OPTICS

The science of light

P. Ewart
Lecture notes: On web site

**NB** outline notes!

Textbooks:
Hecht, *Optics*
Lipson, Lipson and Lipson, *Optical Physics*

Further reading:
Brooker, *Modern Classical Optics*

Problems: Material for four tutorials plus past Finals papers A2

Practical Course: Manuscripts and Experience
Structure of the Course

1. Physical Optics (Interference)
   - *Diffraction Theory (Scalar)*
   - *Fourier Theory*

2. Analysis of light (Interferometers)
   - *Diffraction Gratings*
   - *Michelson (Fourier Transform)*
   - *Fabry-Perot*

3. Polarization of light (Vector)
Oxford Physics: Second Year, **Optics**

Astronomical observatory, Hawaii, 4200m above sea level.
Multi-segment Objective mirror, Keck Observatory
Hubble Space Telescope, HST, In orbit
HST Deep Field

Oldest objects in the Universe:

13 billion years
HST Image: *Gravitational lensing*
SEM Image: Insect head
Coherent Light:

*Laser physics:*

Holography,
Telecommunications
Quantum optics
Quantum computing
Ultra-cold atoms
Laser nuclear ignition
Medical applications
Engineering
Chemistry
Environmental sensing
Metrology ……etc.!
CD/DVD Player: optical tracking assembly
Optics in Physics

- Astronomy and Cosmology
- Microscopy
- Spectroscopy and Atomic Theory
- Quantum Theory
- Relativity Theory
- Lasers
Lecture 1: Waves and Diffraction

- Interference
- Analytical method
- Phasor method
Phase change of $2\pi$
dsin\theta
Phasor diagram

\[ \text{Real} \quad \text{Imaginary} \]

\[ u \quad \delta \]
Phasor diagram for 2-slit interference
Lecture 2: Diffraction theory

- Diffraction at a finite slit
  - Analytical method
  - Phasor method
- 2-D diffraction at apertures
- Fraunhofer diffraction
Diffraction from a single slit

\[
\int_{-a/2}^{+a/2} dy \sin \theta = \int_{r - a/2}^{r + a/2} \sin \theta dr
\]

\[
P = r + y \sin \theta
\]

\[
y = y \sin \theta
\]
Intensity pattern from diffraction at single slit

\[ \text{sinc}^2(\beta) \]
\begin{align*}
+\frac{a}{2} & \\
-a/2 & \quad \text{as} \sin \theta
\end{align*}

\begin{align*}
\theta & \\
r & \\
D & \\
P & \\
\end{align*}
Phasors and resultant at different angles $\theta$
\[ R \sin \frac{\delta}{2} \]
Phasor arc to first minimum

Phasor arc to second minimum
Diffraction from a rectangular aperture
Diffraction pattern from circular aperture

Intensity

Point Spread Function
Diffraction from a circular aperture
Diffraction from circular apertures
Dust pattern

Diffraction pattern

Basis of particle sizing instruments
Lecture 3: Diffraction theory and wave propagation

- Fraunhofer diffraction
- Fresnel’s theory of wave propagation
- Fresnel-Kirchoff diffraction integral
Fraunhofer Diffraction

A diffraction pattern for which the phase of the light at the observation point is a \textit{linear function} of the position for all points in the diffracting aperture is Fraunhofer diffraction.

\textbf{How linear is linear?}
\[ \rho < \frac{\lambda}{20} \]
Fraunhofer Diffraction

A diffraction pattern formed in the image plane of an optical system is Fraunhofer diffraction
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Fraunhofer diffraction: in image plane of system
Equivalent lens system:
*Fraunhofer diffraction is independent of aperture position*
Fresnel’s Theory of wave propagation
Huygens secondary sources on wavefront at \(-z\) radiate to point \(P\) on new wavefront at \(z = 0\)
Construction of elements of equal area on wavefront
Resultant, $R_\pi$, represents amplitude from 1st HPZ
Phase difference of $\lambda/2$ at edge of 1st HPZ
As $n \to \infty$ resultant $\Rightarrow \frac{1}{2}$ diameter of 1st HPZ
\[ u_p = - \frac{i}{\lambda} \int \frac{u_o \, dS}{r} \eta(n, r) \, e^{ikr} \]

Fresnel-Kirchhoff diffraction integral
Babinet’s Principle
Lectures 1 - 3: The story so far

- Scalar diffraction theory: 
  *Analytical methods*
  *Phasor methods*

- Fresnel-Kirchoff diffraction integral: 
  *propagation of plane waves*
Phase at observation is linear function of position in aperture:
\[ \delta = k \sin \theta y \]

\[ u_p = -\frac{i}{\lambda} \int \frac{u_o dS}{r} \eta(n,r) e^{ikr} \]

Fresnel-Kirchoff Diffraction Integral
Lecture 4: Fourier methods

- Fraunhofer diffraction as a Fourier transform
- Convolution theorem – solving difficult diffraction problems
- Useful Fourier transforms and convolutions
Fresnel-Kirchoff diffraction integral:

\[ u_p = -\frac{i}{\lambda} \int \frac{u_0 \, dS}{r} \eta(n \cdot r) e^{ikr} \]

Simplifies to:

\[ u_p \Rightarrow A(\beta) = \alpha \int_{-\infty}^{\infty} u(x) e^{i\beta x} \, dx \]

where \( \beta = k \sin \theta \)

Note: \( A(\beta) \) is the Fourier transform of \( u(x) \)

The Fraunhofer diffraction pattern is proportional to the Fourier transform of the transmission function (amplitude function) of the diffracting aperture
The Convolution function:

\[ h(x) = f(x) \otimes g(x) = \int_{-\infty}^{\infty} f(x').g(x-x').dx' \]

The Convolution Theorem:

The Fourier transform, F.T., of \( f(x) \) is \( F(\beta) \)

F.T., of \( g(x) \) is \( G(\beta) \)

F.T., of \( h(x) \) is \( H(\beta) \)

\[ H(\beta) = F(\beta).G(\beta) \]

The Fourier transform of a convolution of \( f \) and \( g \) is the product of the Fourier transforms of \( f \) and \( g \)
Monochromatic Wave

$$\beta_o = \frac{2\pi}{T}$$

Fourier Transform
\[ V(x) \delta \text{-function} \]

**Fourier transform**

*Power spectrum*

\[ V(\beta) V(\beta)^* = V^2 = \text{constant} \]
Comb of $\delta$-functions

Fourier transform
Constructing a double slit function by convolution

\[ g(x-x') \quad f(x') \]

\[ h(x) \]
Triangle as a convolution of two “top-hat” functions

This is a self-convolution or Autocorrelation function
Lecture 5: Theory of imaging

- Fourier methods in optics
- Abbé theory of imaging
- Resolution of microscopes
- Optical image processing
- Diffraction limited imaging
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• Heat transfer theory: - *greenhouse effect*
• Fourier series
• Fourier synthesis and analysis
• Fourier transform as analysis

Joseph Fourier (1768 – 1830)
Ernst Abbé (1840 -1905)
Abbé theory of imaging (coherent light)
The compound microscope

Objective magnification = $v/u$
Eyepiece magnifies real image of object
Diffracted orders from high spatial frequencies miss the objective lens.

\[ \theta_{\text{max}} \] defines the numerical aperture… and resolution.

So high spatial frequencies are missing from the image.
Fourier plane

Image plane

Image processing
Optical simulation of “X-Ray diffraction”

(a) and (b) show objects: double helix at different angle of view

Diffraction patterns of (a) and (b) observed in Fourier plane

Computer performs Inverse Fourier transform To find object “shape”
amplitude or phase object

\[ u(x) \rightarrow \theta \rightarrow f \rightarrow \text{Fourier plane} \]

\[ \mathbf{D} \]

\[ \mathbf{v(x)} \rightarrow \mathbf{d'} \]
Schlieren photography

Source → Collimating Lens → Refractive index variation → Fourier Plane → Imaging Lens → Image Plane
Schlieren photography of combustion

Schlieren film of ignition
Courtesy of Prof C R Stone
Eng. Science, Oxford University
Diffraction pattern from circular aperture

Point Spread Function, PSF
Lecture 6: Optical instruments and Fringe localisation

• Interference fringes
• What types of fringe?
• Where are fringes located?
• Interferometers
• Interference by:

• Division of wavefront
  - Young’s slits: 2 beams
  - N-slit grating: multiple beams

• Division of amplitude
  - Michelson: 2 beams
  - Fabry-Perot: multiple beams

What kind of interference fringes and where are they?
What kind of interference fringes and where are they?

Depends on:

Type of light source - Point source - Extended source

Division of wavefront - Point sources

Division of amplitude by - Reflection from: Wedged surfaces Parallel surfaces
Division of wavefront

Young’s slits

Usually observed under Fraunhofer conditions - large distance from slits

Plane waves

Non-localized fringes
Plane waves

Fraunhofer condition

Division of wavefront
Diffraction grating

Fringes localized at infinity
Wedged reflecting surfaces

Point source

Non-localized

Fringes of Equal thickness
Point source
Parallel reflecting surfaces

Non-localized
Fringes of Equal inclination
Extended source
Wedged Reflecting surfaces

Localized in plane of wedge, near apex
Equal thickness
Extended source
Parallel reflecting surfaces

Localized at infinity

Fringes of equal inclination

Path difference $x \cos \alpha$

Circular fringe constant $\alpha$
### Summary: fringe type and localisation

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<thead>
<tr>
<th></th>
<th>Wedged</th>
<th>Parallel</th>
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<tr>
<td><strong>Point Source</strong></td>
<td>Non-localised</td>
<td>Non-localised</td>
</tr>
<tr>
<td></td>
<td>Equal thickness</td>
<td>Equal inclination</td>
</tr>
<tr>
<td><strong>Extended Source</strong></td>
<td>Localised in plane</td>
<td>Localised at infinity</td>
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<tr>
<td></td>
<td>of Wedge</td>
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<td>Equal thickness</td>
<td>Equal inclination</td>
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