Wave Optics

Propagation, interference and diffraction of waves

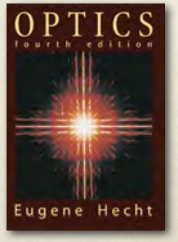
Axel Kuhn, Oxford 2016

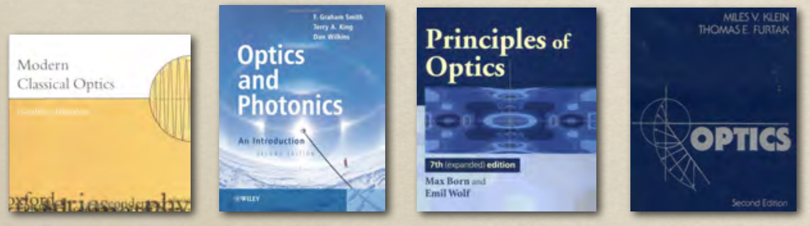
Paul Ewart's lecture notes and problem sets:

https://www2.physics.ox.ac.uk/research/combustion-physics-and-non-linear-optics/teaching

Wave Optics – Literature

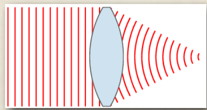
- ***Brooker,*** *Modern Classical Optics*
- ***Hecht,*** *Optics*
- ***Klein and Furtak,*** *Optics*
- ***Smith, King & Wilkins,*** *Optics and Photonics*
- ***Born and Wolf,*** *Principles of Optics*





Wave Optics – Outline

- ***What's it all about?***
- ***Revision of geometrical optics***
- ***Propagation of waves***
- ***Fourier methods***



→ *Fresnel-Kirchhoff integral, theory of imaging*

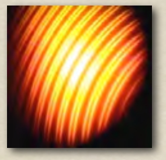
- ***Diffraction-based optical instruments***

→ *2-slit, grating, Michelson and Fabry-Perot Interferometer*

- ***Dielectric surfaces and boundaries***

→ *multilayer (anti)reflection coatings*

- ***Polarized Light***

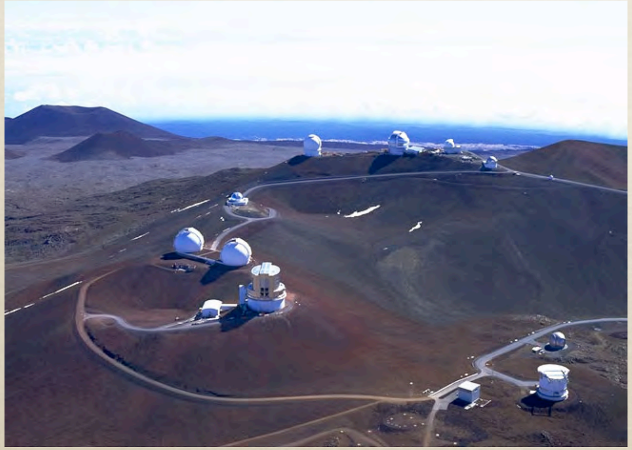


What's it all about?

- ***Imaging***
- ***Visualization (projection, lithography)***
- ***Spectroscopy***
- ***Matter-wave propagation & imaging***
- ***Lasers and applications***
- ***Modern devices***

(opto-electronics, display technology, optical coatings, telecommunication, consumer electronics)

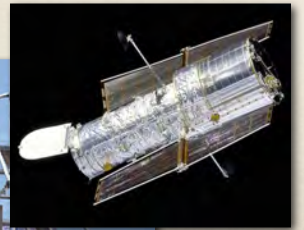
What's it all about?

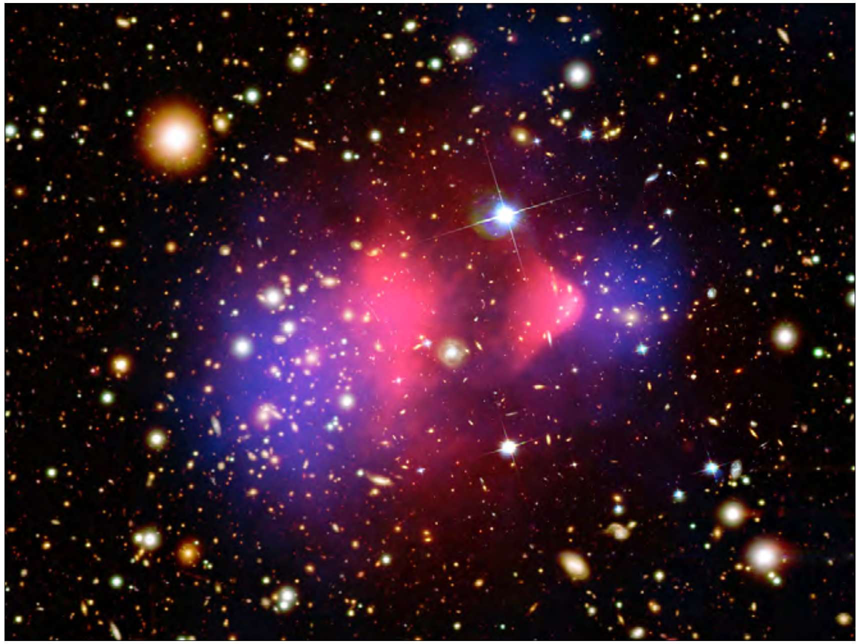


Astronomical observatory, Hawaii, 4200m above sea level.

What's it all about?

Hubble space telescope, 2.4 m mirror



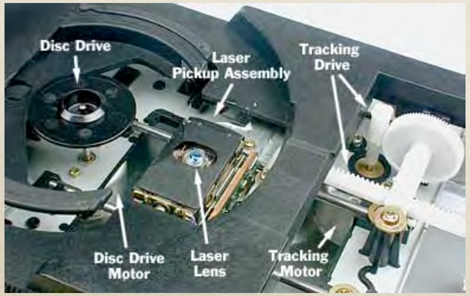


What's it all about?

fruit fly

Optical Microscope

What's it all about?



CD/DVD player optical pickup system

What's it all about?

cutting & welding

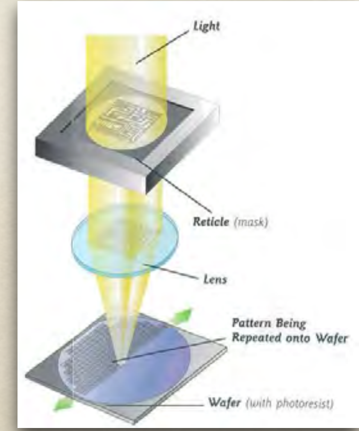
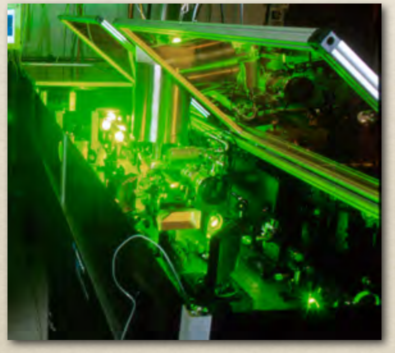


photo lithography

What's it all about?

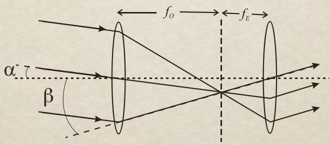
Coherent Light → Laser Physics

- *spectroscopy*
- *metrology (clocks)*
- *quantum optics*
- *quantum computing*
- *laser nuclear ignition*
- *medical applications*
- *engineering*
- *telecommunication*



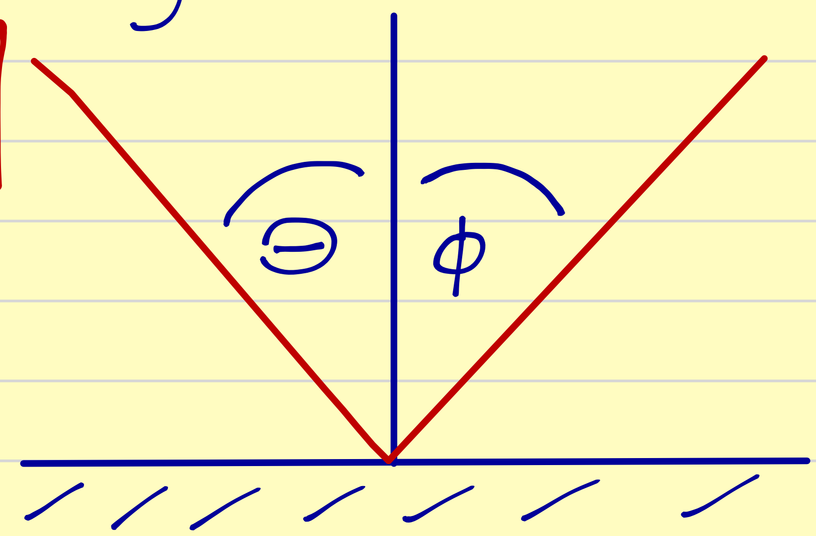
Geometrical Optics – Revision

- *Fermat's principle (shortest path)*
- *reflection & refraction*
- *spherical & thin lenses*
- *paraxial approximation*
- *lensmaker's formula*
- *combining lenses*
- *principal planes*
- *optical instruments*
- *Aperture and field stops*
- *Pinhole camera → wave optics*



L1 Revision of Geometrical Optics

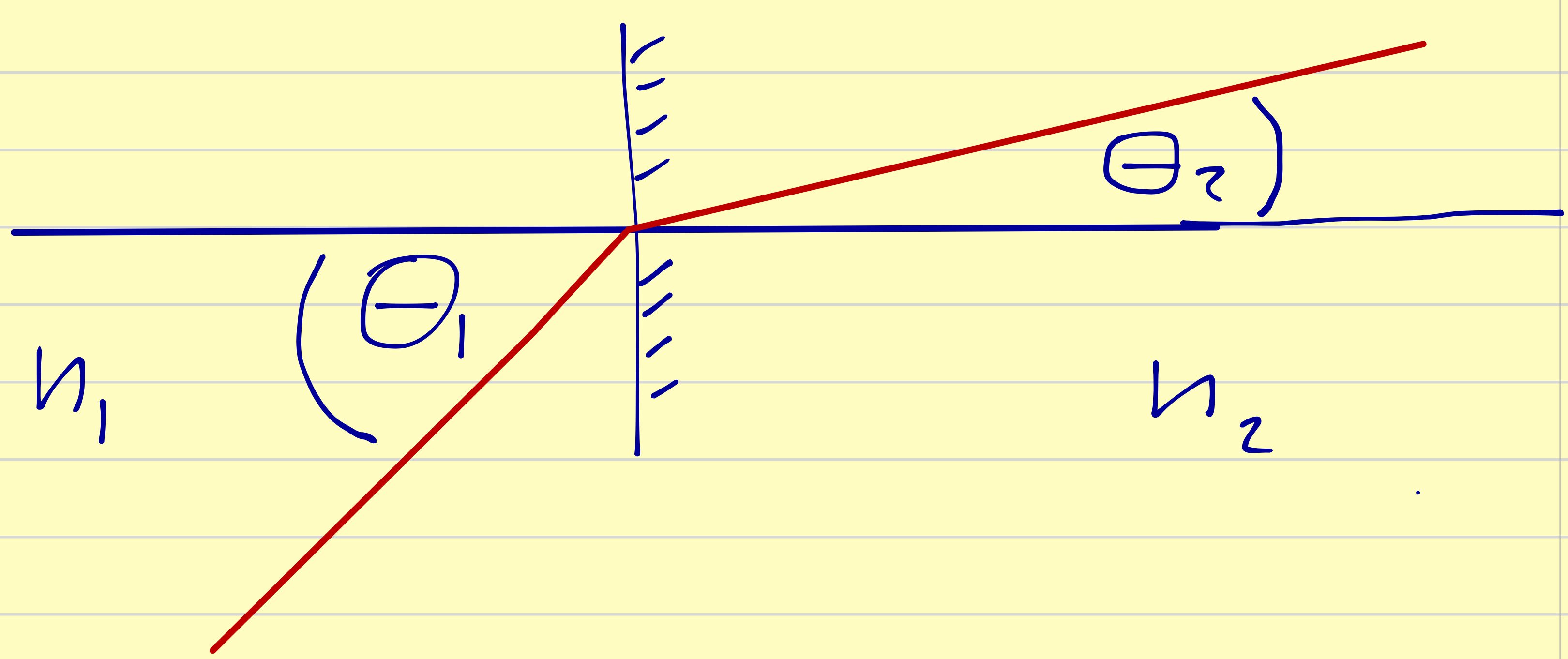
- light rays $\longrightarrow$ straight lines
- reflection : $\boxed{\Theta = \Phi}$



- refraction

Snell's law

$$\boxed{n_1 \sin\Theta_1 = n_2 \sin\Theta_2}$$



Speed of light $v = c/n$

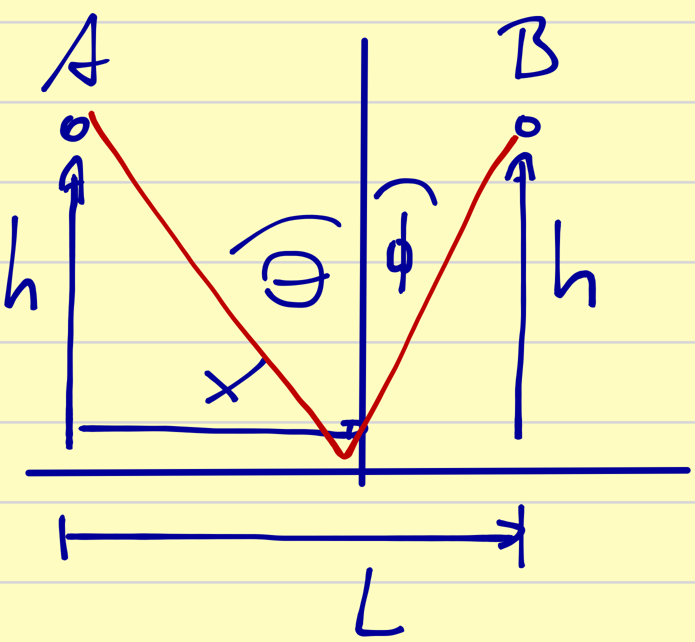
(n = refractive index)

Fermat's principle

$$\boxed{\text{light takes 'shortest' optical path from A to B}}$$

OPL = n × real path length
(optical path length)

→ reflection



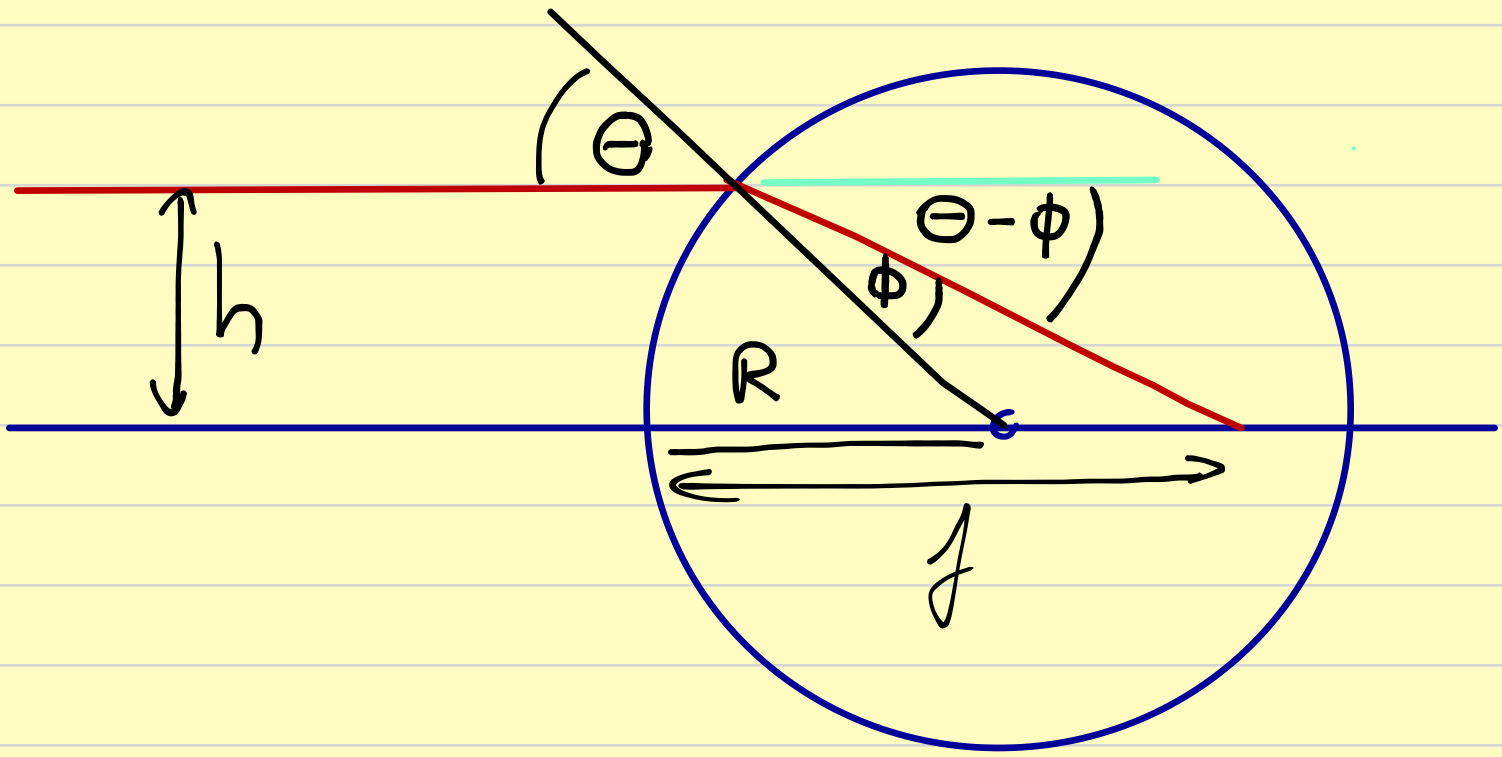
$$OPL = \sqrt{h^2 + x^2} + \sqrt{h^2 + (L-x)^2}$$

$$\frac{d}{dx}(OPL) = 0 \quad \therefore \quad x = L/2 \quad \therefore \quad \theta = \phi$$

→ Snell's law in similar manner

Simple Imaging → lenses

① spherical surface



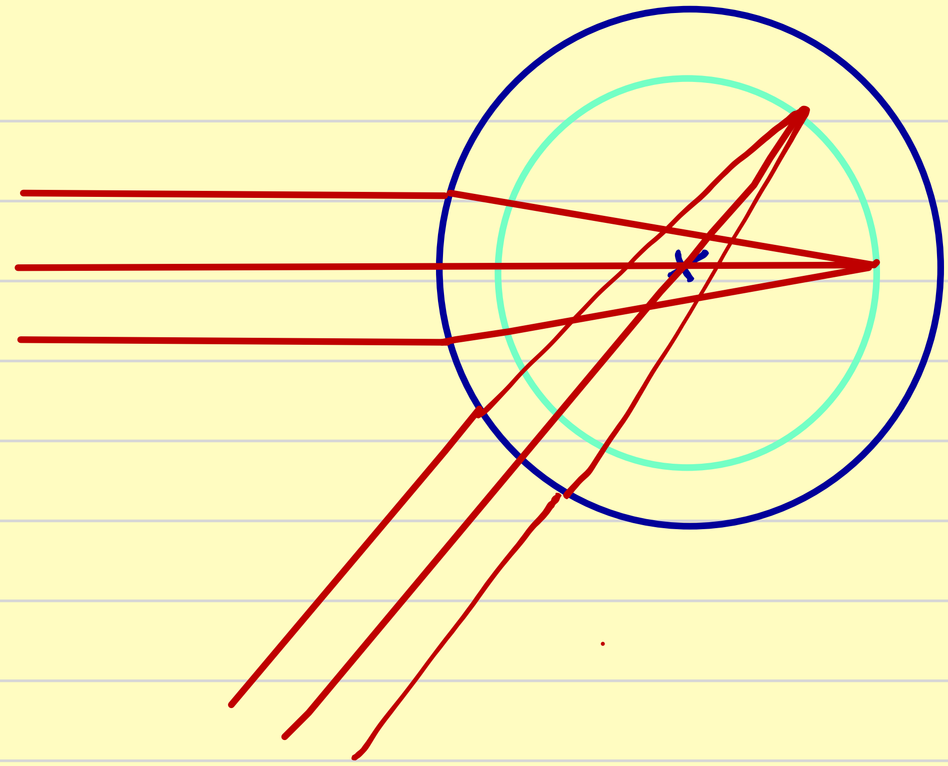
$$\frac{h}{R} = \sin\Theta = n\sin\Phi \simeq n\left(\Theta - \frac{h}{f}\right) \simeq n\left(\frac{h}{R} - \frac{h}{f}\right)$$

$$\Longrightarrow \boxed{\frac{n}{f} = (n-1)\frac{1}{R}}$$ single sph. surface

Note: Symmetrie →

- focal points on a sphere
- image from sphere to sphere

also:
object sphere
↓
image sphere



Paraxial approximation

→ small angles: $\theta \simeq \sin\theta$ etc.

→ neglect curvature of focal or image planes

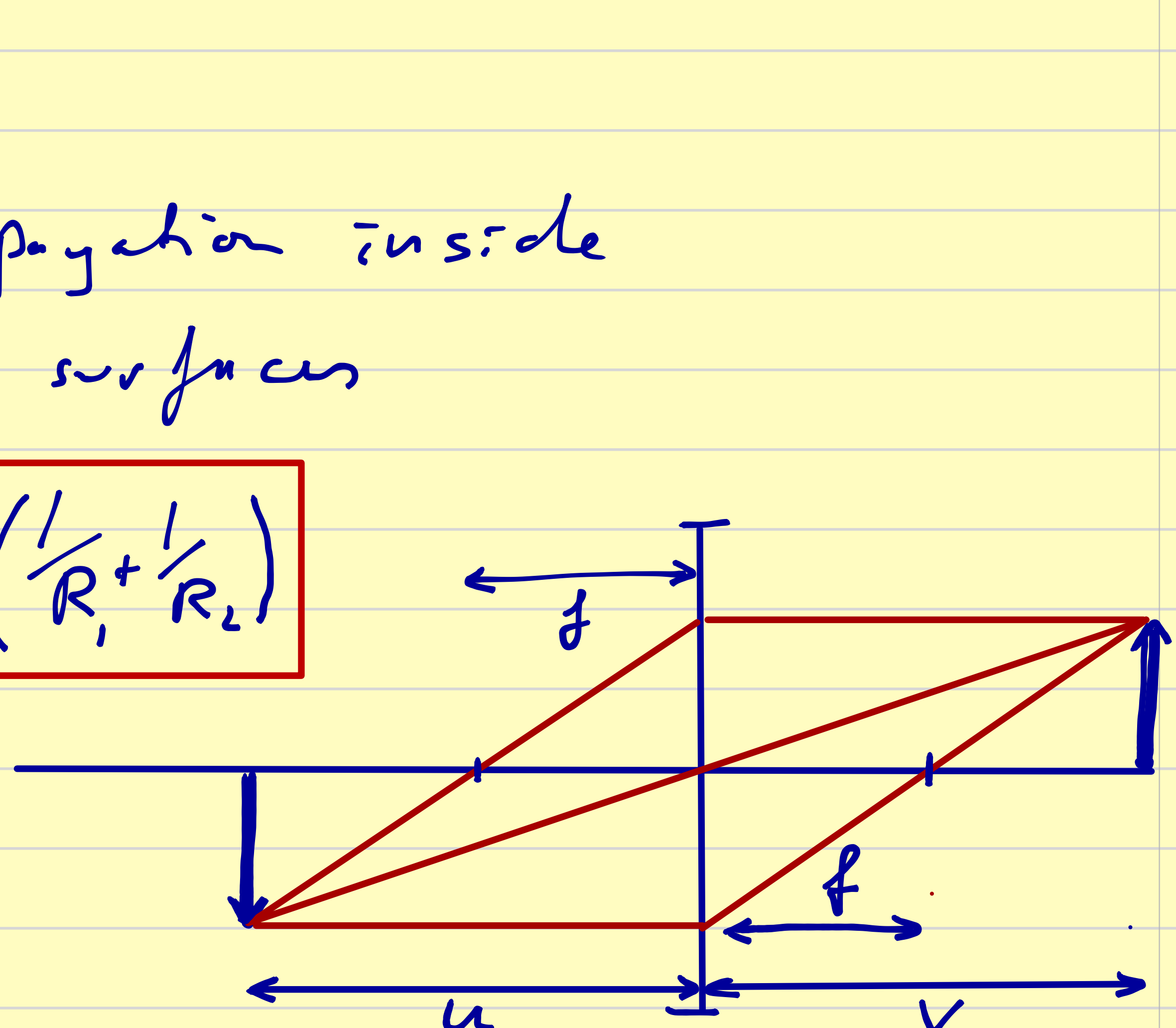
Thin lens

→ neglect propagation inside

→ two curved surfaces

$$\boxed{\frac{1}{u}+\frac{1}{v}=\frac{1}{f}=(n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}$$

Lensmaker's equation



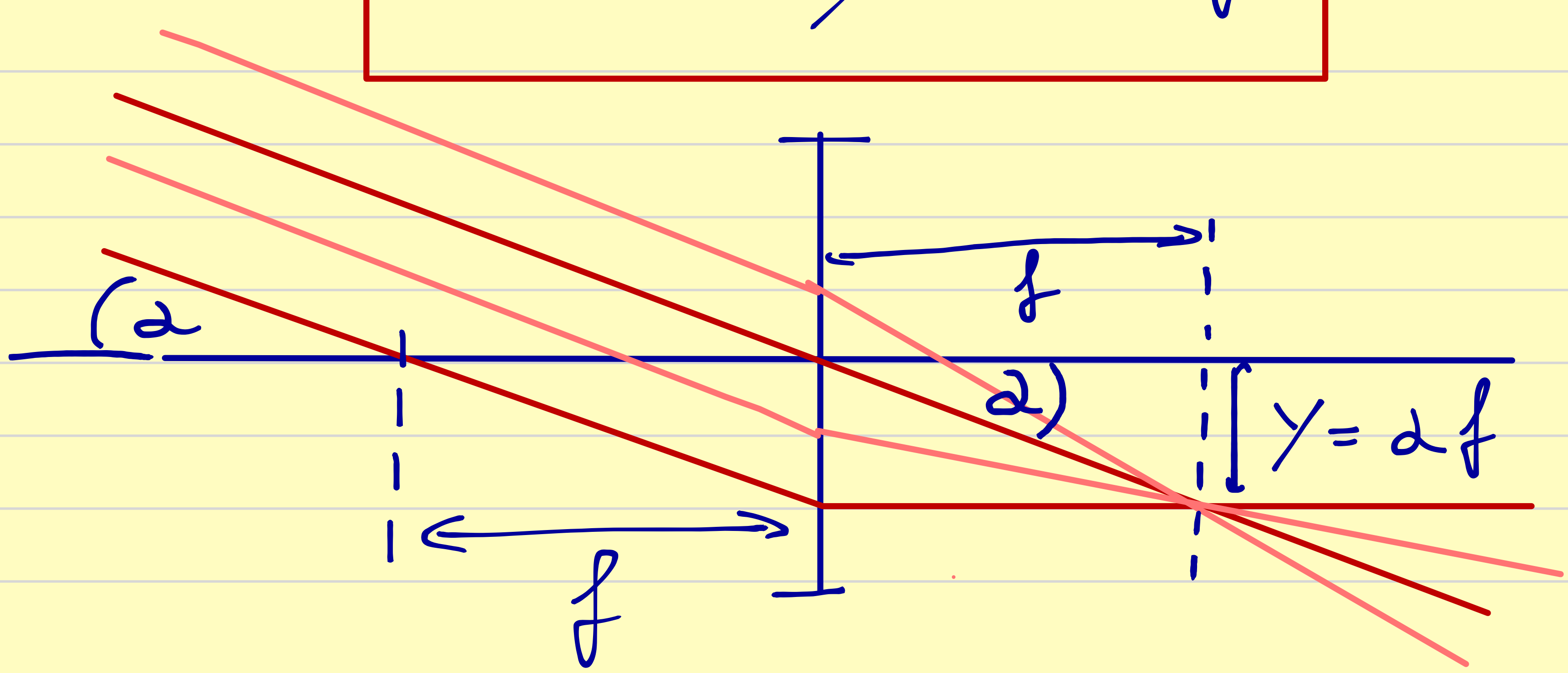
basic lens properties

① central rays not refracted

② rays parallel to opt. axis go through focal point on other side

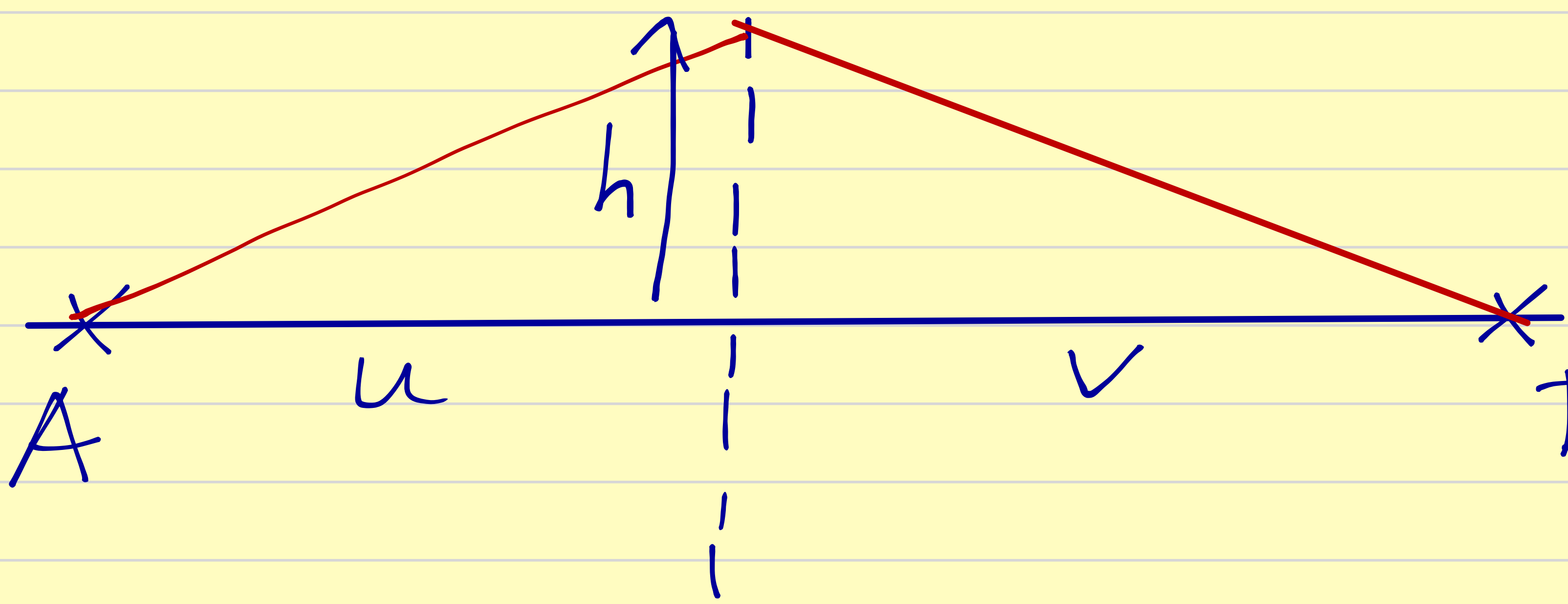
③ parallel bundles of rays travelling under angle α on one side all go through the same point in the focal plane on the other side =

$$\boxed{\alpha \longrightarrow y = \alpha \cdot f}$$



Lens thickness from Fermat's principle

(Form follows function)



→ Fermat: shortest OPL from A to B

→ B image of A if <u>all</u> OPL's of <u>same</u> length

→ lens thickness $d(h) = d_0 - \Delta(h)$

$$OPL(h) = \sqrt{h^2+u^2} + \sqrt{h^2+v^2} + (n-1)\left(d_0 - \Delta(h)\right)$$

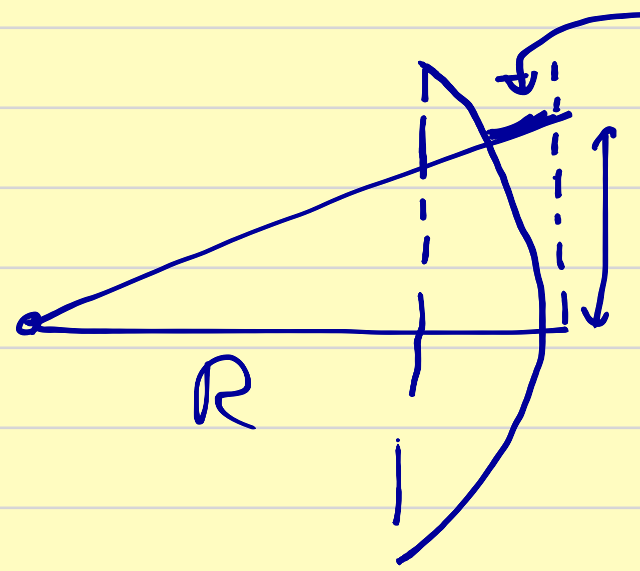
$$= OPL(0) = u + v + (n-1)\, d_0$$

$= \text{const.}$ (h independent)

$$\therefore \quad \Delta(h)\cdot(n-1) = \sqrt{h^2+u^2} + \sqrt{h^2+v^2} - u - v$$

$$\frac{h^2}{2R}(n-1) \simeq \frac{h^2}{2u} + \frac{h^2}{2v}$$

OBIA: plano-convex lens



$$\Delta = \sqrt{h^2+R^2} - R \simeq \frac{h^2}{2R}$$

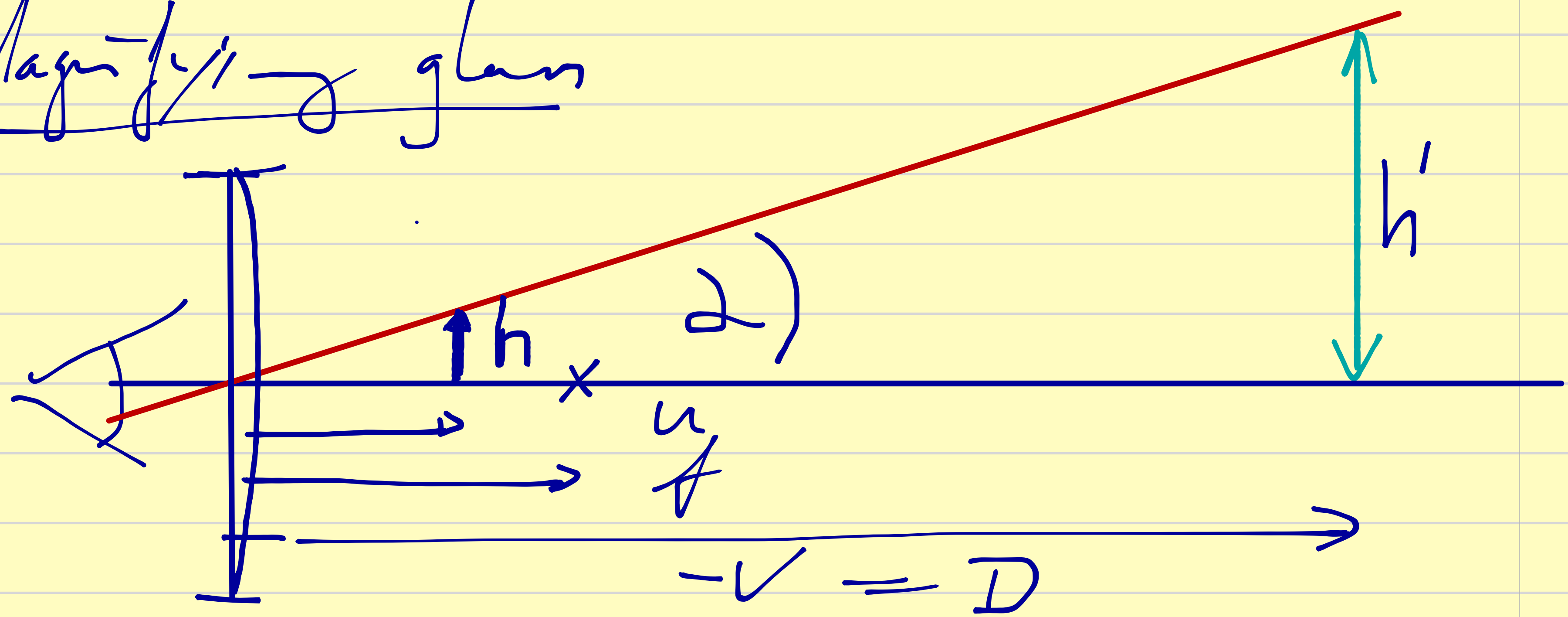
<u>therefore:</u>

$$\boxed{\frac{1}{R}(n-1) = \frac{1}{u} + \frac{1}{v}}$$

<u>Widespread use:</u>

imaging magnification observation lithography	microscopes telescopes cameras + projectors magnifying glasses eyepiece etc.

Magnifying glass



angle subtended by virtual image:

$$\alpha = \frac{h}{u} = \frac{h'}{D}$$

$$\rightarrow \frac{1}{f} = \frac{1}{u} + \frac{1}{v} = \frac{1}{u} + \frac{1}{-D}$$

$$\rightarrow \frac{1}{f} + \frac{1}{D} = \frac{1}{u} \rightarrow \alpha = \frac{h}{f} + \frac{h}{D}$$

naked eye → object at D, angle $\alpha' = \frac{h}{D}$

Magnification $\boxed{M = \frac{\alpha}{\alpha'} = \frac{D}{f} + 1}$

Geometrical Optics – Revision

- *Ignoring the wave nature of light*
- *Basic theory for optical instruments*

- *Fermat's Principle*

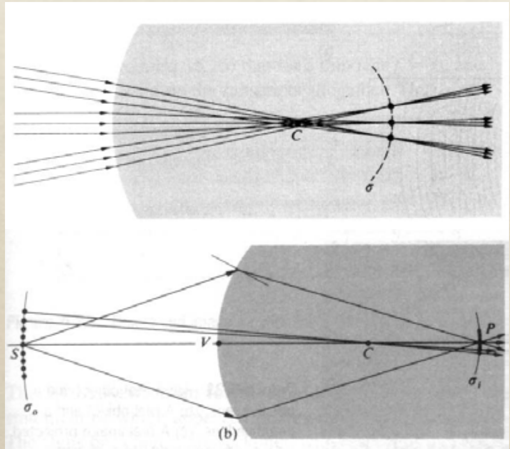
Light propagating between two points follows a path, or paths, for which the time taken is an extremum (minimum)

Geometrical Optics – Revision

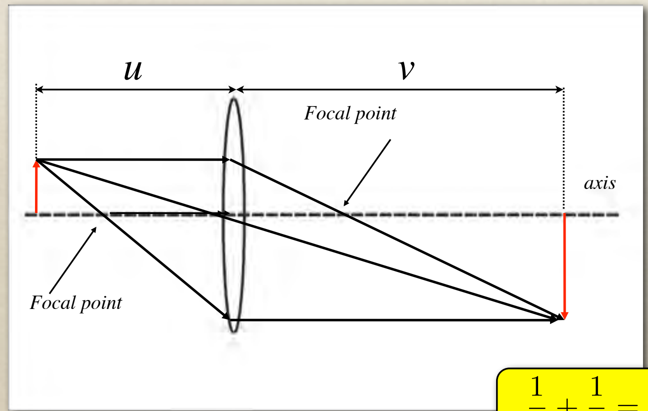
focussing with spherical surfaces

parallel bundles
↓
image sphere

object sphere
↓
image sphere



Geometrical Optics – Revision

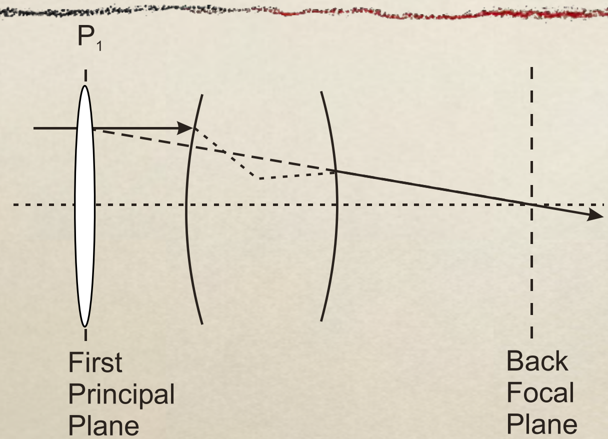


thin lens formula

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

Geometrical Optics – Instruments

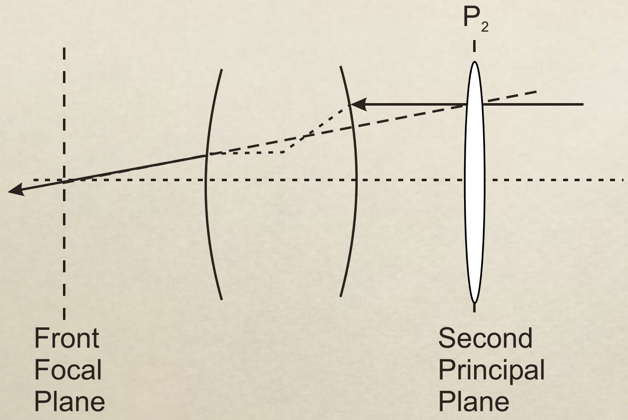
Principal planes



location of equivalent thin lens

Geometrical Optics – Instruments

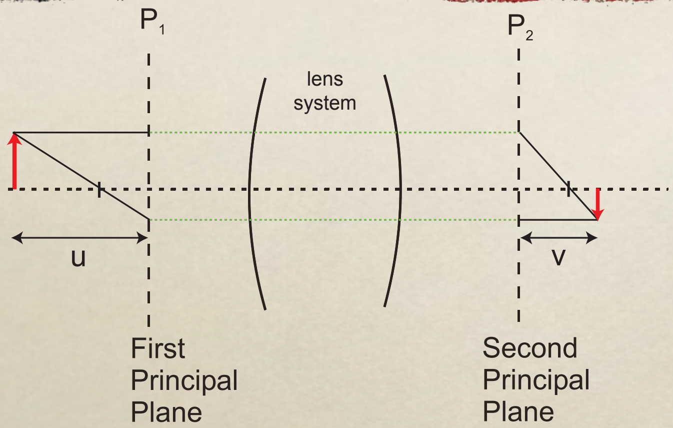
Principal planes



location of equivalent thin lens

Geometrical Optics – Instruments

Principal planes

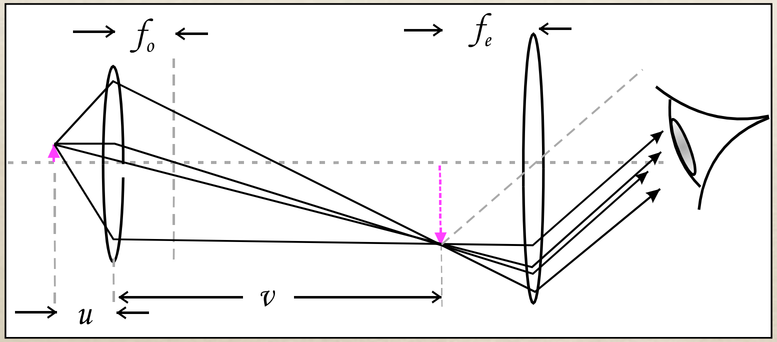


$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

thin lens equation applies with u and v measured from the two principal planes

Geometrical Optics – Instruments

Compound microscope

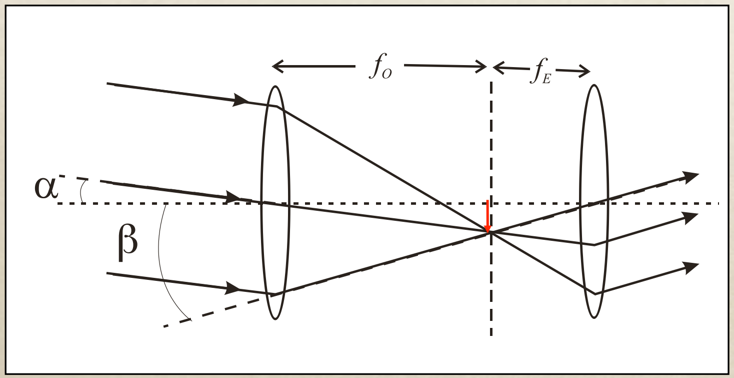


Objective magnification = v/u

Eyepiece magnifies real image of object

Geometrical Optics – Instruments

Astronomical telescope



angular magnification = $\beta/\alpha = f_o/f_E$

Image brightness

light collected $\sim D^2$

(lens area)

object area $\sim \alpha^2$

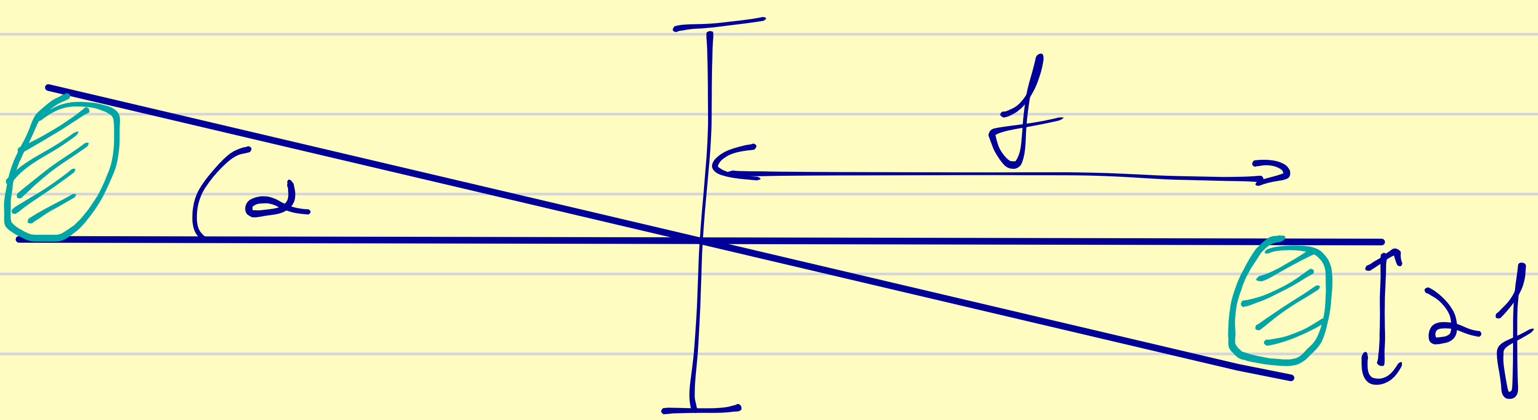


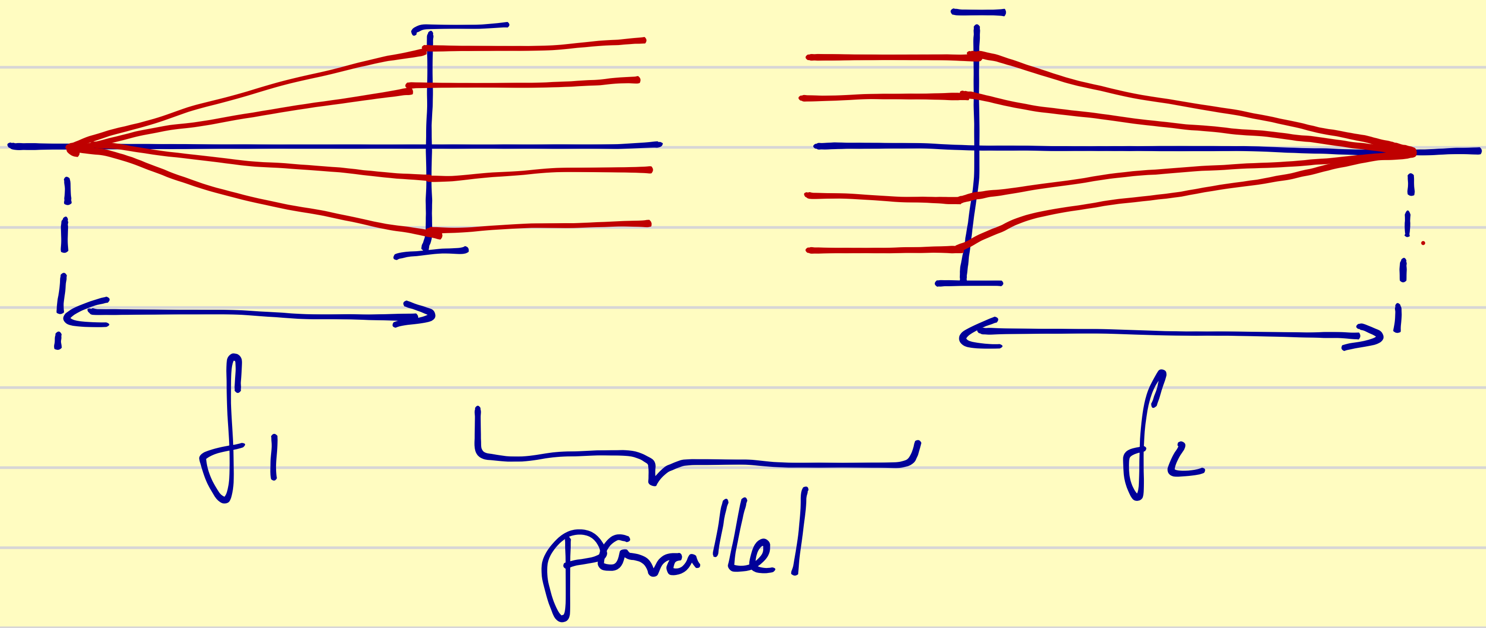
Image area $\sim (\alpha f)^2$

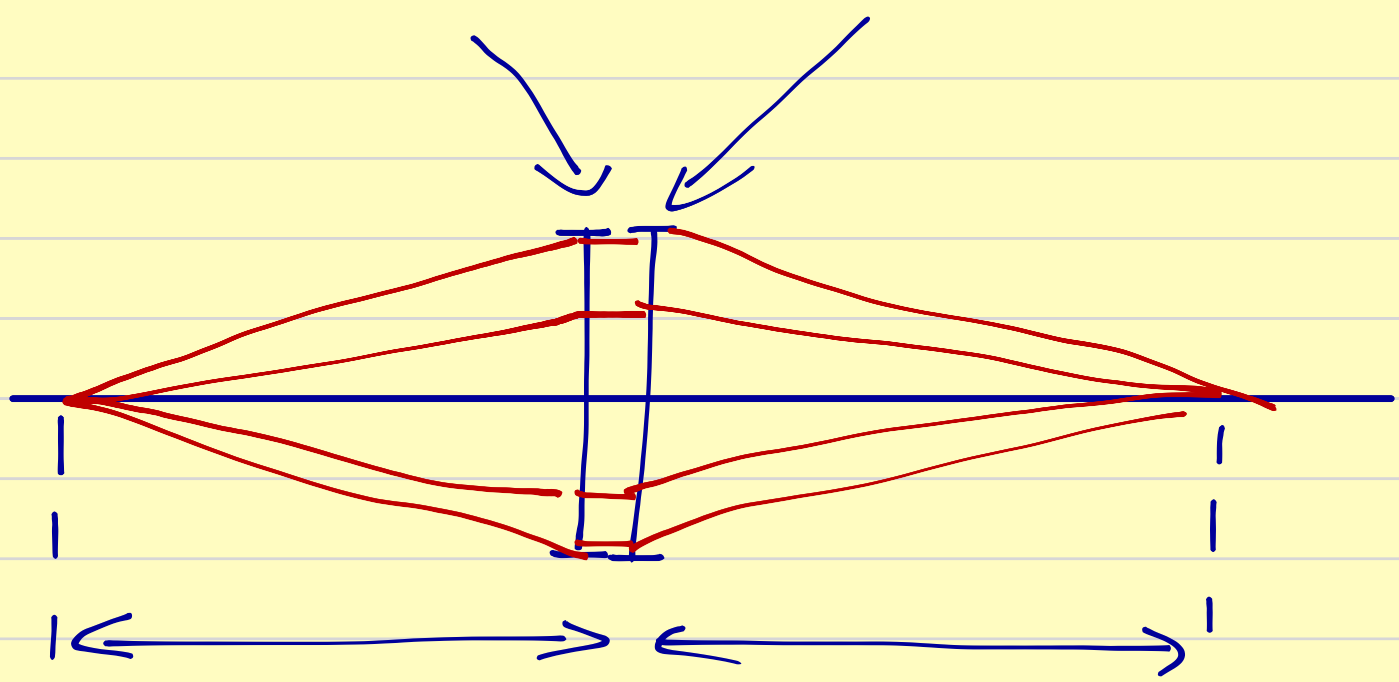
$\therefore$ brightness $\sim$

$$\frac{\text{lens area}}{\text{image area}} \sim \frac{D^2}{(\alpha f)^2} \sim \left(\frac{f}{D}\right)^{-2} = (f\text{-no.})^{-2}$$

$$\boxed{f\text{-number} = \frac{\text{focal length}}{\text{'pupil' diameter}}}$$

Combining two lenses





$$\Rightarrow \boxed{\frac{1}{u} + \frac{1}{v} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f}}$$ if lenses are close

$\frac{1}{f} \rightarrow$ power of the lens, measured in <u>dioptre</u> : [dioptre] = [m^{-1}]

ABCD ray-transfer matrices

ray-vector $\begin{pmatrix} y \\ \theta \end{pmatrix}$ ← dist. from axis
← angle to axis

$$\begin{pmatrix} y_2 \\ \theta_2 \end{pmatrix} = \underbrace{\begin{pmatrix} A & B \\ C & D \end{pmatrix}}\begin{pmatrix} y_1 \\ \theta_1 \end{pmatrix}$$ transfer

propagation by d

$$\boxed{S_d = \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}}$$

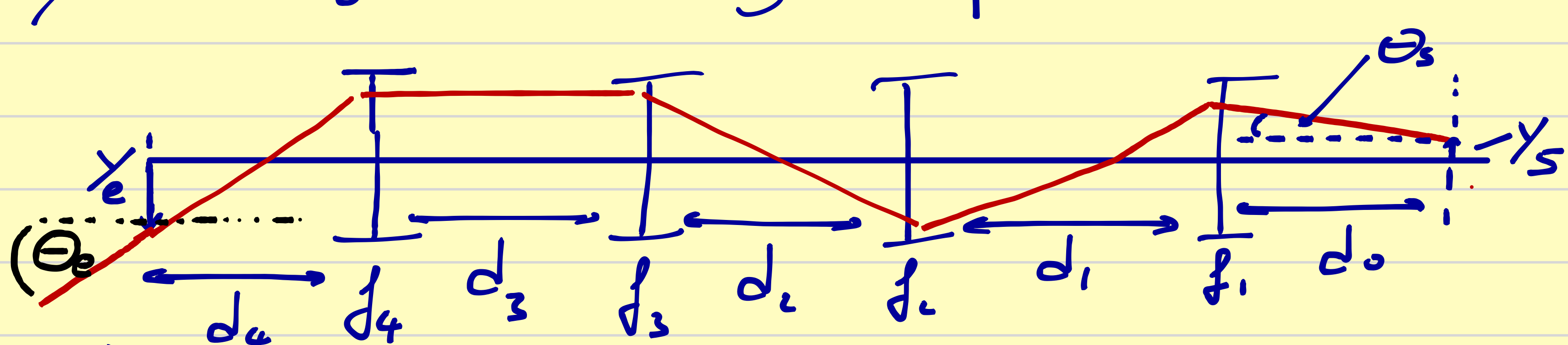
$y_2 = y_1 + \theta_1 d$
$\theta_2 = \theta_1$

lens of f focal length

$$\boxed{S_f = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}}$$

$y_2 = y_1$
$\theta_2 = \theta_1 - y_1/f$

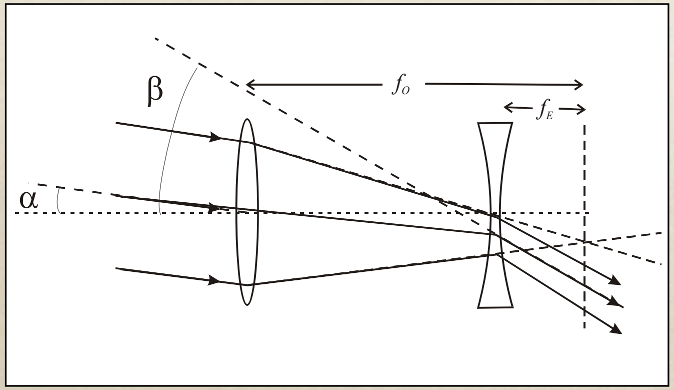
Ray transfer through optical system



$$\begin{pmatrix} y_e \\ \theta_e \end{pmatrix} = \underbrace{S_{d_4} S_{f_4} S_{d_3} S_{f_3} S_{d_2} S_{f_2} S_{d_1} S_{f_1} S_{d_0}}_{\text{system transfer}} \begin{pmatrix} y_s \\ \theta_s \end{pmatrix}$$

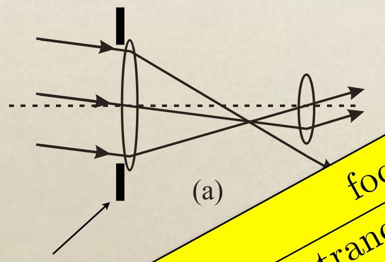
Geometrical Optics – Instruments

Galilean telescope

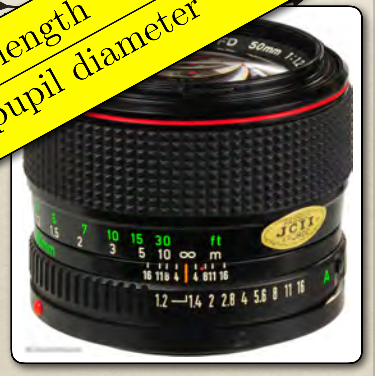


angular magnification $= \beta/\alpha = f_o/f_E$

Apertures and Field Stops



$$f/no. = \frac{\text{focal length}}{\text{entrance pupil diameter}}$$

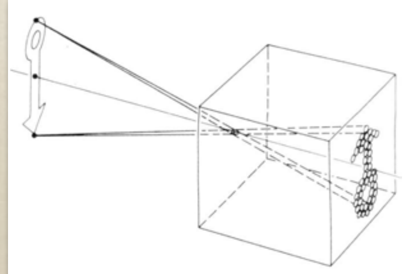


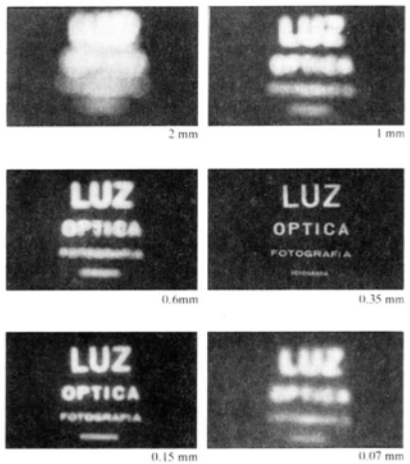
Aper

limi

Intensity

Camera Obscura





optimum pinhole size

contradicts expectations from geometrical optics

from Hecht, Optics

The Wave Nature of Light

- *Maxwell's equations* $\longmapsto$ *waves*
- *equivalence to matter waves*
- *plane and spherical waves*
- *energy flow / intensity*
- *Huygen's principle*

Wave Nature of Light

Maxwell with $\rho = 0$ and $\underline{j} = 0$

$$\boxed{\begin{array}{ll} \vec{\nabla}\cdot\underline{B} = 0 & \vec{\nabla}\times\underline{H} = \frac{d}{dt}\underline{D} \\ \vec{\nabla}\cdot\underline{D} = 0 & \vec{\nabla}\times\underline{E} = -\frac{d}{dt}\underline{B} \end{array}}$$

$\underline{D} = \varepsilon_r\varepsilon_0\,\underline{E}$

$\underline{B} = \mu_r\mu_0\,\underline{H}$

$\frac{d}{dt}$

$$-\underbrace{\vec{\nabla}\times\vec{\nabla}\times\underline{E}}_{-\vec{\nabla}(\vec{\nabla}\cdot\underline{E})+\vec{\nabla}\cdot(\vec{\nabla}\underline{E})} = \underbrace{\mu_0\varepsilon_0}_{1/c^2}\underbrace{\mu_r\varepsilon_r}_{n^2}\frac{\partial^2\underline{E}}{\partial t^2}$$

$$-\vec{\nabla}\underbrace{(\vec{\nabla}\cdot\underline{E})}_{=0}+\vec{\nabla}\cdot(\vec{\nabla}\underline{E}) = 1/c^2\; n^2\;\frac{\partial^2\underline{E}}{\partial t^2}$$

$$\boxed{\vec{\nabla}^2\underline{E} = \left(\frac{n}{c}\right)^2\frac{\partial^2}{\partial t^2}\underline{E}}$$

$\rightarrow$ wave equ. in each component of $\underline{E}$ (or $\underline{H}$)

$c = 1/\sqrt{\mu_0\varepsilon_0}$ and $n = \sqrt{\mu_r\varepsilon_r} \simeq \sqrt{\varepsilon_r}$

Ansatz: $\underline{E} = \underline{E}_{sp}\, e^{-i\omega t}$

(or $\underline{H} = \underline{H}_{sp}\, e^{-i\omega t}$)

$\boxed{\text{note real amplitudes: } \tfrac{1}{2}(\underline{E} + \underline{E}^*)}$

for each E-component:

$$\Delta E = \left(\frac{n}{c}\right)^2 \frac{\partial^2}{\partial t^2} E = -\omega^2 \left(\frac{n}{c}\right)^2 E$$

$$\hookrightarrow \boxed{\begin{aligned} &(\Delta + n^2 k_0^2)\, E = 0 \\ \text{or } &(\Delta + k^2)\, E = 0 \end{aligned}}$$

wavevector $\boxed{k = \frac{2\pi}{\lambda} = n k_0 = n \frac{2\pi}{\lambda_0} = n \frac{\omega}{c}}$

Note: this also holds for matter waves

$$\underline{TISE}: \left(-\frac{\hbar^2}{2m}\Delta + V\right)\psi = E_t\psi$$

$$\hookrightarrow \left(\Delta + \frac{2m\overbrace{(E_t - V)}^{E_{kinetic}}}{\hbar^2}\right)\psi = 0$$

with $2mE_{kinetic} = p^2 = \hbar^2 k^2$

$$\hookrightarrow \boxed{(\Delta + k^2)\psi = 0}$$

- looks familiar
- $\hbar^2 k^2 = 2m(E_t - V)$ for matter waves

<u>back to</u> light $(\Delta + k^2)u = 0$

u: general amplitude $\sim E$ or H

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

<u>Ansatz</u> $u(x,t) = u_0 \, e^{i(\underline{k}\cdot\underline{r} - \omega t)}$

with $|\underline{k}| = n k_0 = n \frac{\omega}{c} = \frac{\omega}{v_p}$

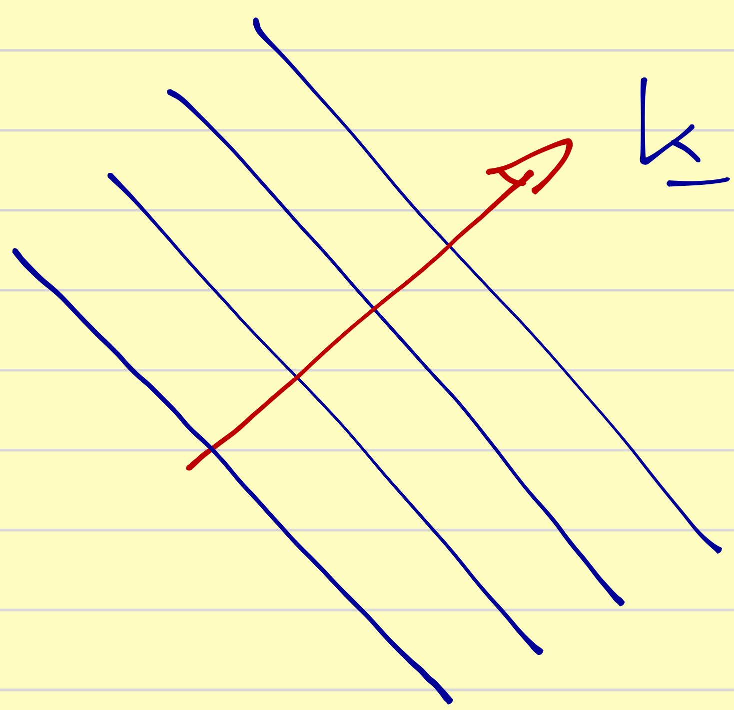
($v_p = c/n$ = phase velocity)

→ solves the wave equation

→ <u>Plane Wave</u>

propagating with v_p =

wavefronts perpendicular to $\underline{k}$



↳ defined by planes of constant phase

$$\frac{d}{dt}(\underline{k}\underline{r} - \omega t) = 0$$

obdA assume $\underline{k} = \begin{pmatrix} 0 \\ 0 \\ k \end{pmatrix}$

$$\Rightarrow \quad k \frac{dz}{dt} = \omega$$

$$\Rightarrow \quad z = t \cdot \frac{\omega}{k} + \text{const.} = t v_p + \text{const.}$$

$\therefore$ Wavefronts propagate with v_p

Field direction?

$$\underline{E}(\underline{r}, t) = \underline{E}_0 \, e^{i(\underline{k}\,\underline{r} - \omega t)}$$

$\longrightarrow$ into Maxwell's equations:

$\omega \underline{B} = \underline{k} \times \underline{E}$ and $-\omega \underline{D} = \underline{k} \times \underline{H}$

$\underline{k} \cdot \underline{B} = 0$ and $\underline{k} \cdot \underline{D} = 0$

$$\Rightarrow \quad \boxed{\begin{matrix} \underline{B} \perp \underline{k} & \quad \underline{D} \perp \underline{k} \\ \underline{B} \perp \underline{E} & \quad \underline{D} \perp \underline{H} \end{matrix}}$$

$\Rightarrow$ EM waves mostly transverse!

Energy density + Intensity

$$\rho_{EM} = \frac{1}{2}\left(\underline{E}\cdot\underline{D} + \underline{B}\cdot\underline{H}\right)$$

Energy flow $\longrightarrow$ Poynting vector

$$\underline{S} = \underline{E} \times \underline{H} \parallel \underline{k}$$

$$[\underline{S}] = W/m^2$$

(Henry Poynting, 1884)

<u>Intensity</u> $\quad \underline{I} = \langle \underline{S} \rangle_t$

$$= \frac{1}{2}\varepsilon_0 E_0^2 \sqrt{\frac{\mu_r \varepsilon_r}{\mu_0 \varepsilon_0}} \simeq \underbrace{\frac{1}{2}\varepsilon_r \varepsilon_0 E_0^2}_{\rho_{el}} \cdot \underbrace{\frac{c}{n}}_{v_p}$$

$\mu_r \simeq 1$

Other Solutions?

→ any superposition ...

→ e.g. spherical waves

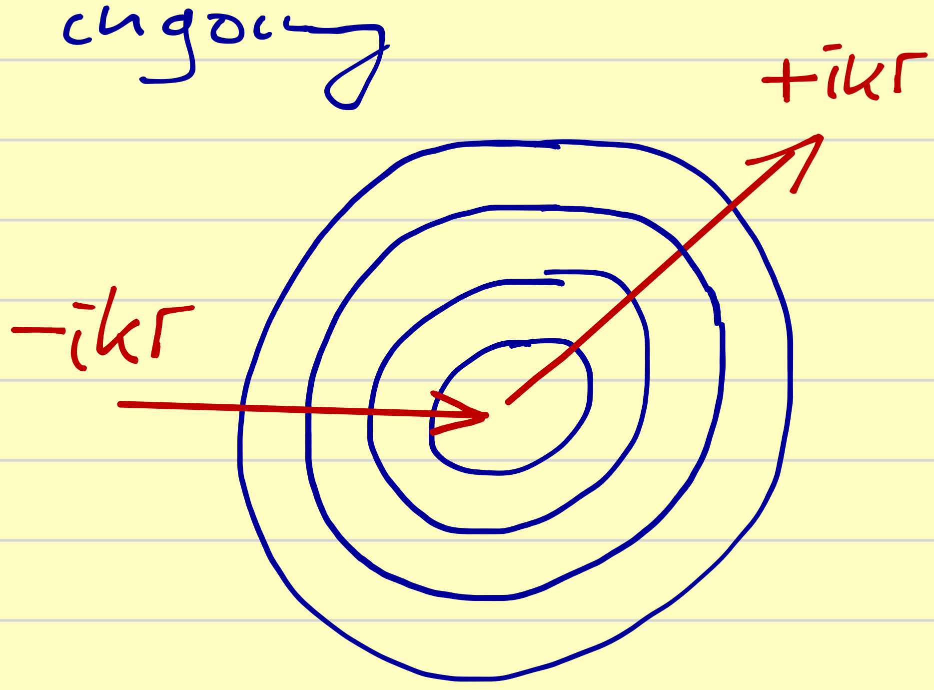
$$\Delta = \frac{1}{r}\frac{\partial^2}{\partial r^2} r + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\Phi^2}$$

this yields

$$\boxed{u(r) = \frac{u_0}{r} e^{\pm ikr - i\omega t}}$$

$\pm ikr$ = out- or ingoing

→ Huygen's wavelets



Maxwell's Equations

$$\vec{\nabla}\cdot\vec{B}=0 \qquad \vec{\nabla}\times\vec{H}=\frac{d}{dt}\vec{D}$$

$$\vec{\nabla}\cdot\vec{D}=0 \qquad \vec{\nabla}\times\vec{E}=-\frac{d}{dt}\vec{B}$$

$$\begin{aligned}\vec{D} &= \epsilon_r\epsilon_0\vec{E}\\ \vec{B} &= \mu_r\mu_0\vec{H}\end{aligned}$$

linear isotropic medium

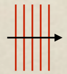
$\rho=0 \qquad \vec{J}=0$

$$\nabla^2\vec{E}=\left(\frac{n}{c}\right)^2\frac{d^2}{dt^2}\vec{E} \quad \text{and} \quad \nabla^2\vec{H}=\left(\frac{n}{c}\right)^2\frac{d^2}{dt^2}\vec{H}$$

Plane and Spherical Waves

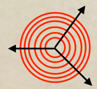
$(\Delta+k^2)u=0$ $\qquad$ $u \rightarrow$ *amplitude of* $E, H, \psi \ldots$

plane wave



$$u(\vec{r},t)=u_0e^{i(\vec{k}\vec{r}-\omega t)}$$

spherical wave

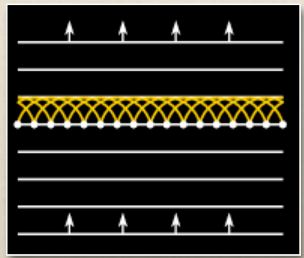


$$u(\vec{r},t)=u_0e^{i(kr-\omega t)}/r$$

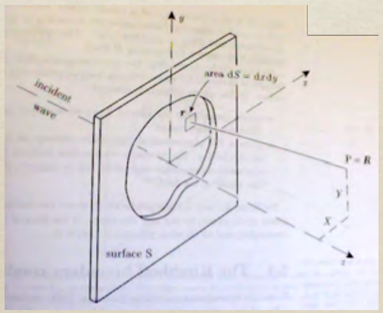
$$\begin{aligned}n &= \sqrt{\epsilon_r\mu_r}\\ c &= 1/\sqrt{\epsilon_0\mu_0}\\ v_p &= c/n=\omega/k=\nu\lambda\\ \vec{S} &= \vec{E}\times\vec{H}\end{aligned}$$

$$k=\frac{2\pi}{\lambda}=nk_0$$

Huygen's Principle

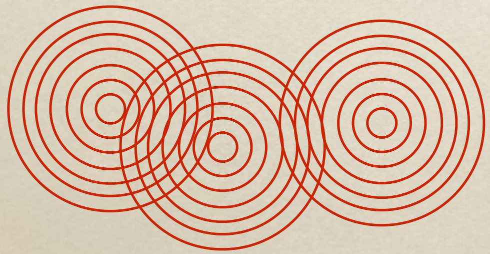


Every point on a wave front can be considered as a source of secondary spherical waves



$$u(\vec{R}) \propto \int u(\vec{r})\frac{e^{ik|\vec{R}-\vec{r}|}}{|\vec{R}-\vec{r}|}dS$$

Interference of Waves



Huygen's principle

- Assume $u_0(\underline{r})$ is known in an optical plane
- $u(\underline{R})$ is the superposition of spherical wavelets emanating from all points of the plane

$$\rightarrow u(\underline{R}) = \int_S \frac{u(\underline{r})}{|\underline{R}-\underline{r}|} e^{ik|\underline{R}-\underline{r}|} dS$$

Special case = Surface S @ $z=0$

$$\therefore \underline{r} = \begin{pmatrix} x \\ y \\ 0 \end{pmatrix} \qquad \underline{R} = \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}$$

$$u(\underline{R}) = \int_{x,y} \frac{u(x,y,0)\, e^{ik\sqrt{\text{sa.}}}}{\sqrt{(x'-x)^2+(y'-y)^2+z'^2}}$$

Fresnell - Kirchoff integral