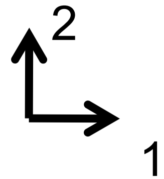
Key Questions about irreps

1. Are they a property of the group or of the space?
2. How many are they?
3. How can we “characterise” them, since for each there is clearly an infinite number of matrix irreps?
4. How can we construct all of them?
5. How can we decompose a *reducible* representation in its irreducible “components”?
6. Once we have an irrep, how can we construct the corresponding basis vectors?

Point group *32* – variant 1

	E	A	B	K	L	M
Γ_1	1	1	1	1	1	1
Γ_2	1	1	1	-1	-1	-1
Γ_3	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ +\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$	$\begin{pmatrix} -\frac{1}{2} & +\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$	$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} +\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$	$\begin{pmatrix} +\frac{1}{2} & +\frac{\sqrt{3}}{2} \\ +\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$

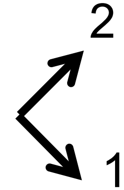


Point group *32* – variant 2

$$U = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

UMU^{-1}

	E	A	B	K	L	M
Γ_1	1	1	1	1	1	1
Γ_2	1	1	1	-1	-1	-1
Γ_3	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ +\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$	$\begin{pmatrix} -\frac{1}{2} & +\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	$\begin{pmatrix} +\frac{\sqrt{3}}{2} & +\frac{1}{2} \\ +\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$	$\begin{pmatrix} -\frac{\sqrt{3}}{2} & +\frac{1}{2} \\ +\frac{1}{2} & +\frac{\sqrt{3}}{2} \end{pmatrix}$

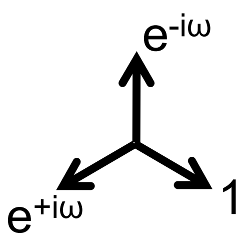


Point group *32* – variant 3

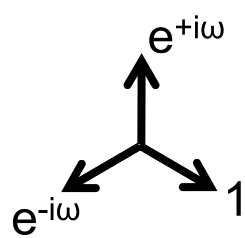
$$U = \frac{\sqrt{2}}{3}\begin{pmatrix} \frac{\sqrt{3}}{2}\left(1-e^{-i\omega}\right) & -\frac{1}{2}\left(1+e^{-i\omega}\right)+e^{+i\omega} \\ \frac{\sqrt{3}}{2}\left(1-e^{+i\omega}\right) & -\frac{1}{2}\left(1+e^{+i\omega}\right)+e^{-i\omega} \end{pmatrix}$$

UMU^{-1}

	E	A	B	K	L	M
Γ_1	1	1	1	1	1	1
Γ_2	1	1	1	-1	-1	-1
Γ_3	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} -\frac{1}{2}+\frac{i\sqrt{3}}{2} & 0 \\ 0 & -\frac{1}{2}-\frac{i\sqrt{3}}{2} \end{pmatrix}$	$\begin{pmatrix} -\frac{1}{2}-\frac{i\sqrt{3}}{2} & 0 \\ 0 & -\frac{1}{2}+\frac{i\sqrt{3}}{2} \end{pmatrix}$	$\begin{pmatrix} 0 & -\frac{1}{2}-\frac{i\sqrt{3}}{2} \\ -\frac{1}{2}+\frac{i\sqrt{3}}{2} & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & -\frac{1}{2}+\frac{i\sqrt{3}}{2} \\ -\frac{1}{2}-\frac{i\sqrt{3}}{2} & 0 \end{pmatrix}$



1



2

Point group *422* (one of the variants)

	E	2_z	4^+	4^-	2_x	2_y	2_{xy}	$2_{x\bar{y}}$
Γ_1	1	1	1	1	1	1	1	1
Γ_2	1	1	1	1	-1	-1	-1	-1
Γ_3	1	1	-1	-1	1	1	-1	-1
Γ_4	1	1	-1	-1	-1	-1	1	1
Γ_5	$\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$	$\begin{pmatrix}-1 & 0\\0 & -1\end{pmatrix}$	$\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}$	$\begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}$	$\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$	$\begin{pmatrix}-1 & 0\\0 & 1\end{pmatrix}$	$\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$	$\begin{pmatrix}0 & -1\\-1 & 0\end{pmatrix}$

Point group *32* – Character Table (full group)

	E	A	B	K	L	M
Γ_1	1	1	1	1	1	1
Γ_2	1	1	1	-1	-1	-1
Γ_3	2	-1	-1	0	0	0

Point group *32* – Character Table(classes)

	E	2A	3K
Γ_1	1	1	1
Γ_2	1	1	-1
Γ_3	2	-1	0

Point group *422* Character Table

	E	2_z	$2\ 4^+$	$2\ 2_x$	$2\ 2_{x\bar{y}}$
Γ_1	1	1	1	1	1
Γ_2	1	1	1	-1	-1
Γ_3	1	1	-1	1	-1
Γ_4	1	1	-1	-1	1
Γ_5	2	-2	0	0	0

$$[A] = \begin{bmatrix} . & . & 1 & . & . & . \\ 1 & . & . & . & . & . \\ . & 1 & . & . & . & . \\ . & . & . & . & . & 1 \\ . & . & . & 1 & . & . \\ . & . & . & . & 1 & . \end{bmatrix} [K] = \begin{bmatrix} . & . & 1 & . & . & . \\ . & 1 & . & . & . & . \\ 1 & . & . & . & . & . \\ . & . & . & . & . & -1 \\ . & . & . & . & -1 & . \\ . & . & . & -1 & . & . \end{bmatrix}$$

$$[B] = \begin{bmatrix} . & 1 & . & . & . & . \\ . & . & 1 & . & . & . \\ 1 & . & . & . & . & . \\ . & . & . & . & 1 & . \\ . & . & . & . & . & 1 \\ . & . & . & 1 & . & . \end{bmatrix} [L] = \begin{bmatrix} 1 & . & . & . & . & . \\ . & . & 1 & . & . & . \\ . & 1 & . & . & . & . \\ . & . & . & -1 & . & . \\ . & . & . & . & . & -1 \\ . & . & . & . & -1 & . \end{bmatrix}$$

$$[E] = \begin{bmatrix} 1 & . & . & . & . & . \\ . & 1 & . & . & . & . \\ . & . & 1 & . & . & . \\ . & . & . & 1 & . & . \\ . & . & . & . & 1 & . \\ . & . & . & . & . & 1 \end{bmatrix} [M] = \begin{bmatrix} . & 1 & . & . & . & . \\ 1 & . & . & . & . & . \\ . & . & 1 & . & . & . \\ . & . & . & . & -1 & . \\ . & . & . & -1 & . & . \\ . & . & . & . & . & -1 \end{bmatrix}$$

D_3 (32)

Applied first →

Applied second ↑

	E	A	B	K	L	M
E	E	A	B	K	L	M
A	A	B	E	L	M	K
B	B	E	A	M	K	L
K	K	M	L	E	B	A
L	L	K	M	A	E	B
M	M	L	K	B	A	E

D_3 (32)

Applied first →

Applied second ↑

	E	B	A	K	L	M
E	E	B	A	K	L	M
A	A	E	B	L	M	K
B	B	A	E	M	K	L
K	K	L	M	E	B	A
L	L	M	K	A	E	B
M	M	K	L	B	A	E