Michaelmas Term 2016 1st Year Relativity - Problems 1

Special Relativity: Professor K M Blundell

*The problems marked with * are somewhat harder. Numerical answers are given to some of the problems.*

SECTION A – Lorentz Transformation and Elementary Consequences

1. The space and time coordinates of two events as measured in a frame S are as follows:

Event 1: $x_1 = x_0, \;\; t_1 = x_0/c \quad (y_1 = 0, \; z_1 = 0)$

Event 2: $x_2 = 2x_0, \; t_2 = x_0/2c \quad (y_2 = 0, \; z_2 = 0)$

Show that there exists an inertial frame in which these events occur at the same time, and find the velocity of this frame relative to S. At what value of time in the new frame do both events occur?

[Ans: $\beta = -1/2$, $t' = \sqrt{3}x_0/c$.]

2. A small, powerful laser is placed on a turntable that rotates at 1200 rev/s. The laser, whose beam makes a 30° angle with the horizontal, shines on clouds 50 km away. Calculate the speed with which the light spot on the clouds moves. Does this speed violate the limitation of the speed of light? Explain.

[Ans: 3.3×10^8 m s^{-1}]

*3. Two observers A and B, in inertial frames, have identical clocks. B passes close to A with a relative velocity 0.5c. t seconds after B has passed, A flashes him a light signal. $2t$ seconds after receiving this signal (as measured on his own clock), B acknowledges by a return light signal to A. When does A receive it?

[Ans: $(3 + 2\sqrt{3})t$.]

4. Proxima Centauri, the star nearest our own, is some 4.2 ly away. (a) If a spaceship could travel at a speed of 0.80c, how long would it take to reach the star, according to the spaceship's pilot? (b) What would someone in the frame that moves with the spaceship measure as the distance to Proxima Centauri?

[Ans: (a) 3.1 yr, (b) 2.5 ly]

5. The Galaxy is about 10^5 light years across and the most energetic cosmic rays known have energies of the order of 10^{19} eV. How long would it take a proton with this energy to cross the Galaxy as measured in the rest frame of (i) the Galaxy and (ii) the proton.

The rest-mass of the proton is 938 MeV/c^2.

[Ans: (i) 10^5 years, (ii) 4.96 min.]

6. The proper mean lifetime of π^+ mesons is 26 ns.

(a) What is the mean lifetime of a burst of π^+ mesons travelling with $\beta = 0.73$?

(b) What distance is travelled at this velocity during one mean lifetime?

(c) What distance would be travelled without the effect of time dilation?

[Ans: (a) 38 ns, (b) 8.32 m, (c) 5.69 m.]

7. A burst of muons is produced by a cosmic ray interacting in the upper atmosphere. They travel towards the Earth's surface with an average speed of $0.99c$. If 1% of the burst survive to reach ground level, estimate the height of the burst and calculate the distance as measured in the muon's frame. The muon mean lifetime is 2.2 μsec.

[Ans: 2.13×10^4 m, 3.01×10^3 m.]

*8. Twin paradox for fun – C. Darwin, Nature **180**, 976 (1957). On New Year's day 2030 an astronaut (A) sets out from Earth at speed $0.8c$ to travel to the nearest star α-Centauri, about 4 light-years away (as measured in the Earth frame of reference). On reaching the star A immediately turns around and returns to Earth arriving back on New Year's day 2040 (Earth time). A has a sibling B who remains on Earth and they agree to send each other greetings by radio every New Year's day until A returns.

(a) Satisfy yourself that A sends only 6 messages (including the one on the last day of the trip, whereas B sends 10.

(b) Draw a space-time diagram of A's journey in the Earth frame (mark the scales of x and ct in light years). Draw the world lines of all the radio signals that B transmits. Use the diagram to verify that A (astronaut) has received only 1 signal up to the moment of reaching the star and turning back, and receives the other 9 during the return half of the trip.

(c) On a second space-time diagram (again in the Earth frame) show the world lines of the astronaut (A) and all the signals that A sends. Verify that B (on Earth) receives one message each 3 years of Earth time for the first 9 years of the journey, then receives 3 more messages in the last year to make the total of 6 - which is the expected total since according to A's time the trip takes 3 years out and 3 years back.

SECTION B – Addition of velocities; Energy & Mass

9. Show that two successive Lorentz transformations with velocities v_1 and v_2 in the same direction are equivalent to a single Lorentz transformation with velocity $(v_1+v_2)/(1+v_1v_2/c^2)$.

10. *Estimate* the mass lost when 1 millon tonnes of TNT explodes. Assume that each chemical reaction between individual molecules involves 10 eV of energy.

11. Energy from the Sun (distance 1.496×10^{11} m from Earth) arrives at the Earth (above the atmosphere) at a rate of about 1400 W m^{-2}. How fast is the sun losing mass due to energy radiation?

[Ans: 4.4×10^9 kg s^{-1}]