

Symmetry in Condensed-Matter Physics Lecture 3

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Table 1: Matrix representation of the representation of point group 422 onto the space of 3-dimensional vectors, using the usual Cartesian basis set $[\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}]$.

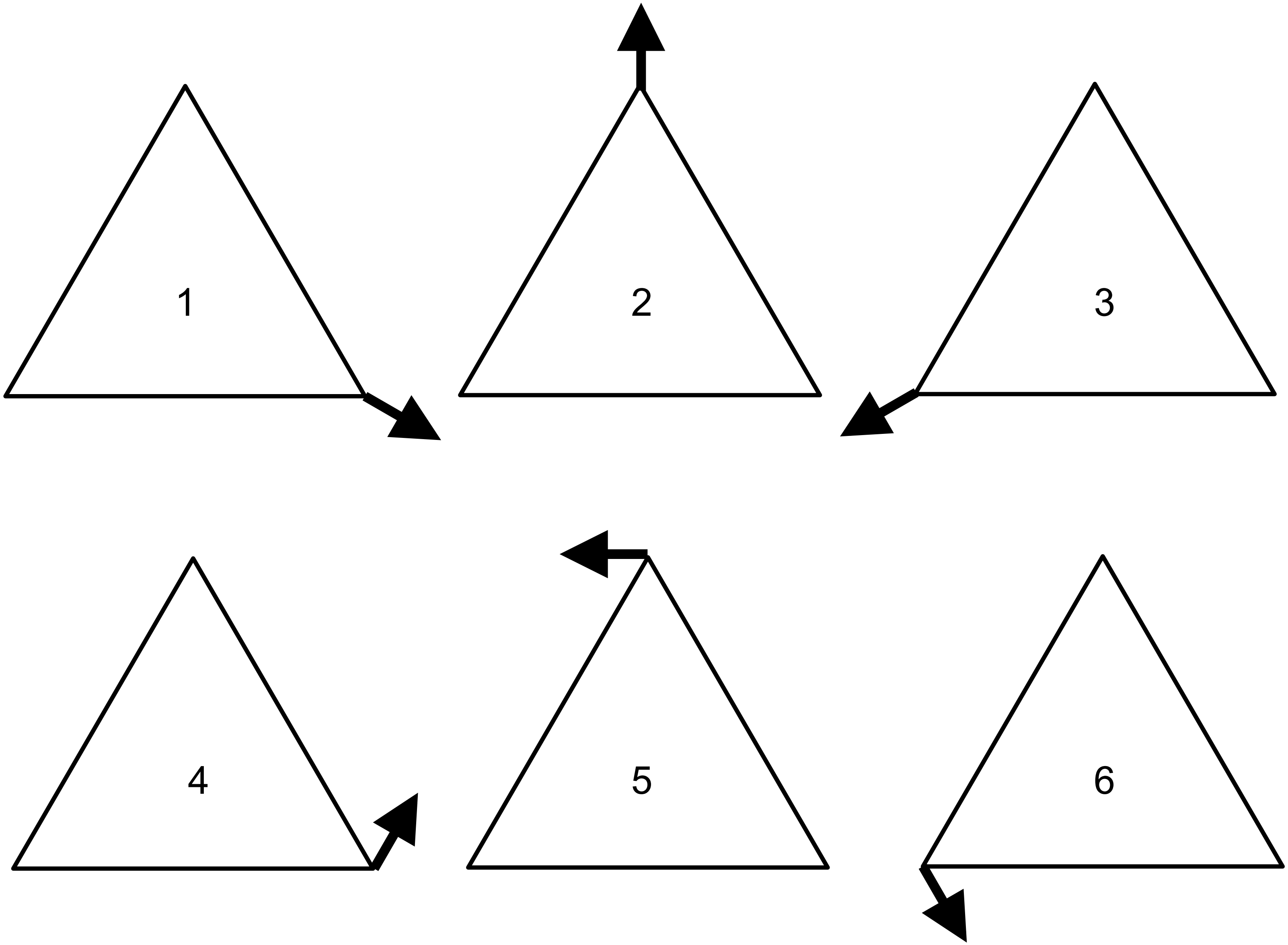
E	2_z	4^+	4^-
$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

2_x	2_y	2_{xy}	$2_{x\bar{y}}$
$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

Table 2: Matrix representation of the representation of point group 422 onto the space of 3-dimensional vectors, using the basis set $[\hat{\mathbf{i}} + \hat{\mathbf{j}}, -\hat{\mathbf{i}} + \hat{\mathbf{j}}, \hat{\mathbf{k}}]$.

E	2_z	4^+	4^-
$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

2_x	2_y	2_{xy}	$2_{x\bar{y}}$
$\begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$


1
2
3
4
5
6

$$[A] = \begin{bmatrix} . & . & 1 & . & . & . \\ 1 & . & . & . & . & . \\ . & 1 & . & . & . & . \\ . & . & . & . & . & 1 \\ . & . & . & 1 & . & . \\ . & . & . & . & 1 & . \end{bmatrix} \quad [K] = \begin{bmatrix} . & . & 1 & . & . & . \\ . & 1 & . & . & . & . \\ 1 & . & . & . & . & . \\ . & . & . & . & . & -1 \\ . & . & . & . & -1 & . \\ . & . & . & -1 & . & . \end{bmatrix}$$

$$[B] = \begin{bmatrix} . & 1 & . & . & . & . \\ . & . & 1 & . & . & . \\ 1 & . & . & . & . & . \\ . & . & . & . & 1 & . \\ . & . & . & . & . & 1 \\ . & . & . & 1 & . & . \end{bmatrix} \quad [L] = \begin{bmatrix} 1 & . & . & . & . & . \\ . & . & 1 & . & . & . \\ . & 1 & . & . & . & . \\ . & . & . & -1 & . & . \\ . & . & . & . & . & -1 \\ . & . & . & . & -1 & . \end{bmatrix}$$

$$[E] = \begin{bmatrix} 1 & . & . & . & . & . \\ . & 1 & . & . & . & . \\ . & . & 1 & . & . & . \\ . & . & . & 1 & . & . \\ . & . & . & . & 1 & . \\ . & . & . & . & . & 1 \end{bmatrix} \quad [M] = \begin{bmatrix} . & 1 & . & . & . & . \\ 1 & . & . & . & . & . \\ . & . & 1 & . & . & . \\ . & . & . & . & -1 & . \\ . & . & . & -1 & . & . \\ . & . & . & . & . & -1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{4} & \frac{3}{4} & 0 & \frac{\sqrt{3}}{4} & 0 & 0 \\ \frac{3}{4} & \frac{1}{4} & 0 & -\frac{\sqrt{3}}{4} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -\frac{\sqrt{3}}{2} & +\frac{\sqrt{3}}{2} & 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{4} & \frac{3}{4} & \frac{\sqrt{3}}{4} & 0 & 0 & 0 \\ \frac{3}{4} & \frac{1}{4} & -\frac{\sqrt{3}}{4} & 0 & 0 & 0 \\ -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$$

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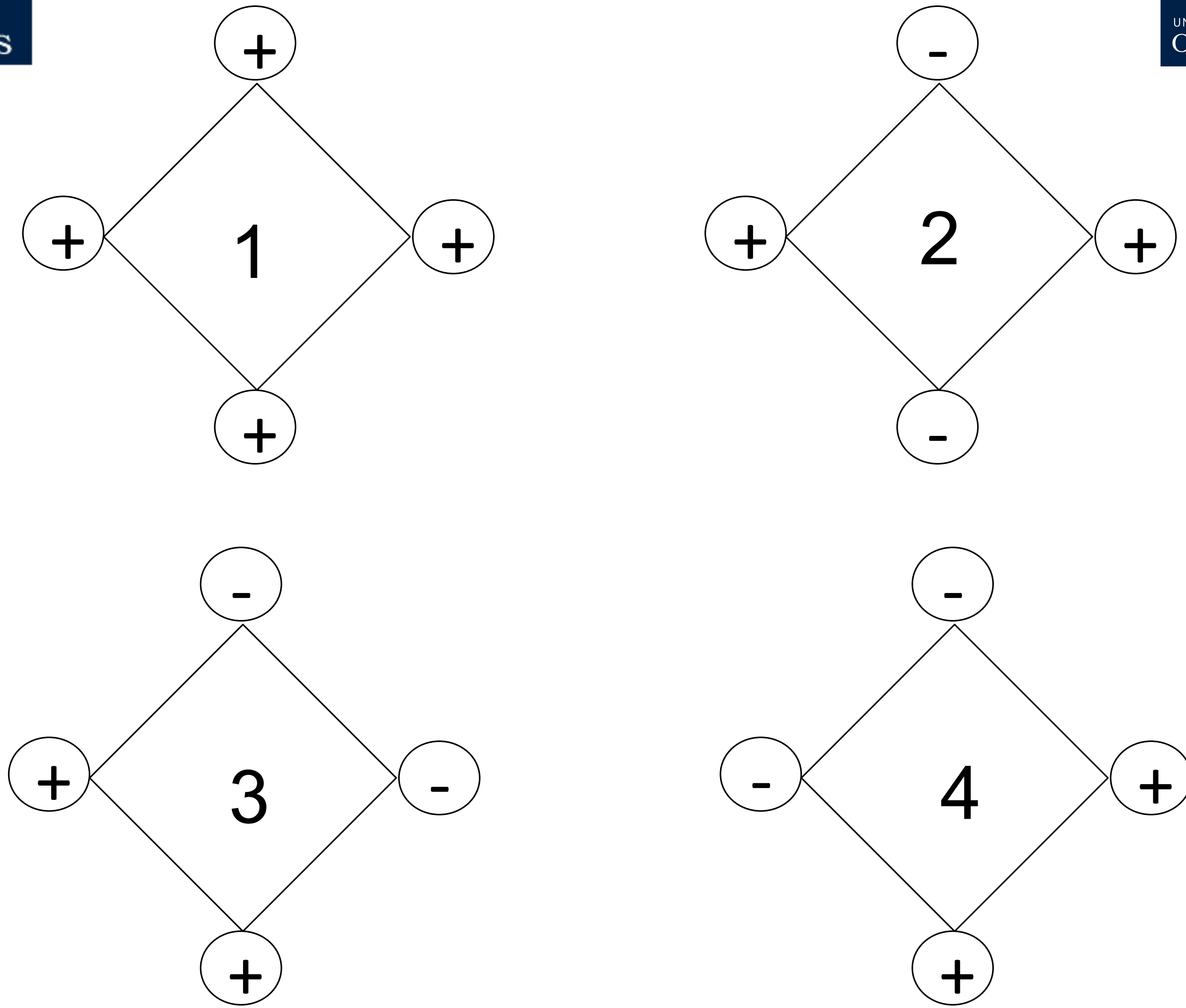
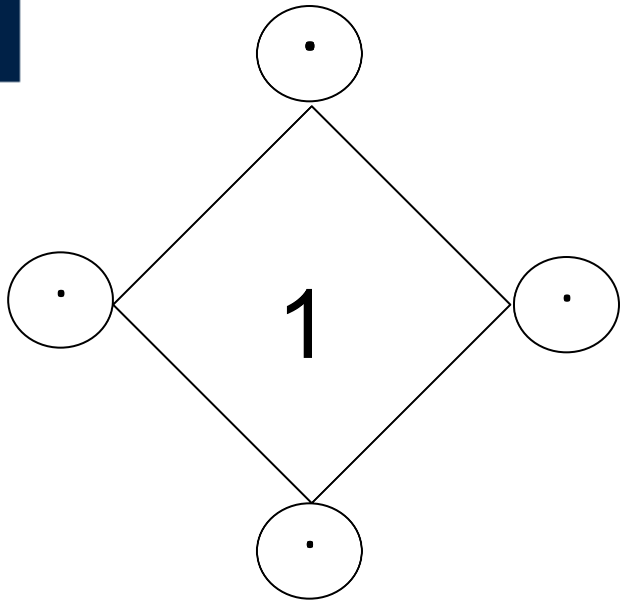
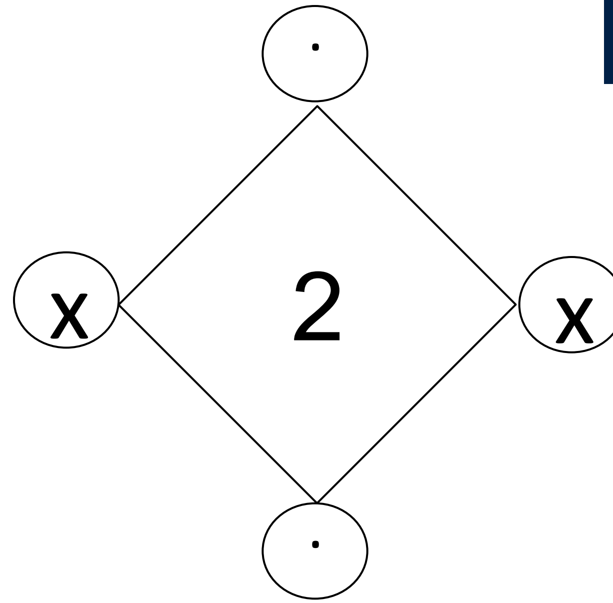
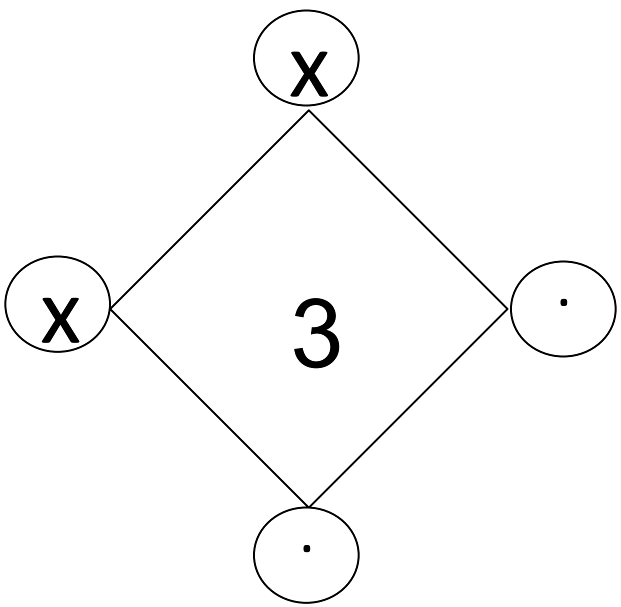


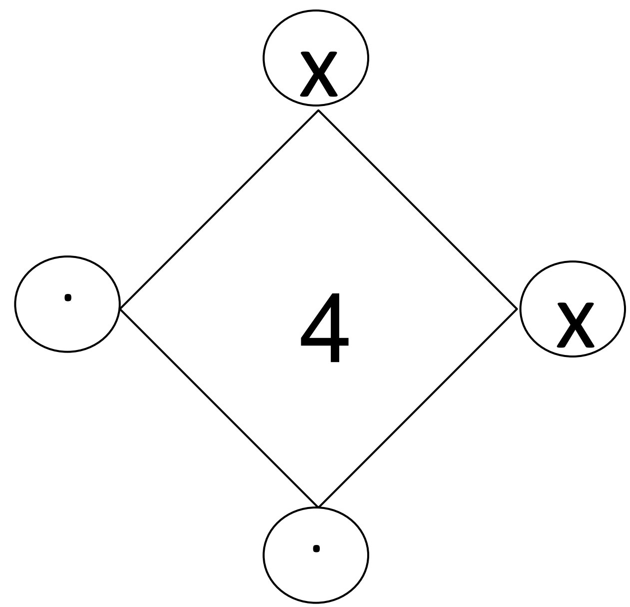
	E	2_z	4^+	4^-
$\vert 1\rangle$	1	1	1	1
$\vert 2\rangle$	1	1	-1	-1
$\begin{bmatrix} \vert 3\rangle \\ \vert 4\rangle \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$	$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

	2_x	2_y	2_{xy}	$2_{x\bar{y}}$
$\vert 1\rangle$	1	1	1	1
$\vert 2\rangle$	1	1	-1	-1
$\begin{bmatrix} \vert 3\rangle \\ \vert 4\rangle \end{bmatrix}$	$\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$


1


x
2
x


x
x
3


x
4
x

	E	2_z	4^+	4^-
$\|1\rangle$	1	1	1	1
$\|2\rangle$	1	1	-1	-1

	2_x	2_y	2_{xy}	$2_{x\bar{y}}$
$\|1\rangle$	-1	-1	-1	-1
$\|2\rangle$	-1	-1	1	1