Turbulent bursts and linear instabilities in rotating channel flow

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Turbulent channel flow with spanwise rotation

$$Re = \frac{U_b h}{\nu}, \quad Ro = \frac{2\Omega h}{U_b}$$

bulk velocity $U_b$, channel half-height $h$, rotation rate $\Omega$

Spectral DNS
box size $8\pi h \times 2h \times 3\pi h$ or larger, constant flow rate
Flow characteristics ($Re = 30 000$)

$U_{mean}$

$\nu^+$

$Ro = 0$
$Ro = 1.2$
$Ro = 2.4$

stable side ($Ro = 1.2$)

unstable side

$\lambda_2$ iso-cont.
Structures at $Re = 30000$: visualizations of $x$-$z$ planes

- **Roll-cells**
  - $Ro = 0.45$
  - (unstable side)

- **Turbulent-laminar patterns**
  - $Ro = 0.45$
  - (stable side)

- **Turbulent spots**
  - $Ro = 0.9$
  - (stable side)

- **Oblique waves**
  - $Ro = 2.7$
  - (unstable side)
Recurring bursts of turbulence ($Re = 30\,000$ and high $Ro$)

- **$Ro = 1.2$**
- **$Ro = 2.4$**

Time series

- **$Ro = 1.5$**
- **$\tau_w$**
- **cyclic bursts in laminar channel flow**
- - Boeck *et al.* (2008), MHD
- - Wallin *et al.* (2013), rotating

- **$K$**
- **$\tau_w$ stable side**
- **$\tau_w$ unstable side**
Burst mechanism

$Re = 31600, Ro = 1.2$

- Exponentially growing TS-wave
- Secondary instability
- Turbulent burst
- Turbulence decay, TS wave re-emerges
- Flow
Linear stability analysis

Orr-Sommerfeld and Squires equations
($\alpha$ streamwise, $\beta$ spanwise wavenumber)

$$\omega \nabla^2 \hat{\upsilon} = \alpha (U \nabla^2 - U'') \hat{\upsilon} + \frac{i}{Re} \nabla^4 \hat{\upsilon} + \beta Ro \hat{\eta}$$

$$\omega \hat{\eta} = \alpha U \hat{\eta} + \frac{i}{Re} \nabla^2 \hat{\eta} + \beta (U' - Ro) \hat{\upsilon}$$

- TS modes ($\beta = 0$) are unaffected by rotation
- Base flow $U(y)$ is the spatially averaged velocity of the DNS
- No eddy viscosity

(Wallin et al. 2013)
Energy equation of the TS-wave

\[
\frac{d}{dt} \int_{-1}^{1} E(k, t) dy = \\
\int_{-1}^{1} \left[ \Re \left( -\hat{u}^*(k) \hat{v}(k) \frac{\partial U}{\partial y} \right) - \nu \left( \frac{\partial}{\partial y} \hat{u}_i(k) \right) \left( \frac{\partial}{\partial y} \hat{u}^*_i(k) \right) \right] dy
\]

\(k\) wavenumber of the TS mode

- Nonlinear triad interactions are neglected
- \(\hat{u}^*(k), \hat{v}^*(k)\) and \(U(y)\) are obtained from DNS
- Energy equation and LSA are consistent for laminar Poiseuille flow, but not necessarily for turbulent flows
Comparison LSA and DNS \((Re = 31600 \text{ and } Ro = 1.2)\)

**LSA of \(\beta = 0\)-modes**

**Energy TS-wave**

\[ U(y) \]

\[ E(t) \]
Comparison LSA and DNS ($Re = 30000$ and $Ro = 2.1$)

Energy TS-wave

- DNS, --- energy eqn.
- LSA
- LSA with eddy visc.

Reynolds stress and prod.

- $Re(\hat{u} \hat{v})$, DNS
- $Re(-\hat{u} \hat{v} \frac{\partial U}{\partial y})$, DNS
- LSA
- LSA with eddy visc.

$v'$ $u'$

- LSA
- DNS
Cyclic burst regime

- **Cyclic bursts**
- No bursts but flow is linearly unstable
- No bursts and flow is linearly stable
Conclusions

- Strong recurring bursts of turbulence caused by a TS-instability can occur in rotating (partly) turbulent channel flow
- TS-wave growth is a linear process in all cases
- In some cases LSA agrees with DNS while in other cases agreement is poor
- Turbulence (or other disturbances) can strongly affect the Reynolds shear stress $Re(\hat{u}\hat{v})$ of the TS-wave
- Including an eddy viscosity hardly improves LSA predictions

Brethouwer et al. (2014) *PRL* 112, 144502.