) = \langle \mathcal{V} | \sigma_q(t, \ell_1) \tilde{\sigma}_q(t, \ell_2) | \mathcal{V} \rangle$

$$\boxed{S(A) = \lim_{q \to 1} \frac{1}{1-q} G_q(t)}$$



crossing points z_{c1} & z_{c2} ↔ refraction at bulk shell

it from id ➙ require trivial monodromy on smile contour

write $z_1 = e^{i\theta_1}, z_2 = e^{i(\theta_1 + L)}$ & continue to Lorentzian time $\theta_1 = t$

maximize S(A) over crossing points ➙ parametric equation for S(t)

entanglement entropy

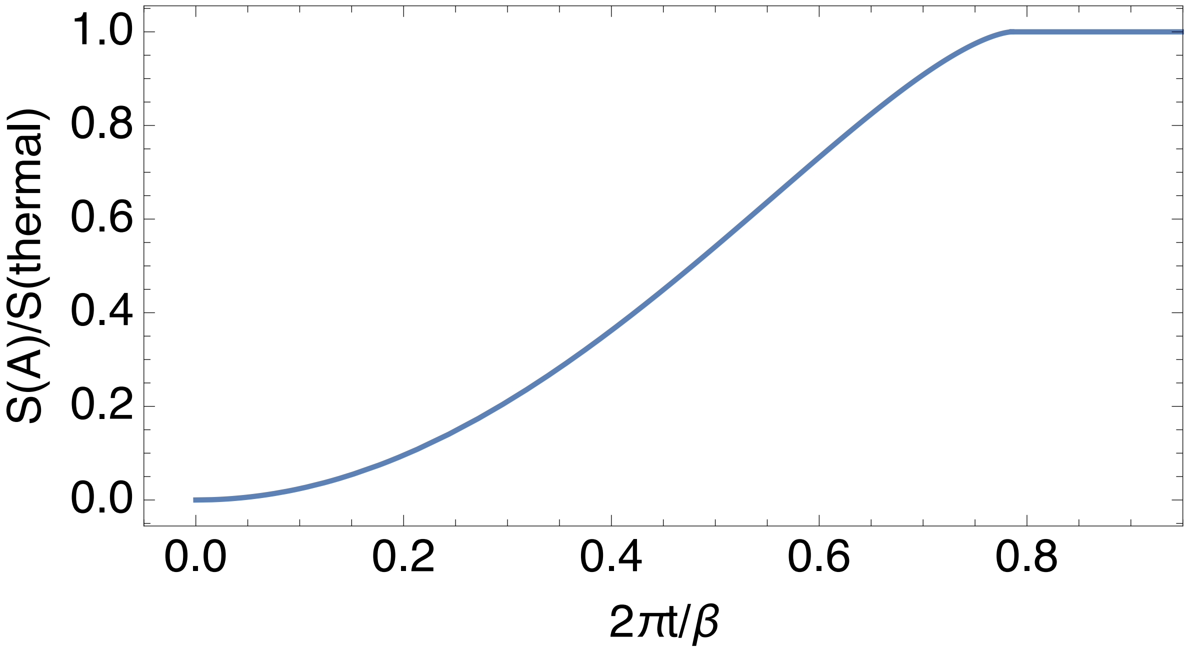
Implicit formula for growth of entanglement entropy:

$$t = \frac{\beta}{2\pi}\cosh^{-1}\left\{\cosh(2\pi T q) + 2\pi T\tan\left(\frac{L}{2}-q\right)\sinh(2\pi T q)\right\}$$

$$S_{EE} = \frac{c}{3}\log\left\{\frac{\sin\left(\frac{L}{2}-q\right)\cosh(2\pi T q) + \frac{1}{2\pi T}\left[1+\frac{1}{2}\left\{1+4\pi^2T^2\right\}\tan^2\left(\frac{L}{2}-q\right)\right]\cos\left(\frac{L}{2}-q\right)\sinh(2\pi T q)}{\epsilon_{UV}/2}\right\}$$

matches **exactly** global AdS_3 Vaidya:

- thermal at late time
- EE growth = change of channel
- sees **beyond horizon**

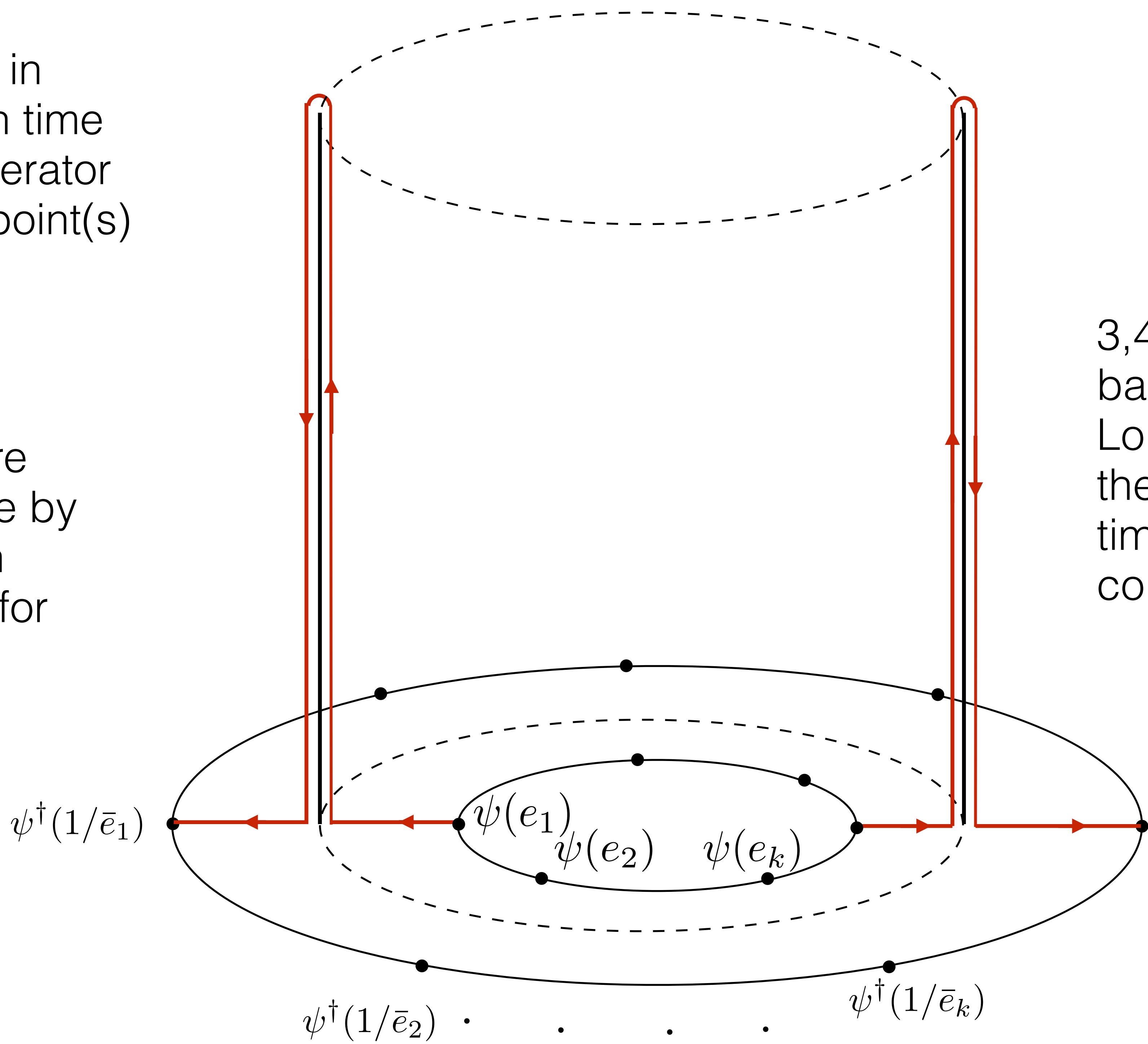


CFT calculation shows that purity of state is preserved: $S(A) = S(A^c)$

alternative picture: IN-IN computation

2.) evolve in Lorentzian time until Q-operator insertion point(s)

1.) prepare initial state by Euclidean evolution for time σ



3,4.) evolve back in Lorentzian time, then Euclidean time to form conjugate