Wave Optics
Propagation, interference and diffraction of waves

Axel Kuhn, Oxford 2016

Paul Ewart’s lecture notes and problem sets:
https://www2.physics.ox.ac.uk/research/combustion-physics-and-non-linear-optics/teaching

Wave Optics – Literature

- **Brooker**, *Modern Classical Optics*
- **Hecht**, *Optics*
- **Klein and Furtak**, *Optics*
- **Smith, King & Wilkins**, *Optics and Photonics*
- **Born and Wolf**, *Principles of Optics*
Wave Optics – Outline

- What’s it all about?
- Revision of geometrical optics
- Propagation of waves
- Fourier methods
  - Fresnel-Kirchhoff integral, theory of imaging
- Diffraction-based optical instruments
  - 2-slit, grating, Michelson and Fabry-Perot Interferometer
- Dielectric surfaces and boundaries
  - multilayer (anti)reflection coatings
- Polarized Light

Intro
What’s it all about?

- Imaging
- Visualization (*projection, lithography*)
- Spectroscopy
- Matter-wave propagation & imaging
- Lasers and applications
- Modern devices
  *(opto-electronics, display technology, optical coatings, telecommunication, consumer electronics)*
What’s it all about?

Hubble space telescope, 2.4 m mirror

Intro
What’s it all about?

Optical Microscope

CD/DVD player optical pickup system
What’s it all about?

cutting & welding

photo lithography

Intro

What’s it all about?

Coherent Light ➔ Laser Physics

- spectroscopy
- metrology (clocks)
- quantum optics
- quantum computing
- laser nuclear ignition
- medical applications
- engineering
- telecommunication

Intro
Fermat’s Principle

Light propagating between two points follows a path, or paths, for which the time taken is an extremum (minimum).
1. Light rays → straight lines

2. Reflection: $\theta = \phi$

3. Refraction: Snell's law

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

4. Speed of light $v = \frac{c}{n}$

($n =$ Refractive index)
Fermat's principle

light takes shortest optical path from A to B

\[ \text{OPL} = n \times \text{real path length} \]

\[ \text{(optical path length)} \]

\[ \text{Reflection} \]

\[ \text{OPL} = \sqrt{h^2 + x^2} + \sqrt{h^2 + (L-x)^2} \]

\[ \frac{d}{dx}(\text{OPL}) = 0 \quad \therefore \quad x = \frac{L}{2} \quad \therefore \quad \theta = \phi \]

→ Snell's law in similar manner
Simple Imaging → lenses

1. Spherical surface

\[ \frac{h}{R} = \sin \Theta = n \sin \phi \approx n (\Theta - \frac{h}{d}) \approx n \left( \frac{h}{R} - \frac{h}{d} \right) \]

\[ \Rightarrow \quad \frac{h}{d} = (n-1) \frac{1}{R} \]

Single spherical surface

Note: Symmetric →

- focal points on a sphere
- image from sphere to sphere
Also:

Object sphere

→ Small angles: \( \Theta \approx \sin \Theta \) etc.

→ Neglect curvature of focal or image planes

Thin lens

→ Neglect propagation inside

→ Two curved surfaces

\[
\frac{1}{u} + \frac{1}{v} = \frac{1}{f} = (n-1) \left( \frac{1}{R_1} + \frac{1}{R_2} \right)
\]

Lensmaker's equation
Geometrical Optics – Revision

focussing with spherical surfaces

parallel bundles

image sphere

object sphere

image sphere

thin lens formula

\[ \frac{1}{u} + \frac{1}{v} = \frac{1}{f} \]
Basic lens properties

1. Central rays not refracted
2. Rays parallel to optic axis go through focal point on other side
3. Parallel bundles of rays travelling under angle 2 on one side all go through the same point in the focal plane on the other side:

\[ 2 \rightarrow \frac{1}{y} = \frac{1}{f} \cdot \frac{1}{d} \]
Long thickness from Fermat's principle

(Form follows function)

\[ \text{OPL}(h) = \sqrt{h^2 + u^2} + \sqrt{h^2 + v^2} + (n-1)(d_0 - \Delta(h)) \]

\[ = \text{OPL}(0) = u + v + (n-1)d_0 \]

\[ = \text{const.} \quad (h \text{ independent}) \]
6. \[ \Delta h \cdot (n-1) = \sqrt{h^2u^2 + h^2v^2} - u - v \]
\[ \frac{1}{2} \frac{h^2}{2R} (n-1) \approx \frac{h^2}{2u} + \frac{h^2}{2v} \]

\[ \text{ORL}A : \text{ plano-convex lens} \]

\[ \Delta = h - \frac{7h^2R^2}{R^2} = \frac{h^2}{2R} \]

Therefore:
\[ \frac{1}{R} (n-1) = \frac{1}{u} + \frac{1}{v} \]

Widespread use:
- Imaging
- Magnification
- Observation
- Lithography
- Microscopes
- Telescopes
- Cameras & projectors
- Magnifying glasses
- Eyepiece etc.
Geometrical Optics – Instruments

Principal planes

Front Focal Plane

Second Principal Plane

Thin lens equation applies with u and v measured from the two principal planes

\( \frac{1}{u} + \frac{1}{v} = \frac{1}{f} \)
Angle subtended by virtual image:

\[ d = \frac{h}{u} = \frac{h'}{D} \]

\[ \Rightarrow \frac{1}{d} = \frac{1}{u} + \frac{1}{v} = \frac{1}{u} + \frac{1}{-D} \]

\[ \Rightarrow \frac{1}{d} + \frac{1}{D} = \frac{1}{u} \rightarrow d = \frac{h}{\frac{h}{D} + \frac{h}{D}} \]

Naked eye - object at D,

angle \( \theta' = \frac{h}{D} \)

**Magnification**

\[ M = \frac{2}{d'} = \frac{D}{\frac{h}{D} + 1} \]
Objective magnification = $v/u$

Eyepiece magnifies real image of object

Geometrical Optics – Instruments

Astronomical telescope

angular magnification = $\beta/\alpha$
angular magnification = $\beta / \alpha$

\[ \frac{f_o}{f_E} \]
Geometrical Optics – Instruments

Galilean telescope

angular magnification = \frac{\beta}{\alpha}

Geometric

21-1

Geometrical Optics – Instruments

Galilean telescope

angular magnification = \frac{\beta}{\alpha} = \frac{f_o}{f_E}

Geometric

21-2
Apertures and Field Stops

Aperture stop
limiting the intensity

Field stop
limiting the field-of-view

Aperture stop = \frac{\text{entrance pupil diameter}}{\text{focal length}}
Image brightness

light collected \sim D^2

(less area)

object area \sim A^2

\text{Image area} \sim (2A)^2

\text{So brightness} \sim

\frac{\text{lens area}}{\text{image area}} \sim \frac{D^2}{(2A)^2} \sim \left(\frac{f}{D}\right)^{-2} = (f\text{-no.})^{-2}

f\text{-no.}^{-5/2} = \frac{\text{focal length}}{\text{pupil diameter}}
Combining two lenses

\[ u = \frac{1}{f_1} \quad v = \frac{1}{f_2} \]

\[ \frac{1}{u} + \frac{1}{v} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f} \quad \text{if lenses are close} \]

\[ \frac{1}{f} = \text{power of the lens, measured in dioptries}: \quad [\text{dioptries}] = [\text{m}^{-1}] \]
Ray transfer matrices

Ray vector \( (Y) \leftarrow \text{dist. from axis} \), \( (\theta) \leftarrow \text{angle to axis} \)

\[
\begin{pmatrix}
Y_2 \\
\theta_2
\end{pmatrix} =
\begin{pmatrix}
AB \\
CD
\end{pmatrix}
\begin{pmatrix}
Y_1 \\
\theta_1
\end{pmatrix}
\]

Transfer

Propagation by \( d \) \quad \begin{pmatrix}
1 & d \\
0 & 1
\end{pmatrix}
\quad \begin{align*}
Y_2 &= Y_1 + \theta_1 d \\
\theta_2 &= \theta_1
\end{align*}

Lens of focal length \( f \) \quad \begin{pmatrix}
1 & 0 \\
-\frac{1}{f} & 1
\end{pmatrix}
\quad \begin{align*}
Y_2 &= Y_1 \\
\theta_2 &= \theta_1 - \frac{Y_1}{f}
\end{align*}

Ray transfer through optical system

\[
\begin{pmatrix}
Y_2 \\
\theta_2
\end{pmatrix} = S_4 S_3 S_2 S_1 S_0 S_5 \begin{pmatrix}
Y_3 \\
\theta_3
\end{pmatrix}
\]

System transfer
Camera Obscura

optimum pinhole size

contradicts expectations from geometrical optics

The Wave Nature of Light

- Maxwell’s equations $\rightarrow$ waves
- equivalence to matter waves
- plane and spherical waves
- energy flow / intensity
- basic interference
- Huygen’s principle
- Fraunhofer diffraction & resolution limit

http://www2.physics.ox.ac.uk/contacts/people/kuhn#fragment-2