

Problem sheet - TT tutorial 1

Main questions

Q1. Simple illustration of the sudden and adiabatic approximation

A particle is in the ground state of a simple box (infinite square potential well) of side L . Describe qualitatively the evolution of the system

- a. If the length of the side of the box is suddenly doubled.
- b. If the length of the side of the box is very slowly increased to $2L$.
- c. What would be the result of an attempt to decrease the box's diameter suddenly to $L/2$?

[JW QM TT Q7.1]

Q2. Application of the sudden approximation to radioactive decay

Sudden approximation In the β decay H^3 (1 proton + 2 neutrons in the nucleus) \rightarrow $(\text{He}^3)^+$ (2 protons + 1 neutron in the nucleus), the emitted electron has a kinetic energy of 16 keV. We will consider the effects on the motion of the atomic electron, i.e. the one orbiting the nucleus, which we assume is initially in the ground state of H^3 .

- a. Show by a brief justification that the perturbation is sudden. What is the state of the atomic electron immediately after the perturbation (e.g. at a time around 5×10^{-17} s after the electron was emitted)?
- b. Give the mean kinetic energy and mean potential energy of the atomic electron before and after the perturbation, expressing your results as multiples of the Rydberg energy (ignore the very slight change in reduced mass.) [Use your knowledge of hydrogen to avoid doing integrals]. Hence find the new mean energy.
- c. Show that the probability for the electron to be left in the ground state of $(\text{He}^3)^+$ is $2^3(2/3)^6 \simeq 0.7$, and comment on how this result relates to the information you obtained in part (b). [Comment: the integral here is one you have done before, but to save time you might like to change variable to $u = 3r/2$ to convert it exactly into a standard one for hydrogen, for which you then know the value].
- d. Describe the main features of the subsequent behaviour of the $(\text{He}^3)^+$ ion when the electron is not left in the ground state.

[JW QM TT Q7.3]

Q3. Book work on deriving central result

Derivation Let $\psi(t)$ be the state of a system subject to a Hamiltonian $H = H^0 + \delta H(t)$ where H^0 is time-independent and has eigenfunctions ψ_n^0 . The basic perturbation theory result, accurate to first order in δH , is

$$d_f(t) = \delta_{fi} - \frac{i}{\hbar} \int_0^t e^{i\omega_{fi}t'} \langle \psi_f^0 | \delta H(t') | \psi_i^0 \rangle dt'.$$

for the amplitude of the state ψ_f^0 in $\psi(t)$ when a system starts in the state ψ_i^0 and is subject to perturbation $\delta H(t)$ between times 0 and t . After first checking your notes, see if you can perform the derivation of this result without returning to them for help.

[JW QM TT Q7.7]

Q4. Application to the harmonic oscillator

A harmonic oscillator starts in its ground state ($n = 0$) at $t = -\infty$. A perturbation $\delta H = -e\mathcal{E}x \exp(-t^2/\tau^2)$ is applied between $t = -\infty$ and $+\infty$.

- What is the probability that the oscillator makes a transition to the state $n = 2$?
- Show that the probability the oscillator makes a transition to the state $n = 1$ is

$$P_{0 \rightarrow 1} = \frac{e^2 \mathcal{E}^2 \pi \tau^2}{2m\omega\hbar} e^{-\omega^2 \tau^2 / 2}$$

[there are two integrals to do here, one for the matrix element, and an integral over time. For the matrix element, you can either perform the integral in the x basis, or use raising/lowering operators.]

- Plot the dependence of $P_{0 \rightarrow 1}$ on the timescale τ of the perturbation, and comment on the behaviour in the limits $\tau \rightarrow 0$ and $\omega\tau \gg 1$.

[JW QM TT Q7.1]

Q5. Dipole interaction of an atom

Dipole radiation For a single atom, in the process called *electric dipole radiation*, what two physical entities are interacting together? Briefly discuss whether or not an isolated atom possesses an electric dipole moment.

[JW QM TT Q7.4]

Q6. Selection rules for dipole induced transitions

Selection rules This question illustrates the way selection rules can come about, by treating a system in a state of well-defined orbital angular momentum, such as the hydrogen atom when spin-orbit coupling is ignored. For such a system (i.e. one whose energy eigenfunctions have the form $R(r)Y_{l,m_l}(\theta, \phi)$) find the electric dipole selection rules on m_l , by considering the components of the electric dipole matrix element $\langle er \rangle$ in the form of three integrals. [Hint: consider z and $x \pm iy$, expressing them in polars.]

[JW QM TT Q7.5]

Q7. Computing a dipole transition matrix element for hydrogen

Calculate a matrix element Here is an example of an electric dipole matrix element. It is for one of the Zeeman components of the 1s–2p transition in hydrogen, ignoring fine structure.

$$|\langle n = 2, l = 1, m = 0 | z | n = 1, l = 0, m = 0 \rangle|^2 = \frac{2^{15}}{3^{10}} a_0^2$$

Use the hydrogen wavefunctions to verify this result. [Use spherical polar coordinates. The radial integral can be looked up, or alternatively by a change of variable to $u = 3r/4$ it can be converted into the same form as $\langle r^2 \rangle$ for hydrogen ground state, for which you can use your previous study of such integrals]

[JW QM TT Q7.8]