EXPERIMENTS ON MAGNETIC TRANSPORT, MAGNETIC TRAPPING, AND BOSE-EINSTEIN CONDENSATION

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A thesis submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy at the University of Oxford
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Min Sung Yoon, The Queen’s College, University of Oxford
D.Phil thesis, Hilary Term 2009

This thesis describes the construction of a new magnetic trapping apparatus and experimental procedures leading to magnetic transport and quantum degeneracy of $^{87}$Rb in an Ioffe type trap. The atoms are collected and pre-cooled in a double magneto-optical trap (MOT) apparatus where a pyramidal MOT acts as a source of atoms for a second MOT in the UHV region. The magnetic trap setup is a modified quadrupole and Ioffe trap configuration (QUIC trap), which consists of two pairs of overlapping quadrupole coils and one Ioffe coil. Magneto-optically trapped atoms are compressed, cooled by the optical molasses technique, and then optically pumped into the $|F = 2, M_F = 2\rangle$ state before being loaded into the quadrupole trap. With a transfer efficiency of 38 percent from the second MOT into the QUIC trap, this setup is capable of magnetic transport of cold atoms over a distance of 20 mm into the QUIC trap. The QUIC trap forms a harmonic potential with a radial trapping frequency of $\omega_r = 2\pi \times (147.43 \pm 0.4)$ Hz and an axial trapping frequency of $\omega_a = 2\pi \times (12.74 \pm 0.01)$ Hz. The atomic cloud released from the magnetic trap is probed by absorption imaging to determine the number of trapped atoms ($N$), temperature ($T$), and phase space density ($\rho$). We demonstrate efficient runaway evaporation characterized by efficiency parameters of $-d\ln \rho/d\ln N = 2.7$ and $d\log N/d\log T = 0.7$. Evaporative cooling is performed as the RF frequency is swept from 22.5 MHz to 0.39 MHz over a period of 59 s. We observe a nearly pure Bose-Einstein condensate at 90 nK with an atom number of $1.7 \times 10^5$. The experimental setup allows for good optical access for further experiments such as direct quantum simulation and neutral atom quantum computing.
Now faith is the substance of things hoped for, the evidence of things not seen. For by it the elders obtained a good report. Through faith we understand that the worlds were framed by the word of God, so that things which are seen were not made of things which do appear. And without faith it is impossible to please God, because anyone who comes to him must believe that he exists and that he rewards those who earnestly seek him. (Hebrews 11)

I tell you the truth, whoever hears my word and believes him who sent me has eternal life and will not be condemned; he has crossed over from death to life. (John 5:24)
Building the experiment for this thesis had not been an easy task, but there are those who worked with me and helped make the endeavour successful. Above all I would like to thank my supervisor Prof. Christopher Foot for giving me the wonderful opportunity to work in atomic and laser physics, and for his kind guidance and warm encouragement throughout my D.Phil course. I also want to thank the post-docs, my colleague and mentor Herbert Crepaz, and Giuseppe Smirne, and Gerald Hechenblaikner, who helped mould me into the scientist I am. My gratitude goes to the people of our group who helped me set up the experiment; Amita Deb, Marcus Gildemeister, Martin Shotter, Eileen Nugent, Andrian Harsono, and C. Chandrashekar. Their contribution towards the experimental research has been vital and priceless. My thanks also to other members of the group who had helped me out in one way or another whenever I was in need; Rachel Godun, Donatella Cassetari, Vincent Boyer, Zhao Yuan Ma, William Heathcote, Benjamin Sheard, Benjamin Sherlock, Sarah Al-Assam, Ross Williams, Ben Fletcher, Marcin Kubasik, David McCabe, and Duncan England. I want to say a big thank you, Dr Partick Baird, Prof. Ian Walmsley for their support throughout my graduate studies, Dr Simon Cornish at Durham for working as an examiner of the thesis, Graham Quelch, Rob Harris, Jaya John, and the Central Electronics staff. There are also other people outside the Clarendon Laboratory who helped me overcome the occasionally stressful moment and gave me the best years of my life at Oxford; my precious families in Sao Paulo, Hwasung Lee and Hason Jeong at the Embassy of Korea in London, Sung Ho Yoon’s family at Cambridge, Dr Jane Mellor and Prof. Jane Langdale who were my moral tutors at the Queen’s College, Seung Woo Lee and Seung Joo Lee who read theoretical physics, Dr Norman Solomon at the Faculty of Oriental Studies, and Prof. Peter Raynes, Wook Sung Kim, and Changhun Lee at the Engineering Science Department. Many thanks to Maurice and Marthe Glover, and Grove Free Evangelical Church (pastor Paul John) for providing spiritual support and hospitality during my life at Oxford. Special thanks to my parents and all my families; Taebok, Oeoak, Daegyung, Heyeon, Yongwon, Jebyeol and Kyu Soon Yoon for their lovely encouragement, patience, and support, and who always stood by my decision to do physics. I love all of you. I acknowledge financial support from the Korean Ministry of Education (KOGOS 2003), the Sarang Community Church and Rev. Hanheom John Oak, the Queen’s College, and the British Engineering and Physical Sciences Research Council (EPSRC). Finally, this doctoral thesis is dedicated to God our eternal Lord.
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1.1 Bose-Einstein Condensation

1.1.1 Bose-Einstein Condensation

Bose-Einstein Condensation (BEC) is a relatively recent experimental research field. Its first experimental achievement in 1995 [1, 2, 3] opened an entirely new era in atomic physics. Since then, this field has drawn the attention of a large portion of the atomic physics community, and the important achievements have been acknowledged with Nobel prizes awarded to the pioneering contributors in 2001. This section, dedicated to BEC, gives a partial review of the techniques and concepts and its experimental realization (see [4, 5] for more details). The phenomenon known as Bose-Einstein condensation was predicted in 1925 for atoms by Einstein [6], following a study of the properties of photons by Bose [7] in 1924. When the temperature of a gas of bosons (integer-spin particles) is low enough, a significant fraction of them spontaneously accumulates in the lowest energy state purely because of statistical mechanics rather than the interactions which cause the phase transition from gas to liquid. In BEC the particles act collectively as a coherent classical wave. The characteristic size of the wave packet for an atom is determined by the thermal de Broglie wavelength at a temperature $T$ of the gas,

$$\lambda_{dB} = \frac{\hbar}{\sqrt{2\pi mk_B T}}$$  \hspace{1cm} (1.1)$$

where $\hbar$ is the Planck constant, $m$ is the mass of one particle, and $k_B$ is the Boltzmann constant. The mean distance between atoms in a gas with number density $n$ is $n^{-1/3}$. For a bosonic gas, the theory predicts that as a gas is cooled under conditions where it does not liquify there is a phase transition to quantum degeneracy when the de Broglie wavelength becomes comparable with the average separation
between the atoms. The phase transition to a Bose-Einstein condensate occurs when \( n\lambda_{dB}^3 \approx 2.612 \). The quantity \( n\lambda_{dB}^3 \), or the phase space density, represents the number of particles within the volume given by the uncertainty in the position of a single particle. When the condition \( \lambda_{dB} \approx n^{-1/3} \) is satisfied, the quantum statistical effects in an atomic gas appear. In a Bose-Einstein condensate the de Broglie waves of the individual atoms interfere constructively such that all the particles can be described by a single macroscopic wave function.

### 1.1.2 Magnetic trapping for BEC

Experimentalists first tried to observe BEC using spin-polarized atomic hydrogen cooled down with cryogenic techniques. Three body recombination prevented the atomic gas from reaching quantum degeneracy and the phase space density did not go above 0.05 in those experiments [8]. The development of laser cooling, magnetic trapping, and forced RF-induced evaporative cooling [9] techniques were the breakthroughs that enabled BEC to be observed with rubidium, sodium, and lithium atoms in a dilute vapour [1, 2, 3]. Atoms were first collected and cooled in a magneto-optical trap (MOT) [10]. They were transferred into a magnetic trap where they were further cooled by evaporation leading to macroscopic quantum degeneracy. Magneto-static traps for atoms with permanent magnetic dipole moments \( \vec{\mu} \) use an inhomogeneous magnetic field \( \vec{B}(r) \). Maxwell’s equations do not allow a local maximum of a static magnetic field in a region devoid of charges and currents [11]. As a result, only a minimum of a static magnetic field can be used to trap atoms in low field seeking states where energy increases as the magnetic field increases. The interaction between the atomic magnetic dipole moment and the non-uniform magnetic field\(^1\) \( \vec{B}(r) \) can be written as

\[
U = -\vec{\mu} \cdot \vec{B}(r) = g_F M_F \mu_B B
\]

where \( g_F \) is the Landé factor, and \( M_F \) is the Zeeman substate of the hyperfine level \( F \), and \( \mu_B \) is the Bohr magneton, and \( B \) is the absolute value of the magnetic field. The first experiment on trapping cold atoms in a static magnetic field was done with a quadrupole magnetic trap for \( ^{23}\text{Na} \) atoms pre-cooled to 17 mK by a counter-propagating laser beam [12]. The spherical quadrupole trap was formed simply by two coaxial coils carrying equal currents flowing in opposite directions. It produces a static magnetic field with a minimum at the trapping centre where the magnetic fields from the two coils cancel. The main shortcoming of magnetic quadrupole traps is the presence of a zero magnetic field minimum at their trapping centre. Atoms approaching this zero field point can undergo nonadiabatic spin flips, \( i.e. \) transitions from trapped states to untrapped states [13]. Thus the

\(^1\)Following common usage, I refer to the magnetic induction \( \vec{B} \) as a magnetic field in this thesis. In vacuum the magnetic field \( \vec{H} \) is related to the magnetic induction by the relationship \( \vec{B} = \mu_0 \vec{H} \) where \( \mu_0 \) is the magnetic permeability of free space.
zero magnetic field minimum effectively leads to the existence of a hole in the trap from which the atoms escape. The rate of spin-flip transitions depends on the temperature of the atoms and prevents atoms from being cooled to low temperature in a quadrupole trap. However, there are several ways to close the hole. One way is to superimpose a rotating uniform magnetic field on a static quadrupole magnetic field. This so called time-averaged orbiting potential (TOP) trap was used to demonstrate the first Bose-Einstein condensate in a rubidium vapour; when the magnetic field is kept away from the centre of the trap, there is no channel for the atoms to escape [14]. Alternatively, an intense laser beam with blue frequency detuning shining through the centre of the trap creates a potential barrier that acts as an optical plug, and the potential produced by the combined quadrupole magnetic field with an optical potential was successfully used for the sodium BEC experiment [2]. An Ioffe-Pritchard (I-P) trap is an example of a static magnetic trap with a nonzero local minimum in the magnetic field. In its original design four straight bars provide a radial quadrupole field and a pair of circular pinch coils produce the axial confinement field [15]. A convenient way to make the Ioffe-Pritchard-typed trap is to bend the wire into one coil that resembles the seams on a baseball [16, 17]. The ease of the baseball coil arises from the fact that applying a current to the single coil produces the same field near the trapping centre as flowing equal current through the four parallel wires and the pinch coils. Another variant, called the cloverleaf trap, was demonstrated by replacing the four bars with a pair of four coils surrounding each pinch coil and providing the radial magnetic field gradient [18] to give improved optical access. A modification of the I-P trap called the QUIC trap was invented by Hänsch and co-workers [19]. It consists of two circular coils in the quadrupole configuration, and a third circular coil perpendicular to the first two coils and located at a distance from them. Atoms initially caught in the quadrupole magnetic trap are transferred into the Ioffe-Pritchard type by smoothly switching on the current of the third coil.

The experimental achievement of BEC from atomic gases was first performed by Anderson et al. in 1995 at JILA [1]. The procedure of their experiment is as follows. They captured about $10^7$ atoms of $^{87}$Rb in a magneto-optical trap (MOT) directly from an atomic vapour at room temperature. The collected atoms were cooled to $\sim 20 \mu$K by adjusting the field gradient and laser frequency. The atoms were optically pumped into the $|F = 2, M_F = 2\rangle$ state with the help of a small magnetic bias field and a short pulse of circularly polarized light. After the MOT was turned off, the spin-polarized atoms were transferred into a TOP trap. Then the current was adiabatically ramped up to the maximum value of the quadrupole field component of the TOP trap. They had about $4 \times 10^6$ atoms in the trap at about 90 $\mu$K and then evaporatively cooled the atomic sample confined in the magnetic trap for 70 s by varying the RF frequency which causes transitions to an untrapped spin state. The evaporation process cools the atom cloud and increases the phase space density. The condensate first appeared at a temperature
of 170 nK and a number density of $2.5 \times 10^{12}$ atoms/cm$^3$. The final number of atoms in the condensate was about $2 \times 10^4$. After its first demonstration with gaseous $^{87}$Rb atoms, the exploration of BEC in atomic gases grew rapidly. Bose-Einstein condensates have been obtained with stable alkali species such as $^{23}$Na [2], $^7$Li [3], $^{85}$Rb [20], $^{41}$K [21], $^{133}$Cs [22] among alkali Bose isotopes [23] and a wide variety of fascinating phenomena have been observed in BEC. The occupation of a single quantum state by a large number of identical bosons means a source of coherent matter waves analogous to the occupation of photons in a single mode of a laser cavity. The evidence of coherence between two condensates was observed by interference between them in a free expansion [24, 25]. Extraction of atoms from a trapped Bose condensate was demonstrated to produce atom lasers [25, 26, 27, 28]. In addition, the superfluid nature of the condensates has been studied such as sound propagation and creation of vortices within the condensate [29, 30, 31, 32]. Furthermore, using BEC loaded into optical lattices was realized a quantum phase transition between the superfluid state and Mott insulator state [33]. Recently properties of atoms in an array of optical traps are being studied with a view to the demonstration of logic gates for quantum information processing [34].

### 1.1.3 Magnetic transport

A widely used way to produce BEC is a combination of a magneto-optical trap (MOT) and a magnetic trap. Typically both traps are in the same place. Since many laser beams and magnetic coils are required to create BEC, the conventional technique imposes major restrictions on the optical and mechanical access to further experiments such as manipulation of condensate atoms using a three dimensional optical lattice [35]. Therefore, it is desirable to transport the atoms between the MOT and final magnetic trap. Three main ways to transport magnetically cold atoms have been demonstrated.

The first method is to use compact lithographic atom microtraps [36]. These traps use miniaturized current-carrying conductors for generating the magnetic field. They typically produce strongly confining traps with high trapping frequencies and high atomic densities. Transport of cold atoms on an atom chip-based magnetic conveyor up to a linear distance of 24 cm in 3 s was demonstrated by Long et al. [37]. During the transfer over an initial 4.4 mm at a speed of 22 mm s$^{-1}$, two thirds of atoms were lost from the conveyor configuration when they started with $1.5 \times 10^5$ atoms. Although evaporation in the small trap can be very fast due to the tight confinement and high atomic density, the small trap volume limits the number of condensed atoms and atom loss is enhanced by three-body collision rates.

An alternative way to transport cold atoms from the MOT into a spatially separated vacuum chamber was accomplished by using a row of overlapping quadrupole coil pairs with time-varying currents [38]. It is possible to maintain a constant
geometry of the trap potential during magnetic transport by regulating suitable currents simultaneously through three neighbouring quadrupole coil pairs. Using the nine overlapping quadrupole coil pairs, magnetically trapped cold atoms were transferred by a distance of 33 cm with a maximum speed of 140 mm s\(^{-1}\) and a maximum acceleration of about 300 mm s\(^{-2}\). This configuration does not need a second MOT in the science cell, but requires sophisticated switching electronics for a chain of multiple quadrupole coils to avoid heating of the cloud during the transport process.

Instead of the macroscopic multiple coils with time-varying currents, a rather simpler method uses mechanically moving macroscopic trap coils. The first demonstration of mechanical magnetic transport from the MOT to the science cell was reported by researchers in JILA [39]. A commercial linear actuator is used to move the quadrupole coils from one end of the vacuum system to the other over a distance of 50 cm with a maximum velocity of 0.6 m s\(^{-1}\) and a maximum acceleration of about 2 m s\(^{-2}\) [40]. This method is well suited for the transport of mixtures of different species and was recently applied to the study of a Bose-Fermi mixture [41, 42]. Moving coils requires reliable mechanical stability and position reproducibility of the bulky coil system during the transport, and the motor of a translation stage on which the trap coils are mounted may generate vibration and RF noise which can perturb the evaporative cooling. Such macroscopic transport systems for cold atoms typically transfer one third of particles to the experimental region [38, 40, 41].

In the thesis, I describe the development of a simple and efficient magnetic transport system, i.e. a modified quadrupole and Ioffe trap configuration (QUIC-type trap) that consists of two pairs of overlapping quadrupole coils and one Ioffe coil to perform magnetic transport over a distance of 20 mm, which is enough to avoid the optical and mechanical access constraints of conventional condensate experiments. The magnetic transport operates as follows. Magneto-optically trapped atoms are initially loaded into a quadrupole trap formed by a first pair of quadrupole coils (the MOT coils). Then the current in a second pair of quadrupole coils is gradually switched on so that they have the same current as the first pair of quadrupole coils, resulting in a transport of the atoms trapped around the field zero position halfway between the centres of the two coil pairs. Then the quadrupole magnetic trap is converted into an Ioffe-type trap by switching on the current in the Ioffe coil. The same current applied to all coils reduces heating of the trapped atoms from current fluctuations. This results in a final trapping position close to the centre of the second quadrupole coils. The great flexibility of this transport scheme gives improved optical access and the implementation of various optical manipulation techniques at the final trapping position. The current direction in one of the second pair of quadrupole coils can be reversed to make a Helmholtz configuration, generating uniform magnetic fields for Feshbach resonance experiments.
1.2. Properties of the Bose gas in a trap

1.2.1 Condensation of the ideal Bose gas in a box

We consider a non-interacting Bose gas in the context of the grand canonical ensemble for temperature $T$ and chemical potential $\mu$ \cite{43}. We first confine our interest to the ideal spinless Bose particles with mass $m$ in a cubic box of volume $V = L^3$. In this case the single-particle Hamiltonian has the form $H^{(1)} = \frac{p^2}{2m}$. The system can be described by the summation of the independent particle Hamiltonians, i.e. $H = \sum_i H^{(1)}_i$. Thus, any energy eigenvalue of an ideal system comes from a sum of single-particle energy levels which are given by $\epsilon_p = \frac{p^2}{2m}$ where $p$ is the momentum eigenvalue of the single particle and $p = |p|$. The momentum becomes $p = \frac{h}{L} n$ when we adopt periodic boundary conditions for the cubic box where $n$ is a vector whose components are 0 or ± integers. For the limit of a large box ($V \to \infty$), a sum over the states with $p \neq 0$ can be replaced by an integration such as $\sum_p \to \frac{V}{h^3} \int 4\pi p^2 dp$ since the allowed spectrum of the single particle states of $p$ forms a quasi-continuum.

The mean occupation number $N_p$ of the non-interacting Bose particles in the energy level $\epsilon_p$ is given by the Bose-Einstein distribution function

$$N_p = [e^{\beta(\epsilon_p - \mu)} - 1]^{-1}$$

and total energy is

$$E = \sum_p \epsilon_p N_p$$

where $\beta = 1/k_B T$, and $\mu = \mu(T, N)$ is the chemical potential of the boson gas system fixed by the condition of normalization $N = \sum_p N_p$. The chemical potential of the system always satisfies the condition $\mu < 0$ since the number of particles in a given state must remain non-negative and the lowest single-particle state has zero energy ($\epsilon_0 = 0$). To count correctly the number of the gaseous boson particles we need to account separately for the occupation number $N_0$ of $p = 0$ that may be very large in the system. Thus total number of particles is determined by

$$N = \sum_p N_p = N_0 + \sum_{p \neq 0} N_p.$$  \hspace{1cm} (1.5)

For the ideal gas in the limit of the large box ($V \to \infty$), the sums for $p \neq 0$ are replaced by an integral;

$$\frac{N}{V} = \frac{N_0}{V} + \frac{1}{h^3} \int \frac{4\pi p^2}{e^{\beta(p^2/2m-\mu)} - 1} \, dp$$  \hspace{1cm} (1.6)

where $\frac{N}{V} = n$ is the total number density of particles and $\frac{N_0}{V} = n_0$ is the number density of particles in the ground state, respectively. Since the number of states
with energy below a given value $\varepsilon$ is $N(\varepsilon) = \frac{4\pi V}{3h^3}(2m\varepsilon)^{3/2}$, the density of states is given by $D(\varepsilon) = \frac{V}{4\pi^2(2mh^2)^{3/2}}\sqrt{\varepsilon}$, and the number of atoms in excited states can be written in the form

$$\sum_{p \neq 0} N_p = \int \frac{D(\varepsilon)}{e^{\beta(\varepsilon - \mu)} - 1} d\varepsilon = \frac{Vg_{3/2}(z)}{\lambda_{dB}^3}$$

(1.7)

where $\lambda_{dB}$ is the thermal de Broglie wavelength given by eq. 1.1. Thus, the total number density of particles is written by

$$n = n_0 + \frac{g_{3/2}(z)}{\lambda_{dB}^3}$$

(1.8)

where we introduce the quantity

$$g_{3/2}(z) = \frac{2}{\sqrt{\pi}} \int_0^\infty \frac{\sqrt{x}}{z^{-1}e^x - 1} dx = \sum_{l=1}^\infty \frac{z^l}{l^{3/2}}$$

(1.9)

and $z = e^{\beta\mu}$ ($0 \leq z \leq 1$) is the fugacity which depends on the temperature for the system. For real values of $z$ between 0 and 1, $g_{3/2}(z)$ is a monotonically increasing function of $z$, but its maximum at $z = 1$ is a finite value, i.e. $g_{3/2}(z = 1) \approx 2.612$. Thus, from eq. 1.8, we see that when the condition $n > \frac{g_{3/2}(1)}{\lambda_{dB}^3}$ is satisfied, a finite fraction of the particles occupies the ground energy level with $p = 0$.

As the fugacity approaches its limiting value of one, or the chemical potential approaches zero from below, the population of the ground state becomes macroscopic. This is the phenomenon known as Bose-Einstein condensation (BEC). For a three-dimensional cubic box, a critical temperature $T_c$ is defined as the temperature where the number of atoms in the excited states is equal to the total number of atoms in the system, so that $N_0 = 0$ at $T = T_c$. The relation of $n\lambda_{dB}^3(T_c) = g_{3/2}(1)$ gives the critical temperature

$$T_c = \frac{\hbar^2}{2\pi mk_B} \left( \frac{n}{g_{3/2}(1)} \right)^{2/3}.$$ 

(1.10)

Below the critical temperature it is impossible to accommodate all the particles in excited states so that a macroscopic number of atoms are forced to accumulate in the lowest energy state. The condition on the phase space density at the critical temperature

$$n\lambda_{dB}^3 \simeq 2.612$$

(1.11)

is consistent with the intuitive picture that condensation occurs when the thermal de Broglie wavelength becomes comparable to or larger than the inter-particle spacing. From eq. 1.8 we obtain the temperature dependence for the number of particles in the condensate as

$$\frac{N_0}{N} = 1 - \left( \frac{T}{T_c} \right)^{3/2}.$$ 

(1.12)
1.2. Properties of the Bose gas in a trap

Hence, the ground state population \( N_0 \) grows continuously from zero to \( N \) as temperature decreases from the critical temperature to zero (see Fig. 1.1 (b)).

1.2.2 The Bose gas in a harmonic trap

In actual experiments we have a trap potential which is a three-dimensional harmonic potential of the form

\[
U(\mathbf{r}) = \frac{1}{2} m (\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2)
\]

(1.13)

where \( \omega_i \) are the oscillator frequencies in the three directions. The corresponding energy levels are

\[
E = \sum_i \hbar \omega_i \left( n_i + \frac{1}{2} \right) = \hbar \omega_x \left( n_x + \frac{1}{2} \right) + \hbar \omega_y \left( n_y + \frac{1}{2} \right) + \hbar \omega_z \left( n_z + \frac{1}{2} \right)
\]

(1.14)

where \( n_i \) denote non-negative integers. To obtain a density of states \( D(\varepsilon) = \frac{\partial N(\varepsilon)}{\partial \varepsilon} \) with energy below a given value \( \varepsilon \), we treat \( n_i \) as continuous variables and introduce a coordinate system defined by the three variables \( \varepsilon_i = n_i \hbar \omega_i \). The number of states \( N(\varepsilon) \) in the harmonic oscillator is determined from an octant bounded by the surface of equal energy of \( \varepsilon = \varepsilon_x + \varepsilon_y + \varepsilon_z \); \( N(\varepsilon) = \frac{\varepsilon^3}{6 \hbar^3 \omega_x \omega_y \omega_z} \) [44]. Thus, we find the density of states of the trapped gas given by

\[
D(\varepsilon) = \frac{\varepsilon^2}{2(\hbar \bar{\omega})^3}
\]

(1.15)

where we introduce the geometric average of the oscillator frequencies in the three spatial directions

\[
\bar{\omega} = \left( \omega_x \omega_y \omega_z \right)^{1/3}.
\]

(1.16)

The central number density for a thermal gas in the harmonic trap is related to the average trap frequency by the expression [45]

\[
n_0 = N \bar{\omega}^3 \left( \frac{m}{2 \pi k_B T} \right)^{3/2}
\]

(1.17)

and so the peak phase space density is given by

\[
\rho_0 = n_0 \lambda^3 dB = N \left( \frac{\hbar \bar{\omega}}{k_B T} \right)^3
\]

(1.18)

When eq. 1.15 is inserted into the relation\(^2\) of \( N = N_0 + \int \frac{D(\varepsilon)}{\varepsilon^3 (\varepsilon - \mu - 1)} d\varepsilon \), we find

\[
N = N_0 + \zeta(3) \left( \frac{k_B T}{\hbar \bar{\omega}} \right)^3
\]

(1.19)

\(^2\)The so-called semiclassical approximation assumes that the temperature of the trapped gas is much larger than the spacing between single-particle energy levels, i.e. \( k_B T \gg \hbar \bar{\omega} \). In the limit of \( V \to \infty \) (see eq. 1.6 and eq. 1.7) the energy levels in the uniform free Bose gas are continuous but they become discrete for the trapped gas. So replacing the sum over the single-particle states with an integral must be treated with caution.
where $\zeta(3) \approx 1.202$ is the Riemann zeta function. Requiring that the equality $N_0 = 0$ at $T = T_c$ for Bose-Einstein condensation, we find the critical temperature

$$T_c = 0.9405 \frac{\hbar \omega}{k_B} N^{1/3}.$$ (1.20)

This relation sets the critical line for the observation of BEC. Using eq. 1.19 and eq. 1.20, we obtain the condensate fraction as a function of temperature for the trapped gas,

$$\frac{N_0}{N} = 1 - \left( \frac{T}{T_c} \right)^3.$$ (1.21)

![Figure 1.1: (a) The density of states as a function of energy for a uniform ideal Bose gas (curve 1) and trapped Bose gas in a harmonic potential (curve 2). (b) Condensate fraction $N_0/N$ as a function of the normalized temperature $T/T_c$ for the homogeneous ideal Bose gas (curve 1) and trapped Bose gas in the harmonic potential (curve 2).](image)

In contrast to the homogeneous ideal Bose gas in eq. 1.12, now the temperature dependence of the condensate fraction in the trapped Bose gas $N_0$ scales with $T^3$. Fig. 1.1 (a) shows that the trapped gas has a smaller density of states near very low energy than the uniform free gas. At a given temperature ($T < T_c$) Bose particles in the harmonic trap that can be occupied in excited states are less than the uniform free particles. Thus, as temperature falls below $T_c$, the condensate fraction in the trapped gas increases more rapidly than in the uniform free gas [46], as plotted in Fig. 1.1 (b).

### 1.2.3 Bose condensation with interacting particles

From the threshold of condensation the interaction energy is not negligible any more, even for a dilute system like an alkali gas at very low pressure. In such
1.2. Properties of the Bose gas in a trap

a condition, the interactions are described mostly by binary collisions at low energy characterized by the $s$-wave scattering length $a$. Introducing a mean-field interaction to the Schrödinger equation gives

$$i\hbar \frac{\partial}{\partial t} \Phi(\mathbf{r}, t) = \left( -\frac{\hbar^2 \nabla^2}{2m} + U(\mathbf{r}) + g|\Phi(\mathbf{r}, t)|^2 \right) \Phi(\mathbf{r}, t). \quad (1.22)$$

This is known as the time-dependent Gross-Pitaevskii (GP) equation [47, 48] which has proved an excellent description of condensate behaviour provided that the system is so dilute that $n|a|^3 \ll 1$ [5]. We choose $U(\mathbf{r})$ as the 3D harmonic potential given by eq. 1.13. For the ground state of the dilute Bose gas system we can write the wavefunction as $\Phi(\mathbf{r}, t) = \phi(\mathbf{r}) e^{-i\mu t/\hbar}$, where the spatial wavefunction is normalized to the number of atoms in the condensate, $\int |\phi(\mathbf{r})|^2 d^3r = N_0$ and $\mu = \mu(T_c)$. Entering the ground state wavefunction into eq. 1.22 we obtain a time-independent Gross-Pitaevskii equation

$$\left( -\frac{\hbar^2 \nabla^2}{2m} + U(\mathbf{r}) + g|\phi(\mathbf{r})|^2 \right) \phi(\mathbf{r}) = \mu \phi(\mathbf{r}). \quad (1.23)$$

In the absence of interactions ($g = 0$) this becomes the normal Schrödinger equation for a single particle in the potential $U(\mathbf{r})$ whose eigenvalue is the chemical potential. A very low density condensate can be described in the ideal gas limit where the interactions are ignored and the wavefunction has the Gaussian profile of a harmonic oscillator ground state

$$\phi(\mathbf{r}) = \phi_0 \prod_i e^{-x_i^2/2a_i^2} \quad (1.24)$$

with a harmonic oscillator width $a_i = \sqrt{\hbar/m\omega_i}$.

For a large cloud and high density the quantum kinetic energy term can be neglected in comparison to the interaction term and eq. 1.23 becomes simply

$$\left( U(\mathbf{r}) + g|\phi(\mathbf{r})|^2 \right) \phi(\mathbf{r}) \simeq \mu \phi(\mathbf{r}). \quad (1.25)$$

In this so-called the Thomas-Fermi regime, the atoms fill the trap up to the level of the chemical potential $\mu$, which is the ground state energy of the condensate. The condensate density profile follows the inverted form of the parabolic trapping potential,

$$n(\mathbf{r}) = |\phi(\mathbf{r})|^2 = \frac{1}{g} \left( \mu - \sum_i \frac{1}{2} m\omega_i^2 x_i^2 \right) \quad (1.26)$$

in the region where $\mu > U(\mathbf{r})$, while $|\phi(\mathbf{r})|^2 = 0$ outside this region. The normalization condition of the ground state wavefunction leads to an expression between the chemical potential and the total number of atoms $N$ in the Thomas-Fermi limit,

$$\mu = \frac{\hbar^2}{2a_{ho}} \left( \frac{15Na}{a_{ho}} \right)^{2/5} \quad (1.27)$$
where \( a_{ho} = \sqrt{\frac{\hbar}{m\omega}} \) is the width of the harmonic oscillator. The widths of the condensate are determined by the positions on the \( x \), \( y \), and \( z \) axes at which the density goes to zero and defined by

\[
R_i = \sqrt{\frac{2\mu}{m\omega_i^2}}, \quad i = x, y, z. \tag{1.28}
\]

Thus, eq. 1.26 can be rewritten as

\[
|\phi(r)|^2 = n_0 \left[ 1 - \sum_i \left( \frac{x_i}{R_i} \right)^2 \right] \tag{1.29}
\]

where \( n_0 = \mu/g \) is the peak condensate density.
Chapter 2

Laser cooling and trapping techniques

This chapter presents the properties of $^{87}\text{Rb}$ atoms used for the project together with the relevant physical quantities for magnetic trapping, and gives an overview of the experimental methods for making a BEC, including laser cooling techniques, the magneto-optical trap, magnetic trap, and evaporative cooling.

2.1 The properties of rubidium atoms

2.1.1 Physical properties

Since the alkali elements have odd atomic numbers ($Z$), their isotopes with odd mass number ($A$) are composite bosons composed of an even number of fermions such as protons, neutrons, and electrons. These bosonic alkali atoms have half integer values of nuclear spin. Many cold atom experiments use alkali metal elements since the strong $D$-lines of these atoms are easily accessible with tunable lasers and offer closed cycling transitions suitable for laser cooling [49]. For this work, we use rubidium\(^1\) with mass number 87 which was the isotope used in the first BEC in a dilute atomic vapour. The element consists of 37 electrons, 37 protons, and 50 neutrons. Table 2.1 shows physical properties of naturally abundant isotopes of rubidium; $^{85}\text{Rb}$ and $^{87}\text{Rb}$. At room temperature rubidium is a soft solid with a density of $1.53 \times 10^3 \text{ kg m}^{-3}$ and vapour pressure of $5.23 \times 10^{-5} \text{ Pa}$ at $25 \text{ oC}$ which makes it convenient for vapour cell spectroscopy and laser cooling and trapping experiments.

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\(^1\)Rubidium was first discovered in 1861 by the German chemists R. Bunsen and G. Kirchhoff while spectroscopically analyzing samples of the mineral lepidolite, and named after the two dominant red lines of its spectrum.
Chapter 2. Laser cooling and trapping techniques

<table>
<thead>
<tr>
<th>Properties</th>
<th>$^{85}$Rb</th>
<th>$^{87}$Rb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative natural abundance</td>
<td>72.17(2) (%)</td>
<td>27.83(2) (%)</td>
</tr>
<tr>
<td>Mass</td>
<td>84.911 789 732(14) u</td>
<td>86.909 180 520(15) u</td>
</tr>
<tr>
<td>Nuclear spin</td>
<td>5/2</td>
<td>3/2</td>
</tr>
<tr>
<td>Nuclear lifetime</td>
<td>stable</td>
<td>4.88 × 10^{10} year</td>
</tr>
</tbody>
</table>

Table 2.1: Physical properties of naturally abundant rubidium isotopes [50, 51].

2.1.2 Spectral properties of $^{87}$Rb

The ground electronic configuration of $^{87}$Rb is $5^2S_{1/2}$. The spin-orbit coupling causes the first excited configuration ($5^2P$) to be split into two levels $5^2P_{1/2}$ and $5^2P_{3/2}$ specified by the angular momentum quantum number $J = L + S$. The hyperfine structure arises from the interaction between the nuclear magnetic moment proportional to $\vec{I}$ and the magnetic field created by the electrons which is proportional to their total angular momentum $\vec{J}$. The Hamiltonian for the hyperfine interaction is

$$H_{hf} = A \vec{I} \cdot \vec{J} \quad (2.1)$$

where $A$ is a constant. The appropriate conserved quantity is the total angular momentum $\vec{F} = \vec{I} + \vec{J}$. We express the expectation value of $\vec{I} \cdot \vec{J}$ in the terms of the quantum numbers $I, J,$ and $F$ as $(\vec{I} \cdot \vec{J}) = (1/2)[F(F+1) - I(I+1) - J(J+1)]$. A neutral alkali atom has a single valence electron outside one or more closed shells. This valence electron has no orbital angular momentum, and $S = 1/2$ which combines with the nuclear spin $I$ yields the quantum number of total spin, $F = I \pm 1/2$. The hyperfine energy splitting in the ground state is given by

$$\Delta E_{hf} = \hbar \nu_{hf} = (I + 1/2)A \quad (2.2)$$

For a $^{87}$Rb atom in the ground state ($S = J = 1/2$) with a nuclear spin $I = 3/2$, the ground state splits into two hyperfine levels with the quantum numbers $F = 1$ and $F = 2$ with an energy difference of $\Delta E_{hf} = 2A$. The $^{87}$Rb level structure including the fine structure and hyperfine energy is shown in Fig. 2.1.

For laser cooling we use the hyperfine transition $F = 2 \rightarrow F' = 3$ of the $D_2$ line at a wavelength of $\lambda = 780$ nm, where the unprimed state $F$ refers to the $5^2S_{1/2}$ manifold and the primed state $F'$ refers to the $5^2P_{3/2}$ manifold. The main cooling laser beam excites the $F = 2 \rightarrow F' = 3$ transition which is a cyclic transition because of the electric-dipole selection rule: $\Delta F = 0, \pm 1$. This laser also gives rise to non-resonant excitation to the $F' = 2$ state, from which the atoms may decay to the lower hyperfine state ($F = 1$). Atoms in this level are no longer excited by the cooling light, because the 6.8 GHz energy splitting of the ground states is much bigger than the natural linewidth. Therefore, a separate laser beam (repumping laser) resonant with the $F = 1 \rightarrow F' = 2$ transition is used to bring atoms back
2.1. The properties of rubidium atoms

Figure 2.1: The level structure and laser cooling scheme of $^{87}$Rb [52, 53, 54]. Level spacing is not to scale. The process of cooling and trapping operates on the closed $F = 2 \rightarrow F' = 3$ transition. The repumping utilizes the $F = 1 \rightarrow F' = 2$ transition to put atoms back into the upper hyperfine ground state.
into the cooling cycle. To prepare atoms for loading into a magnetic trap, we use an optical pumping beam. Optical pumping into the $|F = 2, M_F = 2\rangle$ state is achieved by the $\sigma^+$ polarized laser beam resonant with the $F = 2 \rightarrow F' = 2$ transition (see section 4.3). Relevant spectral properties for laser cooling and magnetic trapping of $^{87}$Rb are given in Appendix A.

### 2.1.3 Zeeman energy splitting by an external magnetic field

![Zeeman energy diagram](image)

Figure 2.2: Zeeman energy diagram of the $^{87}$Rb ground state hyperfine structure in an external magnetic field. Magnetic trapping is achievable with the three weak field seeking states of $g_F M_F > 0$ (see section 2.1.4) at low magnetic fields that are indicated as red color curves.

The Zeeman splitting on hyperfine ground states arises from the interaction of the magnetic moment of the electron with an external magnetic field and is described by the Hamiltonian

$$ H = A \vec{I} \cdot \vec{J} + g_J \mu_B \vec{B} \cdot \vec{J}. $$

(2.3)

For a weak field ($\mu_B B \ll A$), the Zeeman effect splits each level into $(2F + 1)$ states labelled by the quantum numbers $M_F$. For the case of $J = 1/2$, the Zeeman energies as a function of the magnetic field are described by the Breit-Rabi formula [55, 56],

$$ E(F_\pm, M_F) = -\frac{\hbar \nu_{hfs}}{2(2I + 1)} - g_I M_F \mu_B B \pm \frac{\hbar \nu_{hfs}}{2} \left( 1 + \frac{4M_F}{(2I + 1)} x + x^2 \right)^{1/2} $$

(2.4)

where $x = (g_I + g_S) \frac{B}{\nu_{hfs}}$, and $\hbar \nu_{hfs}$ is the separation between the hyperfine ground levels $F_+ = I + 1/2$ and $F_- = I - 1/2$ at zero magnetic field. $g_I$ and $g_S$ are
2.1. The properties of rubidium atoms

The nuclear g-factor and electronic spin g-factor respectively. Atomic structure constants of $^{87}$Rb required for the Breit-Rabi diagram are given in Table 2.2. The energy levels for $^{87}$Rb as a function of the magnetic field are plotted in Fig. 2.2. The frequency spacing is 0.7 MHz/G between Zeeman sub-levels in the ground state $^3\!S_{1/2}$ for low magnetic fields.

<table>
<thead>
<tr>
<th>Physical quantity</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nuclear spin</td>
<td>$I$</td>
<td>$3/2$</td>
</tr>
<tr>
<td>Bohr magneton</td>
<td>$\mu_B$</td>
<td>$9.274,008,99(37)\times10^{-24}$ J T$^{-1}$</td>
</tr>
<tr>
<td>Nuclear g-factor</td>
<td>$g_I$</td>
<td>$-0.0009951414(10)$</td>
</tr>
<tr>
<td>Electron spin g-factor</td>
<td>$g_S$</td>
<td>$2.002,319,304,373,7(80)$</td>
</tr>
<tr>
<td>Hyperfine splitting frequency</td>
<td>$\nu_{hfs}$</td>
<td>$6.834,682,610,904,29(9)$ GHz</td>
</tr>
</tbody>
</table>

Table 2.2: Atomic structure constants of $^{87}$Rb used for the Breit-Rabi diagram [50].

2.1.4 Magnetic dipole interaction in a weak external magnetic field

The magnetic force on an atom with magnetic moment $\vec{\mu}$ in an inhomogeneous weak magnetic field $\vec{B}(r)$ is given by

$$\vec{F} = -\nabla U = \nabla (\vec{\mu} \cdot \vec{B}). \quad (2.5)$$

For an atom in the state $|F, M_F\rangle$ the interaction energy $U$ corresponds to the Zeeman energy

$$U(F, M_F) \simeq U(F) + g_F M_F \mu_B B, \quad (2.6)$$

Figure 2.3: Zeeman splitting of the hyperfine ground state of $^{87}$Rb in a local weak magnetic field. Zeeman sublevels with a green circle indicate the weak field seeking states.
where the quantum number $M_F$ is the projection of the total angular momentum $F$ on the magnetic field axis and $U(F)$ is the energy for $B = 0$. In the weak field region, each hyperfine level $F$ splits into its substates as shown in Fig. 2.3. The Landé $g$-factor [55] for each state is given by

$$g_F = g_J \left\{ \frac{F(F+1)+J(J+1)-I(I+1)}{2F(F+1)} \right\} - \frac{g_I \mu_n}{\mu_B} \left\{ \frac{F(F+1)+I(I+1)-J(J+1)}{2F(F+1)} \right\} \quad (2.7)$$

where $\mu_n$ is the nuclear magneton. Since $\frac{\mu_n}{\mu_B} = 1/1836$, the second term can be neglected in most cases [57]. For $F = 2$, the electron spin of $^{87}\text{Rb}$ is aligned parallel to the total spin with positive $g_F$-factor. Consequently the state of the highest energy is $|F = 2, M_F = 2\rangle$ in which the electronic and nuclear spin components have maximum values along the direction of the magnetic field, i.e. the doubly polarized state. On the other hand, for $F = 1$, the electron spin of $^{87}\text{Rb}$ is aligned antiparallel to the total spin giving a negative $g_F$-factor, and so the $|F = 1, M_F = -1\rangle$ state has the highest energy which is called the maximally stretched state [44].

The orientation of the atomic magnetic moment is quantized relative to the direction of the magnetic field. The rate of precession of the magnetic dipole moment about the local magnetic field is described by the Larmor frequency,$$
\omega_{\text{Larmor}} = \frac{\mu B}{\hbar} \quad (2.8)
$$

where $\mu = g_F M_F \mu_B$ is the effective magnetic moment. Once an atom is polarized in one of the Zeeman states, it is driven towards either high or low magnetic field regions depending on the sign of the $g_F$ and $M_F$. As indicated in Fig. 2.2, states of $g_F M_F > 0$ are the weak field seeking states whose energy increases with increasing magnetic field. It has been pointed out by Wing [11] that in a spatial region devoid of charge and electric current, the strength of a static electric or magnetic field can have local minima but no local maxima. This is a version of Earnshaw’s theorem in electrostatics [58] which states that it is impossible to trap a charged particle by using only electrostatic forces. As a result, only atoms in weak field seeking states can be trapped by the static magnetic field that produces a local minimum of the magnetic potential energy $U(F, M_F)$. Configurations for magnetic trapping will be discussed in more details in section 2.6 and 2.7.

### 2.2 Laser cooling techniques

#### 2.2.1 The light forces

Radiation forces on atoms can be classified into two main types; those arising from absorption and refraction respectively. A dissipative force arises from the impulse an atom experiences when it absorbs a photon (or spontaneously emits
2.2. Laser cooling techniques

a fluorescence photon) [59]. Each photon absorbed by an atom transfers momentum to the atom in the direction of photon’s propagation. The recoil from the subsequent spontaneous emission occurs in a random direction and so averages to zero over many absorption-emission cycles. As a result, a net force acts on the atom in the direction of the light propagation, whose magnitude equals the recoil momentum of one photon times the rate of absorption-spontaneous emission cycles. The magnitude of the dissipative force saturates at high intensity. A dipole force is simply the time-averaged force arising from the interaction of the dipole induced by the oscillating electric field of the light with the gradient of electric field magnitude. It derives from an effective potential which is the spatially dependent light shift of the atomic ground state. The magnitude of the force acting on the atom is proportional to the gradient of the beam intensity, while the sign of the force is determined by detuning of the optical frequency below or above the atomic transition. For instance, when a Gaussian laser beam is tuned below resonance frequency, the light shift of the atomic ground state in the beam is negative. This light shift leads to an attractive force on the atoms towards the maximum intensity and by making the detuning large enough the heating effect of spontaneous emission can be made negligible.

In the application of these forces to trapping of atoms, it was initially thought to be limited by the optical Earnshaw theorem [58, 60] which states that it is impossible to trap a small dielectric particle at a point of stable equilibrium in free space by using only the scattering force of radiation pressure. Pritchard originally suggested that the spontaneous light force can be used to trap neutral atoms by using the internal degrees of freedom of the atoms [60]. In 1987 Raab et al. demonstrated a trap configuration with quadrupole field gradient and three retro-reflected laser beams propagating along orthogonal axes [10] (see section 2.4).

2.2.2 Doppler cooling

The process called Doppler cooling is based on the atomic velocity dependence of the radiation pressure due to the Doppler effect. This cooling scheme was first proposed by Hänsch and Schawlow [61] for free atoms. Atoms moving with velocity \( \vec{v} \) towards a laser source are excited if the laser frequency \( \omega \) is slightly detuned to the red with respect to the optical transition \( \omega_0 \) (\( \delta = \omega - \omega_0 < 0 \)). When a photon with the wave vector \( \vec{k} \) is absorbed, the momentum transfer is \( \hbar \vec{k} \) in the direction of the beam, and spontaneous emission averages to zero as stated above. The photon's scattering rate is given by

\[
R = \frac{\Gamma}{2} \frac{I/I_{sat}}{1 + I/I_{sat} + \left[2(\delta - \vec{k} \cdot \vec{v})/\Gamma\right]^2}
\]  

(2.9)

where \( \Gamma \) is the decay rate of the excited atomic state, \( \vec{v} \) is the velocity of the atom, \( I \) is the laser intensity, and \( I_{sat} \) is the saturation intensity for the transition from
ground state to the excited state. The mean damping force on the atom in the direction of the light propagation. The magnitude of the radiative force exerted by the laser beam is equal to the photon’s scattering rate multiplied by the momentum transfer $\hbar k$ per absorption-emission cycle [62], i.e.

$$F^s(\delta - \vec{k} \cdot \vec{v}) = \frac{\hbar k \Gamma}{2} \frac{I/I_{\text{sat}}}{1 + I/I_{\text{sat}} + [2(\delta - \vec{k} \cdot \vec{v})/\Gamma]^2}. \quad (2.10)$$

In an arrangement of two identical counter-propagating beams along the direction ($z$ direction) as illustrated in Fig. 2.4 (a), for an atom at rest the radiation pressures from two laser beams balance each other and there is no force. However for an atom with velocity $\vec{v}$, the co-propagating beam of the wave vector $\vec{k}$ ($\vec{k} \cdot \vec{v} > 0$) with the atom has an apparent frequency of $\delta - kv_z$ by the Doppler effect which is moved away from the resonance. The counter-propagating beam ($\vec{k} \cdot \vec{v} < 0$) has an apparent frequency of $\delta + kv_z$ which is closer to resonance. Thus the atom preferentially absorbs photons from the counter-propagating beam, resulting in an imbalance of the scattering forces. In the case of low beam intensity, the net force on the atom from the two laser beams is

$$F_z = F_z(\delta - kv_z) - F_z(\delta + kv_z) \simeq \frac{8\hbar k^2 v_z \delta}{\Gamma} \frac{I/I_{\text{sat}}}{1 + I/I_{\text{sat}} + (2\delta/\Gamma)^2}. \quad (2.11)$$

For red-detuned frequency ($\delta < 0$), the net force varies linearly with velocity, $F_z = -\alpha v_z$ where $\alpha$ is the friction coefficient given by

$$\alpha = -\frac{8\hbar k^2 \delta}{\Gamma} \frac{I/I_{\text{sat}}}{1 + I/I_{\text{sat}} + (2\delta/\Gamma)^2}. \quad (2.12)$$

For two counter-propagating laser beams of $I/I_{\text{sat}} = 1/4$ and wavelength of $\lambda = 780$ nm, the sum of the two forces that a $^{87}$Rb atom feels is plotted as a function of velocity in Fig. 2.4 for $\delta = -\Gamma/2$ and $\delta = -\Gamma$. The net force has a negative gradient near zero velocity and maxima where $kv_z = \pm \delta$. The Doppler cooling can be extended to other directions by additional standing waves along $x$ and $y$ directions. This is called the optical molasses since it acts like a viscous fluid on atomic motion [63].

While spontaneous emission allows the dissipation of energy by emission of fluorescence photons, the effect of the random momentum recoil of the atoms after spontaneous emission induces an increase in the kinetic energy. For the two-level atom model, we can obtain the equilibrium temperature by equating the cooling rate due to the damping force with the heating rate due to momentum diffusion. Diffusion is characterized by a momentum diffusion constant given by $D_p = \frac{1}{2} \frac{d}{dt} < p_z^2 > = \frac{(2\hbar k)^2 R}{2}$ and in the limit where $|kv_z| \ll \delta$,

$$D_p \simeq \frac{(\hbar k)^2 \Gamma I/I_{\text{sat}}}{1 + I/I_{\text{sat}} + (2\delta/\Gamma)^2}. \quad (2.13)$$
2.2. Laser cooling techniques

Figure 2.4: Doppler cooling scheme. (a) The Doppler cooling along a given direction (z direction) is obtained by using the superposition of two counter-propagating, red-detuned beams along the same direction. The velocity-dependent force is plotted for $\delta = -\Gamma/2$ in (b) and $\delta = -\Gamma$ in (c), assuming $I/I_{\text{sat}} = 0.25$ and $\lambda = 780$ nm, respectively. The dotted traces represent the force from each beam and the solid curve is the sum of the two forces. The net force displays a viscous damping over a restricted velocity range.
Chapter 2. Laser cooling and trapping techniques

From the relation of \( k_B T/2 = m v_z^2/2 = D_p/2\alpha \), the equilibrium temperature [64] is given by

\[
k_B T = -\frac{\hbar \Gamma^2}{8\delta} \left[ 1 + \frac{I}{I_{\text{sat}}} + \left( \frac{2\delta}{\Gamma} \right)^2 \right]. \tag{2.14}
\]

For \( I \ll I_{\text{sat}} \), the minimum achievable temperature called the Doppler cooling limit is given by \( T_D = \hbar \Gamma/2k_B \) when \( \delta = -\Gamma/2 \) [65]. For example, it corresponds to \( T_D = 240 \, \mu K \) for sodium atoms, \( T_D = 146 \, \mu K \) for rubidium atoms, and \( T_D = 120 \, \mu K \) for cesium atoms, respectively.

### 2.3 Sub-Doppler cooling

In 1988, precise temperature measurements of optical molasses found the temperature to be around 43 \( \mu K \) which is far below the predicted Doppler cooling limit of 240 \( \mu K \) [66]. The measurements of temperatures below \( T_D \) were experimental evidence of very efficient cooling mechanisms that required a new model for their explanation. The previous theoretical models dealt only with two level atoms, but sub-Doppler cooling arises from the multiplicity of sublevels of the ground state and the different light shifts of the sublevels. A second key point is that the polarization of the laser light varies on an optical wavelength scale. Thus the position dependent light shifts of the sublevels appear and behave as effective potentials for a moving atom. As in Doppler cooling, the equilibrium is when the energy loss by the new cooling process is balanced by the recoil energy obtained from spontaneous emission. The minimum temperature is set by the recoil limit \( E_R = \hbar^2 k^2/2m \) where \( m \) is mass of the atom, \( i.e. \) the mean increase of the atomic kinetic energy after the spontaneous emission of a single photon.

Dalibard and Cohen-Tannoudji developed a theoretical model of the sub-Doppler cooling mechanism. There are two cases; the linear \( \perp \) linear configuration and the \( \sigma^+ - \sigma^- \) configuration. The \( \sigma^+ - \sigma^- \) configuration uses two counter-propagating, oppositely circularly polarized, and red-detuned laser beams. The total electric field is linearly polarized but the direction of the polarization rotates to form a helix with a period of \( \lambda/2 \) along the propagation axis of the beam pair [64]. An atom moving with speed \( v \) along the beam experiences the rotating polarization. In the rotating frame that follows the polarization axis an extra inertial term is added to the Hamiltonian of the atom [67]. The perturbation term causes an asymmetrical mixing of the ground state sub-levels in a way that preferentially promotes light absorption from the laser beam towards which the atom moves. Thus, an imbalance appears in the absorption rate of photons from the two counter-propagating beams, causing a force that opposes the atoms motion. This model provides the equilibrium temperature for the sub-Doppler cooling mechanism, which in the limit
Sub-Doppler cooling

\[ |\delta| \gg \Gamma/2 \] is given by

\[ k_B T \propto \frac{\hbar \Omega^2}{|\delta|}. \tag{2.15} \]

where \( \Omega \) is the Rabi frequency for the transition between excited and ground states. A detailed discussion of the sub-Doppler cooling mechanism in the \( \sigma^+ - \sigma^- \) configuration is given in references [67, 68].

2.3.1 Sisyphus cooling

For the two counter-propagating plane waves along the \( z \)-axis with orthogonally linear polarizations (linear \( \perp \) linear configuration), we consider the cooling mechanism for a \( J_g = 1/2 \rightarrow J_e = 3/2 \) atomic transition as shown in Fig. 2.5. The internal atomic cooling dynamics involves a polarization gradient and a given atom shows a tendency to ascend more often than descend in its potential energy diagram because the spatially modulated light shifts and the optical pumping rate are correlated. The electric field resulting from the superposition of two counter-propagating light beams with respective \( \hat{x} \) and \( \hat{y} \) polarization, respective phases of 0 and \(-\pi/2\) at \( z = 0 \), with amplitude \( E_0 \) [68] can be written as

\[
E(z, t) = E_0(\hat{x}e^{ikz} - i\hat{y}e^{-ikz})e^{-i\omegaLt} + c.c.
\]

\[
= \sqrt{2}E_0\left[ \cos(kz)\hat{x} - i\hat{y}\sqrt{2} + i\sin(kz)\hat{x} + i\hat{y}\sqrt{2} \right] e^{-i\omegaLt} + c.c.
\tag{2.16}
\]

Therefore, the total electric field along the \( z \) direction is the superposition of two fields with \( \sigma^+ \) and \( \sigma^- \) polarization and amplitudes \( \sqrt{2}E_0 \cos(kz) \) and \( \sqrt{2}E_0 \sin(kz) \), respectively. The polarization of the resulting field alternates from linear to circular polarization with a period of \( \lambda/8 \). Between the circularly polarized positions with a period of \( \lambda/4 \), the electric field forms linear or elliptical polarization. For examples, at \( z = 0 \) the radiation field is circularly \( \sigma^- \) polarized, and it is linear polarized along \( \frac{1}{\sqrt{2}}(\hat{x} - \hat{y}) \) at \( z = \lambda/8 \) as shown in Fig. 2.5 (c). The polarization gradient causes periodic modulation of the light shift of the two ground substates along the standing wave. When \( |\delta| \gg \Gamma/2 \), the light shift is proportional to \( \hbar \Omega^2 / |\delta| \), where \( \Omega \) is the optical Rabi frequency for the transition, \( \Gamma \) the natural width of the excited state and \( \delta \) the detuning [68]. The Rabi frequency is proportional to the electric field amplitude of the light. The magnitude of the light shift also depends on the strength of the transition between the two levels which is characterized by the relevant Clebsch-Gordan coefficient. Hence, in the position of circularly polarized light field, the substates have different light shifts due to different coupling strengths. The interference pattern of the two lasers leads to a periodic modulation of the energies of the ground state levels through the AC stark effect. For the \( J = 1/2 \) ground state, the energies of the \( |g, M_J = \pm 1/2\rangle \) sublevels vary periodically and are 180° out of phase with each other as shown in Fig. 2.5 (b). The detuning here is
Figure 2.5: Sisyphus cooling mechanism for a $J = 1/2 \rightarrow J = 3/2$ transition. (a) Atomic level structure of the transition. The squared Clebsch-Gordan coefficients are indicated along the lines joining various transitions. (b) The optical potential experienced by sublevels. The arrows indicate the path followed by the atom being cooled. The cooling mechanism maximizes its effect when the atom travels a distance of the order of $\lambda/4$ in the mean time lag $\tau_p$ and then undergoes an optical pumping. Thus, the net effect is that the atom moving in a potential jumps preferentially from the top of one potential hill to the bottom of the other potential valley at the expense of its kinetic energy. (c) The polarization of the resulting electric field in counter-propagating lin $\perp$ lin laser configuration. (d) Relative light shifts for left circular, linear, and right circular polarization.
negative so that the light shifts are negative. At a position of $\sigma^+$ polarization, the light shift of $|g, M_J = +1/2\rangle$ is three times larger than that of $|g, M_J = -1/2\rangle$. Similarly, for the $\sigma^-$ polarized position the previous conclusion is reversed. In between where there is linear polarization, both sublevels undergo the same light shifts, which are 2/3 times the maximum light shift of circular polarization. In this way, the spatially periodic variation of polarization causes the shift of the two ground substates to vary as a function of position as shown in Fig. 2.5 (b). As long as the atom stays in the same state, it moves up and down potential hills and valleys with no change of its total energy. However, for a red-detuning of the laser beam, atoms undergo an optical pumping process at the top of the potential hills; selection rules allow the atom to fall back to the same sublevel, or to the lower one with the latter process having higher probability. Hence energy is lost and this is called the Sisyphus effect. In the linear $\perp$ linear polarization configuration there is a strong correlation between the spatial modulation of light shifts and the optical pumping rate as illustrated in Fig. 2.5.

Dissipation of energy in this atom-laser configuration originates from the spontaneously emitted anti-Stokes Raman photons. To understand the damping effect, consider an atom moving with velocity $v$ to the right, for instance, starting from the position $A$ in the state of $|g, M_J = +1/2\rangle$. Suppose that the optical pumping takes a finite time $\tau_p$ between the two ground sublevels and the atom remaining on the same substate travels over a distance of the order of $\lambda/4$ during the mean time lag $\tau_p$. The characteristic time $\tau_p$ for pumping is of order of the time required for the radiation field to excite an atom. If the optical pumping time $\tau_p$ from $|g, M_J = +1/2\rangle$ to $|g, M_J = -1/2\rangle$ is long enough, the atom can climb along a hill of the potential $U_+ (z)$ and arrive at the top of the potential (at the position $B$ in Fig. 2.5 (b)) before changing its internal state. During the journey, the total external energy $E = mv^2/2 + U_+ (z)$ remains constant but the atom converts part of its kinetic energy into potential energy. In the vicinity of the maximum of the potential $U_+ (z)$, the laser light becomes $\sigma^-$ polarized and the atom undergoes the maximum probability to be optically pumped into the $|e, M_J = -1/2\rangle$ sublevel. The atom of the $|g, M_J = +1/2\rangle$ state absorbs a $\sigma^-$ polarized laser photon with energy $\hbar \omega_a$ that transfers it into an excited state of $|e, M_J = -1/2\rangle$. Then, the excited atom will preferentially decay down into the $|g, M_J = -1/2\rangle$ ground state, i.e. the bottom of a valley of the potential $U_-(z)$ in the position $C$, by spontaneously emitting a photon of higher energy $\hbar \omega_e$ ($\hbar \omega_e > \hbar \omega_a$). In this optical pumping process, energy is taken out of the atom-laser system because the energy of the absorbed light is lower than that of the fluorescence light by approximately $U_0$, i.e. the height of the potential wells. Atoms keep climbing potential hills until their energy becomes too small to climb the next top of the potential hill. By this argument the minimum atomic kinetic energy is of the order of the modulation depth $U_0$ in the light shift, i.e. the limit of Sisyphus cooling is $k_B T \approx U_0$. 
2.4 Magneto-optical trap

2.4.1 Magneto-optical trap (MOT)

In optical molasses atomic gases are cooled. However, they diffuse out of the region where the beams intersect and so this is not a trap. To trap actually atoms in real space a spatially dependent radiative force is required. A position-dependent force is created by a combination of appropriately polarized laser beams and an inhomogeneous magnetic field in the widely used magneto-optical trap (MOT) [10]. As shown in Fig. 2.6 (a), a typical MOT consists of six counter-propagating laser beams of opposite circular polarizations with each beam slightly detuned to the red from the atomic resonance frequency, and a pair of anti-Helmholtz coils that produce a spatially-varying magnetic field. The quadrupole magnetic field plotted in Fig. 2.6 (b) is calculated from the parameters used for our pyramidal MOT (see section 3.5.1). The diagram in Fig. 2.7 illustrates the operation of the MOT along the z direction. When atoms are subject to a gradient of magnetic field along the laser axis, the Zeeman effect leads to an imbalance in the radiation force in the following way.

Consider a two-level atom with a $J_g = 0 \rightarrow J_e = 1$ transition in an external magnetic field which is zero at the trap centre ($z = 0$) and has a uniform gradient ($b'$). The magnetic field is directed along the laser beam axis (z-axis) and defines the quantization direction. The excited level of the atom has three Zeeman substates $|J_e = 1, M_J = -1\rangle$, $|J_e = 1, M_J = 0\rangle$, and $|J_e = 1, M_J = +1\rangle$. Two counter-propagating laser beams of opposite circular polarization have a frequency detuning slightly below that of the atomic resonance ($\delta = \omega - \omega_0 < 0$). The $\sigma^+$ polarization drives only $\Delta M_J = +1$ transitions while the $\sigma^-$ polarization induces $\Delta M_J = -1$ transitions. These sublevels undergo Zeeman shifts such that the $M_J = 1$ ($M_J = -1$) magnetic substate has higher (lower) energy at position $z > 0$. For an atom displaced from the trap centre to positive $z$, the $\Delta M_J = -1$ transition moves closer to resonance with the light frequency because of the Zeeman shift of $M_J = -1$. This increases the absorption rate of photons from the $\sigma^-$ polarized beam, and produces a force pushing the atom back towards the centre of the trap. This is reversed on the other side of the trap, i.e. for an atom with negative displacement along z direction, more light scatters from the $\sigma^+$ beam, because the $M_J = +1$ state is shifted downward, closer to resonance with the light. Thus, an atom away from the centre where the two sublevels cross experiences an imbalance between the radiation forces from the opposing beams arising from the position-dependent Zeeman shifts, leading to a restoring force towards the centre of the trap.

To describe mathematically the MOT, first consider the frequency detuning written as

$$\delta_\pm = (\omega \pm kv_z) - (\omega_0 \pm \beta z) \quad (2.17)$$
2.4. Magneto-optical trap

Figure 2.6: (a) A schematic setup of a MOT trap with 3 pairs of counter-propagating laser beams and the two coils to create a quadrupole field. (b) The magnetic field along the symmetry axis is calculated for a current of \( I = 390 \, \text{A} \) in the quadrupole coils where each coil radius is 59 mm and the distance between the quadrupole coils is 50 mm. The geometry matches that of our pyramidal MOT.

where \( kv \) is the term of the Doppler shift which is added or subtracted from the laser frequency \( \omega_L \), and \( \beta z = (g_e M_e - g_g M_g) \mu_B \theta' z / \hbar \) is the Zeeman shift of the atomic transition at displacement \( z \). The Doppler shift and the Zeeman shift both have opposite signs for opposite beams and this is crucial for trapping. The total radiative pressure force on an atom is \( F_{\text{MOT}} = F_{\sigma^+} + F_{\sigma^-} \), which can be expanded for small velocities and small Zeeman shifts, leading to

\[
F_{\text{MOT}} = F(\delta - kv_z - \beta z) - F(\delta + kv_z + \beta z) \quad (2.18)
\]

\[
\approx -\left(2k \frac{\partial F}{\partial \delta}\right) v_z - \left(2\beta \frac{\partial F}{\partial \delta}\right) z.
\]

Both the position-dependent and velocity-dependent forces arise from an imbalance of radiation forces. Atoms that enter the region of intersection of the laser beams
are slowed and pushed towards the trap centre. Physically, this can be understood simply; for red frequency detuning $\delta < 0$, the imbalance in the radiation force caused by the Zeeman shift leads to a restoring force pushing the atoms toward the centre of the trap, as well as a velocity-dependent damping force. The atomic motion is governed by the damped harmonic oscillator equation with mass $m$, i.e.

$$m\ddot{z} = -\alpha \dot{z} - Kz$$  \hspace{1cm} (2.19)$$

with the associated friction coefficient $\alpha = 2k\frac{\partial F}{\partial \delta}$ and spring constant $K = m\omega_{\text{MOT}}^2 = 2\beta\frac{\partial F}{\partial \delta}$.

Typically a MOT operates in the over-damped regime especially when the higher damping produced by sub-Doppler cooling mechanism is taken into account. The scheme can be extended to three dimensions, taking into account that the quadrupole field gradient in the $z$ direction has twice the gradient in the $x$ or $y$ direction so that the spring constants are $K_z = 2K_x = 2K_y$. This makes the MOT anisotropic since the restoring force along the $z$-axis of the quadrupole is twice the restoring force in the $x$-$y$ plane. The 3-D configuration of the MOT using three orthogonal pairs of counter-propagating, circularly polarized laser beams is illustrated in Fig. 2.6 (a). The three orthogonal pairs intersect at the centre of the quadrupole field. Note that the laser beams propagating along $z$ and perpendicularly to $z$ have opposite helicities due to the different signs in field gradients along the $z$ and $x$-$y$ plane.
2.4.2 Fluorescence measurement from the MOT

Figure 2.8: Scheme to measure the fluorescence signal from the cloud of atoms.

The number $N$ of $^{87}$Rb atoms trapped in a MOT can be estimated by measuring the power $P$ of the fluorescence, emitted from the atoms, that falls on a photodiode. This power is related to parameters of the MOT [69, 70] and given by the relationship

$$P = N \frac{\Omega_S \hbar \omega_L \Gamma}{4\pi} \frac{0.73I/I_{sat}}{2\left(1 + 0.73I/I_{sat} + 4\delta^2/\Gamma^2\right)}$$

(2.20)

where $\Omega_S$ is the solid angle subtended by the lens, $\omega_L$ the laser frequency, $I$ the beam intensity, $I_{sat}$ saturation intensity, $\delta$ the laser detuning from resonance, and $\Gamma$ the natural linewidth, respectively. The solid angle subtended by the collection lens is included since the power incident on the photodiode is a fraction of the actual power radiated by the atoms. A photodiode signal $S$ [Volt] is given by

$$S = \rho_{pd} R P$$

(2.21)

where $\rho_{pd}$ is the responsivity, or the current produced for a given power incident on the photodiode and $R$ the load resistance. Thus, the number of atoms $N$ is given as

$$N = \frac{8\pi S}{\rho_{pd} \Omega_S \hbar \omega_L \Gamma} \frac{(1 + 0.73I/I_{sat} + 4\delta^2/\Gamma^2)}{0.73I/I_{sat}}.$$  

(2.22)

When $d$ is the distance between the position of trapped atoms and the collection lens of radius $r$ as illustrated in Fig. 2.8, the solid angle is equal to $\pi r^2/d^2$.

In dilute gases, the phenomenological equation for the evolution of the number of the trapped atoms during the loading of the MOT is given by [71]

$$\frac{dN}{dt} = R_L - \gamma N - \frac{\beta N^2}{V_{eff}}.$$

(2.23)

The first term $R_L$ of eq. 2.23 indicates the loading rate at which slow atoms are captured by the trap in the trapping volume during loading. The second term of eq. 2.23 represents the one-body loss rate due to collisions between trapped atoms
and background gas and its constant $\gamma$ is the inverse of the lifetime $\tau$ of the trap, i.e. $\gamma = 1/\tau$. The third term of eq. 2.23 describes the two-body losses resulting from collisions between the trapped atoms. Here $\beta$ is the two-body decay rate constant and $V_{\text{eff}}$ is the effective volume of the trap. In the presence of a loading rate ($R_L \neq 0$) with the initial condition of $N(t = 0) = 0$, the solution of eq. 2.23 is

$$N(t) = N_s \frac{1 - e^{-t/\tau_0}}{1 + ae^{-t/\tau_0}}$$

(2.24)

where $N_s$ is the steady state value of the atom number, $a = N_s^2 \beta/R_L V_{\text{eff}}$, and $\tau_0 = \tau/\sqrt{1 + (4\beta R_L \tau^2/V_{\text{eff}})}$. From the steady state solution for eq. 2.23 we find a relation for the two-body decay rate constant,

$$\beta = \frac{V_{\text{eff}}}{N_s^2} \left( R_L - \frac{N_s}{\tau} \right).$$

(2.25)

In the absence of a loading source ($R_L = 0$) and assuming that the initial number of atoms is equal to the steady state value $N_s$, the decay behaviour of $N$ is given by

$$N(t) = N_s \frac{e^{-t/\tau_0}}{1 + b(1 - e^{-t/\tau_0})}$$

(2.26)

with $b = N_s \beta \tau / V_{\text{eff}}$.

For sufficiently low density, the two-body losses term can be neglected and eq. 2.23 becomes simply $\frac{dN}{dt} = R_L - \gamma N$ and its solution is given by

$$N(t) = \frac{R_L}{\gamma} (1 - e^{-\gamma t})$$

(2.27)

where we assume a loss coefficient $\gamma$ independent of trapped atoms [64]. Thus the steady state number of trapped atoms in the MOT is caused by a balance between the loading and loss rates. From measuring the fluorescence of the trapped atoms as a function of time during loading, we can obtain the filling rate $R_L$ that is characterized by the initial slope of the loading curve, and the loss coefficient $\gamma$ that is obtained by the inverse of the curve’s time constant. The quantity $N_s$ can also be measured from the number of trapped atoms observed in steady state.

### 2.5 Optical dipole trap

The radiative force used for laser cooling is described in section 2.2 and 2.3. The other main kind of radiation force, called the optical dipole force [72], comes from the potential that an atom feels when the oscillating electric dipole moment of the atom induced by the oscillating electric field of the laser light interacts with the field. Two important quantities for optical dipole traps are the depth of the
2.5. Optical dipole trap

Figure 2.9: Schematic of a dipole trap formed by a focused Gaussian laser beam with a red frequency detuning. Since the potential energy of atoms is lowered in the presence of a strong light intensity, the trap creates weak axial and strong radial confinement.

potential $U_{\text{dip}}$ resulting from light shift of atomic ground state, and the scattering rate $\Gamma_{\text{sc}}$ in photons per second. The expressions for these quantities are derived in reference [72]. In terms of the decay rate of the excited state $\Gamma$ they can be expressed as

$$U_{\text{dip}}(r) = \frac{3\pi c^2 \Gamma}{2\omega_0^3 \delta} I(r), \quad (2.28)$$
$$\Gamma_{\text{sc}}(r) = \frac{3\pi c^2}{2\hbar \omega_0^3} \left( \frac{\Gamma}{\delta} \right)^2 I(r) \quad (2.29)$$

where $\delta = \omega - \omega_0$ is the frequency detuning of the laser from the frequency of the optical transition $\omega_0$, and $I(r)$ is the intensity of the laser beam. The dipole potential is attractive for red detuning ($\delta < 0$) and repulsive for blue detuning ($\delta > 0$). Atoms are attracted towards or repelled from an intensity maximum by controlling the values of the detuning. In the limit $\Gamma \ll \delta$, the ratio of the scattering rate to the optical potential vanishes.

The simplest example of optical dipole traps for cold atoms is provided by a strongly focused beam formed by the TEM$_{00}$ Gaussian mode, detuned far below the atomic resonance as illustrated in Fig. 2.9. For a total power $P$ of a Gaussian beam propagating along the $z$-direction with its focus at the origin of coordinates,
the beam intensity profile is given by
\[ I(r, z) = \frac{2P}{\pi w^2(z)} e^{-2r^2/w^2(z)} \]  
(2.30)

where \( w(z) = w_0 \sqrt{1 + (z/z_R)^2} \) is the 1/e\(^2\) beam radius. This radius is characterized by a minimum radius \( w_0 \) which is called the beam waist. The beam width increases with \( z \) on the scale of the Rayleigh length \( z_R = \pi w_0^2/\lambda \). The peak intensity is given by \( I_0 = 2P/\pi w_0^2 \). For a red-detuned laser beam, since the atoms are attracted towards the regions of maximum laser intensity, a potential depth minimum occurs at the intensity maximum generated by the focussed beam. The trap depth is defined as \( U_0 = |U(r = 0, z = 0)| \) and linearly proportional to the beam intensity. Expanding around the position of maximum intensity leads to a harmonic potential
\[ U_{dip}(r, z) \approx -U_0 \left[ 1 - 2 \left( \frac{r}{w_0} \right)^2 - \left( \frac{z}{z_R} \right)^2 \right]. \]  
(2.31)

In this cylindrically symmetric dipole trap, atoms have a radial trap frequency \( \omega_r = (4U_0/mw_0^2)^{1/2} \) and axial trap frequency \( \omega_z = (2U_0/mz_R^2)^{1/2} \). By using two counter-propagating laser beams of the same frequency and polarization one can create a 1-dimensional periodic potential given by
\[ U_{SW}(r, z) \approx -U_0 \cos^2(kz) \left[ 1 - 2 \left( \frac{r}{w_0} \right)^2 - \left( \frac{z}{z_R} \right)^2 \right] \]  
(2.32)

where \( k = 2\pi/\lambda \) is the absolute value of the wave vector of the laser light. The potential depth \( U_0 \) of the optical lattice has four times that of each individual beam of the same power and waist size, because of the constructive interference between the electric fields of the two counter-propagating laser beams. The modulation in the \( z \) direction gives a high intensity gradient. Keeping only the second order terms in \( r \) and \( z \), the trapping frequencies of the standing wave potential are obtained as \( \omega_r = (4U_0/mw_0^2)^{1/2} \) and \( \omega_z = k(2U_0/m)^{1/2} \). For the red-detuned frequency, atoms are confined at the anti-nodes of the standing wave. Using three pairs of such beams propagating in mutually orthogonal directions produces a three-dimensional optical lattice.

### 2.6 Magnetic fields of circular current loops

#### 2.6.1 Exact B field expressions for a circular current loop

Each infinitesimal current element of a wire makes a contribution to the magnetic field at the observation point \( r \). The relationship between the magnetic field and its source current element is given by the Biot-Savart Law [73],
\[ dB = \frac{\mu_0 l d\mathbf{l} \times \hat{r}}{4\pi |\mathbf{r}|^2} \]  
(3.33)
2.6. Magnetic fields of circular current loops

where $dl$ is the infinitesimal length vector of the current source element carrying electric current $I$ and $\hat{r}$ is the unit vector to specify the direction of the displacement vector $r$ from the current source segment to the field point. For a vacuum, $\mu_0 = \mu = 4\pi \times 10^{-7}$ N A$^{-2}$ in SI units. For a single circular coil of radius $R$ perpendicular to the $z$-axis and centred at $z = A$, the magnetic field in cylindrical polar coordinates is given by [15]

$$B_\phi = 0,$$

$$B_z = \frac{\mu I}{2\pi \{(R + \rho)^2 + (z - A)^2\}^{1/2}} \left\{ K(k^2) + \frac{R^2 - \rho^2 - (z - A)^2}{(R - \rho)^2 + (z - A)^2} E(k^2) \right\},$$

$$B_\rho = \frac{\mu I (z - A)}{2\pi \rho \{(R + \rho)^2 + (z - A)^2\}^{1/2}} \left\{ -K(k^2) + \frac{R^2 + \rho^2 + (z - A)^2}{(R - \rho)^2 + (z - A)^2} E(k^2) \right\},$$

where $K(k^2)$ and $E(k^2)$ are the complete elliptic integrals for the first and second kind respectively [74] with the argument $k^2 = \frac{4R\rho}{(R+\rho)^2+(z-A)^2}$, and $I$ is the current of the coil. In this section, $I$ represents the total current in A-turns, $I = NI_0$ where $N$ is the number of turns in the electromagnets and $I_0$ is the actual current per turn. Near the origin, we can expand the field components in eq. 2.34 along $z$ and $\rho$ as a power series to the third order;

$$B_\phi = 0,$$

$$B_z = \frac{\mu IR^2}{2\{(R^2 + A^2)^{3/2}\}} + \frac{3\mu IAR^2}{2\{(R^2 + A^2)^{5/2}\}} z + \frac{3\mu I R^2(4A^2 - R^2)}{4\{(R^2 + A^2)^{7/2}\}} (z^2 - \rho^2/2)$$

$$+ \frac{15\mu I R^2(4A^2 - 3R^2)}{48\{(R^2 + A^2)^{9/2}\}} (4z^3 - 6z\rho^2) + \cdots,$$

$$B_\rho = -\frac{3\mu I AR^2}{4\{(R^2 + A^2)^{5/2}\}} \rho - \frac{3\mu I R^2(4A^2 - R^2)}{4\{(R^2 + A^2)^{7/2}\}} z\rho + \frac{15\mu I R^2(4A^2 - 3R^2)}{32\{(R^2 + A^2)^{9/2}\}} (\rho^3 - 4\rho z^2) + \cdots.$$

These equations are needed to estimate numerically the total magnetic field of a thick circular coil by summing the fields produced by many single loops which make up the large coil.

2.6.2 Helmholtz coils

A standard technique for generating a uniform magnetic field is to use a pair of circular coaxial coils with equal currents flowing in the same sense. The magnetic field on the axis of a circular current loop of radius $R$ carrying a steady current $I$ is

$$B(z) = \frac{\mu_0 I R^2}{2(R^2 + z^2)^{3/2}}$$
at a distance \( z \) above the centre of the coil. We can produce a nearly uniform field near the point midway between them \( (z = 0) \) by using two such loops with a distance \( d \) apart. The field as a function of \( z \) is

\[
B(z) = \frac{\mu_0 IR^2}{2} \left[ \frac{1}{\{R^2 + (z - \frac{d}{2})^2\}^{3/2}} + \frac{1}{\{R^2 + (z + \frac{d}{2})^2\}^{3/2}} \right],
\]

(2.37)

and its first order derivative \( \frac{dB(z)}{dz} \) and third order derivative \( \frac{d^3B(z)}{dz^3} \) vanish at the centre. If the separation of two coils is equal to the radius \( (d = R) \), its second order derivative \( \frac{d^2B(z)}{dz^2} \) is zero at the centre. This gives a uniform field over a wide region around the centre where the magnetic field is \( B(0) = \frac{8\mu_0 I}{5\pi^2 R} \). This is known as a Helmholtz arrangement. For BEC experiments, the Helmholtz configuration is used for bias coils that provide a uniform axial field at the centre of an Ioffe-Prichard trap and adjust the offset magnetic field of the trap.

### 2.6.3 Anti-Helmholtz coils

For trapping a cloud in a MOT system, we need a locally uniform field gradient to provide the spatially dependent Zeeman shift. To do this, one simply reverses the current in one of the Helmholtz coils. Such coils were used in the first experimental observation of magnetically trapped neutral atoms by Migdall et al. [12] in 1985. Alkali metal MOTs have field gradients of order 10 G/cm along the symmetry direction. There is an anti-Helmholtz condition \( d = \sqrt{3}R \) for which the third order derivative of its magnetic field \( \frac{d^3B(z)}{dz^3} \) vanishes at the origin. Typically, one or more pairs of anti-Helmholtz coils are wrapped around the vacuum cell in which atoms are trapped and the variation of their size depends on the experimental setup. A calculation of the magnetic field \( B \) along the \( z \)-axis (the axis of symmetry) of a pair of anti-Helmholtz coils yields

\[
B(z) = \frac{\mu_0 IR^2}{2} \left[ \frac{1}{\{R^2 + (z - \frac{d}{2})^2\}^{3/2}} - \frac{1}{\{R^2 + (z + \frac{d}{2})^2\}^{3/2}} \right]
\]

(2.38)

The current in each coil is flowing in opposite directions with respect to one another giving rise to the minus sign between the two terms. From eq. 2.34, the field components of anti-Helmholtz coils near the origin to the third order are written as

\[
B_z = \frac{3\mu_0 dR^2}{2\{R^2 + (d/2)^2\}^{5/2}} + \frac{15\mu_0 IR^2(d^2 - 3R^2)}{24\{R^2 + (d/2)^2\}^{9/2}}(4z^3 - 6z\rho^2) + \cdots,
\]

(2.39)

\[
B_\rho = -\frac{3\mu_0 dR^2}{4\{R^2 + (d/2)^2\}^{5/2}\rho} + \frac{15\mu_0 IR^2(d^2 - 3R^2)}{16\{R^2 + (d/2)^2\}^{9/2}}(\rho^3 - 4\rho z^2) + \cdots.
\]
The maximum field gradient of $B_z$ is given when $d = R$, which is

$$\frac{\partial B_z(0)}{\partial z} = \frac{4\mu_0 I}{2\sqrt{2} R^2} = \frac{2}{\partial B_z(0)}.$$ 

Thus, for both Helmholtz and anti-Helmholtz configurations to be operated from the same coaxial coils it is good to separate the coils with their spacing equal to radius, $d = R$.

![Figure 2.10: (a) A pair of anti-Helmholtz coils for the quadrupole trap. (b) Contour plot of the strength of the magnetic field produced by the two coils on the $x$-$z$ plane. The four dark regions indicate the places where the coils emerge on the $x$-$z$ plane, and the region of the strongest magnetic field. The calculated magnetic fields are generated with a current of $I = 200$ A flowing through each coil and with the separation between coils equal to the radius of $r = 30$ mm.](image)

### 2.6.4 Spherical quadrupole coils

For magnetic trapping, the simplest magnetic field geometry with a local minimum is the quadrupole field generated by two coaxial current loops (along the $z$-axis) with currents flowing in opposite directions. In this configuration, near the quadrupole trap centre, the magnetic field is given by

$$\vec{B}_q = b'_q(x, y, -2z) \quad (2.40)$$

where $b'_q$ is the field gradient along the radial direction of the coils. This satisfies the conditions $\nabla \cdot \vec{B}_q = 0$ and $\nabla \times \vec{B}_q = 0$. The magnetic field is zero at the trap centre and grows linearly with distance in all directions away from the centre. The linear dependence of the field distribution means that the quadrupole trap is quite stiff compared with the quadratic dependence of those discussed in section 2.7. The quadrupole coil configuration and field distribution are illustrated in Fig. 2.10.
2.6.5 Non-adiabatic loss from quadrupole traps

The quadrupole trap is the simplest way to trap magnetically atoms in weak field seeking states \((g_F M_F > 0)\). For an atom in a weak field seeking state, the magnetic potential in the quadrupole trap increases linearly with distance from the trap centre where the magnetic field vanishes. The magnetic dipole moment of the atom precesses around the external magnetic field at the Larmor frequency (see Fig. 2.3 (b)). To maintain its polarization the atom must follow the external magnetic field adiabatically. This condition requires that the Larmor frequency should be much larger than the variation rate of the angle \(\theta\) of the magnetic field seen in the atomic frame, i.e.

\[
\frac{d\theta}{dt} \ll \omega_{\text{Larmor}} = g_F M_F \mu_B B / \hbar.
\]

This ensures that the atom remains in the same quantum state relative to the instantaneous field direction. However, the adiabatic condition in eq. 2.41 becomes violated in the vicinity of the zero magnetic field point. If the local magnetic field becomes very small, atoms traversing that region will experience a substantial change in the instantaneous direction of the magnetic field due to their motion. When atoms congregate near the quadrupole trap centre, the separation of the Zeeman sub-levels becomes very small and they can no longer stay aligned to the magnetic field direction so that they may undergo transitions into untrapped states \((g_F M_F < 0)\), which are known as Majorana spin flips [13]. The Majorana transitions allow atoms near the bottom of the quadrupole trap to escape just like water dropping from the bottom of a conical funnel, leading to loss from the quadrupole magnetic trap. As the trapped cloud is colder, the loss rate becomes larger.

To circumvent the inherent shortcoming of the quadrupole trap there are a variety of configurations of magnetic traps. The first BEC experiment carried out at JILA employed the time-averaged orbiting potential (TOP) trap where a rapidly rotating, uniform bias field was superimposed onto the quadrupole magnetic field. The time-average of the resulting instantaneous potential yielded a harmonic potential [1]. An alternative approach taken at MIT was to plug the hole with a blue-detuned laser beam which repelled atoms from the centre of the quadrupole trap [2]. The Ioffe-Prichard (IP) trap and quadrupole-Ioffe configuration (QUIC) trap related to this experiment are discussed below.

2.7 Ioffe magnetic trapping

2.7.1 Ioffe-Pritchard trap

An Ioffe-Pritchard (IP) trap consists of four parallel straight wires called the Ioffe bars, and a pair of co-axial circular coils called the pinch coils as illustrated in
2.7. Ioffe magnetic trapping

Fig. 2.11. The Ioffe-Pritchard trap is designed to have a static, nonzero field minimum in order to trap neutral atomic clouds and to avoid Majorana transitions near the centre of the trap. The atoms are confined by the linear quadrupole field in the radial direction and by the curvature of the axial field. For radial confinement the four Ioffe bars at the corners of a square with sides of length $A$ create the magnetic field

$$\vec{B}_L = b'(x, -y, 0)$$

(2.42)

with the radial quadrupole field gradient of $b' = \frac{2\mu_0 I_b}{\pi A^2}$ where currents in adjacent bars flow in opposite directions with the same magnitude. For axial confinement of the atoms there are a pair of coils with radius $R$ and separation $2a$. These pinch coils produce a magnetic field along the $z$-axis but give no field gradient at $z = 0$. For small displacements from the trapping centre, the resulting magnetic field is given by

$$\vec{B}_p = B_0(0, 0, 1) + \frac{b''}{2}(-xz, -yz, z^2 - \frac{r^2}{2})$$

(2.43)

where $B_0 = \frac{\mu_0 NI_p R^2}{(R^2 + a^2)^{3/2}}$ and $b'' = \frac{3\mu_0 NI_p R^2(4a^2 - R^2)}{(R^2 + a^2)^{7/2}}$ are the axial bias field and field curvature, respectively, for coils with $N$ turns and current $I_p$. Therefore, adding the radial quadrupole field in eq. 2.42, the total magnetic field becomes

$$\vec{B} = \vec{B}_L + \vec{B}_p.$$  

(2.44)

The magnitude of the total magnetic field around the origin is approximated up to quadratic terms as

$$B = \left[\left(\frac{b''}{2} \left(z^2 - \frac{r^2}{2}\right) + B_0\right)^2 + (b'r)^2\right]^{1/2} \approx B_0 + \frac{1}{2} \left(\frac{(b')^2}{B_0} - \frac{b''}{2}\right)r^2 + \frac{1}{2} b''z^2.$$  

(2.45)

For small deviations, the potential energy of an atom in this magnetic field is given by

$$U(r, z) = g_F M_F \mu_B B(r, z) \approx U_0 + \frac{1}{2} m(\omega_r^2 r^2 + \omega_z^2 z^2$$

(2.46)

where $m$ is the mass of the trapped atom, $U_0 = g_F M_F \mu_B B_0$, the radial angular frequency

$$\omega_r = \sqrt{\left(\frac{(b')^2}{B_0} - \frac{b''}{2}\right) \frac{g_F M_F \mu_B}{m}}.$$  

(2.47)

and the axial angular frequency

$$\omega_z = \sqrt{\frac{b'' g_F M_F \mu_B}{m}}.$$  

(2.48)

It is important to note that the radial frequency is inversely proportional to the
square root of the bias field $B_0$ but proportional to current running along the Ioffe bars. On the other hand, the axial frequency is proportional to the square root of current flowing in the pinch coils. The shape of the IP trap potential is controlled by changing the bias field\(^2\) $B_0$. When \(\frac{(b')^2}{B_0} > \frac{3b''}{2}\), the radial curvature becomes larger than the axial curvature, and the resulting potential is that of an anisotropic harmonic oscillator near the trap centre. Fig. 2.12 shows an example of the harmonic potential of an Ioffe-Pritchard trap near the origin along both radial and axial directions with the parametric ratio \(\frac{(b')^2}{B_0} = \frac{16b''}{5}\). In the IP trap atoms oscillate more slowly along the axial direction than in the radial direction perpendicular to the four Ioffe bars. The Ioffe-Prichard trap is a common magnetic trap, but it suffers from the difficulty of aligning the centre of the atomic cloud in the MOT with the centre of the magnetic trap.

### 2.7.2 Quadrupole-Ioffe Configuration (QUIC) trap

A type of magnetic trap with a simple structure is the QUIC trap. The QUIC trap is formed by two quadrupole coils and one Ioffe (or curvature) coil as shown in Fig. 2.13, which was first developed by Hänsch’s group in Munich [19]. The MOT uses the same coils as the quadrupole trap, so the transfer of atoms from the MOT into the magnetic trap is straightforward. Atoms are loaded into a quadrupole trap and subsequently transferred to an Ioffe type trap. The trap is converted

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\(^2\)To change conveniently the constant component $B_0$ a second pair of coils, the so-called bias coils are included. They have a larger radius compared to the pinch coils and are close to the Helmholtz configuration to create an almost uniform magnetic field near the origin.
2.7. Ioffe magnetic trapping

Figure 2.12: (a) Potential of an Ioffe-Pritchard trap that shows a harmonic potential near the origin along both radial ($r$) and axial ($z$) directions. (b) Contour plot of the potential in the $r$-$z$ plane. The magnetic potential is plotted with the parametric ratio \( \frac{(\delta')^2}{B_0} = \frac{16\delta^2}{5}. \)

into the Ioffe configuration by turning on the current $I_3$ through the Ioffe coil. The QUIC trap has a small nonzero magnetic field minimum which suppresses Majorana transitions. The axial confinement comes from the non-uniform field varying along the axis of the Ioffe coil. The axial magnetic fields of the Ioffe coil and the radial field of the quadrupole pair are applied in opposite directions along the $x$-axis, so that they almost cancel. If the Ioffe coil produces a higher magnetic field in the $x$ direction than the quadrupole pair, this coil geometry results in a harmonic single well. On the other hand, if there are points in the $x$ direction where the magnetic field of the Ioffe coil is less than that of the quadrupole pair, a double-well potential is created and atoms in the trap can be lost at the two field zeros. An example of the evolution of magnetic trap transfer from a spherical quadrupole trap to a QUIC trap along the $x$-axis is illustrated in Fig. 2.14. After the current in the two quadrupole coils is switched on to $I_1 = I_2 = 4500$ A·turns, the current in the Ioffe coil $I_3$ is ramped up from 0 to 5000 A-turns. As the process is carried out, the spatial centre of the trap on the $x$-axis is shifted toward the Ioffe coil (Fig. 2.14 (b)). Before the Ioffe configuration is reached, the second zero point of magnetic field appears in the vicinity of the Ioffe coil, leading the potential to take the form of a double well (Fig. 2.14 (c) and (d)). When the two minima have merged (Fig. 2.14 (e)), the potential barrier disappears and then the combined field of the three coils produces a nonzero minimum in the absolute field on the symmetry ($x$) axis of the Ioffe coil. During this whole process the magnetic trap is transformed from the quadrupole trap into the QUIC trap and the constructed trapping potential is harmonic near the field minimum position. The magnetic
Chapter 2. Laser cooling and trapping techniques

Figure 2.13: QUIC trap setup. The current \( I_1 \) and \( I_2 \) create a quadrupole field. The Ioffe coil with current \( I_3 \) provides the curvature along the \( x \)-axis, the symmetry axis of the QUIC trap. Turning up the current in the Ioffe coil converts a spherical quadrupole trap into an Ioffe trap with its magnetic trap centre moving toward the Ioffe coil along the \( x \)-axis.

Field near the bottom of the QUIC trap generated by all the coils is

\[
B(r, x) \approx B_0 + \frac{1}{2} B_r'' r^2 + \frac{1}{2} B_a'' x^2 \tag{2.49}
\]

where \( B_0 \) is the magnetic offset field, \( r = \sqrt{y^2 + z^2} \), and \( B_r'' \) and \( B_a'' \) are the curvatures of the field in the radial and axial direction respectively. The potential energy of an atom in this magnetic field is

\[
U(r, x) = g_F M \mu_B B(r, x) = U_0 + \frac{1}{2} m (\omega_r^2 r^2 + \omega_a^2 x^2) \tag{2.50}
\]

where \( m \) is the mass of the trapped atom, \( U_0 = g_F M \mu_B B_0 \), \( \omega_r = \sqrt{g_F M \mu_B B_r'' / m} \) and \( \omega_a = \sqrt{g_F M \mu_B B_a'' / m} \) are the circular trap frequencies in the radial and axial direction respectively. Usually the radial curvature is much larger than the axial curvature so that atomic clouds in the QUIC trap are cigar-shaped with their long axis in the \( x \) direction. Note that the axis labeled with \( x \) for the QUIC trap is the symmetry axis of the Ioffe coil and is perpendicular to the \( z \)-axis of the quadrupole system.

2.8 Evaporative cooling

The method of evaporative cooling is based on the preferential removal of atoms with an energy higher than the average energy from a trapped sample and rether-
Figure 2.14: The absolute values of the total magnetic field along the $x$ axis of the QUIC trap are shown for a fixed current magnitude of $I_1 = I_2 = 4500$ A-turns in the quadrupole coils and (a) $I_3 = 0$ A-turns, (b) 1000 A-turns, (c) 3000 A-turns, (d) 4000 A-turns, and (e) 5000 A-turns in the Ioffe coil. The following dimensions used for the field calculation are adopted from reference [19]. The quadrupole coils diameter: 25.5 mm. The Ioffe coil diameter: 9 mm. The centre to centre distance between the quadrupole coils: 66 mm. The distance between the centre of the Ioffe coil and the symmetry axis of the quadrupole coils: 34 mm.
malization of the remaining atoms by elastic collisions. A comprehensive review of the technique applied to trapped atoms can be found in reference [75]. An experiment with RF-spectroscopy of magnetically trapped atoms was first performed in 1988 by Pritchard and his collaborators [76]. The principle of evaporative cooling of atoms is as follows. The energy of an atom within the magnetic trap is the sum of the potential energy and kinetic energy. As the atom moves up the sides of the trap, it exchanges kinetic energy for potential energy. The hottest atoms that have turning points of their orbits are in the regions of the highest potential energy and so driving atomic transitions from trapped states to untrapped states in these regions selectively removes the hottest atoms of an ensemble from a magnetic trap. The system must rethermalize at a lower average temperature before iterating the procedure. Forced evaporative cooling refers to such a process in which a continuous decrease of the temperature is achieved. The process is implemented by removing all atoms above some threshold energy in order to truncate the Boltzmann energy distribution of a thermal gas. The truncation energy is continuously swept down, enforcing an ongoing cooling effect. An obvious drawback of the method is that the size of the sample gets drastically reduced during the evaporation. Forced evaporation is experimentally performed by applying a radio frequency ($\nu_{rf}$) radiation and lowering its frequency continuously. The truncation energy $\epsilon_t$ for atoms in the state $M_F$ to be ejected from the trap is

$$\epsilon_t = M_F h (\nu_{rf} - \nu_0)$$

(2.51)

where $\nu_0 = g_F \mu_B |B(0)|/h$ is the resonance frequency at the centre of the trap. For the evaporation process one of the most important requirements on the system is that the lifetime of the trap is long compared with the time required for thermalization. While the trap lifetime is mainly determined by the rate of inelastic collisions\(^3\), the time necessary to rethermalize the sample depends on the rate of elastic collisions. It is important that the ratio of the elastic to inelastic collisions is sufficiently large for successful evaporative cooling. The efficient evaporation limit in alkalis requires that a ratio between elastic (good) and inelastic (bad) collisions is about $> 10^2$ [75, 77]. Also for continued evaporative cooling the elastic collision rate must increase or at least be maintained despite the reduction in the atom number. The mean elastic collision rate [45, 78] is defined as

$$\gamma_{el} = n \sigma_{el} v_r$$

(2.52)

where $n$ is the mean atom number density, $\sigma_{el}$ is the elastic collision cross-section, and $v_r = 4 \sqrt{k_B T/\pi m}$ is the average relative velocity between two atoms. At

\(^3\)Inelastic collision occurs in such a way that part of the internal energy of the colliding atoms is exchanged with their kinetic energy in the collision. The internal energy released in inelastic collisions can heat up the atoms and change their internal states, causing trap loss. Inelastic processes are two-body losses caused by spin exchange or dipolar relaxation between two trapped atoms that changes the internal state of the atoms, and three-body loss caused by molecule formation.
sufficiently low temperatures $s$-wave elastic collisions are dominant and the energy dependent $s$-wave cross section [79, 80] varies as

$$
\sigma_{el} = \frac{8\pi a^2}{1 + (ka)^2}
$$

(2.53)

where $k = mv_r/2\hbar$ and $a$ is the $s$-wave scattering length. The approximation $\sigma_{el} \approx 8\pi/k^2$ is valid when $(ka)^2 \gg 1$ while the approximation $\sigma_{el} \approx 8\pi a^2$ is valid in the low temperature limit $(ka)^2 \ll 1$, i.e. $T \ll \frac{h^2}{16\pi mk_B a^2}$. For $^{87}$Rb the $s$-wave scattering length is $a = 104.5a_0$ and the $s$-wave cross section$^4$ is approximately $\sigma_{el} \approx 8\pi a^2 = 7.68 \times 10^{-12}$ cm$^2$ when $T \ll 143.4$ µK [81].

### 2.9 Imaging and data acquisition

#### 2.9.1 Magnification calibration

The magnification of the imaging system is determined by releasing a cloud from the trap and measuring its central position on the CCD camera as a function of time as it freely falls under gravity. The result of the measurement is fitted with the equation of the displacement of the cloud under the gravitational acceleration $(g)$ given by

$$
z(t) = z_0 + v_0 t - \frac{1}{2} Mgt^2
$$

(2.54)

where $M$ is the magnification of the lens system, $z_0$ is the cloud’s initial position, and $v_0$ is the initial velocity along the $z$-axis. Fig. 2.15 shows a typical fit to the data, which gives $M = 0.502 \pm 0.005$.

#### 2.9.2 Absorption image analysis

Absorption imaging is the most common imaging technique for cold atomic clouds. Absorption imaging is performed by illuminating an atomic cloud with a pulse of circularly polarized light resonant with the $F = 2 \rightarrow F' = 3$ transition in the presence of a bias magnetic field along the beam axis and then detecting the fraction of the beam intensity transmitted. The absorption has a maximum at a certain detuning of the probe beam for which the Zeeman shift makes the transition resonant (see Fig. 3.29). Before taking the image, the magnetic trap is switched off so that the cloud expands. The imaging beam is absorbed by the atomic cloud and the shadow cast by the cloud is detected on a CCD (Charged Couple Device) camera. For quantitative imaging, the absorption imaging procedure is composed

$^4$Through a derivation of elastic cross sections for various temperatures Burke et al. in JILA showed that the elastic cross section between $^{87}$Rb atoms in the $|2, 2\rangle$ ground state is approximately constant at $0 < T < 200$ µK.
of three consecutive images at the end of each experiment cycle (see Fig 2.16). The first image (image I) contains the shadow picture of the probe beam transmitted by the atoms. The second image (image II), taken long after the atoms have dispersed, contains just the probe beam itself without any atoms. The third image (image III) gives the inherent background signal seen by the camera without any atoms or imaging beam. The data from the absorption imaging process are recorded as a 2D array and are used to extract physical information on the cloud such as the number of trapped atoms ($N$), temperature ($T$), and phase space density ($\rho$). The spatial resolution of the imaging technique is typically of the order of a few micrometers.

For the analysis of the three pictures we consider the trapped cloud to be in a harmonic potential (see eq. 1.13). After propagating through the atomic cloud along the probe beam axis ($y$ axis in our case), the initial intensity $I_0^i$ of probe beam arriving at the $i^{th}$ pixel in image I is attenuated according to Beer-Lambert’s law given by

$$ I_1^i = I_0^i e^{-\overline{OD}_i} = I_0^i e^{-\int n'(y) \sigma dy} \tag{2.55} $$

where $I_1^i$ is the transmitted intensity after absorption, $\overline{OD}_i$ is the optical density, $n'(y)$ is the number density of the cloud along the column of real space imaged onto the $i^{th}$ pixel, and $\sigma$ is the absorption cross section. The imaging $x$-$z$ plane in the CCD chip is perpendicular to the $y$-axis. Assuming that the cloud is optically
thin and the intensity does not change significantly across it, the absorption cross section is given by

$$\sigma = \sigma_0 \frac{1}{1 + I_0/I_{sat} + (2\delta/\Gamma)^2}$$  \hspace{1cm} (2.56)

where $I_{sat}$ is saturation intensity and $\sigma_0 = 3\lambda^2/2\pi$ is the unsaturated resonant cross-section that in our case is $2.9 \times 10^{-9}$ cm$^2$. Optical properties of $^{87}$Rb are given in the table of appendix A. After the probe beam passes through the atomic cloud, the number of counts recorded on the $i^{th}$ pixel is $C_i = \alpha_i I_i$ where $\alpha_i$ is the efficiency of the pixel. The intensity at the pixel in the absence of the cloud $I_i$ is recorded in image II so that $C_i = \alpha_i I_i$. The dark count $C_i$ of the pixel is obtained from image III and subtracted from both $C_i$ and $C_i$ to minimize the contribution of the background light taken in the absence of the detection beam. From three camera images we obtain the optical density at the $i^{th}$ pixel of the CCD array, using the relationship

$$OD_i = \ln \frac{C_i - C_i}{C_i - C_i}$$  \hspace{1cm} (2.57)

The total optical density comes from summing eq. 2.57 over all pixels. Fig 2.16 shows an example of the optical density picture obtained from the consecutive three images method using a cloud with $3.2 \times 10^5$ atoms. The higher the optical density of the atomic cloud, the higher the absorption in the cloud. Analysis of the optical density structure provides us with most physical quantities that can be experimentally determined about the cloud. From eq. 2.55 we find the total number of atoms recorded at the $i^{th}$ pixel

$$N_i = A \int n^i(y)dy = \frac{A}{\sigma} \ln \frac{C_i - C_i}{C_i - C_i}$$  \hspace{1cm} (2.58)

where $A = \frac{16 \times 16 \mu m^2}{M^2}$ is the effective area imaged onto each pixel with magnification $M$. Therefore, the total number of atoms in the sample is

$$N = \sum_i N_i = \frac{A}{\sigma} \sum_i \ln \frac{C_i - C_i}{C_i - C_i}$$  \hspace{1cm} (2.59)

where the sum is extended over all the pixels within a selected region of interest. To obtain the axial ($x$) and radial ($z$) size of the atomic cloud, the rows and columns of the 2D image array are added to produce two 1D density distributions and the resulting profiles are fitted with Gaussian curves

$$n(x_i) = n_0 e^{-x_i^2/\sigma^2}.$$  \hspace{1cm} (2.60)

Thus, the radii $\sigma_x$ and $\sigma_z$ which are the half-widths of the atomic cloud in the axial and radial directions are extracted from the $1/e$ radii of the Gaussian fit on
Figure 2.16: Absorption imaging. An example of the optical density picture (d) using a cloud of $3.2 \times 10^6$ atoms, which is obtained from consecutive image I (a), image II (b), and image III (c). The magnification of the CCD camera is 0.5 and the atomic cloud image (a) is taken after 4 ms time-of-flight from the magnetic trap.
the two profiles given by summing in the two directions. Then they are multiplied by 16 µm/M in order to be converted from pixel unit to µm. The density of the cloud in the harmonic potential is calculated from the relationship

\[ n = \prod_j \frac{N}{\pi^{3/2} \sigma_j} = \frac{N}{\pi^{3/2} \sigma_x \sigma_y \sigma_z}. \]  

(2.61)

From the equipartition theorem, the temperature of the cloud in the harmonic potential is obtained using the following equation

\[ T_j = \frac{m \omega_j^2 \sigma_j^2}{2k_B}. \]  

(2.62)

From equation 2.61 and equation 2.62, we find the phase space density

\[ \rho = n \left( \frac{\hbar^2}{2\pi mk_B T} \right)^{3/2}. \]  

(2.63)

Most of the images on the CCD camera in our experiment are taken after a time-of-flight expansion of a cloud. After suddenly switching off the current, the cloud undergoes a ballistic expansion for time \( \tau \). The initial size of the cloud \( \sigma_j(t = 0) \) is calculated from

\[ \sigma_j(t = 0) = \sigma_j(\tau) / \sqrt{1 + (\omega_j \tau)^2}. \]  

(2.64)
Chapter 3

Experimental apparatus

This chapter describes in detail our experimental apparatus comprising an ultra-high vacuum (UHV) system, the optics for laser cooling, electronics for magnetic trapping, and imaging. A double MOT system where a pyramidal MOT acts as a source of atoms for a second MOT in the UHV region is also presented along with its setup necessary to create optimized MOTs with working conditions suitable for magnetic trapping. As a major part of the chapter I describe the design and construction of a new QUIC trap system used as a tool for both magnetic transport and magnetic trapping for BEC experiments.

3.1 Vacuum system

We use a double MOT system which was first proposed in 1996 [82] with two separate vacuum chambers with a differential pressure. A higher Rb vapour pressure in the first chamber allows faster collection of atoms for a pyramidal MOT, and thus quicker loading of a science MOT in the second chamber where the pressure is two orders of magnitude lower.

The vacuum system comprises the pyramidal MOT section and science MOT section. A 40 l/s ion pump (Varian VacIon 40) maintains a pressure of about $10^{-7}$ Pa inside the pyramidal MOT part. Low pressure of the second section is maintained by a 55 l/s ion pump (VacIon Plus 55 by Varian) and a non-evaporable getter (NEG) pump (SAES Capacitorr-CF35, Cartridge C-400-DSK-St.172). The NEG pump helps to reach and preserve a pressure of $10^{-9}$ Pa inside the science cell. The NEG pump works through a chemical reaction, adsorbing active molecules impinging on the getter. The distance between the pyramidal MOT and experimental MOT is about 30 cm. A schematic of the vacuum system is shown in Fig. 3.1.

The first chamber is placed in a stainless steel cylinder with a 10 cm diameter glass window on the front side where the laser beam enters, and a stainless steel
3.1. Vacuum system

The feedthrough to the getter source is connected to a flange on top of the custom-made piece for the pyramidal MOT pipe that connects this chamber with the glass cell vacuum chamber on the other side. Differential pumping is achieved by a 17 mm long cylindrical opening with a 5 mm hole between the two chambers. The conductance of flow between the two chambers is proportional to the number of atoms entering the tube ($\propto D^2$), and inversely proportional to the average number of collisions with the wall as the atom goes through the tube ($\propto L/D$) under the assumption of free gases moving at very low pressure. This leads to the background gas conductance of the tube [83] given by

$$F = \frac{65}{\sqrt{M}} \times \frac{D^3}{L} \text{ l/s} \quad (3.1)$$

where $D$ is tube diameter and $L$ its length, both expressed in cm. $M$ indicates the relative gas weight of the gas. Therefore, the length (diameter) of pipework connecting the two MOTs needs to be reduced (enlarged) to increase conductance. To achieve a large collection region, the pyramid mirror has a base of 60 mm that is the largest size which would fit inside the vacuum system. The differential pumping tube in our vacuum system has an inner diameter $D = 0.5$ cm and length $L = 1.7$ cm. For $^{87}\text{Rb}$, the background gas conductance of the tube connecting the two vacuum regions becomes $F = 0.5$ l/s. The vacuum system is mounted on an independent breadboard and can be wheeled on a track attached to the optical table. This allows the vacuum system to slide out of the surrounding coils and gives access to the magnetic trap centre.
Chapter 3. Experimental apparatus

3.1.1 Rubidium source

The rubidium source consists of two dispensers (SAES Getters, Rb/NF/7/25 FT10+10) located above the pyramid mirrors as shown in Fig. 3.2, which can be independently operated. The resistively heated Rb dispenser is designed for rapid deposition of large amounts of Rb, at an operating current of 6 A. In all the experiments the Rb dispenser is heated with currents ranging from initially 2.4 A to 5.5 A at the end of the operational lifetime. The current is applied to the getter inside the vacuum through a high current feedthrough attached to the top flange of the chamber. The current passing through the dispenser generates heat and activates a chemical reaction to release rubidium atoms. The DC current is provided by a regulated power supply (Thurlby Thandar TSX 3510 35V, 10A). With a fixed current applied to the Rb dispenser, the Rb background vapour pressure is constant and so the loading rate is uniform during experiments, allowing the collection of about $2 \times 10^8$ atoms in the science MOT.

3.1.2 Glass cell

The fused silica science cell made by Optiglass Ltd. has a square cross-section of $20 \times 20$ mm$^2$, a wall thickness of 2 mm, and is about 70 mm long. The fringe patterns made by the glass cell indicate that its thickness is uniform as shown in Fig. 3.3. The open end is fused to a cylindrical glass-metal part with a stainless steel flange allowing easy connection to the vacuum system. The length of the
circular cross-section part from the glass tube to the metal sealing adaptor is approximately 100 mm, and its diameter is 38 mm.

3.2 Laser sources

All our light sources are based on semiconductor diode lasers. Laser diodes were first used for the cooling of caesium atomic beams [84] in 1986. In our optics setup, the laser light (Master and repumping lasers) is generated by external cavity diode laser (ECDL) units [85], designed to operate at 780 nm. The diode lasers in a Littrow configuration [86] use optical feedback from a 1200 lines/mm diffraction grating to reduce frequency excursions to a level suitable for laser cooling (< 1 MHz linewidth). The Littrow configuration consists of a mounting base serving as a heat sink, a Peltier cooler, a base plate for a laser diode holder and collimator, a grating mount with a piezo actuator, and a mirror to direct the output beam. The first-order diffracted beam from the grating is fed back into the laser diode, while the zeroth order reflection beam from the grating is coupled out for the experiment. The optical feedback into the laser diode by the grating establishes a new extended resonator between the rear facet of the laser diode and the diffraction grating; the external cavity length determines the individual mode frequencies, while the range of frequencies of the laser diode is set by tilting the angle of the grating. A piezoelectric transducer (PZT) is mounted behind the grating. As the PZT expands and contracts, the grating angle is varied and the length of the laser cavity also changes. Over time the output frequency of a diode laser with grating feedback can drift because of external mechanical vibrations, fluctuations in injection current, and temperature.
3.2.1 Master and slave laser

The Master laser is an external cavity diode laser (DC 100) made by TUI Optics. For the Master laser an active temperature controller (DTC 100/30 W) via a Peltier cooler (TEC) working as a heat sink is used to minimize thermal drifts of the diode laser cavity. In addition, for low noise operation the current control driver (DCC 100/500 mA) is used. The power from the Master laser is partly injected into a temperature stabilized slave laser to boost optical output power (master-slave arrangement). The Master laser also supplies the light for saturation spectroscopy. We use two home-made slave lasers. Each of them consists of a laser diode (Sanyo DL7140-201), an objective collimating the output beam, a thermistor, and a temperature controller (Newport Model 350) with a Peltier effect heat pump. The temperature of the whole diode mount is stabilized to within 0.1 °C. The slave lasers are optically injected from the Master laser and their output frequency follows the behaviour of the Master laser’s frequency as shown in Fig. 3.4. It is monitored by sending the slave light through a Rb vapour cell for saturation spectroscopy. Output power from each slave laser is used as cooling light for the MOT.

3.2.2 Repumping laser

A second laser system (repumping laser) provides laser light tuned to the \( |F = 1\rangle \rightarrow |F' = 2\rangle \) transition and repumps atoms that end up in the \( F = 1 \) ground state into the cooling cycle. To generate light which is locked to the repumping transition, we use a home-made external cavity diode laser. The diode used for the repumping laser is Sanyo DL7140-201 (785 nm, 70 mW) and stabilized via a frequency selective external cavity formed between a diffraction grating (1200 lines/mm) and the rear facet of the diode. The resulting beam power is divided into 2.4 mW for the pyramidal MOT and 2.3 mW for the science MOT.

3.2.3 Stabilization and control of the laser frequencies

Stabilization of the laser frequencies

For the Master laser stabilization, the crossover line between the \( |F = 2\rangle \rightarrow |F' = 1\rangle \) and \( |F = 2\rangle \rightarrow |F' = 3\rangle \) transition is frequency-locked. The locking system consists of a home-built scan generator with a PI servo circuit (EW1225) and a high voltage piezo driver (EW 1145). The laser frequency stabilization is based on Doppler-free spectroscopy. The servo electronics employs a dual feedback loop for the laser diode current and the piezo transducer. Fig. 3.5 illustrates the scheme for stabilizing the frequency of two semiconductor diode lasers (Master and repumping lasers). The saturated absorption spectrum and its error signal, obtained using Rb vapour cells, are shown in Fig. 3.4. The error signal of a selected absorption signal is generated by modulating the laser current at 100 kHz and feeding the
3.3 Optical bench layout

The light paths from the diode lasers to the optical fibres are shown in a schematic of the optical table in Fig. 3.6. The profile of the output beam from the diode laser is elliptical. Its shape is made more circular by a pair of anamorphic prisms to...
Chapter 3. Experimental apparatus

Figure 3.5: (a) The scheme for stabilizing the frequency of semiconductor diode lasers. (b) A block diagram of the frequency-locking electronics (EW 1225). The laser diode stabilizer consists of amplifier board, oscillator board, phase sensitive detector board, and integrator board.

improve the coupling into the optical elements. Then it is sent through an optical isolator that prevents reflected light from upsetting the laser frequency stability. After the optical isolator, the power available from the Master (repumping) laser is 25 mW (20 mW). A polarizing beamsplitter cube divides the beam into two parts. A small fraction of the beam (13 µW) goes to the Rb vapour cell for saturation spectroscopy. The fraction of the beam power is controlled by the orientation of a half-wave plate in front of the cube.

Three Acousto-Optic Modulators (AOMs, Crystal Technology 3080-122) are used to tune finely the laser frequency (see Appendix B). An oscillating electric signal drives a piezo-electric transducer to vibrate at a frequency of $\Omega = (\Omega_0 + \delta)/2$ where $\Omega_0 = 2\pi \times 211.8$ MHz for the MOT beam and $\Omega_0 = 2\pi \times 266.65$ MHz for the optical pumping beam. This frequency is controlled by a DC voltage from an analogue output of the computer which goes to a Voltage Controlled Oscillator (VCO). The acoustic wave generated from the VCO is amplified up to 2 W via a RF linear amplifier (Motorola CA2832C) before going into the AOM where a transducer creates a sound wave. The beam from the Master laser splits in two at PBS2. One of these beams is passed through AOM1. The other goes towards AOM2 after being reflected by PBS3. As illustrated in Fig. 3.7, AOM1 frequency-
shifts the incident beam with a frequency detuning $\delta = \delta_{\text{pyr}}$ with respect to the $|F = 2 \rangle \rightarrow |F' = 3 \rangle$ transition. Also AOM2 frequency-shifts the incident beam with a frequency detuning $\delta = \delta_{\text{MOT}}$ with respect to the $|F = 2 \rangle \rightarrow |F' = 3 \rangle$ transition. Downstream from each AOM is a quarter-wave plate to give circular polarization. A convex lens and a mirror with their spacing equal to the focal length of the lens form a cats eye retro-reflector. The 0th and $-1$st diffracted orders are blocked and the $+1$st order beam is retro-reflected back through the AOM so that it undergoes another frequency shift of $\Omega$. The beam which passes AOM1 twice is injected into the Slave 1 laser. The output (about 60 mW) from Slave 1 laser passes a pair of anamorphic prisms, and optical isolator, and is then directed towards the pyramidal MOT section. The beam which is double-passed through AOM2 splits at PBS4. 1.5 mW of the beam is selected to inject the Slave 2 laser and the rest of the power (6.4 mW) is sent to AOM3 for imaging and optical pumping. The injected light into the Slave 2 laser is amplified and then sent into a polarization maintaining optical fibre. For the imaging beam, we choose the 0th
order beam out of AOM3 after we set AOM2 to make the light be nearly resonant with the $|F = 2\rangle \rightarrow |F' = 3\rangle$ transition. For optical pumping, we select the -1st order of AOM3 to produce a frequency detuning $\delta_{OP}$ from the $|F = 2\rangle \rightarrow |F' = 2\rangle$ transition. Both the probe light for imaging and the optical pumping light are coupled into another fibre and delivered to the experiment section.

The repumping light splits into two paths by a polarizing cube (PBS1) so that a fraction (2.4 mW) of the beam is superimposed with the MOT1 cooling light at PBS7 while the rest (2.4 mW) is superimposed with the MOT2 cooling light at PBS6 before going to the optical fibre. Shutters along the various beam paths are used to extinguish completely the light during the magnetic trapping and evaporation phase.

### 3.4 QUIC trap coils

This section describes the design, simulation, and construction of a novel magnetic trapping system which combines MOT coils with a quadrupole and Ioffe trap configuration in order to provide magnetic transport and magnetic trapping.
3.4. QUIC trap coils

Figure 3.8: Geometry of the QUIC trap coils. (a) Cross section of the QUIC trap coils around the science cell with mm scale. $d$ is the distance between the centre of the first quadrupole coils and the centre of the Ioffe coil along the x-axis. (b) The QUIC trap coils consist of A coils (green) which are the first quadrupole coils (QD1), B coils (blue) which are the second quadrupole coils (QD2), and C coil which is the Ioffe (curvature) coil (yellow). Arrows indicate the directions of currents flowing through each coil.
3.4.1 Coils for the science MOT and quadrupole trap

Our magnetic trap incorporates the quadrupole and Ioffe configuration. The magnetic trap’s design is inspired by the scheme first introduced by Esslinger et al. [19]. To trap magnetically and transport the $^{87}\text{Rb}$ atoms, we construct five individual coils as shown in Fig. 3.8. All of the wound coils are fabricated using copper tube of squared cross section (wire outer sizes of $3.3 \times 3.3 \text{ mm}^2$) from S & W Wire Company. The material is insulated with 0.15 mm thick Kapton layer. Thus, the part of pure copper is a hollow square conductor of outer size of 3.15 mm with inner circular hole of 1.8 mm mean diameter. Its electrical resistance is 2.3 m$\Omega$ per metre at 20 °C. Power dissipation of 3 kW generated from resistive heating in the magnetic coils is cooled by forcing water through the hollow core of the copper wires [87]. For cooling water, a differential pressure of 60 psi (1 psi = 6.89 kPa) is applied, which leads to a flow rate of 13 litre/min. Three types of coils are wound by aluminium formers especially made according to the geometry of each coil. The coils are wound with the help of a dedicated winding machine (MAXEL) that holds the formers to tighten the wire and easily wind each turn. Wires are glued together by applying 2011 Araldite adhesive between the wire layers. The first set is a pair of quadrupole coils to generate the magnetic field for the MOT. The second set is a pair of circular coils designed for the Helmholtz/anti-Helmholtz configuration and its centre is shifted 20 mm away from the MOT coils along the $x$-axis. The last set is an Ioffe coil with 8 windings. The mounts to hold these coils consist of three circular pieces of cotton fabric Tufnol (6F/45). A pair of large mounts with four grooves house the two sets of quadrupole coils. Another small mount is made for encasing the single Ioffe coil. An image of their geometry is shown in Fig. 3.16.

The first pair of quadrupole coils (A coils, QD1) are used as MOT coils which initially collects large samples of cold atoms. Each coil is composed of 24 windings with centre diameter of 100 mm. Centre to centre separation between the quadrupole coils is 100 mm. At the centre between the coils, the configuration for MOT coils produces a field gradient of 10 G/cm along their symmetry (z) axis with a current of 15.7 A per turn as shown in Fig. 3.9 (a) and (b). For the calculation we choose the $x$-axis along the symmetry axis of the glass cell and assume that each wire has no thickness and is considered as a line located at its geometric centre. The first pair of quadrupole coils are also used for initial magnetic trapping. For loading atoms into the magnetic trap with the first quadrupole coils, when we apply a current of $I_A = 78.24$ A per turn, its corresponding field gradient of $\frac{\partial B_z}{\partial z} = 50 \text{ G/cm}$ is generated at the coils’ centre as shown in Fig. 3.9 (c) and (d).

The second pair of quadrupole coils (B coils, QD2) are designed for position transfer of the trapped atoms. It consists of two circular coils each with 24 windings and centre diameter of 120 mm. The centre to centre distance between the coils is 60 mm as shown in Fig. 3.8. When the current of $I_A = I_B = 138$ A per turn is applied on two pairs of quadrupole coils at the same time, they produce a magnetic field of
Figure 3.9: First pair of quadrupole coils create a magnetic field for the MOT configuration; (a) magnetic field and (b) field gradient of $\frac{\partial B_z}{\partial z} = 10 \text{ G/cm}$ at the coils centre along the symmetry ($z$) axis are generated by applying a current of 15.7 A per turn. The first pair of quadrupole coils (QD1) are also used to create a magnetic field for the quadrupole trap configuration; (c) magnetic field and (d) field gradient of $\frac{\partial B_z}{\partial z} = 50 \text{ G/cm}$ at the coils centre along the symmetry ($z$) axis are generated by applying a current of $I_A = 78.24 \text{ A}$ per turn.
Figure 3.10: Two pairs of quadrupole coils (QD1 and QD2) are used to generate a magnetic field and field gradient for the effective quadrupole magnetic trap. With each current of \( I_A = I_B = 138 \) A per turn, they produce a magnetic field of 90.5 G (a) and field gradient \( \frac{\partial B}{\partial x} = 88.64 \) G/cm at \( x = 20.57 \) mm (b). The Ioffe coil consisting of 8 circular loops with each current of \( I_C = 138 \) A produces a magnetic field of 91.5 G at \( x = 20.57 \) mm (c) and its axial curvature of \( \frac{\partial^2 B}{\partial x^2} = 106.2 \) G/cm\(^2\) at \( x = 20.57 \) mm is obtained from the field gradient plot in (d)
3.4. \textit{QUIC} trap coils

3.4.1 \textit{QUIC} trap coils

Figure 3.11: Magnetic field calibrations of the first quadrupole coils (a), and second quadrupole coils (b) along their symmetry axis by applying a current of 9.1 A per turn.

90.5 G and a gradient of \( \frac{\partial B_x}{\partial x} = 88.64 \text{ G/cm} \) at the position of \( x = 20.57 \text{ mm} \), and the magnetic field minimum is displaced at \( x = 10.69 \text{ mm} \) as shown in Fig. 3.10 (a) and (b).

Fig. 3.11 shows magnetic field calibrations of the first quadrupole coils and second quadrupole coils along their symmetry axis, respectively. By linear fits to the data we found that the magnetic field gradient which the first quadrupole coils generate is 5.61 ± 0.06 G/cm at a current of 9.1 A while the magnetic field gradient which the second quadrupole coils create is 6.9 ± 0.11 G/cm at a current of 9.1 A. Thus the first quadrupole coils produce a field gradient of 0.616 ± 0.007 G/cm per Ampere near the coils’ centre, and the second quadrupole coils produce a field gradient of 0.758 ± 0.01 G/cm per Ampere near the coils’ centre. Our numerical calculation deviates 3.3\% from the calibration result for the field gradient of 10 G/cm in the science MOT configuration. This magnetic field calibrations are useful in the course of implementing the science MOT optimization and initial loading of the quadrupole magnetic trap.

3.4.2 \textit{Ioffe} type magnetic trap

The Ioffe coil (C coil, curvature coil) is used to convert the quadrupole trap into an Ioffe-Prichard trap so that the minimum of the magnetic field is shifted towards the Ioffe coil. The Ioffe coil consists of 8 turns; 3 windings with a diameter of 23.4 mm, 3 windings with a diameter of 30.2 mm, and 2 windings with a diameter of 37 mm as shown in Fig. 3.8. So the mean diameter of the Ioffe coil is 30.2 mm. The distance between the centre of the Ioffe coil and the symmetry axis of the first quadrupole coils is chosen to be \( d = 41 \text{ mm} \). The geometry of all the \textit{QUIC} trap coils is summarized in Table 3.1.
As shown in Fig. 3.10 (a), the two pairs of quadrupole coils (A coils and B coils) generate a quadrupole field and its field minimum is located at 10.69 mm when the current of 138 A per turn flows in the two pairs. In addition, when the same current flows in the curvature coil, it produces a magnetic field that opposes the field produced by these two pairs of quadrupole coils along the $x$-axis. With a current of $I_C = 138$ A per turn, the Ioffe coil produces a magnetic field of 91.5 G with axial curvature of $\frac{\partial^2 B_x}{\partial x^2} = 106.2$ G/cm$^2$ at $x = 20.57$ mm as shown in Fig. 3.10 (c) and (d). Fig. 3.12 shows that the minimum in the magnetic field formed by the combination of all coils is displaced at 20.57 mm from the centre of the first quadrupole coils towards the curvature coil. Fig. 3.13 illustrates the evolution of the currents and magnetic fields during the transfer from the quadrupole magnetic trap into the QUIC trap.

<table>
<thead>
<tr>
<th>Coil name</th>
<th>Turns per coil</th>
<th>Radius (mm)</th>
<th>Separation (mm)</th>
<th>Distance from A coils (mm)</th>
<th>Current (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A coils</td>
<td>24 (top)/ 24 (bottom)</td>
<td>50</td>
<td>100</td>
<td></td>
<td>138</td>
</tr>
<tr>
<td>B coils</td>
<td>24 (top)/ 24 (bottom)</td>
<td>60</td>
<td>60</td>
<td>20</td>
<td>138</td>
</tr>
<tr>
<td>C coil</td>
<td>8</td>
<td>15.1</td>
<td>41</td>
<td>138±3</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.1: Summary of the geometry of the QUIC trap coils.
3.4. QUIC trap coils

Figure 3.13: Evolution of the currents and magnetic fields for the magnetic transport process. The Ioffe coil (C coil) converts the quadrupole traps-(a) and (b)-into an Ioffe-Prichard trap-(c)-so that the minimum of the magnetic field is shifted towards the Ioffe coil. Each arrow in the left-side figures indicates the direction of the current of 138 A per turn flowing through each coil. The right-side figures at different stage are contour plots in the $x$-$z$ plane, showing the absolute value of the magnetic field at different stages.
Figure 3.14: Dependence of the magnetic field minimum $B_0$ and magnetic trap’s centre position $x$ (a), magnetic field gradient and curvature (b), and trap frequencies (c) of the QUIC trap on variation of the position $d$ of the curvature coil with a current of $I_C = 138$ A per turn ($I_A = I_B = 138$ A per turn). Displacement $d$ is the centre position of the Ioffe coil along the $x$-axis from the centre of the first pair of the quadrupole coils.
Figure 3.15: Dependence of the trap frequencies of the QUIC trap on variation of the position of the curvature coil with a current of $I_C = 135$ A per turn (a), and $I_C = 141$ A per turn (b) ($I_A = I_B = 138$ A per turn). Displacement $d$ is the centre position of the Ioffe coil along the $x$-axis from the centre of the first pair of the quadrupole coils.
From the calculated bias field of $B_0 = 0.992$ G, field gradient of $88.64$ G/cm, and curvature of $106.2$ G/cm$^2$ by all coils with $I_A = I_B = I_C = 138$ A per turn and $d = 41$ mm, we expect the trapping frequencies of $\omega_a = 2\pi \times 13.3$ Hz in the axial direction and $\omega_r = 2\pi \times 113.2$ Hz in the radial direction, for $^{87}$Rb atoms of the $|F = 2, M_F = 2\rangle$ state. The radial frequency $\omega_r$ can be adjusted by finely tuning the field $B_0$ at the bottom of the trap. Typical operating values for $B_0$ range within a few Gauss. Fig. 3.14 shows dependence of the magnetic field minimum and magnetic trap’s centre position along the $x$-axis, magnetic field gradient and curvature, and trap frequencies of the QUIC trap on variation of the position of the curvature coil with its fixed current of $I_C = 138$ A per turn. As the position $d$ of the Ioffe coil—which is the distance between the centre of the QD1 and the centre of the Ioffe coil (see Fig 3.8 (a))-increases, the QUIC trap’s centre along the $x$-axis moves towards the curvature coil, while the magnetic field minimum, field gradient, and curvature decrease as plotted in Fig. 3.14 (a) and (b). By a linear fit to the data in Fig. 3.14 (a), the position variation of the Ioffe coil leads to the change of the magnetic field minimum $B_0$ with a ratio of $8.93 \pm 0.01$ G/mm. Fig. 3.14 (c) shows that the radial trap frequency is very sensitive to the position variation of the Ioffe coil. As the position $d$ of the Ioffe coil increases, the radial trap frequency increases while the axial trap frequency decreases. The bottom field of the QUIC trap $B_0$ or trap frequencies can be also adjusted by varying the current of the Ioffe coil. Fig. 3.15 shows dependence of the trap frequencies of
3.5 Auxiliary coil systems

3.5.1 Quadrupole coils for the pyramidal MOT

To make the pyramidal MOT (MOT1), a pair of quadrupole coils each with 13 turns are wound around the steel chamber that contains the pyramidal mirrors, respectively. The coils are made of a hollow copper pipe, encased in a glass-fibre cladding to ensure electrical isolation between the individual turns. The two coils have a central radius of about 59 mm (inner one of 50 mm and outer of 68 mm) respectively and the mean separation between the two coils is 50 mm along their axis whose centre coincides with the MOT region. This geometry gives an anti-Helmholtz configuration and is able to generate a magnetic field gradient of about 10 G/cm at the centre of the trap at a current of 30 A (see Fig. 2.6 (b)), which is delivered by a Kepco CPS (15 V, 30 A) power supply. The resistive heat is dissipated by sending cooling water through the copper tube.

3.5.2 Shim coils

The shim coils are used to cancel the effect of external magnetic fields in both MOT regions. The main contribution to spurious magnetic fields in the laboratory comes from the Earth. Its field is of the order of 0.5 G. Hence, distant coils with a large diameter can be used without requiring too much current.

**Shim coils for the pyramidal MOT**

To control the centre of the pyramidal MOT three shim coils are mounted in orthogonal directions around the pyramidal MOT. The three coils have identical circular shape with a diameter of 17 cm. Each coil has six layers of 18 turns for a total of 107 turns per coil. We drive them with home-made power supplies capable
of delivering 2 A for each coil. Details of the geometry of the pyramidal MOT shim coils are given in Table 3.2. The current of each shim coil is adjusted to nullify the background magnetic field. They are also used to shift the magnetic field centre of the pyramidal MOT generated from the quadrupole coils so that we can optimize the number of atoms transferred through the hole of the pyramid into the science MOT.

<table>
<thead>
<tr>
<th>Spatial direction</th>
<th>Turns per coil</th>
<th>Diameter (cm)</th>
<th>Distance from MOT1 (cm)</th>
<th>Current (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>107</td>
<td>17</td>
<td>10</td>
<td>0.24</td>
</tr>
<tr>
<td>$y$</td>
<td>107</td>
<td>17</td>
<td>9.5</td>
<td>0.64</td>
</tr>
<tr>
<td>$z$</td>
<td>107</td>
<td>17</td>
<td>13.5</td>
<td>0.60</td>
</tr>
</tbody>
</table>

Table 3.2: Summary of the geometry of the MOT1 shim coils.

**Shim coils for the science MOT**

To nullify external fields around the science MOT centre, we use 3 pairs of Helmholtz-type coils (see Fig. 3.16). They are set up at different distances from the trapping centre of the science MOT. We measure the background field in the MOT centre with a gaussmeter along the symmetry axis of each shim pair and the current applied on each shim pair is adjusted to cancel the background field. We drive them with the same power supplies as the shim coils of the pyramidal MOT. Details of the geometry of the science MOT shim coils are presented in Table 3.3.

<table>
<thead>
<tr>
<th>Spatial direction</th>
<th>Turns per coil</th>
<th>Side length (cm x cm)</th>
<th>Distance from MOT2 (cm)</th>
<th>Current (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>35</td>
<td>$8 \times 8$</td>
<td>$\pm 11$</td>
<td>0.48</td>
</tr>
<tr>
<td>$y$</td>
<td>35</td>
<td>$17 \times 17$</td>
<td>$\pm 14$</td>
<td>0.88</td>
</tr>
<tr>
<td>$z$</td>
<td>115</td>
<td>$20 \times 20$</td>
<td>$\pm 13$</td>
<td>0.12</td>
</tr>
</tbody>
</table>

Table 3.3: Summary of the geometry of the MOT2 shim coils.

Figure 3.17: RF system for evaporative cooling.
3.5. Auxiliary coil systems

3.5.3 RF signal-driving coil

![Characteristic of the RF coil. The plot shows the amplitude of the oscillatory peak-to-peak current ($I_{pp}$) as a function of input frequency by the RF generator.](image)

Figure 3.18: Characteristic of the RF coil. The plot shows the amplitude of the oscillatory peak-to-peak current ($I_{pp}$) as a function of input frequency by the RF generator.

We use a circular RF (radio frequency) coil which sends RF signals to the atomic cloud during evaporative cooling. The coil with 18.5 mm average radius has two windings and is located at a distance of 1.2 cm from the centre of the magnetic trap. Our RF source is a synthesized function generator (Stanford Research Systems Model DS345, 30 MHz, 10 V$_{pp}$) which is connected to the coil as shown in Fig. 3.17. The sequence of the RF waveforms is programmed and triggered by LabVIEW and transmitted to the RF generator through a NI PCI-GPIB (General Purpose Interface Bus) connection. Fig. 3.18 plots the characteristic oscillatory current amplitude ($I_{pp}/2$) of the RF coil as a function of input frequency from the RF generator. The coil current is measured with a Hall probe attached to a digital oscilloscope (Tektronix TDS 2024B). In order to maximize the coupling with atoms and induce transitions between the Zeeman states, we placed the coil directly outside the glass cell bottom where the axis of the coil is perpendicular to the bias field the Ioffe coil produces. A resistor (RS 155-914, 7 W 47 Ω, 5%) is placed in series to a terminal of the RF coil in order to improve impedance matching between the load and the RF source at low frequencies.
3.5.4 Electronics for current control

Power supply and current stabilization

The control of the magnetic fields during the switching of the initial quadrupole trap and magnetic transport from the quadrupole magnetic trap to QUIC trap is important for the efficiency of the process. The switching time needs to be fast enough so that the atomic cloud does not expand extensively or fall under the influence of gravity. For the fast switching on of the initial quadrupole magnetic trap, we use an ESS DC power supply (Lambda America Inc. 40 V, 375 A, 15 kW). The power supply unit is able to increase the current to the working value of 81 A within 1.3 ms. During magnetic transport and evaporation the power supply is operated in constant current mode where an output voltage load changes. Constant current mode causes current variation of less than 0.1 percent. Another DC power supply (PSD3510A) is directly connected to two terminals of the curvature coil and serves as an adjustable current source for the bias field of the QUIC trap. For precise timing and current control the power supply receives voltage and current settings via an optically isolated analogue interface from the experiment control computer. All coils are connected to the switching electronics and power supply by means of a low resistance welding cable with 25 mm$^2$ cross sectional area. A closed loop Hall current sensor (Honeywell CSNJ481) allows us to measure the total current flowing through the coil arrangement. To check directly the stability of the high current, the current monitor output is connected to a digital oscilloscope. A high current diode (International Rectifier, 85HF40) is placed in series with the positive terminal of the power supply to avoid any reverse current damaging the power supply. The electronics for the high current control circuit is schematically shown in the Fig. 3.19.

MOSFET Banks

Both the switching and control of the current are managed by MOSFETs (Metal Oxide Semiconductor Field Effect Transistor). We construct electronics for the high current control, which is composed of 7 MOSFET banks. Each MOSFET bank consists of 5 MOSFETs (N-Channel STE180NE10, STMicroelectronics) which are connected in parallel to increase power handling. Each MOSFET bank serves as a variable resistor by regulating its gate voltage. The MOSFETs are rated for a maximum drain current of 180 A and a maximum power dissipation of 360 W at 25 °C. Hence, each MOSFET bank is theoretically capable of sustaining a current of 900 A. We control the amount of current flowing through the coils by including a by-pass loop at each set of coils. One MOSFET bank is used for overall current regulation and three pairs of MOSFET banks are used to adjust the current ratio between each coil set and by-pass. A so-called voltage floating circuit between the source and the gate of each MOSFET bank is included to sense the source voltage
Figure 3.19: Schematics of the electronic circuits employed to control the magnetic trap current.
Chapter 3. Experimental apparatus

Figure 3.20: The voltage-floating circuit for each MOSFET bank. \( V_G \) is the voltage value sent by the analogue signal of LabVIEW and \( V_S \) is the voltage at the source of the MOSFET with respect to the ground of the power supply.

and add it to the gate voltage (see Fig. 3.20). The circuit is required because a control voltage at the gate of a MOSFET bank should be in the range of 0 and 10 V with respect to the bank’s source rather than the ground of the power supply. To protect the MOSFET banks from high voltage spikes varistors (V33ZA70P) are connected between source and drain terminals of each MOSFET bank. Since the MOSFETs may dissipate a large amount of heat, each MOSFET bank is mounted on a copper tube that is water-cooled.

Figure 3.21: The reverse control circuit.

By-pass controller

A reverse control circuit and a pair of MOSFET banks are used to adjust the ratio between the by-pass current and the coil current in each set. The reverse control circuit is illustrated in Fig. 3.21. As the gate voltage in one MOSFET bank is ramped up from 0 to 10 V, the gate voltage in the other MOSFET bank is ramped down from 10 to 0 V. This means that the by-pass channel is turned on as the coil is turned off and vice versa.
3.6 Pyramidal MOT

The pyramidal MOT has two purposes; firstly to pre-cool and collect atoms from a $^{87}$Rb vapour source, secondly to send them to the region of the science MOT for further experimental manipulation. After Jhe and co-workers [89] introduced a compact MOT with a single beam reflected by mirrors in a corner cube configuration, a pyramid-shaped MOT was constructed in Oxford to make a continuous and robust source of slow atoms [90].

The pyramidal MOT consists of four separate mirrors in the shape of an inverted pyramid with a hole left at its vertex and a pair of anti-Helmholtz coils to generate about 10 G/cm at the trapping centre. The arrangement of the pyramid mirrors is shown in Fig. 3.2 and Fig. 3.23. Two (upper and lower) square mirrors are placed

Figure 3.22: (a) The response time of an opto-coupler is 100 $\mu$s for a 10 V input signal. (b) The opto-coupler produces a linear gain with respect to the input signal. The channel 1 on each screen of the oscilloscope represents the input signal and the channel 2 the output signal.

Opto-coupler

Without a direct electrical connection analogue signals need to be transferred from one terminal to another in such situations as the source of a signal and its destination are at very different voltage levels. To protect the experiment control computer from damage of overvoltage spikes and to avoid ground loops, a home-built circuit based on precision analogue opto-couplers (HCNR201) is used to link the analogue control signals and the high current control electronics. The transfer characteristic of the opto-couplers circuit [88] is linear with a gain equal to one over a range between 0 and 10 V, and its response time is 100 $\mu$s for a 10 V input signal as shown in Fig. 3.22.
Chapter 3. Experimental apparatus

Figure 3.23: Pyramidal MOT. (a) A cross section of the pyramidal MOT schematically illustrating how the required MOT beams are achieved. (b) The CCD image of the fluorescence from the atom cloud loaded in the pyramidal MOT where about $1.2 \times 10^8$ atoms are captured. The arrow indicates the true cloud and there are four others made by reflection of mirrors.

at 90° to form a right angle on which two (left and right) prisms sit in this angle. They are surrounded by three shim coils positioned in three orthogonal directions. The pyramidal MOT is mounted inside the vacuum chamber and a $1 \times 2$ mm$^2$ rectangular hole exists at the vertex of its pyramidal shape to allow cooled atoms to be sent into the region of the second MOT. The side of the pyramid base is 60 mm long. Each reflecting surface of the pyramid to serve as a mirror is coated with broad-band dielectric to achieve the same reflectivity for both $s$-polarization and $p$-polarization. When a wide incoming laser beam is sent around the pinnacle area of the pyramid, three pairs of counter-propagating beams are automatically produced therein that realize the polarization and beam configuration equivalent of a conventional six-beam MOT. The operational parameters are set by optimizing the fluorescence signals. Fluorescence from the trapped atoms in the pyramidal MOT is measured on a photodiode after the centre of the trap is shifted vertically a few mm. A CCD camera monitors the fluorescence of the pyramid MOT. Fig. 3.23 illustrates the configuration of the pyramidal MOT and a CCD image of the fluorescence from the atom cloud loaded in it.

A $\sigma^-$-polarized beam containing both cooling light (60 mW) with a beam waist ($1/e^2$) of 0.73 mm and a repumping light (2.2 mW) with a beam waist of 1 mm is magnified (magnification 15.6) and collimated towards a mirror of diameter 100 mm. The enlarged beam goes through a glass window with diameter 95 mm. A pair of oppositely polarized, counter-propagating beams are generated from the
first reflection on one of the pyramidal mirrors (see Fig. 3.23). Then a σ⁺ polarized, retro-reflected beam appears from the second reflection on the other mirror. The small hole at the vertex allows the cooled atoms to be continuously pushed into a second part of the vacuum chamber where they are captured by a second MOT. The transport distance from the pyramid apex to the trap centre of the second MOT is 300 mm.

The direction of the incident laser beam on the axis of the pyramid is controlled by a pair of steering mirrors. The part of the pyramid beam which passes the apex can push atoms away from the science MOT centre and so reduce overall loading efficiency in the science MOT. Therefore the pyramid beam is adjusted to point slightly in an upward direction. Fig. 3.24 shows a series of measurements taken for the optimization process with respect to frequency detuning of the cooling laser and magnetic field gradient, which lead to the choice of a field gradient of 9.26 G/cm and a detuning of −15 MHz (see also Fig. 3.25 (a) and (b)). The power dependence on the cooling laser and the repumping laser of the MOT is shown in Fig. 3.25 (c) and (d). The atom number captured in the MOT is saturated at
Figure 3.25: Optimization of the pyramidal MOT parameters. (a) The measurement of atom number vs frequency detuning of the cooling laser at $\frac{\partial B_z}{\partial z} = 9.26$ G/cm. (b) The measurement of atom number vs magnetic field gradient at detuning of $-15$ MHz. Dependence of number of atoms loaded in the MOT on the cooling laser power (c) and on the repumping laser power (d). (e) MOT loading rate vs frequency detuning of the cooling laser. (f) Dependence of number of atoms loaded in the MOT on the beam polarization controlled by a quarter-wave plate.
cooling beam power more than 60 mW and repumping power more than 2 mW. Fig. 3.25 (e) shows the loading rate of atoms in the pyramidal MOT over time (during 0.3 s) just after the atoms load in an empty trap. We found that there exists an angle tolerance of about $\pm 5^o$ from an optimized angle when the single incident beam with linear polarization is passed through the quarter-wave plate as shown in Fig. 3.25 (f).

### 3.7 Science MOT

The beam of slow $^{87}$Rb atoms collected by the pyramidal MOT travels for 30 cm along the direction of the symmetry axis of the pyramid, and then is trapped in a second MOT (MOT2) which is called the science MOT within the glass cell. A large atom number is necessary to achieve good starting conditions for evaporative cooling. The main objective of the second MOT is to collect the largest possible number of atoms before loading the magnetic trap. The science MOT is a 6-beam MOT. The three pairs of counter-propagating beams are formed from the light coming out of the MOT2 optical fibre. 40 mW of power, of which 2.3 mW is from the repumping beam, is transferred through the fibre. After the trapping beam is expanded from the fibre and collimated by a doublet lens (focal length = 100 mm), it is split into six beams by using polarizing beam splitters and half-wave plates. As a result, each beam has a Gaussian profile with a radius $(1/e^2)$ of 9.1 mm. Two of these beams counter-propagate along the vertical direction through the centre of the glass cell. The other four beams form two counter-propagating pairs in the horizontal plane, which merge at a 70$^o$ angle towards the trapping centre. The power of each beam pair is balanced by adjusting a half-wave plate placed before each of the beam splitter cubes. A quarter-wave plate in each of the six beams makes their polarization circular just before they are sent into the science cell. For diagnostic purposes fluorescence is measured with a calibrated photodiode (Thorlabs-DET110, responsivity of 0.5 A/W at 780 nm) to estimate the number of atoms. The frequency detuning of the cooling light and the magnetic field gradient are adjusted to maximize atom number loaded in the second MOT. The measurements taken during the optimization process are plotted in Fig. 3.26. Fig. 3.26 (a) illustrates the dependence of atom number in the science MOT on various axial field gradients at the cooling light detuning of $-12.5$ MHz. Fig. 3.26 (b) shows the dependence of atom number in the science MOT on various detunings of the cooling light at the magnetic field gradient $\frac{\partial B_z}{\partial z} = 11$ G/cm along the symmetry axis of the MOT coils. Fig. 3.26 (c) shows a typical loading curve obtained from the fluorescence measurement of the science MOT. From fitting the loading curve with eq. 2.27, we found that the filling rate of the science MOT is $R_L = 3.9 \times 10^6$ s$^{-1}$ and the loss coefficient is $\gamma = 1.9 \times 10^{-2}$ s$^{-1}$. The science MOT is initially loaded with a rate of about $4 \times 10^6$ atoms per second, and reaches $1.0 \times 10^8$ atoms within 31 s, and arrives at an equilibrium of $2 \times 10^8$ atoms after...
Figure 3.26: Optimization of the science MOT loading parameters. (a) Dependence of atom number in the MOT on various axial field gradients at a cooling light detuning of $-12.5 \text{ MHz}$ (b) Dependence of atom number in the MOT on various detunings of the cooling light at the magnetic field gradient $\frac{\partial B_z}{\partial z} = 11 \text{ G/cm}$ along the axial direction. (c) A loading curve of atoms in the MOT as a function of time. Inset: A fluorescence image of the cloud of $1 \times 10^8$ atoms trapped in the science MOT which is taken by the CCD camera. (d) An example showing the exponential decay of the MOT after switching off the pyramidal MOT beam. The atom number in each plot is obtained by measuring the fluorescence with a calibrated photodiode. Red curves are fits to the data points.
180 s as shown in Fig. 3.26 (c). Inset of Fig. 3.26 (c) is a fluorescence image of the cloud of $1 \times 10^8$ atoms trapped in the science MOT taken by the CCD camera. The decaying behaviour of the atom number in the science MOT is also obtained from the measurement of the fluorescence signal after blocking the pyramidal MOT beam. Fig. 3.26 (d) shows a typical decay curve of the science MOT with an initial atom number of $N_s = 2.4 \times 10^8$. By fitting the curve with an exponential decay function (red curve), we found that the cloud of the science MOT decays with the lifetime of $\tau = 62.7 \pm 0.3$ s. These measurements are accomplished under optimal conditions determined from the data in Fig. 3.26 (a) and (b).

### 3.8 Imaging system

#### 3.8.1 CCD camera and optics for imaging

![Figure 3.27: Schematic diagram of the imaging system.](image)

Our charge coupled device (CCD) camera for imaging is an Andor IXON DV 887. A square chip on the detector consists of an array of small pixels each with pixel size of $16 \times 16$ $\mu$m$^2$. The entire array is $512 \times 512$ pixels and the overall size of the chip is $8.2 \times 8.2$ mm$^2$. The well depth is $2 \times 10^5$ electrons/pixel and the quantum efficiency is 44% at a wavelength of 780 nm. The readout noise is 62 electrons per pixel in the fastest readout mode of 10 MHz. Each pixel converts incident photons to electrons with the quantum efficiency and stores the resulting electrons during a given exposure time.

When we consider a dynamic range to allow 50% filling ($1 \times 10^5$ $e^-$) of each pixel well and with the given quantum efficiency of 44% at $-20$ °C, on average, each pixel receives a maximum of $2.27 \times 10^5$ photons. From the photon numbers and one pixel area of $2.56 \times 10^{-6}$ cm$^2$, the energy density per pixel on the chip is $2.26 \times 10^{-8}$ J/cm$^2$ for the wavelength of 780 nm. Let us assume that we are using a resonant intensity of $0.3 \times I_{sat} = 0.5$ mW/cm$^2$ in the imaging beam. Considering a magnification of $M = 3.5$ leads to an effective intensity of $40.8 \mu$W/cm$^2$ impinging on the CCD chip. Therefore, the maximum exposure time is 0.55 ms.
Figure 3.28: The magnification of our imaging system determined by using the fitting function given in eq. 2.54, which is found to be $M = 3.55 \pm 0.03$.

The optics for imaging is illustrated in Fig. 3.27. A probing beam resonant with the $|F = 2\rangle \rightarrow |F' = 3\rangle$ transition is spatially filtered and coupled out by a single-mode polarization-preserving optical fibre. The beam is then allowed to expand before being collimated by a doublet lens of focal length 40 mm. The collimated beam has a radius $(1/e^2)$ of 3.48 mm. There is a quarter-wave plate placed before the glass cell to produce a circular polarization of the probe beam so that it may drive the appropriate resonant transition in the imaging process. To get the circular polarization, we choose the quantization axis parallel to the propagation direction of the probe beam. The appropriate imaging beam intensity is adjusted through the absorption imaging process. We initially set the intensity of the beam to be about one-fourth saturation intensity of the imaging transition, $I_0/I_{sat} = 0.25$. For the current beam size, this requires the total beam power to be 79 $\mu$W. The imaging pulse duration is set to 0.1 ms to reduce interference fringe artifacts in the absorption images due to mechanical vibrations of the camera during exposure. The atoms have to remain in the depth of focus during exposure of the imaging light. For the observation of atomic clouds, an achromatic doublet lens (LINOS: effective focal length 80.57 mm, diameter 25.4 mm, thickness 7 mm) is used to magnify the image by a factor of 3.55. The lens is placed at a distance of 103 mm from the centre of the atomic cloud. An image with magnification of 3.55 is formed at a distance of 366 mm from the lens. From the numerical aperture $NA = 0.12$
3.9. Computer control

Our experiment requires precise timing controls of a variety of components. The electronic devices are controlled by a LabVIEW program via two PCI cards, i.e. PCI-6713 and PCIe-6259 from National Instruments. The first card has 8 analogue outputs and 8 digital input/outputs, while the second card has 4 analogue outputs,
Figure 3.30: A control panel of the LabVIEW user-interface.
3.9. Computer control

32 analogue inputs, and 48 digital input/outputs including 16 PFI (Programmable Function input/output) channels. The two cards are connected by a RTSI (Real Time Synchronization Integration) cable for sharing system timing and synchronization. They both have two gated counters which are used for the TTL signals triggering the optical pumping pulses. An analogue to digital converter is used to provide 8 more buffered digital outputs at the expense of an analogue output channel from the first card. We also use GPIB as the main mode of communication for programming the RF synthesizer (SRS 354 DS). LabVIEW programs are developed to give precise control over several devices and to provide accurate synchronization over digital and analogue devices. The programs regulate the control voltages of the VCOs for the frequency of the AOMs, the state of the mechanical shutters, and provide control voltages to the power supply for generating and switching each coil current. After the science MOT is fully loaded, the program also starts a sequence that triggers and regulates all the analogue and digital devices that are needed to load the magnetic trap, perform the evaporative cooling stage, and trigger the probe pulses for image acquisition. We use two computers in order to control the experiment. While one is used to handle the analogue/digital signals and GPIB commands, the other is a camera computer dedicated to control the CCD camera, image acquisition and data analysis. Fig. 3.30 shows one of the front panels of our LabVIEW interface used for optimization of the MOT stage and transfer to the magnetic trap.
Magnetic transport, evaporation in the magnetic trap, and BEC

This chapter describes the experimental procedure of magnetic transport, the QUIC trap, and then evaporative cooling of $^{87}\text{Rb}$ atoms in the $|F = 2, M_F = 2\rangle$ state. One of the most critical points of our experiment is the transfer of the atoms from the science MOT into the magnetic quadrupole trap and converting the quadrupole trap into a QUIC trap. Here I discuss the optimized transfer process designed to achieve high atomic number loading in the magnetic trap, which is crucial for the following final step of evaporative cooling. Secondly this chapter outlines the general strategy that consists in first preparing a large sample of cold atoms, then optically pumping the atoms to the appropriate Zeeman state followed by magnetic trapping and transport into the QUIC trap. The final part of this chapter presents measurements performed during the evaporative cooling stage leading to the production of a $^{87}\text{Rb}$ condensate.

4.1 Compression of the cloud in the MOT

While the low field gradient optimized for a MOT is suitable for capturing a large number of atoms and reducing losses due to reabsorption of photons, it is not adequate for the following sequences towards condensate production such as loading a magnetic trap and evaporative cooling. The smaller size of the cloud makes transfer into a magnetic trap easier. The higher density of the cloud is favorable for the process of evaporative cooling. Hence, one of the typical sequences to prepare for loading into the magnetic trap is the compression of the magneto-optical trap (CMOT) which was first demonstrated in 1994 by Petrich et al. [91]. The CMOT stage is usually optimized by considering the following conditions; the density of the atomic cloud is to be maximized while the total atom number
4.1. Compression of the cloud in the MOT

Figure 4.1: The recaptured percentage of number of atoms against the final field gradient of the compressed MOT stage at the final frequency detuning of $-19$ MHz and duration of 55 ms.

should not decrease significantly during the compression. It is usually beneficial to compress the cloud in order to match the center of the MOT with the center of the magnetic trap and to improve the collision rate in the magnetic trap. Since it helps match the size of the cloud with the optical pumping beam, the CMOT is also effective for the process of optical pumping (see section 4.3).

In practice, the optimization parameters to be varied during the CMOT are the final frequency detuning of the cooling laser, duration, and the magnetic field gradient at the end of the compression stage. In our case we optimize the CMOT stage with the aim of effectively capturing the largest number of atoms in the magnetic trap. Fig. 4.1 and Fig. 4.2 illustrate the dependence of the loading efficiency in the magnetic trap on the final field gradient, final frequency detuning of the cooling laser, and duration of the compression stage. The recapture measurements of the atom number against these parameters in the CMOT stage are carried out by first recording the fluorescence of the cloud in the MOT, loading the magnetic trap for 210 ms, and then recapturing the atoms into the MOT in order to obtain the fluorescence level. From these observations we determined the optimized parameter values by which we could maximize the number of trapped atoms. First, the field gradient ($\frac{\partial B}{\partial z}$) of the MOT is linearly ramped from 11 G/cm to 31 G/cm during the compression phase in order to reduce the overall spatial extent of the atoms in the MOT. Second, the duration of the CMOT stage is chosen to be 70 ms which is a balance between an increased collision rate by the compression and an increase in loss rate. Third, to avoid losing atoms due to the multiple scattering
between the atoms and laser photons in the course of increasing magnetic field, the final frequency detuning of the cooling beam is readjusted [92]; the cooling laser frequency detuning is linearly ramped down from $-11 \text{ MHz}$ to $-19 \text{ MHz}$ for the CMOT stage.

### 4.2 Optical molasses

As the CMOT stage causes heating of the cloud, optical molasses or polarization gradient cooling is a process often used to lower the temperatures of the cloud prior to loading the magnetic trap. Cooling of optical molasses is provided by the sub-Doppler cooling mechanism described in section 2.2 which gives a final temperature of the cloud inversely proportional to the frequency detuning. The sub-Doppler cooling mechanism requires the ground state to have degenerate magnetic sublevels and so operates efficiently in zero magnetic field [93]. Thus, the quadrupole magnetic field for the science MOT is switched off during the optical molasses and the earth’s magnetic field is cancelled using shim coils which are centered around the science cell. The balance of the laser beam intensity is optimized by observing that the cloud from the MOT expands evenly in the radial direction when the quadrupole field of the MOT is suddenly turned off. Two infrared cameras installed in orthogonal directions are used to monitor the expansion behaviour.
of the cloud after the end of the optical molasses. As we vary the duration and final frequency detuning of the molasses phase, respectively, we measure the number of atoms recaptured just after being held in the quadrupole trap for 210 ms in order to see how these parameters have an effect on the atomic cloud. Fig. 4.3 shows the results taken from the recapture ratio measurements for the optimization of the optical molasses stage. We observed that the dependence of the recapture ratio on the duration of the molasses stage is nearly constant in the range between 20 ms and 55 ms (see Fig. 4.3 (b)). We also found that there exists an optimum among the final detuned-frequencies by which the molasses stage is obviously affected. We choose to maintain the process of optical molasses for 25 ms. For the fixed molasses duration of 25 ms the cooling laser detuning is varied and the fraction of recaptured atoms recorded (see Fig. 4.3 (a)). The optimum value for the detuning is set to $-25$ MHz.

### 4.3 Optical pumping

For our setup the weak field seeking $|F = 2, M_F = 2\rangle$ state is selected because of the higher magnetic moment and hence stronger confinement in magnetic traps compared with the $|F = 1, M_F = -1\rangle$ state. After optical molasses, the atoms are distributed over all five magnetic substates of the $F = 2$ ground state manifold. Thus atoms that stay in undesired states are transferred into the $|F = 2, M_F = 2\rangle$ state by optical pumping carried out immediately after the molasses phase. For an optical pumping beam, we use a pulse of $\sigma^+$ polarized light resonant with the $|F = 2\rangle \rightarrow |F' = 2\rangle$ transition. When the atoms reach the $|F = 2, M_F = 2\rangle$ state,
i.e. dark state as illustrated in Fig. 4.4, further $\sigma^+$ polarized light absorption is forbidden, which prevents unnecessary heating. The optical pumping beam’s frequency and amplitude are controlled by AOM3. After the cooling laser is extinguished, a pulse of $\sigma^+$ polarized light with an intensity of 2 mW/cm$^2$ passes the atoms in the $y$ direction. The repumping laser is left on to transfer atoms which have decayed to the $F = 1$ ground state back into the optical pumping cycle. Fig. 4.6 (a) shows pulse timing of an optical pumping beam which begins 0.3 ms before the current of the magnetic trap is switched on. Simultaneously the bias field with a magnitude of 1.55 G along the direction of the pumping beam is applied at the position of the cloud by reversing the direction of the current flow in one of the $y$-axis shim coil pairs. The small uniform magnetic field produced by the shim coils in Helmholtz configuration provides the atoms with a quantization axis. If there is an even population distribution across the $\mid F = 2, M_F \rangle$ levels, we expect to trap magnetically a maximum of 40% of the $\mid F = 2, M_F = 2 \rangle$ state atoms without the optical pumping process. We observed that the recapture percentage is typically $(43 \pm 2)\%$ in the absence of the optical pumping pulse. This could be attributed to a slightly uneven population distribution at the end of the molasses phase. To check for optical pumping efficiency, we carry out measurements of the number of atoms recaptured immediately after holding atoms in the magnetic trap for 210 ms. Fig. 4.5, plotted from the recaptured percentage measurements, shows that there are optimum values for the optical pumping pulse length and frequency detuning. For this stage we set the pulse length to 0.15 ms with a frequency detuning of $-15$ MHz as longer pumping times induce noticeable heating. With these
4.4 Loading the quadrupole magnetic trap

In order to transfer the optically pumped atoms into a quadrupole magnetic trap, all of the laser beams are switched off after the optical pumping stage. Then the current of the first quadrupole magnetic coils is switched on as fast as possible. The rise time to a current of 81 A of the coils is about 1.3 ms (see Fig. 4.6 (a)). This current value corresponds to a magnetic field gradient of 50 G/cm along the symmetry (z) axis of the coils. To maintain the orientation of the atomic magnetic moment, the magnetic field of the quadrupole magnetic coils is turned on just before the bias field in the y direction is turned off. The quadrupole magnetic trap is formed by the same magnetic coils which are used for the science MOT. Thus, the center of the quadrupole magnetic trap is automatically aligned with the center of the science MOT when all external magnetic fields are compensated. Fig. 4.6 (b) shows the transfer process from the science MOT into the quadrupole trap and the fraction of atoms that remains in the trap after a hold time of 210 ms. After compression of the MOT, optical molasses, and optical pumping, we achieve a recapture ratio from the quadrupole trap of 73% for an initial number of $1 \times 10^8$
Figure 4.6: Stages for the transfer process in the quadrupole magnetic trap. (a) Pulse timing for the optical pumping beam (black curve), and current ramp (blue curve) as a function of time for the first pair of quadrupole coils. The pumping pulse begins 0.3 ms before switching on the current of the magnetic trap. The rise time for an initial current of 81 A is 1.3 ms. (b) Plots showing the ratio of fluorescence levels (black curve) and evolution of the current (blue curve) of the first quadrupole coils for the recapture measurement. A recapture ratio of 73% is determined by comparing the initial MOT’s fluorescence level with the recaptured level.
4.5 Magnetic transport and the QUIC trap

After the large cloud of cold atoms is loaded into the quadrupole magnetic trap, we move the quadrupole magnetic trap’s center about a centimeter along the symmetry axis of the science cell and then convert the quadrupole magnetic trap into an Ioffe-type trap (QUIC trap). By controlling the gate voltage in the bypass MOSFET bank for each coil we can split the current between the coil and bypass and so control the final current value in each of the coils. At the end of the transport, the final value of current is set to 138 A and identically flows through each coil with all bypasses closed (see Fig. 4.9). First, the current in the first quadrupole magnetic coils (QD1) is ramped up adiabatically from 80 A per turn to $I_A = 138$ A per turn (85 G/cm) within 800 ms. The magnetic transport process is then performed by slowly turning on the second quadrupole coils (QD2) and ramping up within 800 ms with the same current of $I_B = 138$ A per turn as illustrated in Fig. 4.9, so that the magnetic trap’s centre is shifted to the midpoint between the first quadrupole coils and second quadrupole coils. Then, the current of the Ioffe coil is ramped up to $I_C = 138$ A per turn within 1000 ms to move the atoms to the final trapping position in the QUIC trap where we can monitor the cloud through a CCD camera and apply evaporative cooling. The compression involved in ramping up the current from 80 A to 138 A increases the trap stiffness and
Figure 4.7: (a) Measurement of the lifetime in the quadrupole magnetic trap by the recapture method and fit with an exponential decay curve that yields the trap lifetime of $(124 \pm 9)$ seconds. (b) Measurement of the lifetime in the QUIC trap by absorption imaging and fit with an exponential decay curve that yields the QUIC trap lifetime of $(224 \pm 7)$ seconds.
4.5. Magnetic transport and the QUIC trap

Figure 4.8: Optimization of ramping time for loading the QUIC trap. By changing the ramp time of each coil we measure the number of atoms and peak optical density of the cloud loaded in the trap. Plots (a) and (b) are for the first quadrupole coils (QD1). Plots (c) and (d) are for the second quadrupole coils (QD2). Plots (e) and (f) are for the curvature coil (Ioffe). For the measurements we use an initial MOT with $1.5 \times 10^8$ atoms.
elastic collision rate before evaporation. We hold the QUIC trap stage for 20 ms just before we start to trigger the evaporative cooling stage.

All these current-ramping times of coils are experimentally determined by a series of optimization processes as seen in Fig. 4.8. The plot shows that fast current ramps in the B and C coils (less than 500 ms) induce strong loss of the cloud during transport and an efficient ramp time for each coil appears between 500 ms and 1400 ms. We observed that we typically lose 62% of the cloud through the transport process from the second MOT into the QUIC trap. The results of a simulation of the magnetic transport according to the current evolution given in Fig. 4.9 is illustrated in Fig. 4.10. The plot shows both the position (black squares) of magnetic field minimum along the symmetry ($x$) axis of the curvature coil and the calculated transport speed (blue circles) of the atomic cloud as a
Figure 4.10: Simulation of the transport from the quadrupole trap to the QUIC trap according with the current evolution given in Fig. 4.9. The plot shows the position (Black square) of magnetic field minimum along the symmetry ($x$) axis of the curvature coil and the transport speed (Blue circle) as a function of time. Inset pictures show an initially MOT-trapped cloud (left-fluorescence image) with $1.5 \times 10^8$ atoms and a finally transported cloud (right-absorption image) with $5.6 \times 10^7$ atoms.

function of time. As the currents of the two pairs of the quadrupole coils ramp up, the speed along the $x$-axis increases from zero to 33.7 mm s$^{-1}$ with a maximum acceleration of 175 mm s$^{-2}$ at 0.75 s and then the speed decreases until the current of the curvature coil turns on. As the current of the curvature coil ramps up, the second maximal speed (19.2 mm s$^{-1}$) appears at 2.44 s, i.e. the moment of the transition from a quadrupole magnetic potential into a harmonic potential and then a maximum deceleration ($-156$ mm s$^{-2}$) occurs at 2.5 s. Inset pictures in Fig. 4.10 show an initial MOT cloud with $1.5 \times 10^8$ atoms (left figure-fluorescence image) and a final transported cloud with $5.5 \times 10^7$ atoms and $x$-size of 2.30 mm and $z$-size of 1.88 mm after 6 ms of time of flight (right figure-absorption image).

A cloud in a harmonic trap can undergo simple harmonic motion about the minimum of the magnetic trapping potential. Sloshing motion of the centre of mass of the cloud occurs when the cloud loaded into the magnetic trap possesses some velocity initially. As long as the amplitude of the oscillation is small enough, the cloud remains in a harmonic region of the magnetic trap. We calculated the trap-
Figure 4.11: The centre of mass oscillations of the atomic cloud along the axial (a) and radial (b) direction of the QUIC trap, induced by placing the atomic cloud initially off-centre. The camera magnification used for the measurements is $M = 0.5$. From these plots we extract the axial and radial trapping frequencies. Axial frequency is $\omega_a = 2\pi \times (12.74 \pm 0.01)$ Hz while radial frequency is $\omega_r = 2\pi \times (147.43 \pm 0.4)$ Hz.
ping frequencies when designing the QUIC trap coils (see section 3.4). However, the imprecision in constructing the coils makes it impossible to determine the trap frequencies accurately. One way to determine the precise trap frequencies in the harmonic magnetic trap is to induce sloshing motion upon loading the magnetic trap by modifying the trapping potential faster than the atoms can follow. To induce the radial sloshing motion we apply a 2 V pulse to the modulation input of the \( z \)-axis shim field power supply lasting for 0.7 ms. The same method could not be used to excite a sloshing motion in the axial direction as the modulated shim field is not sufficiently strong. Instead, the bypass of the curvature coil is slightly opened for 1 ms by applying 3 V to the gates of the MOSFET bank in order to induce the \( x \)-axis sloshing. The perturbing field’s strength is sufficiently low that the amplitude of the harmonic oscillations remains smaller than the cloud size. After the perturbation, the cloud oscillating in the magnetic trap for a given hold time is abruptly released from the trap. Then we measure the cloud position as a function of hold time in the magnetic trap as plotted in Fig. 4.11, leading to the extraction of the exact trapping frequencies. For the measurements of the centre of mass oscillations we use clouds with \( 7 \times 10^5 \) atoms at \( T = 3 \mu \text{K} \). The cloud images are taken after 12 ms time of flight and the camera magnification is \( M = 0.5 \).

The current applied to the whole QUIC coils is 138 A (from a ESS DC power supply), and an additional current of 0.17 A (from a PSD3510A DC power supply) is applied to the Ioffe coil in the same direction as the current by the ESS power supply. Sine function fit to the data yields that the axial trap frequency is \( \omega_a = 2\pi \times (12.74 \pm 0.01) \text{ Hz} \) and the radial trapping frequency \( \omega_r = 2\pi \times (147.43 \pm 0.4) \text{ Hz} \), respectively. Table 4.2 shows the result of calculation of trap parameters with \( I_A = I_B = 138 \text{ A per turn} \) and \( I_C = 138.17 \text{ A per turn} \). The central position \( d \) of the curvature coil in Table 4.2 is determined through a series of numerical calculations, which fits well within 0.47% deviation from the offset field \( B_0 = 0.554 \text{ G} \) obtained from Fig. 4.13 (b). We expect the centre position of the curvature coil to be at \( d = 41.063 \text{ mm} \) from the centre of the first quadrupole coils. We found the resulting calculation has a deviation of 0.43% from the result of the sloshing measurements for the axial trap frequency and 3.1% for the radial trap frequency.

<table>
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<th>Distance ( d ) (mm)</th>
<th>QUIC trap centre (mm)</th>
<th>( B_0 ) (G)</th>
<th>Gradient (G/cm)</th>
<th>Curvature (G/cm²)</th>
<th>( \omega_a/2\pi ) (Hz)</th>
<th>( \omega_r/2\pi ) (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>41.06</td>
<td>20.64</td>
<td>0.552</td>
<td>88.61</td>
<td>100.57</td>
<td>12.8</td>
<td>151.9</td>
</tr>
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</table>

Table 4.2: Numerical calculation of the QUIC trap frequencies. The calculation shows a deviation of 0.43% for the axial trap frequency and 3.1% for the radial trap frequency compared with the measured ones.

We also measure the storage time in the QUIC trap. The atom numbers in Fig. 4.7 (b) are determined by the absorption imaging method. The data fit well with
an exponential decay curve that yields a trap lifetime of \((224 \pm 7)\) seconds. An exponential decay curve is expected for trap losses dominated by background gas collisions. The trap lifetime is an important factor for magnetic trapping as it limits the time available for evaporative cooling.

![Figure 4.12: A functional schematic of the RF evaporative cooling process.](image)

Figure 4.12: A functional schematic of the RF evaporative cooling process. Radio frequency photons can create a position-dependent resonance condition between the \(M_F\) states, which enforces atoms with higher energy to flip their spins into untrapped states.

### 4.6 Evaporative cooling

#### 4.6.1 RF-induced evaporation

The purpose of evaporative cooling is to reduce further the temperature of the cloud down to the critical temperature \(T_c\) at which the quantum phase transition appears. For further cooling of the atoms we use RF-induced evaporation that proceeds by lowering the depth of the confining potential. While the process of evaporation allows atoms with high kinetic energy to escape from the potential, the remaining atoms acquire a lower temperature and simultaneously higher phase space density through rethermalization. RF radiation frequency \(\nu_{rf}\) is adjusted to be resonant with the Zeeman splitting of the \(|F = 2\rangle\) state. Only the \(|F = 2, M_F = 2\rangle\) state atoms with energy \(E > h(\nu_{rf} - \nu_0)\) can undergo a RF-induced spin flip transition, where \(\nu_0\) denotes the radio frequency corresponding to the bottom energy of the magnetic trap. These atoms eventually reach the untrapped states \(|F = 2, M_F = 0, -1, -2\rangle\) through the \(|F = 2, M_F = 1\rangle\) state and are ejected from the trap. Continued efficient evaporative cooling requires that the elastic collision rate responsible for the rethermalization of the cloud should overcome the inelastic collision rate which leads to trap loss. In general, the larger the ratio of
4.6. Evaporative cooling

the rethermalization rate to loss rate, the more efficient evaporation can be [94] (see section 2.8). The other important quantity in evaporative cooling is the phase space density $\rho = n\lambda_d^3$. From eq. 2.61 the phase space density for $^{87}\text{Rb}$ can be expressed as

$$\rho = \frac{6.5675 \times 10^{-30} n}{T^{3/2}} \quad (4.1)$$

where $n$ is the density expressed in particles/m$^3$ and $T$ the temperature in Kelvin. For a dilute gas in a harmonic trap, the peak phase space density is given by $\rho_0 = N\left(\frac{\hbar \omega}{k_B T}\right)^3$ as described in section 1.2. For evaporative cooling to remain efficient as the number of atoms $N$ decreases, the elastic collision rate ($\gamma_{el} \propto N/T$) should increase, which is called runaway evaporation [75]. This gives a condition on the exponent in a relation of $N \propto T^s$ [95] as

$$s = d \frac{\log_{10} N}{\log_{10} T} < 1. \quad (4.2)$$

When this condition is fulfilled, there can be runaway evaporative cooling over a large range of $N$ and $T$. The smaller the parameter $s$ is, the better the evaporation efficiency achieved [96, 97]. For programming in labVIEW, the sequence of evaporative cooling is divided into several stages. For each evaporation phase, the time dependent form of RF frequency is

$$\nu(t) = \left(\nu_{\text{start}} - \nu_0\right)\left(\nu_{\text{end}} - \nu_0\right)^{t/t_{\text{end}}} + \nu_0 \quad (4.3)$$

where $\nu_{\text{start}}$ and $\nu_{\text{end}}$ are the beginning and end frequency of the given phase respectively, and $t_{\text{end}}$ is the duration of that phase.

4.6.2 RF signal ramp and the results of evaporative cooling

The RF field is generated by a Stanford Research Systems function generator and transferred to the two-turned RF coil which is mounted just outside the glass cell. As preparation for the evaporative cooling, we need to estimate the range of RF cut frequencies applied during evaporation by determining the top and bottom position of atoms within the magnetic potential. Fig. 4.13 (a) shows a plot of the number of atoms as a function of the end frequency of the RF cut. The data indicate that the top RF frequency is near 22.5 MHz, below which a trapped cloud starts to be cut by the evaporation process. We also observed that atoms completely disappear from the QUIC trap at 0.388 MHz as shown in Fig. 4.13 (b), which is measured by varying the end frequency of the evaporation ramp. This frequency corresponds to 0.554 G, which is the bias field of our QUIC trap. During evaporation we observed that the current instability in the curvature coil is 4.8 mA, which induces a variation of about 3.1 mG of the bias field. So the measurement of the bottom frequency has an error of about 1%. The RF cut ramp used during evaporative
Figure 4.13: The number of atoms in the QUIC trap as a function of the end RF frequency. (a) Observation of a top RF frequency below which a cloud starts to be evaporated. (b) Bottom frequency at which a cloud completely disappears from the QUIC trap. The measured bottom frequency $\nu_0 = 0.388$ MHz corresponds to the field of the trap bottom $B_0 = 0.554$ G.

cooling is composed of 10 phases with exponential frequency modulation that lead to the BEC. Each evaporation phase is optimized by observing the changes in the phase space density and the exponent $s$ to achieve runaway evaporation as RF evaporation parameters of the given phase are varied. The typical values of the parameters used at various stages of evaporation are given in Fig. 4.14 (a). The size of a magnetically transported cloud is too large in comparison with our CCD camera chip size under the imaging system of $M = 3.55$. Thus, to find the optimization paths for runaway evaporation we use two magnifications; one part (scheme I) of evaporative cooling with $M = 0.5$ is performed from 22.5 MHz to 2 MHz whereas the other part (scheme II) of evaporative cooling with $M = 3.55$ is performed from 2 MHz to 0.38 MHz.

For scheme I, with an objective camera lens of $M = 0.5$ and initial MOT atom number of $1 \times 10^8$, we try to optimize each RF evaporation step. The first RF sweep phase starts from 22.5 MHz and ramps down exponentially to 11 MHz with an amplitude of $V_{pp} = 4.48$ V during 23 s. Then a series of exponential ramps are performed to 2 MHz according to the variations of parameters given in Fig. 4.14 (a). The evaporation results are plotted in Fig. 4.15. Fig. 4.15 (a) shows the steady decrease of the temperature ($T$) of the cloud as evaporation progresses. Also Fig. 4.15 (b) represents the evolution of atom number against evaporation time. Fig. 4.15 (c) shows dependence of $\log_{10}(\rho)$ on evaporation time. As evaporation
4.6. Evaporative cooling

Figure 4.14: RF signal of evaporation. According to the table in (a), final frequency (b) and amplitude (c) of each RF cut phase are varied as a function of time.
Figure 4.15: Evolution of characteristic parameters during the initial evaporation ramps (scheme I) between 22.5 and 2 MHz: (a) temperature \( T \) versus evaporation time, (b) atom number \( N \) versus evaporation time, and (c) \( \log_{10}(\text{phase space density}) \) versus evaporation time. (d) Plot of phase space density versus atom number.
4.6. Evaporative cooling

Figure 4.16: (a) Plot of $\ln \rho$ versus $\ln N$, showing the increase in the phase space density against the decrease in the number of trapped atoms. (b) Plot of $\log_{10} N$ versus $\log_{10} T$, showing evolution of the number $(N)$ of trapped atoms as the temperature $(T)$ of the cloud decreases.

progresses, the phase space density $(\rho)$ increases steadily. The plot in Fig. 4.15 (d) shows a general trend of the phase space density against the atom number of the evaporative cooling ramps in scheme I. The phase space density starts from $\rho = 1 \times 10^{-6}$. It continuously increases and reaches $\rho = 6 \times 10^{-5}$ in the sixth RF sweep phase. We estimate the evaporation efficiency [94] from the relative increase in the phase space density with the decrease of atom number as shown in Fig. 4.16 (a). A linear fit to the data gives a slope of $\frac{d \ln \rho}{d \ln N} = -2.1 \pm 0.2$, representing the efficiency value [75] of our RF evaporation sweep in scheme I. Fig. 4.16 (b) illustrates the phase diagram of $\log_{10} N$ versus $\log_{10} T$. A linear fit to the data with a slope of $s = \frac{d \log_{10} N}{d \log_{10} T} = 0.79 \pm 0.05$ indicates that the temperature decreases faster than linear with the number of atoms and runaway evaporation is clearly maintained.

4.6.3 BEC signatures

For scheme II, with an imaging magnification of $M = 3.55$ and initial MOT atom number of $1.5 \times 10^8$, we optimize RF ramp phases between 2 and 0.39 MHz during 10 s, leading to the quantum degeneracy regime. Each RF sweep phase ramps down exponentially according to the variations of frequency and amplitude as given in Fig. 4.14 (b) and (c). The results of the evaporation ramp are given in Fig. 4.17. Fig. 4.17 (a) illustrates evolution of the temperature against evaporation time,
Figure 4.17: Evolution of characteristic parameters during the evaporation ramps (scheme II) between 2.2 and 0.39 MHz: (a) temperature ($T$) versus evaporation time, (b) atom number ($N$) versus evaporation time, (c) elastic collision rate versus evaporation time, and (d) $\log_{10}$ (phase space density) versus evaporation time. (e) Plot of phase space density versus atom number.
4.6. Evaporative cooling

showing the continuous decrease in temperature. Dependence of the number \( N \) of trapped atoms on the evaporation time is shown in Fig. 4.17 (b). Fig. 4.17 (c) and (d) show dependence of the elastic collision rate and \( \log_{10}(\rho) \) on the evaporative cooling time, respectively. As evaporative cooling progresses further, the elastic collision rate and phase space density increase rapidly after an evaporation time of 55 s. A plot of the phase space density against the number of atoms during the evaporation process is shown in Fig 4.17 (e).

Figure 4.18: (a) Plot of \( \ln \rho \) versus \( \ln N \) showing continuous increase in the phase space density against decrease in the number of trapped atoms. (b) Phase diagram of the number \( N \) of trapped atoms versus the temperature \( T \) of the cloud at different evaporation ramps, showing an efficient evaporation and the crossing of the BEC transition boundary.

Fig. 4.18 (a) shows a graph of \( \ln \rho \) versus \( \ln N \). A linear fit to the data with a slope of \( d \ln \rho / d \ln N = -3.4 \pm 0.2 \) shows that scheme II gives better evaporative cooling efficiency than scheme I. The evaporation efficiency is similar to optimized values from other groups [98]. Furthermore, Fig. 4.18 (b) illustrates the phase diagram of \( \log_{10} N \) versus \( \log_{10} T \) where the shaded area represents the quantum degenerate regime. A linear fit to the data with a slope of \( s = d \log_{10} N / d \log_{10} T = 0.69 \pm 0.04 \) indicates that efficient runaway evaporation is fulfilled. This slope is similar to ones reported by other groups [96, 97]. The phase space density steadily approaches and crosses the BEC transition boundary near a temperature 200 nK and atom number of \( 2.8 \times 10^5 \).

In the final stage of the RF ramp we observed a dominant increase of the phase space density near 0.4 MHz and a reversal of shape characteristic for a BEC through
Figure 4.19: Absorption images (left) and corresponding optical density plots (right) showing BEC formation during the last stage of RF evaporation. (a) and (b) show that the cloud is close to the condensation threshold at a final frequency of 0.41 MHz. (c) and (d) show that the cloud is below the condensation threshold at a final frequency of 0.40 MHz. (e) and (f) show a nearly pure condensate at a final frequency of 0.39 MHz. Each image is taken after 16 ms time of flight.
4.6. Evaporative cooling

absorption imaging taken after 16 ms time of flight. The spatial distribution pictures of atoms in Fig. 4.19 show the emergence of a condensate as the final RF frequency is lowered during the last phase of RF evaporation. Final RF frequencies from top to bottom are 0.41 MHz, 0.40 MHz, and 0.39 MHz. Pictures in (a) and (b) for the final frequency of 0.41 MHz show a very cold thermal cloud ($N = 6.2 \times 10^5$, $T = 400$ nK, $n = 1.2 \times 10^{13}$ cm$^{-3}$, and $\rho = 0.3$), which is close to the condensation threshold. Pictures in (c) and (d) for 0.40 MHz show that a condensate appears in the centre of the thermal cloud. Pictures in (e) and (f) for 0.39 MHz are well below the BEC threshold and show that most of the atoms are in the condensate. For the final RF frequency of 0.39 MHz, the number of atoms ($N$), number density ($n$), critical temperature ($T_c$), temperature ($T$), and phase space density ($\rho$) are given in Table 4.3.

<table>
<thead>
<tr>
<th>Final frequency</th>
<th>$N$</th>
<th>$n$</th>
<th>$T_c$</th>
<th>$T$</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.39 MHz</td>
<td>$1.7 \times 10^5$</td>
<td>$2.9 \times 10^{13}$ cm$^{-3}$</td>
<td>160 nK</td>
<td>90 nK</td>
<td>7</td>
</tr>
</tbody>
</table>

Table 4.3: Characteristic parameters at the final RF frequency of 0.39 MHz.

The dramatic shape change at the BEC phase transition is further demonstrated in Fig. 4.20, showing cross sections of the clouds. The broad distribution corresponds to the thermal cloud, and the dense narrow distribution corresponds to the condensate. The thermal cloud is well fitted by a single Gaussian profile above the BEC transition temperature as shown in Fig. 4.20 (a). Below the BEC transition temperature, a bimodal distribution is clearly seen in the cross section of the expanded cloud and is well fitted by a combination of a Gaussian profile for the thermal cloud and an inverted parabola profile for the condensate as shown in Fig. 4.20 (b). The observed narrow peak is due to the dominance of interaction in the sample (see eq. 1.26). The narrow density distribution as shown in Fig. 4.20 (c) shows that only a small thermal cloud remains and is well below the critical temperature. The narrowing of the density distribution and changing from the Gaussian profile into the inverted parabolic profile is a behaviour characteristic of the BEC phase transition.
Figure 4.20: Cross sections of images given in Fig. 4.19, showing curves fitted by a Gaussian profile (green curve) and an inverted parabola (red curve). These are the curves expected for the thermal cloud and the condensate respectively.
Chapter 5

Conclusion and outlook

5.1 Summary

The fundamental goal of this thesis was to build a new apparatus in which the atoms are moved efficiently out of the region of the laser beams used to form the second magneto-optical trap (MOT). In order to do that, we studied the evolution of the trapped atomic cloud in the $|F = 2, M_F = 2\rangle$ state as the magnetic potential changes through the different stages of magnetic transfer and magnetic trapping, both experimentally and by numerical calculation. We use a double MOT system where a pyramidal MOT acts as a source of cold atoms for a second MOT in the UHV region. The magnetic trap setup is a modified quadrupole and Ioffe trap configuration (QUIC trap), which consists of two pairs of overlapping quadrupole coils and one Ioffe coil. With a transfer efficiency of 38% from the second MOT to the QUIC trap, this setup is capable of magnetic transport of cold atoms over a distance of 20 mm into the QUIC trap. The QUIC trap forms a harmonic potential with a radial trapping frequency of $\omega_r = 2\pi \times (147.43 \pm 0.4)$ Hz and an axial trapping frequency of $\omega_a = 2\pi \times (12.74 \pm 0.01)$ Hz, respectively. The atomic cloud released from the magnetic trap is probed by absorption imaging from which we derive the number ($N$) of trapped atoms, temperature ($T$), and phase space density ($\rho$). Evaporative cooling is performed as the RF frequency is swept from 22.5 MHz to 0.39 MHz over a period of 59 s. We demonstrated efficient runaway evaporation characterized by efficiency parameters of $-d\ln \rho/d\ln N = 2.7$ and $d\log_{10} N/d\log_{10} T = 0.7$. Through absorption images taken after 16 ms time of flight we observed a dramatic increase in the phase space density near a final evaporation frequency of 0.4 MHz and a condensate peak which appears in the centre of the thermal cloud. At a final frequency of 0.39 MHz we produced a nearly pure Bose-Einstein condensate at 90 nK with $1.7 \times 10^5$ atoms.
The main goals of magnetic transport and BEC have been successfully demonstrated in the new apparatus. The experimental setup allows for good optical access, so we expect this could be an avenue for further experiments aiming to generate a quantum state in an optical lattice potential with precisely one atom in the ground level of each potential well. The quantum state will be a good starting point for direct quantum simulation and neutral atom quantum computation.

New ideas have been developed in the course of building our system. In the light of experimental experience, things to be modified and schemes proposed for future work are as follows.

1. **The source of cold atoms**

   Although it has not been widely adopted, the pyramidal MOT is a reliable apparatus and has worked well in Oxford. The main efforts were in the initial design and construction of the glass blocks of the pyramid mirrors, but the plans are now available commercially to make them straightforwardly. The dielectric coating of the blocks was carried out in the thin film coating facility of the physics department in Oxford. The flux of cold atoms from the pyramidal MOT can be increased by increasing the vapour pressure of rubidium in the source region. The physical limit to what can be achieved in this way arises when the time between collisions is comparable with the time necessary to capture the atoms and to push them through the differential pumping hole into the region of UHV. There is some evidence from our experiment that the Rb getter sources last several years when starting at a heating current of 2.2 A which gradually increased to 4.5 A. In the actual experiments at Oxford we do not use a maximum possible flux because of concerns about rubidium building up in the UHV region and also shortening of the lifetime of the rubidium getter sources by running at higher currents. There has been little incentive to decrease the loading time to be much less than the time for evaporation. However, the apparatus described below should enable us to achieve a much shorter experimental cycle time in the future. Only a single beam is used in the pyramidal MOT set up and this can be scaled up to a larger size than the pyramid with the 6 cm base used in this work. To exploit the potential of the larger MOT requires a higher total power from the lasers which will be accomplished by using the master-oscillator power amplifier (MOPA) chips with output powers of 1 W rather than 100 mW slave lasers. With this order of magnitude increase in laser power and larger size in capture region it is expected to achieve at least an order of magnitude improvement in the cold atom flux for the same vapour pressure in the source region and hence to reduce the loading time by the same factor. This should enable the cycle time between each Bose-Einstein condensate to be about 10 s which will greatly increase the rate at which data can be taken, and should also improve the reproducibility since drifts are less likely during the experimental runs.

2. **Magnetic transport scheme**

   This subsection considers a moving-magnetic trap scheme in which the atoms are
moved while they are magnetically trapped. In this magnetic transport system the coils generating a quadrupole field are physically moved on a translation stage. This method was first used in JILA [39] and is being set up by other members of our group in Oxford. In implementation of the transport scheme the atoms have a journey time of about 3 s and generally the experiments repeat every minute giving a low duty cycle so that there is time for the magnetic coils to cool down. If the time between repetitions is reduced to 10 s, then the magnetic transport coils would require more efficient cooling systems, and would probably need to be made more robust to achieve an acceptable cooling time. Such issues do not arise in the transfer scheme which is a simple extension of the double-MOT arrangement used in the first BEC experiment at JILA and used since the beginning of this work in Oxford. Thus, reduction of the experimental cycle time can be obtained by making a larger pyramidal MOT and using a more powerful laser amplifier while keeping the source vapour pressure the same. It seems unlikely that the background pressure of rubidium in the final trapping region will be significantly higher than in present experiments, assuming that the majority of atoms which find their way from the source to the other end of the apparatus are thermal atoms rather than the cold atoms.

3. Imaging

Figure 5.1: Schematic diagram to detect a single atom in the atomic lattice.

The plan for future work is to use a commercial microscope objective from Nikon (CFI Plan Fluor ELWD 40). This has a numerical aperture $NA = 0.6$ corresponding to an optical resolution of 1 wavelength, i.e. 780 nm in our case. Importantly this lens can be adjusted to compensate for up to 2 mm of glass thickness which is the wall thickness of our current glass cell. Higher $NA$ lenses are commercially available in this range but these require a thinner window. Interestingly the UK company that fabricated our quartz cell, which trades both as Optiglass
and Starna, has developed a technique to put a small porthole into a glass wall. They developed this specifically for microscopy and thicknesses down to 0.2 mm and are available for flow cells suitable for studying biological specimens in water. However, a thickness of around 0.5 mm for a 5 mm diameter circular window is probably the realistic minimum for an evacuated cell. With a thinner cell, we could use a Nikon microscope objective with $NA = 0.85$ giving a theoretical resolution of 550 nm which is comparable with the spacing in an optical lattice. The diffraction-limited optical system with high numerical aperture (e.g. $NA = 0.7$) gives light fields with features on the 1 µm scale at the position of the atoms. There are two main features of this arrangement. First, interference of beams produces a standing light wave resulting in a periodic potential that confines atoms [99]. Second, by using the lens to focus light from a diffractive-optical element, we can tailor light fields with structured intensity distributions [100, 99]. The efficient collection of the fluorescence gives sufficient signal-to-noise to detect single atoms with an electron multiplying CCD camera (e.g. Andor IXON). The finest optical lattice that can be formed by laser beams passing through the high $NA$ lens will have a spacing comparable with the optical resolution. Therefore, the high-resolution optical system will enable in-situ imaging of these multi-particle systems as shown in Fig. 5.1.

5.2 Outlook

There are a number of challenging experiments that can be realized when high-resolution imaging is installed on the new system. The priority will be to obtain a physical implementation of the famous Feynman’s quantum simulator\(^1\) [102] with a two dimensional optical lattice of ultra-cold atoms. This arrangement requires to load precisely one atom per minimum of an optical potential, which is figuratively similar to the shape of egg boxes. The optical lattice will be formed by laser beams transmitted through the objective lens and intersecting at the focal point. The spacing of the optical lattice potential formed by the interference of these beams can be varied by changing the angle between the beams. There is a compromise between having a larger spacing which makes it easier to resolve the atoms and having tight confinement of the atoms at each lattice site to maintain a reasonable value of the on-site interaction energy $U$ (see Appendix D). A method of making a lattice by sending 4 laser beams through a lens has already been used in Oxford’s research group to create a rotating lattice [99]. However, that experiment only has weak confinement from the magnetic trap along the optical axis of the lens. For the arrangement which we plan to make, it is necessary to have much tighter confinement such as that provided by an optical dipole force potential in a standing

\(^1\)In 1981 Feynman [101] conjectured that quantum computers are able to simulate other quantum systems more efficiently than classical computers.
wave light field. This should be possible and over 1.5 W of single-frequency light at 850 nm will be available for this experiment from a titanium-doped sapphire laser.

Atoms from BEC are so cold that, when they are loaded into an optical lattice, they occupy the lowest vibrational state. But, the statistical fluctuations of atom number per site is proportional to $\sqrt{N}$, where $N$ is the mean atom number [103, 104]. Quantum mechanical tunnelling allows the atoms to make a more uniform distribution through the lattice. In a Bose Einstein condensate, the tunnelling process dominates the behaviour of the atoms. This behaviour arises from competition between the atom’s tendency to tunnel through barriers of finite height and the repulsive interactions between two or more atoms when they occupy the same site. As the tunneling rate is reduced, a quantum phase transition to a Mott insulator (a sort of frustrated conductor which occurs because of strong interactions among particles) occurs, which was theoretically predicted by Jaksch et al. [35] in Innsbruck (see also Appendix D). They suggested that it should be possible to convert a weakly interacting Bose gas into a Mott insulator by loading neutral atoms into an optical lattice. The Mott insulator phase which corresponds to having a definite number of atoms at each lattice site was experimentally observed by a group in Munich [33]. They were able to convert a superfluid Bose Einstein condensate into a Mott insulator by controlling the depth of the optical lattice. However, nonideal conditions such as missing atoms and overloaded sites resulted in site occupation defects in that phase. To prepare deterministically a single atom in each well of the optical lattice potential, an alternative method is to use a strong repulsive on-site interaction between atoms. This idea was theoretically suggested by Rabl et al. in Innsbruck [105] (see Appendix E). This so-called Rabl scheme begins with an uncertain number of atoms in an initial state $|a\rangle$ at each optical lattice site. Then an adiabatic transfer process is applied to drive atoms to a different state $|b\rangle$ where the on-site interaction energy $U$ between two atoms is so strong that it is energetically unfavorable to have more than one atom in the final state. The on-site interaction can be made large by exploiting a Feshbach resonance. Feshbach resonances are the resonant enhancements of the interactions between pairs of atoms at certain magnetic fields (see Appendix C). For example, the $|F = 1, M_F = 1\rangle$ state of $^{87}$Rb has a resonance at 1007.4 G. Note that a state with precisely one atom in the ground level of each potential well in the optical lattice potential, i.e. an atomic crystal structure containing exactly one atom per lattice site, is an ideal initialization for direct quantum simulation of 2-dimensional condensed matter systems and neutral atom quantum computing.
APPENDIX A

The physical properties of a $^{87}\text{Rb}$ atom

Physical properties of $^{87}\text{Rb}$ relevant to this experiment are given in below Table [50, 81]. All the spectroscopic quantities are quoted for the $D_2$ ($5^2S_{1/2} \rightarrow 5^2P_{3/2}$) transition of $^{87}\text{Rb}$ used in this experiment.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atomic number / Nuclear spin</td>
<td>$Z / I$</td>
<td>$37 / \frac{3}{2}$</td>
</tr>
<tr>
<td>Mass</td>
<td>$m$</td>
<td>$1.443,160,60(11)\times10^{-25}$ kg</td>
</tr>
<tr>
<td>Wavelength (vacuum)</td>
<td>$\lambda$</td>
<td>$780.241,209,686(13)$ nm</td>
</tr>
<tr>
<td>Wave number (vacuum)</td>
<td>$k/2\pi$</td>
<td>$12,816.549,389,93(21)$ cm$^{-1}$</td>
</tr>
<tr>
<td>Natural linewidth (FWHM) /</td>
<td>$\Gamma$</td>
<td>$2\pi \times 6.065(9)$ MHz</td>
</tr>
<tr>
<td>Decay rate</td>
<td></td>
<td>$3.811(6) \times 10^{6}$ s$^{-1}$</td>
</tr>
<tr>
<td>Lifetime</td>
<td>$\tau$</td>
<td>$26.24(4)$ ns</td>
</tr>
<tr>
<td>Saturation intensity ($\pi$-polarized)</td>
<td>$I_{sat}$</td>
<td>$2.503$ mW cm$^{-2}$</td>
</tr>
<tr>
<td>($F = 2 \rightarrow F' = 3$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Resonant cross section ($\pi$-polarized)</td>
<td>$\sigma_0$</td>
<td>$1.938 \times 10^{-9}$ cm$^2$</td>
</tr>
<tr>
<td>Saturation intensity ($\sigma^\pm$-polarized)</td>
<td>$I_{sat}$</td>
<td>$1.669(2)$ mW cm$^{-2}$</td>
</tr>
<tr>
<td>($</td>
<td>F = 2, \pm 2\rangle \rightarrow</td>
<td>F' = 3, \pm 3\rangle$)</td>
</tr>
<tr>
<td>Resonant cross section ($\sigma^\pm$-polarized)</td>
<td>$\sigma_0$</td>
<td>$2.907 \times 10^{-9}$ cm$^2$</td>
</tr>
<tr>
<td>Recoil velocity</td>
<td>$v_r = \hbar k/m$</td>
<td>$5.8845$ mm s$^{-1}$</td>
</tr>
<tr>
<td>Recoil temperature</td>
<td>$T_r = m v_r^2 / k_B$</td>
<td>$361.96$ nK</td>
</tr>
<tr>
<td>Zeeman shift of $5^2S_{1/2}$ $F = 2$</td>
<td>$1/2 \mu_B \times M_F$</td>
<td>$700$ kHz G$^{-1} \times M_F$</td>
</tr>
<tr>
<td>Scattering length at $B = 0$</td>
<td>$a_s$</td>
<td>$104.5a_0$</td>
</tr>
<tr>
<td>($</td>
<td>F = 2, M_F = 2\rangle$)</td>
<td></td>
</tr>
<tr>
<td>Frequency</td>
<td>$\omega_0/2\pi$</td>
<td>$384.230,484,468,5(62)$ THz</td>
</tr>
<tr>
<td>Transition energy</td>
<td>$\hbar\omega_0$</td>
<td>$1.589,049,439(13)$ eV</td>
</tr>
</tbody>
</table>
The acousto-optic modulators (AOMs, Crystal Technology 3080-122) are used to modulate, or control the intensity and frequency of laser beams, and to achieve fast switching. The AOM contains an optical transparent material (TeO₂) to which a piezo-electric transducer is attached. A RF signal generator drives the transducer to vibrate, which generates traveling acoustic waves in the material. The oscillating mechanical pressure of the sound waves induces variation in the index of refraction of the medium, so the AOM acts as a moving diffraction grating as depicted in Fig. B.1. The diffraction can be considered as a collision process between photons and phonons. From momentum conservation, we find \( k_{\pm 1} = k_0 \pm K \) where \( k_{+1}(k_{-1}) \) is the wave vector of the +1(-1) order diffracted light, \( k_0 \) the wave vector of the incident light, and \( K \) the wave vector of the acoustic wave. Similarly application of energy conservation yields \( \omega_{\pm 1} = \omega_0 \pm \Omega \) where \( \omega_{+1}(\omega_{-1}) \) is the angular frequency of the +1(-1) order diffracted light, \( \omega_0 \) the angular frequency of the incident light, and \( \Omega \) the angular frequency of the acoustic wave. In general the incident beam is diffracted into several orders and these parameters are related by the Bragg angle \( \Theta_B \) as shown in Fig. B.1, such that

\[
\sin \Theta_B = \frac{mK}{2k_0} = \frac{m\lambda}{2\Lambda_0} \tag{B.1}
\]

where \( m = 0, \pm \) integer is the diffraction order, \( \lambda = \frac{2\pi}{k_0} \) the wavelength of the incident light and \( \Lambda = \frac{2\pi}{K} \) the wavelength of the acoustic wave. When the amplitude of the RF driver is reduced to zero, the incident power is transmitted into the zeroth order light with no deflection or frequency shift. We observed that about 80 percent of the incident light is diffracted into the first order by the AOM and its switching time is less than 5 \( \mu s \).
Appendix B. Acousto-Optic Modulator

Figure B.1: The operation of an AOM can be described in terms of photon-phonon collisions. The first order output beam is a diffraction order from a Bragg diffraction grating created by spatially periodic density variations. (a) The $m = +1$ diffraction order is Bragg-diffracted up. (b) The $m = -1$ diffraction order is Bragg-diffracted down.
A useful way to reach the strong interaction regime in cold alkali atoms is via magnetically tunable Feshbach resonances, leading to singularities of the s-wave scattering length $a_s$ between two colliding atoms. The resonant enhancement of the inter-particle collisional cross section is due to the existence of metastable molecular states. Whenever the interatomic interaction supports a weakly bound state, the scattering length $a_s$ is positive. Near a Feshbach resonance the scattering length [106, 107] is described by

$$a_s(B) = a_{bg} \left[1 - \frac{\Delta B}{B - B_0}\right]$$  \hspace{1cm} (C.1)

where $a_{bg}$ is the off-resonant background scattering length, $\Delta B$ is the field width of the resonance, and $B_0$ is the field position of the resonance (see Fig. C.1).

A Feshbach resonance in a two-particle collision in cold gases occurs when a quasi-bound state in a closed channel has an energy close to the scattering continuum of an entrance channel of two colliding atoms as illustrated in Fig. C.2. Consider a collision between two atoms. Before collision, the two atoms are prepared in the entrance channel, whose interaction potential $V_{en}(r)$ gives rise to the background scattering length $a_{bg}$. Here the zero of energy is chosen as the dissociation threshold of the entrance channel such that $V_{en}(\infty) = 0$. They undergo a collision at low incident energy. In the course of the collision, the entrance channel is coupled to the closed channel $V_{cl}(r)$ ($V_{cl}(\infty) > 0$). When a bound state of the closed channel has an energy close to zero, a scattering resonance occurs. The scattered particles are then temporarily captured in the quasi-bound state. The overall position of the closed channel can be shifted with respect to the dissociation threshold of the entrance one by varying the external magnetic field $B$. 

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In 2002, Marte et al. [108] observed 43 resonances in $^{87}$Rb atoms through a magnetic field range between 0.5 G and 1260 G. The widest resonance was found to be centred at 1007.4 G with a width of $\Delta B = 0.21$ G [107, 109]. The dispersive behaviour of the scattering length around the resonance described in eq. C.1 suggests that to be able to tune the magnetic field across the resonances gives an experimental handle on the scattering length, and hence on the interaction energy $U$. In the future it leaves open the possibility of exploring number squeezing experiments by acting on the magnitude of the interaction instead of the depth of the periodic potential in order to tune the parameter $U/J$. The second quadrupole coils (QD2) are also designed to convert a Helmholtz to an anti-Helmholtz configuration that generates a uniform magnetic field. For the anti-Helmholtz configuration, calculation shows that the magnetic field of 1007.4 G at the centre of the coils results from applying a current of 280.4 A per turn (see Fig. C.3) and it is quite homogeneous since its field variation is 1.1 mG/mm near the centre. By reversing the current in one of the two coils, the second quadrupole coils in our magnetic trapping apparatus will be able to provide the field for the Feshbach resonance experiments, assuming that we overcome the issues of current instability and excessive heating caused by high current.
Figure C.2: Scheme of the entrance (solid curve) and closed channel model for a Feshbach resonance (dashed curve).

Figure C.3: Magnetic field of $B = 1047$ G is produced from the configuration of Helmholtz coils with a current of 280.4 A per turn at the centre of B coils.
Quantum phase transition from a superfluid to a Mott insulator

Classical phase transitions are well known, the most familiar example being the melting of ice. At the melting point, thermal fluctuations drive the system from the liquid to the solid phase, or vice versa. An intriguing aspect of quantum mechanics is that quantum fluctuations prevail in a system even at a temperature of absolute zero because of Heisenberg’s uncertainty relation. Instead all thermal fluctuations are frozen out at zero temperature. These quantum fluctuations induce a macroscopic phase transition in the ground state of a many-body system. A quantum phase transition from a superfluid state to a Mott insulator state in an ultra-cold gas was first observed by Greiner et al. [33] through controlling the ratio between the interaction strength and the tunnel coupling in a three-dimensional optical lattice potential [33, 110]. In a superfluid, the atoms move in phase with one another, all part of a single macroscopic wave function. In a Mott insulator each lattice site is occupied by a single atom and the atoms are then isolated from each other. These two phases are described by the same Hamiltonian and are characterized by competition between two parameters: the tendency of the particles to hop into adjacent wells, and the inter-particle forces that keep them in separate wells. Depending on the relative strength of these two parameters, the system can go from superfluid to an insulator and back again much as ice melts and refreezes as the air gets warmer or cooler.

Let us consider a system where ultra-cold bosonic atoms populate the lowest energy band of an optical lattice. The bosonic system with repulsive interaction is qualitatively described by the Bose-Hubbard Hamiltonian

\[ H = -J \sum_{\langle i,j \rangle} a_i^\dagger a_j + \frac{1}{2} U \sum_i n_i(n_i - 1) + \sum_i (\varepsilon_i - \mu)n_i \]  

(D.1)
where $a_i^\dagger$ and $a_i$ are the creation and annihilation operators for a boson on the lattice site $i$, respectively, obeying the canonical commutation relations $[a_i, a_j^\dagger] = \delta_{ij}$. $n_i$ is the number operator counting the number of bosons at the $i$th lattice site. $J$ is the tunnel matrix element describing the strength of the tunnelling coupling between neighbouring sites and $U$ is the on-site matrix element describing the repulsion between two atoms on a single lattice site. The interaction between atoms is a short-range interaction and is thus well characterized by a pure on-site interaction term.

- The first term in the Bose-Hubbard Hamiltonian is the hopping term, describing the tunnelling of bosons between adjacent potential wells. The notation $<i,j>$ in the summation denotes all pairs of nearest neighbours. The tunnelling matrix element is given by $J = -\int w(r - r_i)[-\hbar^2 \nabla^2 / 2m + V_{Lat}(r)]w(r - r_j) d\mathbf{r}$, where $w(r - r_i)$ is the single particle Wannier function localized to the $i$th lattice site [111]. This kinetic energy term of the Hamiltonian characterizes the strength of the tunnelling coupling and tends to delocalize each atom over the periodic lattice potential $V_{Lat}(r)$.

- For $n_i$ atoms on the same site $i$, the second term of the Bose-Hubbard Hamiltonian describes the situation that each atom undergoes a purely on-site interaction with $n_i - 1$ other atoms. $U = \frac{4\pi \hbar^2 a_s}{m} \int |w(r - r_i)|^4 d\mathbf{r}$ quantifies the repulsive on-site interaction between two atoms, where $\hbar$ is Planck’s constant divided by $2\pi$, $m$ is the atomic mass, and $a_s$ is the $s$-wave scattering length. The interaction term is well described for ultra-cold neutral atoms in periodic potentials as the interactions between atoms are very short-ranged compared to the lattice spacing and no long-range Coulomb forces exist between the particles. When two atoms are placed on the same lattice site, they can interact with each other, leading to an interaction energy $U$.

- The third term describes an external confinement which gives rise to an energy offset $\varepsilon_i = V_{ext}(r_i)$ on the $i$th lattice site arising from an external trapping potential. Thus $\varepsilon_i$ is zero for a homogenous system. The last term also introduces the chemical potential $\mu$ which serves as a Lagrangian multiplier fixing the mean number of particles in the grand canonical ensemble. The effective local chemical potential $\mu_i = \mu - \varepsilon_i$ determines the filling factor on the individual lattice site $i$ varying locally as $\text{Mod}(\mu_i/U)$ where $\text{Mod}$ the modulo.

The Bose-Hubbard Hamiltonian of eq. D.1 has two distinct ground states depending on the strength of the interactions $U$ relative to the tunnelling coupling $J$. The parameter $U/J$ that characterizes the strength of the interaction relative to the tunnelling coupling between neighbouring lattice sites can be varied by changing the potential depth of the optical lattice. When the potential depth is increased,
Appendix D. Quantum phase transition from a superfluid to a Mott insulator

The tunnelling barrier between neighbouring lattice sites is raised and therefore the tunnelling matrix element $J$ decreases exponentially. The on-site interaction $U$ on the other hand is slightly increased in a deeper lattice due to a tighter confinement of the wave function on a lattice site. Therefore the ratio $U/J$ can be continuously adjusted over a wide range by changing the strength of the lattice potential.

D.1 Superfluid ground state limit ($U \ll J$)

For weak interactions relative to the kinetic energy ($U/J \ll 1$) such as a shallow optical lattice potential $U$, the system forms a Bose-Einstein condensed state of matter, where each atom is delocalized over the entire lattice. Such a state is favored as the kinetic energy term is minimized and single-particle wave functions spread out throughout the lattice. If the tunnelling matrix element $J$ is much larger than the on-site repulsion $U$, the Bose Hubbard Hamiltonian is dominated by the tunnelling term. In the superfluid regime each atom is then delocalized over the entire lattice and the many-body ground state can be described by the product of identical single particle Bloch wave functions with zero quasi-momentum of the lowest Bloch band from the periodic potential. For $N$ bosonic atoms with $M$ lattice sites with the vacuum, it reads as

$$|\Psi_{SF}\rangle \propto (\sum_{i=1}^{M} a_i^\dagger)^N |0\rangle = (a_1^\dagger + a_2^\dagger + \cdots + a_M^\dagger)^N |0\rangle. \tag{D.2}$$

The total system can be also described by a macroscopic matter wavefunction with a well defined and constant macroscopic phase at each lattice site. As a consequence, the atom number per site is uncertain and each site is filled with a random number of atoms according to Poisson statistics for the shallow depth lattice. The macroscopic phase coherence across the optical lattice can be detected by a time of flight imaging which reveals the momentum distribution in the lattice. In the case of a superfluid ensemble, upon sudden release of the matter wave from the optical lattice potential, this measurement yields a multiple matter wave interference pattern with discrete and narrow interference peaks.

D.2 Mott Insulator state limit ($U \gg J$)

A totally different regime is entered when the potential depth of the lattice is increased. For the case of the large on-site interactions relative to the kinetic energy, since the interaction energy of two particles increases with the square of the number of particle per well, particles minimize their interaction energy by staying away from each other over the whole lattice. Thus, the system enters the strongly correlated state of a Mott insulator, in which the atoms are localized, with a fixed particle number per site. Then fluctuations in the atom number on a single
lattice site become energetically costly and the ground state of the system instead consists of localized atomic wave functions to single lattice sites that minimize the interaction energy. For a commensurate filling of \( N/M \) atoms per lattice site, the ground state of the many-body system is a product of local Fock states with the atom number for each lattice site and is given by

\[
| \Psi_{MI} \rangle \propto \prod_{i=1}^{M} (a_i^\dagger)^{N/M} | 0 \rangle = (a_1^\dagger a_2^\dagger \ldots a_M^\dagger)^{N/M} | 0 \rangle. \tag{D.3}
\]

The atom number at each lattice site is well-defined and the particle number fluctuation is zero. On the other hand, the macroscopic phase has become maximally uncertain. Therefore, while macroscopic phase coherence between atoms on neighbouring lattice sites is completely lost in the system, perfect correlations in the atom number exist between lattice sites. In this regime of a strongly correlated Bose gas it is not possible to describe the many-body state as a macroscopic wave function anymore, and the system cannot be treated by the theories of Gross, Pitaevskii and Bogoliubov for a weakly interacting Bose gas. Therefore, in the Mott insulator system, no matterwave interference pattern can be observed upon releasing the particles from the lattice.
Rabl’s coherent filtering scheme

While it is possible to load a BEC into an optical lattice in the Mott Insulator phase, imperfections such as missing atoms and overloaded sites still exist and lead to occupation number defects in this phase. Besides technical difficulties and finite temperature effects, there are other defects of a more fundamental nature like non-commensurate particle and lattice site numbers or the additional harmonic trapping potential caused by the Gaussian shape of the laser beam. This section presents a coherent filtering scheme which dramatically reduces such site occupation number defects for atoms in an optical lattice by transferring exactly one particle per lattice site into a different internal state via adiabatic passage \cite{105}.

We begin by considering a system of bosonic atoms with two internal states $|a\rangle$ and $|b\rangle$ loaded into an optical lattice such that they do not tunnel between neighbouring sites. The atoms are initially in an internal state $|a\rangle$. They have on-site interaction strength $U = 4\pi \hbar^2 a_s \int dr|w(r)|^4/m$, where $a_s$ is the $s$-wave scattering length and $w(r)$ is a Wannier function. Here the interaction $U$ is assumed to be less than the separation of the two lowest motional states at each site. We then couple the atoms into a second internal state $|b\rangle$, which is trapped by a second lattice potential, via an off-resonant Raman transition of Rabi frequency $\Omega(t)$, which is detuned from state $|b\rangle$ by $\delta(t)$ as illustrated in Fig. E.1. If we denote the on-site interaction between particles in state $|a\rangle$ ($|b\rangle$) as $U_a$ ($U_b$) and the on-site interaction between particles in different internal states as $U_{ab}$ we can write the Hamiltonian as

$$H = U_a n_a (n_a - 1)/2 + U_b n_b (n_b - 1)/2 + U_{ab} n_a n_b - \Omega(t)[a^\dagger b + b^\dagger a]/2 - \delta(t)n_b \quad (E.1)$$

where $a$ and $b$ are the annihilation operators for particles in states $|a\rangle$ and $|b\rangle$, respectively, and $n_a = a^\dagger a$, $n_b = b^\dagger b$. We can then write the state of the site as $|n_a, n_b\rangle$, where $n_a$ and $n_b$ are the number of particles in the internal states $|a\rangle$ and
We know that the initial state at a particular lattice site is $|N, 0\rangle$. Our goal is to transfer exactly one particle from state $|a\rangle$ to state $n_b$, so that for any $N$, the final state is $|N - 1, 1\rangle$. In order to do this we must choose the initial and final values of the detunings, $\delta_1$ and $\delta_2$ so that the system evolves along the avoided crossing in the energy eigenvalues, undergoing an adiabatic passage from $|N, 0\rangle \to |N - 1, 1\rangle$, and does not evolve along any other such avoided crossings. These values have to be simultaneously chosen for all values of $N$ [112]. Once the system is in state $|N - 1, 1\rangle$, we can turn off the lattice trapping state $|a\rangle$. After the remaining particles are released, a pure state is left with a crystal-like structure containing exactly one atom per lattice site. These features will greatly enhance the applicability of atoms in optical lattices to quantum computation and to the precise modeling of condensed matter systems.

Figure E.1: Plot showing avoided crossings in the energy eigenvalues for an example of $N = 3$ as a function of the detuning $\delta$ (top). Arrows indicate the passage of the state transfer. Appropriate $\delta_1$ and $\delta_2$ (gray region) are selected so that we cross only from $|N, 0\rangle$ to $|N - 1, 1\rangle$. Rabl’s scheme of a single atom transfer (bottom).
Appendix F

Timing for the experiment

Figure F.1: Timing sequence of the experiment.
Bibliography


