The Strong Interaction and LHC phenomenology

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Theoretical Physics Graduate School course

Lecture 3:

The renormalization group and asymptotic freedom in QCD

The Renormalization Group

A key property of any **renormalizable Quantum Field Theory** is the **ultraviolet divergences** that arise in calculations beyond the Born level can be absorbed into a **redefinition of the couplings and fields** in the Lagrangian, which then turn to depend on the energy scale of the process

The dependence of the **renormalized couplings** with the energy is determined by the so-called **Renormalization Group Equations**

As an illustration, let's consider a quantum field theory with a **single dimensionless coupling constant.** Any physical quantity in this theory can be written

$$G = G(\alpha, M, s_1, \ldots, s_n)$$

coupling **UV cut-off** **kinematical invariants**

Formally, that this **theory is renormalizable** means that I can define a **renormalized coupling** as

$$\alpha_{\rm ren} = \alpha + \sum_{l=2} c_l\,(M/\mu)\,\alpha^l$$

Finite arbitrary scale from dimensional arguments

then any physical quantity in the theory can be expressed in terms of the **renormalized coupling,** the **finite scale μ** and other kinematical invariants, without the need anymore of the **UV cutoff**

$$G\left(\alpha\left(\alpha_{\rm ren}, M/\mu\right), M, s_1, \ldots, s_n\right) = \widetilde{G}\left(\alpha_{\rm ren}, \mu, s_1, \ldots, s_n\right)$$

Independent of UV cutoff M

The Renormalization Group

The crucial aspect is that the **very same redefinition of the coupling** then makes **all physical observables in the theory** finite and independent of the cutoff

$$G = G(\alpha, M, s_1, \ldots, s_n)$$

Bare coupling, UV divergent coefficients

$$\alpha_{\text{ren}} = \alpha + \sum_{l=2} c_l \left(M/\mu\right) \alpha^l$$

Renormalized coupling

Need to introduce **arbitrary scale μ** to make coefficients dimensionless

$$G\left(\alpha\left(\alpha_{\text{ren}}, M/\mu\right), M, s_1, \ldots, s_n\right) = \widetilde{G}\left(\alpha_{\text{ren}}, \mu, s_1, \ldots, s_n\right)$$

Renormalized, **finite theory for all observables**

The sensitivity of observables with respect to μ is reduced if the computation is performed with more terms in the series expansion, and can only be completely removed when whole series is computed

The dependence of the renormalized coupling with scale is determined by the **renormalization group**

The Renormalization Group

The renormalization group equations arise from the observation that if we vary **μ** and **α_{ren}**, while keeping **M** and **α** fixed, physical observables should be invariant.

This is so because before renormalization, physical quantities depend only on **M** and **α.** Therefore, the same property must hold true **after** renormalization, which implies:

$$G\left(\alpha\left(\alpha_{\text{ren}}, M/\mu\right), M, s_1, \ldots, s_n\right) = \widetilde{G}\left(\alpha_{\text{ren}}, \mu, s_1, \ldots, s_n\right)$$

$$\frac{\partial\,\alpha\left(\alpha_{\text{ren}}, M/\mu\right)}{\partial\,\alpha_{\text{ren}}} d\alpha_{\text{ren}} + \frac{\partial\,\alpha\left(\alpha_{\text{ren}}, M/\mu\right)}{\partial\,\mu^2} d\mu^2 = 0$$

$$\frac{\partial\widetilde{G}\left(\alpha_{\text{ren}}, \mu, s_1, \ldots, s_n\right)}{\partial\alpha_{\text{ren}}} d\alpha_{\text{ren}} + \frac{\partial\widetilde{G}\left(\alpha_{\text{ren}}, \mu, s_1, \ldots, s_n\right)}{\partial\mu} d\mu = 0$$

Follows from the result that μ and α_{ren} enter G via the bare coupling only

Now, combining the two equations, we find that

$$\mu^2 \frac{d\alpha_{\text{ren}}}{d\mu^2} = -\frac{\mu^2 \partial\alpha\left(\alpha_{\text{ren}}, M/\mu\right)/\partial\mu^2}{\partial\alpha\left(\alpha_{\text{ren}}, M/\mu\right)/\partial\alpha_{\text{ren}}} = -\frac{\mu^2 \partial\widetilde{G}\left(\alpha_{\text{ren}}, \mu, s_1, \ldots, s_n\right)/\partial\mu^2}{\partial\widetilde{G}\left(\alpha_{\text{ren}}, \mu, s_1, \ldots, s_n\right)/\alpha_{\text{ren}}}$$

The Renormalization Group

$$\mu^2 \frac{d\alpha_{\rm ren}}{d\mu^2} = -\frac{\mu^2 \partial \alpha\left(\alpha_{\rm ren}, M/\mu\right)/\partial \mu^2}{\partial \alpha\left(\alpha_{\rm ren}, M/\mu\right)/\partial \alpha_{\rm ren}} = -\frac{\mu^2 \partial \widetilde{G}\left(\alpha_{\rm ren}, \mu, s_1, \ldots, s_n\right)/\partial \mu^2}{\partial \widetilde{G}\left(\alpha_{\rm ren}, \mu, s_1, \ldots, s_n\right)/\alpha_{\rm ren}}$$

Therefore, the functional dependence of the **renormalized coupling** with the scale is fixed by:

☑ Cannot depend on ***M***, since RHS does not

☑ Cannot depend on s1, ... , sn, since LHS does not

☑ Cannot depend on μ, since we don't have other dimensionful variables available

$$\mu^2 \frac{d\alpha_{\rm ren}}{d\mu^2} = \beta\left(\alpha_{\rm ren}\right)$$

Therefore, the **scale dependence** of the renormalized coupling is only a function of the coupling itself

This is known as the Renormalization Group equation, which can also be written as

$$\frac{d}{d\log\mu^2}\alpha_{\rm ren} = -b_0\alpha_{\rm ren}^2$$

This equation is completely general, but its coefficients depend on the theory

The RG in QCD

The structure of the renormalization group equation is general, but the **value of b_0** depends on the theory

$$\frac{d}{d\log\mu^2}\alpha_{\rm ren} = -b_0\alpha^2_{\rm ren}$$

This equation can be easily solved to give (**exercise**)

$$\alpha_{\rm ren}(\mu^2) = \frac{1}{b_0\log\mu^2/\Lambda^2}$$

where Λ is an **integration constant**

One would be tempted to say that Λ is the scale at which the **running coupling** becomes infinite, but when the coupling constant is large perturbation theory breaks down

A better definition of Λ is the **scale parameter** which defines the running coupling when **$\mu >> \Lambda$**

This phenomenon is known as **dimensional transmutation:** a dimensionful scale **Λ** appears in a **scale-invariant theory** upon renormalization

Of course, this integration constant is nothing but **$\Lambda = \Lambda_{QCD}$** , the typical s**cale of the strong interactions** in the non-perturbative regime

The structure of the renormalization group equation is general, but the **value of b₀** depends on the theory

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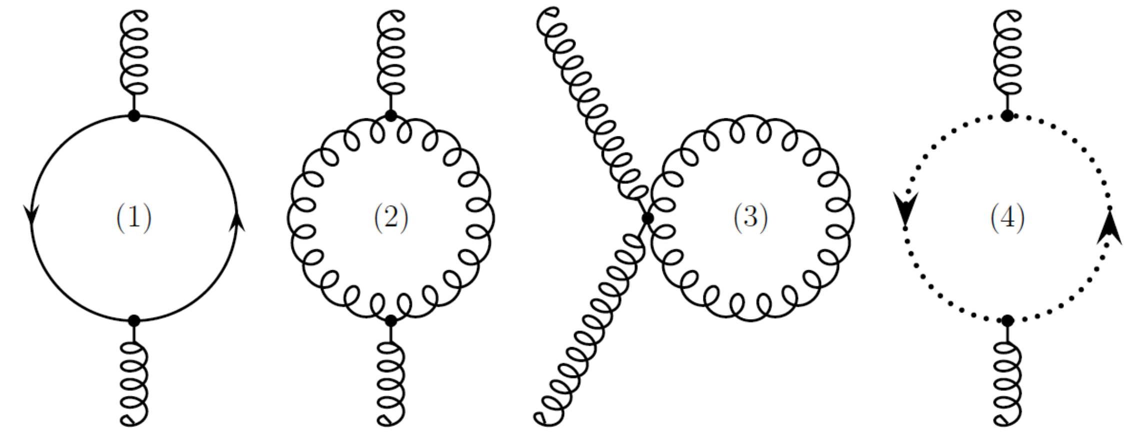
In QCD the computation of **b_0** requires the computation of the **gluon self-energy at the one-loop level**

The answer is

$$b_0 = \frac{33-2n_f}{12\pi}$$



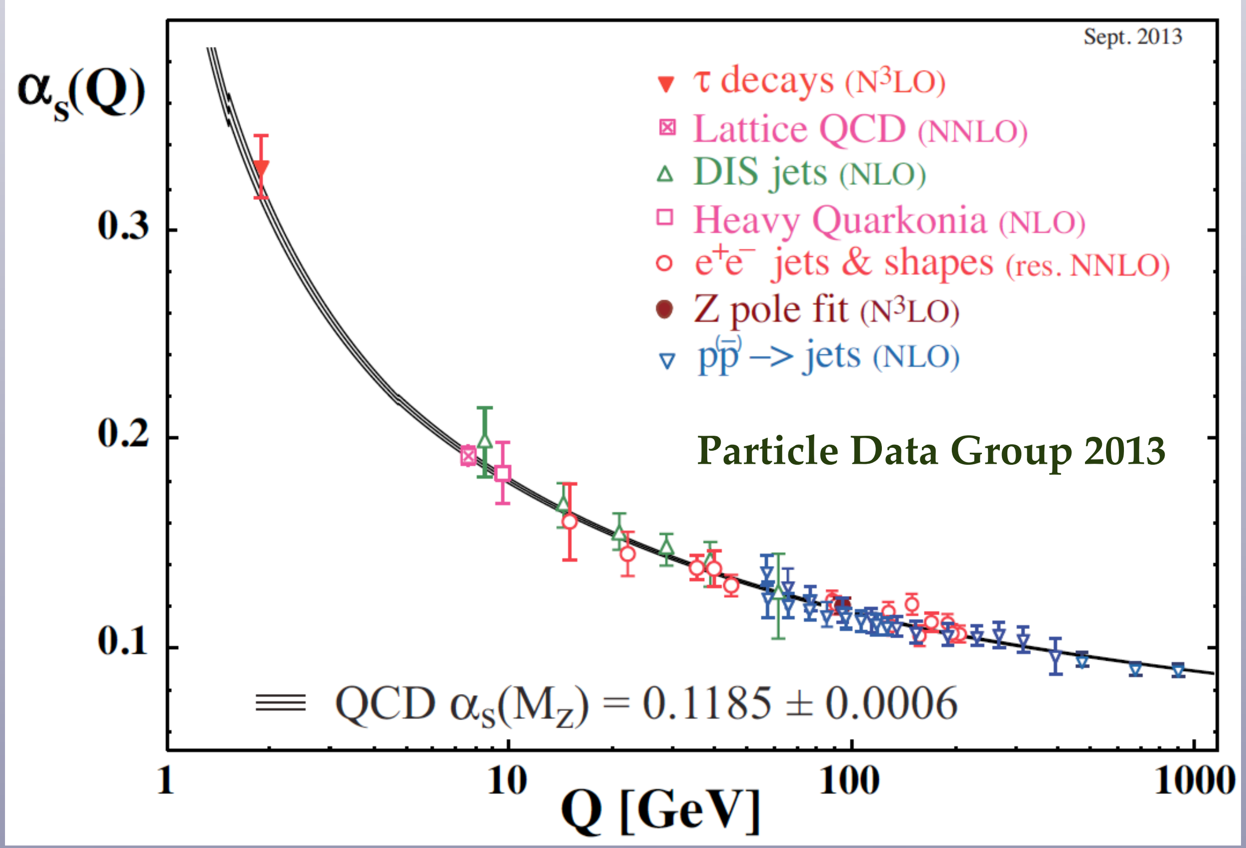
Asymptotic freedom

Therefore, **QCD** is asymptotically free: the coupling constant **decreases** with the scale

$$\alpha_{\text{ren}}(\mu^2) = \frac{1}{b_0 \log \mu^2/\Lambda^2} \qquad b_0 = \frac{33-2n_f}{12\pi}$$

Higher-order computations (up to four loops) do not change this behavior: QCD becomes a **free theory** in the asymptotic limit

Running of the QCD coupling has been verified in a wide variety of experiments, from low to high energy



The result that the QCD coupling becomes large at low scales hints towards **confinement,** which is however a purely perturbative phenomenon

Asymptotic freedom: QCD vs QED

As opposed to this, in QED the beta function is **negative** (due to absence of photon self-coupling)

$$\alpha_{\rm ren}(\mu^2) = \frac{1}{b_0 \log \mu^2/\Lambda^2}$$

$$b_0 = \frac{33 - 2n_f}{12\pi} \qquad b_0^{\rm QED} = -\frac{4n_f}{12\pi}$$

From gluon self-interactions **From gluon(photon)/quark interaction**

QCD: renormalization group equation valid when **μ >> Λ** (about 400 MeV)

QED: renormalization group equation valid when **μ << Λ**

The scale where QED ceases to be trustable is the **Landau pole**, given by (**exercise: check**)

$$\Lambda = m_e \exp\left(-\frac{1}{2b_0\alpha_{\rm ren}(m_e)}\right) \sim 10^{90}\,{\rm GeV} \gg M_{Pl}$$

This is the reason we can always safely think of QED as **purely perturbative theory**

Here we see the crucial role of **SU(3) as a non-abelian theory:** the **self-interactions between gluons** are responsible for the radically different behaviors of the running couplings in QCD and QED

Measurements of α_S

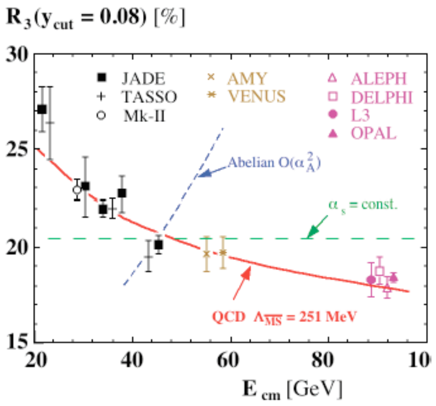
The accurate **measurement of the QCD running coupling** in an important topic for particle physics, and a crucial ingredient for **LHC phenomenology**

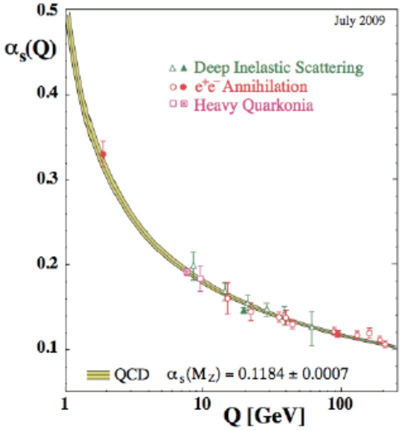
For instance, at the LHC the Higgs is produced in the gluon fusion channel, and we have

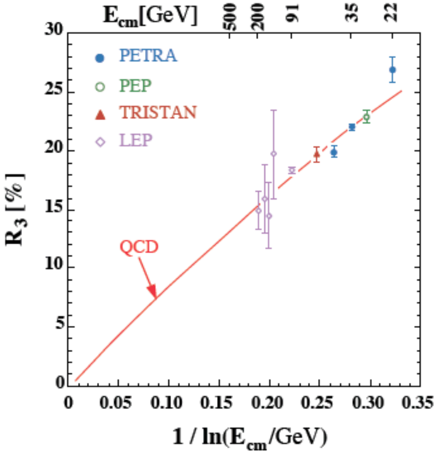
$$\sigma_H \sim \alpha_S^2 \quad \rightarrow \quad \frac{\delta_\alpha \sigma_H}{\sigma_H} \sim 2\frac{\delta\alpha_s}{\alpha_s}$$

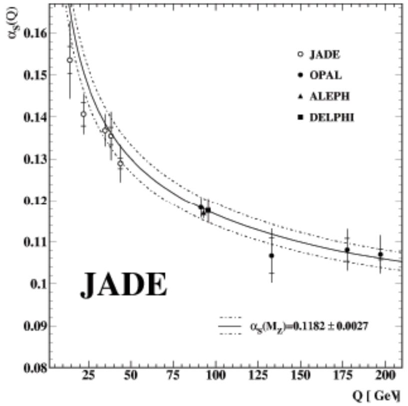
So a 2% error on the coupling translates into 4% in the Higgs cross-section (actually more with higher orders)

The **measurement of the strong coupling** can be achieved in a wide variety of processes (see later in the course)



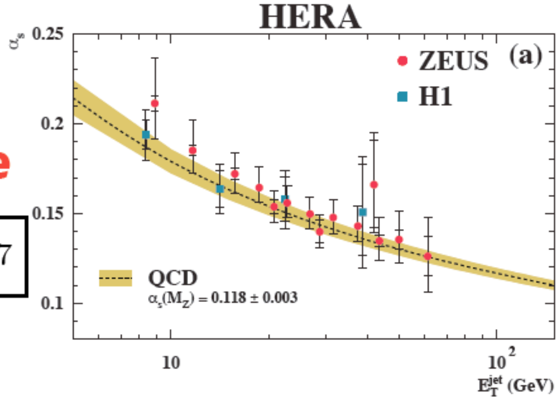






World average

$$\boxed{\alpha_s(M_{Z^0}) = 0.1184 \pm 0.007}$$



Key for **precise α_S measurements**: processes with high sensitivity to α_S variations and small theory and experimental uncertainties

Once α_S determined at some scale, it is customary to evolve it to **$\alpha_S(M_Z)$** using the **renormalization group equations**

RGE, asymptotic freedom and confinement

Asymptotic freedom is one of the central properties in **QCD**

☑ QCD becomes a **free theory** at very large scales

☑ Gluon self-interactions, arising from the **SU(3) invariance** requirement, responsible for this behavior

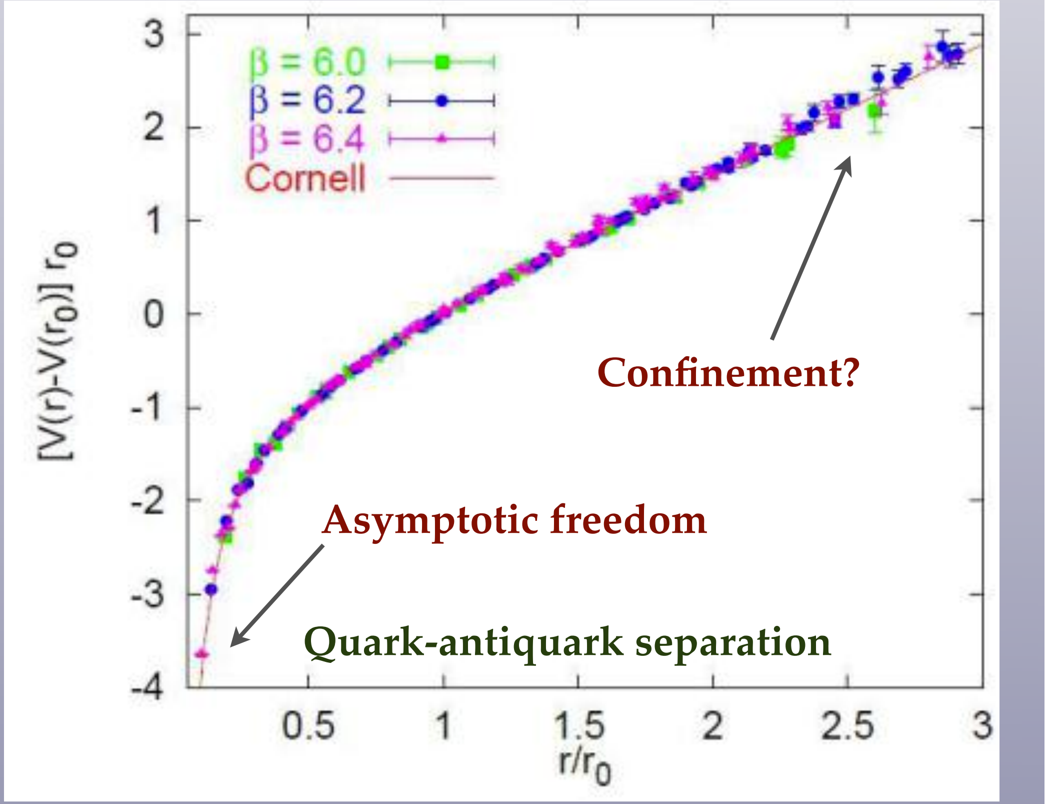
☑ The renormalization group allows to determine the dependence of the running coupling on the scale

On the other hand, we still **don't understand** what are the dynamics that lead to **confinement**

☑ The QCD coupling is large at low scales, but there perturbation theory breaks down

☑ Confinement is an experimental observation (hadrons are always **color singlets**) but it is a purely non-perturbative phenomenon

☑ Non-perturbative methods are required, like **lattice QCD**, which shows that the **quark-antiquark potential increases with the separation**, as expected in a confining theory



Confinement?

Asymptotic freedom

Quark-antiquark separation