

**Q1 (Revision of first year material):** Two long slits of negligible width, separated by a distance  $d$  are illuminated by monochromatic light of wavelength  $\lambda$  from a point source at a distance  $s$  on an axis perpendicular to the plane of the slits and mid-way between the slits:

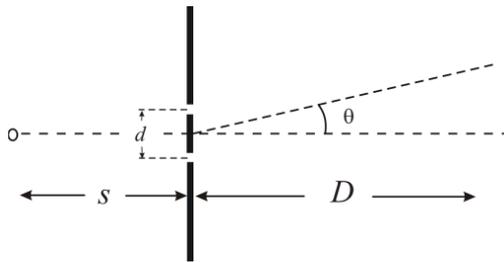


Figure 1(a)

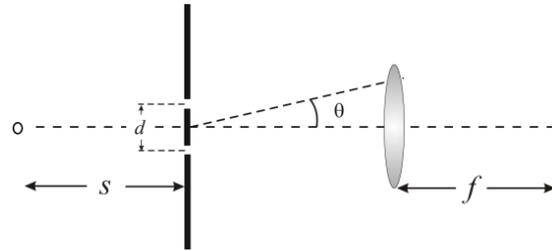


Figure 1(b)

By assuming an amplitude  $u_0$  of light emanating from each slit, calculate the intensity as a function of the angle  $\theta$  to the axis at a distance  $D$  on the opposite side of the slits from the source when  $D \gg d$  (Fig. 1(a)),

- (a) using the analytical method, and
- (b) by the method of phasors.
- (c) What is the angular separation  $\Delta\theta$  between the central maximum and the first maximum on either side?

A large aperture lens of focal length  $f$ , on the axis, is used to form an image of the fringes on a screen placed at a distance  $f$  from the lens (Figure 1(b)).

- (d) Derive an expression for  $y$ , the separation on the screen of the first fringe maximum from the central fringe maximum.
- (e) Make a sketch of the interference pattern seen on the screen and indicate quantitatively the positions of the principle maxima of the pattern.
- (f) One of the slits is now covered with a thin attenuating film to reduce the amplitude to one half that emitted by the uncovered slit. Use either the analytical or phasor method to find the angular distribution of the intensity in this case. Sketch the form of the interference pattern seen on the screen in this case, on the same axes as for part (e), indicating quantitatively the intensity variation in the pattern.
- (g) The film is now replaced by a 0.5 mm thick glass plate of refractive index 1.5 on the input side of the slit. Explain quantitatively any change that may occur in the fringe pattern.
- (h) Explain what would happen to the pattern, when both slits are uncovered, if the source is moved a distance  $x$  perpendicularly from the axis in the plane of the diagram above.
- (i) The original system, with unobstructed slits, is set up with a source emitting light of wavelength 500 nm at  $s = 20$  mm from the slits which are separated by  $d = 0.5$  mm. Explain what would be observed on the screen if a second, independent i.e. incoherent, source of equal intensity was placed beside the original source at a distance  $x$  off axis and  $x$  is gradually increased from zero perpendicularly to the axis. What will be the minimum value of  $x$  at which fringes can no longer be observed? Estimate the maximum size for a single source, of incoherent emitters, if fringes are to be visible.

**Q2:** Explain what is meant by Fraunhofer diffraction. [Your answer should contain no reference to parallel light or observation at infinity.] Why is it of such great practical importance?

Plane waves of monochromatic light of wavelength  $\lambda$  fall on a slit of width  $a$ :

- (a) Estimate the minimum distance from the slit of the plane of observation if the observed diffraction pattern is to fulfil the Fraunhofer condition.
- (b) Derive an expression for the angular distribution of the intensity in the Fraunhofer pattern, and sketch it, indicating the relative distances from the centre to any minima in intensity.

**Q3:** Viewed through a thin layer of cloud, the Moon sometimes appears to be covered by a translucent disk of light of angular diameter  $2^\circ$ . The disk is white at its centre but reddish at its periphery. Explain this phenomenon and estimate the size of water droplets in the cloud.

**Q4:** The Fresnel-Kirchhoff diffraction integral reads 
$$u_p = -\frac{i}{\lambda} \int_S \frac{u_o dS}{r} \eta(\underline{n}, \underline{r}) \exp(ikr)$$

where  $u_p$  is the amplitude of the diffracted wave at position  $r$  from the centre of a diffracting aperture of area  $S$ , in which the amplitude is  $u_o$  per unit area and the other symbols have their usual meanings.

- (a) Explain the physical significance of the factor  $1/r$  in the integral and the quantity  $\eta(\underline{n}, \underline{r})$ .
- (b) Using this integral show that the angular distribution of the Fraunhofer diffraction pattern of a single long narrow slit, width  $a$ , may be represented by the Fourier transform of the amplitude distribution  $u(x)$  in the aperture, making clear any assumptions and approximations required.  $x$  is the dimension transverse to the long axis of the slit.

**Q5:** The amplitude distribution in a slit of width  $a$  in the transverse,  $x$ , direction may be represented by the top-hat function:

$$v(x) = 1 \text{ for } |x| < \frac{a}{2} \text{ and } v(x) = 0 \text{ for } |x| > \frac{a}{2}$$

Find an expression for the Fraunhofer diffraction pattern of such a single slit.

**Q6:** State the Convolution Theorem.

Describe quantitatively how an aperture function  $u(x)$  representing the amplitude transmission of two long slits of width,  $a$ , parallel to the  $y$ -axis and centred at positions  $x = \pm d/2$ , in the  $x$ - direction and where  $d > a$ , may be represented as a convolution of two simpler functions.

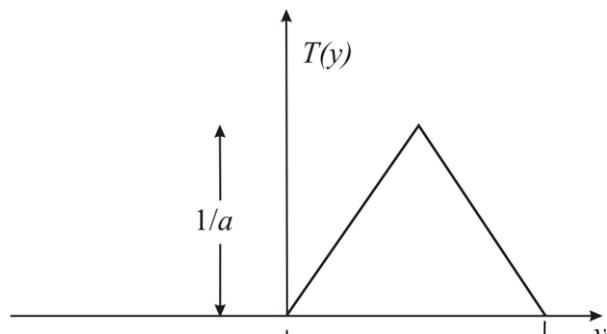
Use the Convolution Theorem to derive an expression for the Fraunhofer diffraction pattern of such a pair of slits illuminated by a plane monochromatic wave of wavelength  $\lambda$ .

**Q7:** An aperture lies in the plane  $z = 0$  and has amplitude transmission function  $T(y)$  independent of  $x$ . It is illuminated by coherent, monochromatic light of wavelength  $\lambda$  at normal incidence.

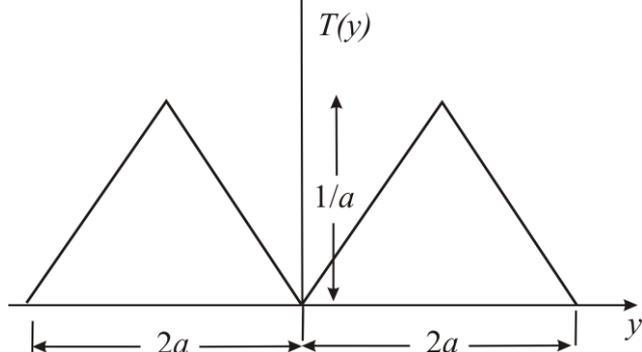
Show that, in the Fraunhofer case, the diffracted intensity is proportional to  $|A(\theta)|^2$ , where

$$A(\theta) = \int_{-\infty}^{+\infty} T(y) \exp[-iky \sin \theta] dy$$

**Figure 2(a)** shows a particular amplitude transmission function. Calculate and sketch the diffracted intensity as a function of  $\sin \theta$ .



**Figure 2(b)** shows another transmission function. Calculate and sketch the diffracted intensity as a function of  $\sin \theta$ . Comment on the location of the first zero of the diffracted intensity with regard to the spatial-frequency content of  $T(y)$ .



**Q8:** An opaque plane has two long, narrow and parallel slits of negligible width spaced by a distance  $d$ . The slits are illuminated by a plane wave of monochromatic light of wavelength  $\lambda$ .

- Make a quantitative sketch of the form of the *intensity* diffraction pattern observed on a screen at distance  $D$  from the plane of the slits where  $D \gg d$ , i.e. label the angular separation of the fringe maxima or minima.
- On the same scale, sketch the pattern obtained if two extra slits are added, one on each side of the original pair of slits at spaced at the same distance,  $d$ .
- Explain what happens to the general features of these patterns as more slits are added (keeping the same value of  $d$ ).
- In the case of the four slit arrangement, the central two slits are covered by a film transmitting 25% of the incident intensity. By using phasors, or otherwise, estimate the change in intensity distribution and sketch the new intensity distribution on the screen.

**Q9:** A grating with  $N$  slits of spacing  $d$  is illuminated normally with monochromatic light of wavelength  $\lambda$ , and the  $p^{\text{th}}$  order principal maximum is observed at an angle  $\theta_p$ .

- Show that the intensity at angle  $\theta$  is given by

$$I(\delta) = \frac{\sin^2(N\delta/2)}{\sin^2(\delta/2)} \quad \text{where} \quad \delta = \frac{2\pi}{\lambda} \sin \theta$$

Hence show that the adjacent minima fall at angles  $\theta_p \pm \delta\theta_p$  where  $\delta\theta_p = \frac{\lambda}{Nd \cos \theta_p}$

- Show that the  $\frac{\partial \theta}{\partial \lambda}$  dispersion of the grating in the  $p^{\text{th}}$  order of interference is  $\frac{\partial \theta}{\partial \lambda} = \frac{p}{d \cos \theta}$

- Hence show that the instrumental width of the grating (in terms of  $\lambda$ ) is  $\Delta\lambda_{\text{INST}} = \frac{\lambda}{Np}$ .

- Deduce that the resolving power of the grating is  $Np$ .

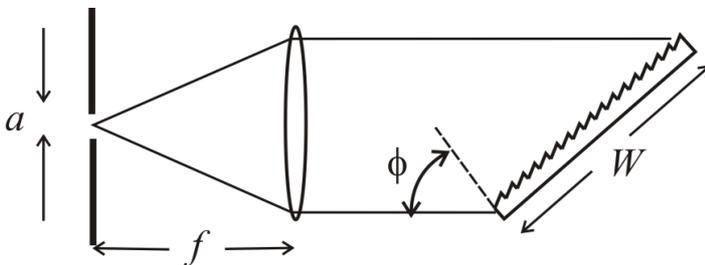
- Does the resolving power of a grating change if it is used in a medium of high refractive index?

**Q10:** A pair of slits, each of width  $a$ , and separated centre-to-centre by a distance  $d$  is illuminated normally by a plane monochromatic wave. Light transmitted by the slits is observed in the focal plane of a lens of focal length  $f$ . Draw sketch graphs showing the variation of the observed intensity in the focal plane of the lens for the cases (i)  $d \gg a$  (ii)  $d = 3a$  (iii)  $d = 2a$ . In each case your sketch graph should show quantitatively the scales being used on the co-ordinate axes.

**Q11:** A collimated beam from a white-light source is incident normally on a transmission grating with 500 lines per mm. The transmitted light then passes through a lens which is used to project the visible spectrum (380–780 nm) of the light source on to a strip of photographic film and to just cover its length of 35 mm. Calculate the focal length of the lens.

**Q12:** A plane wave of light of wavelength  $\lambda$  is incident at angle  $\phi$  on a reflection grating of width  $W$  having  $\delta$  lines per unit length. Derive an expression for the intensity of the reflected light as a function of the diffraction angle  $\theta$ . Show that a maximum intensity is obtained at an angle  $\theta$  whenever  $\sin \theta = \lambda \delta \pm \sin \phi$ .

Find an expression for the minimum wavelength difference  $\Delta\lambda$  that may be resolved with this arrangement.



Such a reflection grating is used in an optical arrangement shown in the diagram and is similar to that used to select the operating wavelength of a laser. Light from a source is incident on the slit of width  $a$ , is collimated by the lens of focal length  $f$  and falls on the reflection grating at angle  $\phi$ . The grating has 2400 lines per mm and a total width  $W$  of 50 mm.

- At what value of  $\phi$  will a maximum intensity of light of wavelength 600 nm from the source return through the slit?
- What factors influence the choice of slit width  $a$  to obtain the narrowest possible range of wavelengths returning to the light source?
- Estimate the optimum value of  $a$  when the lens has a focal length of 200 mm.

**Q13:** Give a brief account of the principal features of a Michelson interferometer used with visual observation and illuminated by an extended monochromatic source. Explain what configurations of the mirrors will give (i) circular fringes (ii) straight, equally spaced fringes. State where the fringes are localised and why. Why in case (ii) does the mirror spacing have to be small?

**Q14:** A Michelson interferometer is set up to give circular fringes when illuminated by light of wavelength  $\lambda$ .

- What will be the order of interference,  $p_0$ , for the fringe on the axis when the difference in the lengths of the arms of the interferometer is  $t$ ?
- What condition must be fulfilled to give a bright circular fringe at angle  $\alpha$  to the axis when the interferometer is illuminated by an extended source of wavelength  $\lambda$ ?
- Explain why the order of interference  $p$  of a fringe at angle  $\alpha$  to the axis is less than  $p_0$ .
- Derive an expression for the radius,  $r_p$ , of the  $p^{\text{th}}$ -order fringe formed in the focal plane of a lens of focal length  $f$ . Hence show that the difference in radii of the  $p^{\text{th}}$ -order and  $(p+1)^{\text{th}}$ -order fringes is given by

$$r_p^2 - r_{p+1}^2 = \frac{f^2 \lambda}{t}$$

- The path length of the arm containing the moving mirror is initially larger than the path in the arm containing the fixed mirror. The path difference  $t$  is slowly decreased towards the zero path difference condition. Describe what happens to the fringes as the path difference decreases and then increases as the moving mirror moves past the position of zero path difference.

**Q15:** A Michelson interferometer is set up to give circular fringes and illuminated with light in the spectral range 800–900 nm from an atomic caesium  $^{133}\text{Cs}$  discharge source. The intensity  $I(x)$  at the centre of the interference pattern is recorded as a function of the distance  $x$  of the moving mirror from that corresponding to zero path difference. It is found to have the form

$$I(x) = 3I_0 + 3I_0 \cos K_1 x \cos K_2 x - I_0 \sin K_1 x \sin K_2 x$$

for  $0 \leq x \leq 5 \text{ mm}$ , where  $K_1 = 1.44 \times 10^7 \text{ m}^{-1}$ ,  $K_2 = 3.48 \times 10^5 \text{ m}^{-1}$  and  $I_0$  is a constant. Show that this can be written as the sum of the patterns due to two monochromatic spectral components. Hence determine, for the two caesium lines in this wavelength range:

- their mean wavenumber,  $\bar{\nu}$ ,
- their wavenumber separation,  $\Delta\bar{\nu}$ ,
- their relative intensities.

When the interference pattern is recorded over a larger range of path difference it is found that the periodic terms in the pattern are in fact multiplied by a function

$$f(x) \propto \exp(-K_3^2 x^2)$$

where  $K_3 = 5.02 \text{ m}^{-1}$ . Make a rough (but reasonably realistic) sketch of the interference pattern as a function of  $x$  over a range large enough to show the effect of this term.

[*Optional extra.* This factor arises from the Doppler broadening of spectral lines. This broadening is due to the different Doppler shift of the frequency,  $f$ , radiated by atoms moving at different velocities within a Maxwell-Boltzmann distribution. The Doppler shift in frequency is  $\Delta f = f(v/c)$ , where  $v$  is the speed of the atom.

Taking an average speed to be approximately  $\sqrt{3kT/M}$ , where  $k$  is Boltzmann's constant,  $M$  is the atomic mass and  $T$  is the temperature, the width of the frequency spread may be used to estimate the temperature. Use the values in the question to estimate the temperature of the gas of Cs atoms.]

**Q16:** Fabry-Perot interferometer

- (a) Show that monochromatic light of wavelength  $\lambda$  from an extended source incident on two parallel reflecting surfaces, separated by a distance  $d$ , will form fringes of equal inclination at angle  $\theta$  and order  $p$  where  $p\lambda = 2d \cos\theta$ .
- (b) Prove that the transmission function of a Fabry-Perot interferometer for monochromatic light of wavelength  $\lambda$  incident at an angle  $\theta$  to the normal is

$$I(\phi) = I_0 \left[ 1 + \left( \frac{4F^2}{\pi^2} \right) \sin^2(\delta/2) \right]^{-1}$$

where  $I_0$  is the incident intensity,  $\delta = (2\pi/\lambda)2nt \cos\theta$ ,  $t$  is the plate separation, and  $n$  is the refractive index of the medium between the reflecting surfaces.

- (c) Obtain an expression for  $F$  in terms of the intensity reflectivity of the plates,  $R$ , and explain how it determines the sharpness of the fringes.

**Q17:** Find the instrument width and resolving power of the Fabry-Perot etalon described in Q16, as follows.

- (a) Show that the values of  $\delta$  at which the intensity has dropped to 50% of maximum near the  $p^{\text{th}}$  order of interference are

$$\delta = 2\pi(p \pm 1/(2F))$$

- (b) Show that two spectral lines for which the  $p^{\text{th}}$  maximum of one falls on the  $(p+1)^{\text{th}}$  maximum of the other, the Free Spectral Range, are separated by

$$\delta\bar{\nu}_{\text{FSR}} = \frac{1}{2nt}$$

- (c) From (a) and (b), show that the instrumental width,  $\Delta\bar{\nu}_{\text{INST}}$ , (FWHM, full width at half maximum intensity) of the etalon is given by

$$\Delta\bar{\nu}_{\text{INST}} = \frac{1}{F} \times \frac{1}{2nt}$$

- (d) Hence show that the resolving power is  $Fp$ .

**Q18:** A certain spectral line is known to consist of two equally intense components with a wavenumber separation  $\Delta\bar{\nu}_s$  less than  $20 \text{ m}^{-1}$ . The Fabry-Perot fringes produced by this line are photographed, using a plate separation of 25 mm. The diameters of the smallest rings are found to be, in mm:

1.82, 3.30, 4.84, 5.57, 6.60, 7.15.

Explain why this experiment does not allow  $\Delta\bar{\nu}_s$  to be determined uniquely, but gives two possible values. What are they? Suggest a further experiment which could be carried out to resolve the ambiguity.

**Q19:** Light of wavelength,  $\lambda$ , falls at normal incidence from air onto a glass plate of refractive index  $n_G$  covered by a film of dielectric material with refractive index  $n_d$  and thickness  $\ell$ . Show that the ratio,  $r$ , of the reflected,  $E'$ , to incident amplitude,  $E$ , is given by

$$\frac{E'_0}{E_0} = r = \frac{\cos k_1 \ell - i(n_G/n_d) \sin k_1 \ell + i n_d \sin k_1 \ell - n_G \cos k_1 \ell}{\cos k_1 \ell - i(n_G/n_d) \sin k_1 \ell - i n_d \sin k_1 \ell + n_G \cos k_1 \ell}$$

where  $k_1 = 2\pi n_d/\lambda$

**Q20:** A layer of a non-absorbing homogeneous dielectric of refractive index  $n_2$ , is deposited on the surface of glass with refractive index  $n_1$ . In the case where the thickness of the layer,  $\ell$  is given by  $\ell = \lambda / 4n_2$ , a quarter wave layer, show that the condition for all the energy to be transmitted is  $n_2^2 = n_1$ .

**Q21:** Polarised light:

- Explain what is meant by *polarised light* and describe the different types of polarisation that are possible.
- Explain how a uniaxial birefringent material of length  $\ell$ , with ordinary and extra-ordinary refractive indices,  $n_o$  and  $n_e$ , respectively, can be used to change the relative phase of two orthogonal components of the electric field in a light wave of wavelength  $\lambda$ . Describe briefly the principle of operation of a polarising device made from a birefringent material.
- Demonstrate analytically that unpolarised light can be converted to circularly polarised light by passing the radiation through a linear polariser followed by a quarter-wave plate at a suitable orientation.
- A beam of initially unpolarised light passes through this system, and is then reflected back normally through it from a plane mirror. Describe the state of polarisation of the beam in each part of its path through the system.
- Show that any elliptically polarised light can be converted to plane polarised light using a suitably oriented quarter-wave plate. Is the converse also true? Show also that plane polarised light remains plane polarised after passing through a half-wave plate, but with the direction of the electric vector altered. How would you use a half-wave plate to change this direction by  $40^\circ$ ?

**Q22:** A beam of light is elliptically polarised with the major axis vertical. The ratio of the major and minor axes of the ellipse is  $a : b$ . Explain how you would use a quarter-wave plate to obtain linearly polarised light, and determine the angle of the plane of polarisation to the vertical.

A beam of light consisting of a mixture of elliptically polarised and unpolarised light is passed through a linearly polarising filter. The maximum of the transmitted light intensity is observed when the transmission axis of the filter is vertical, and is twice the minimum intensity. In a second experiment, the beam is passed through a quarter-wave plate with the fast axis vertical followed by the polarising filter. The maximum is now observed when the transmission axis is at  $33.21^\circ$  to the vertical. Calculate the ratio of the intensities of the polarised to unpolarised components of the light.

How could the handedness of the elliptically polarised component be determined?

**Q23:** A beam of initially unpolarised light is incident normally on a piece of polarising sheet (polaroid) **A**. The transmitted light is then falling on a second similar sheet **B**. **A** and **B** are set with their axes at right angles. A third polaroid sheet **C** is placed between the first two with its axis at an angle  $\phi$  to that of sheet **A**.

- What fraction of the incident energy passes through the system?
- In a Young's double slit experiment, both slits are covered by polarising sheets, one with its axis horizontal, the other with its axis vertical. Explain why no fringes are seen in the plane of observation.
- Given a third polarising sheet and a half-wave mica plate, explain where you would place them, and in what orientation, so that fringes would appear. Could the pattern be produced using only the half-wave plate?