C. MAGNETIZABLE MATERIALS

1. Magnetization: definition and physical origins

When a magnetizable material is placed in a magnetic field $\underline{B}$ it acquires a magnetic dipole moment. This is measured by the magnetization $\underline{M}$ defined as the magnetic dipole moment per unit volume averaged over a length scale d

($\text{Å} \ll d \ll$ sample size)

Why does the field induce a magnetic dipole moment?

(i) all materials

the field changes the shape of the electron orbits by a small amount to give an extra dipole moment $\underline{M} \propto -\underline{B}$

this is diamagnetism ; typically a very small effect

(ii) materials with unpaired electrons

some atoms have an intrinsic magnetic dipole moment due to the angular momentum and spin of the unpaired electron. in a field a small excess point along $\underline{B}$

this is paramagnetism (intrinsically a QM effect)
$\underline{M} \propto \underline{B}$ unless field is very large (Curie's law)

(i), (ii) linear materials ie $\underline{M} \propto \underline{B}$

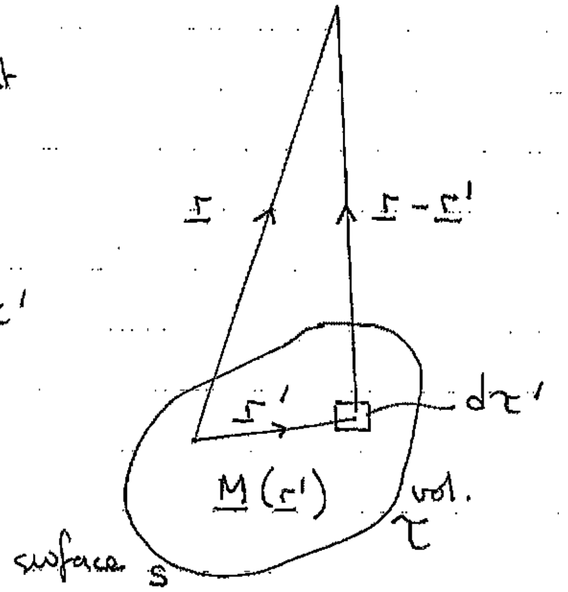
(iii) ferromagnets (also a QM effect)

non-linear – see section C7

2. bound currents

> The vector potential ($\therefore$ the $\underline{B}$-field) of an object with magnetization $\underline{M}$ is the same as the vector potential produced by
>
> a volume current density $\underline{J}_b = \text{curl } \underline{M}$
>
> plus a surface current density $\underline{K}_b = \underline{M} \wedge \hat{\underline{n}}$

vector potential due to a dipole moment per unit volume $\underline{M}$



$$\underline{A}(\underline{r}) = \frac{\mu_0}{4\pi} \int_\tau \frac{\underline{M}(\underline{r}') \wedge (\widehat{\underline{r}-\underline{r}'})}{|\underline{r}-\underline{r}'|^2} \, d\tau'$$

vector algebra using the following steps:

1. write $\dfrac{(\widehat{\underline{r}-\underline{r}'})}{|\underline{r}-\underline{r}'|^2}$ as $\nabla' \dfrac{1}{|\underline{r}-\underline{r}'|}$

2. use $\text{curl}' f\underline{A} = f \,\text{curl}' \underline{A} - \underbrace{\underline{A}}_{\underline{M}(\underline{r}')} \wedge \text{grad}' \underbrace{f}_{\frac{1}{|\underline{r}-\underline{r}'|}}$

3. use divergence thm.

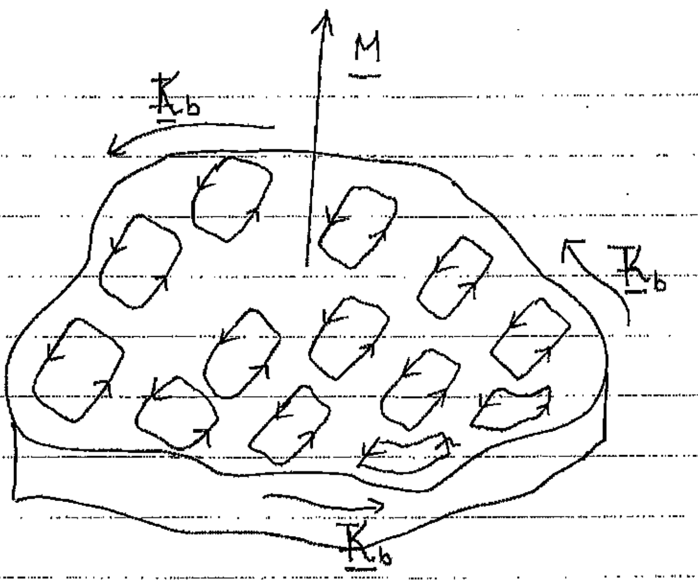
$$\int_\tau \text{curl } \underline{R} \, d\tau = -\int_S (\underline{R} \wedge \hat{\underline{n}}) \, dS$$

$$\frac{\mu_0}{4\pi}\int_\tau \frac{\text{curl}'\{\underline{M}(\underline{r}')\}}{|\underline{r}-\underline{r}'|}\,d\tau' + \frac{\mu_0}{4\pi}\int_S \frac{\underline{M}(\underline{r}')\wedge\hat{\underline{n}}}{|\underline{r}-\underline{r}'|}\,dS'$$

$\downarrow$ vector potential of a volume current density $\underline{J_b} \equiv \text{curl}\,\underline{M}$

$\downarrow$ vector potential of a surface current density $\underline{K_b} \equiv \underline{M}\wedge\hat{\underline{n}}$

cartoon of bound current distribution:



$$\underline{K_b} = \underline{M}\wedge\underline{n}$$

$$\underline{J_b} = \text{curl}\,\underline{M}$$

3. Ampère's law in magnetised materials and $\underline{H}$

$$\text{curl } \underline{B} = \mu_0 (\underline{J}_f + \underline{J}_b) = \mu_0 (\underline{J}_f + \text{curl } \underline{M})$$

$$\text{curl}\left(\frac{\underline{B}}{\mu_0} - \underline{M}\right) = \underline{J}_f$$

define $$\boxed{\underline{H} = \frac{\underline{B}}{\mu_0} - \underline{M}} \qquad (1)$$

$$\text{curl } \underline{H} = \underline{J}_f \qquad (2)$$

4. Linear materials and the relative permeability μ_r

linear means $\underline{M} \propto \underline{B}$ (equivalently $\underline{M} \propto \underline{H}$, $\underline{B} \propto \underline{H}$)

write $\underline{M} = \chi_m \underline{H}$ ← choice of $\underline{H}$ not $\underline{B}$ a matter of definition

χ_m : magnetic susceptibility

from (1) $$\underbrace{\underline{H}(1+\chi_m)}_{\text{define as relative permeability } \mu_r} = \frac{\underline{B}}{\mu_0}$$

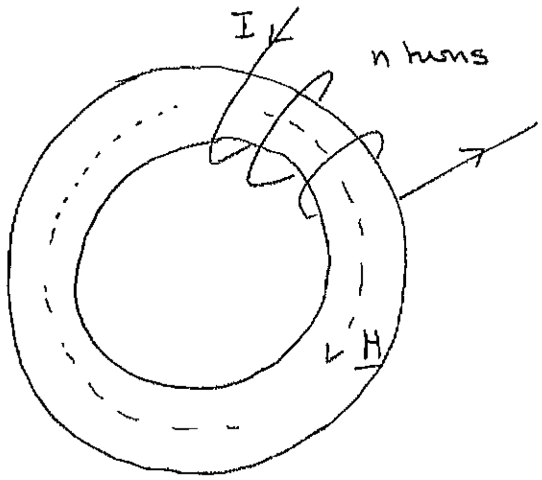
$$\boxed{\underline{B} = \mu_r \mu_0 \underline{H}} \qquad (3)$$

using (2) and (3)

$$\text{curl } \underline{B} = \mu_r \mu_0 \underline{J}_f$$

N.B 1: $\underline{H}$ is sometimes called the 'magnetic field' and $\underline{B}$ the 'magnetic flux density'.

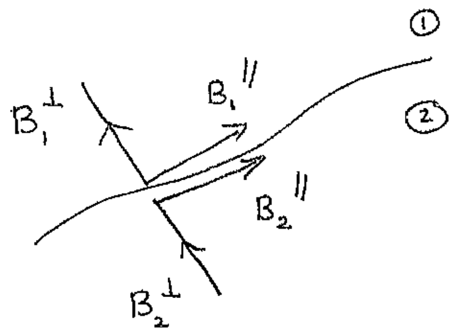
N.B 2: $\underline{H}$ is directly accessible experimentally:



$$\int \underline{H} \cdot d\underline{\ell} = nI$$

5. Most convenient boundary conditions on $\underline{B}$, $\underline{H}$

general magnetostatic boundary conditions are



$$\underline{B_1^{\perp} = B_2^{\perp}}$$ fine

$$B_1^{\parallel} - B_2^{\parallel} = \mu_0 (I_f^s + I_b^s)$$ inconvenient

but

$$H_1^{\parallel} - H_2^{\parallel} = I_f^s$$ and, if there are no free surface currents,

$$\underline{H_1^{\parallel} = H_2^{\parallel}}$$

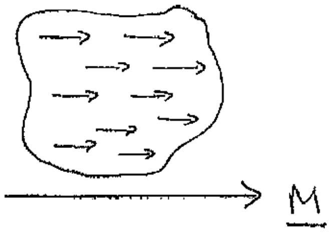
for a boundary between two magnetizable materials with no free currents

$$\left.\begin{array}{ll} B^{\perp} & (\text{'B normal'}) \\ H^{\parallel} & (\text{'H tangential'}) \end{array}\right\} \text{continuous}$$

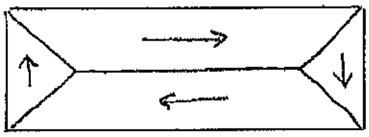
Ferromagnets

(i) microscopic picture

Atomic dipoles want to align because of the quantum mechanical exchange interaction : short range and strong locally a small ferromagnetic particle looks like



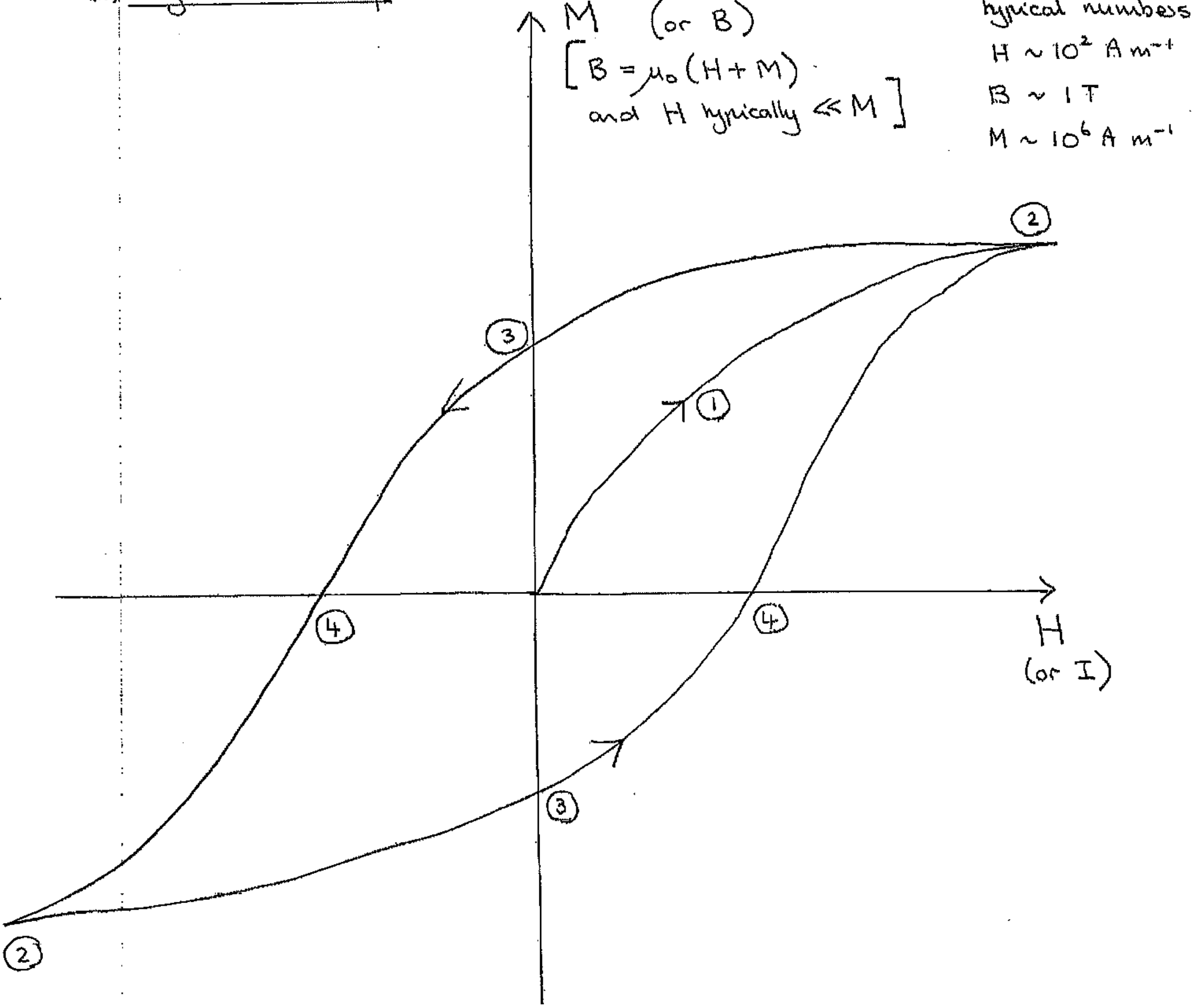
but there is also the dipolar interaction : weak, but long range. to minimise their dipolar energy the atomic dipoles form domains with $\underline{M}$ in different directions



For the whole sample $\underline{M} = 0$

In an applied field the domains along the field grow at the expense of those not $\parallel$ to the field.

(ii) Hysteresis loop



(1) H increases from zero; domains tend to align along field

(2) all domains aligned SATURATION

(3) $M \neq 0$ even when H returned to zero REMANENCE
useful for magnetic memories

(4) field in opposite direction needed to reduce M to zero
COERCIVE FORCE

NB (i) called hysteresis loop because M depends not only on H but also on the history of the sample – memory effects

(ii) example of a non-linear constitutive relation B not $\propto H$

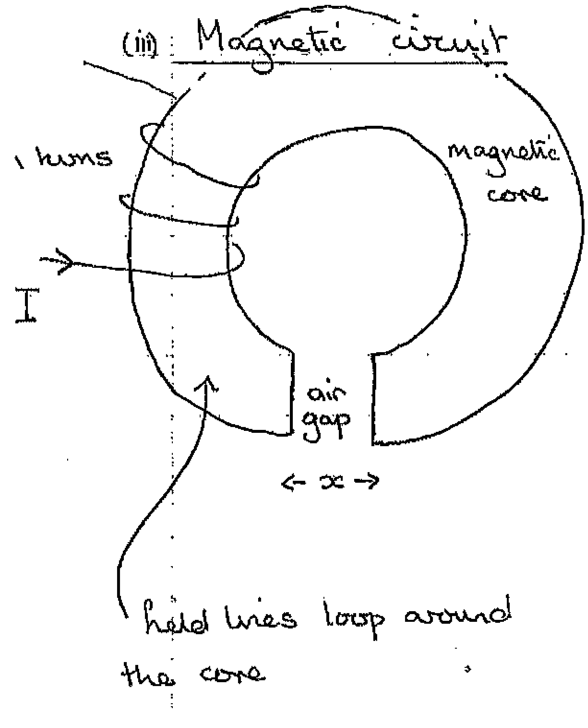
how B depends on H

($B = \mu\mu_0 H$ still written but μ depends on H)

(iii) HARD material; large remanence, large coercive force; hard to move domain walls; useful for permanent magnets

(iv) SOFT material; small remanence, small coercive force; easy to move domain walls; useful for electromagnets, transformers, motors.

(iii) Magnetic circuit



simple design of an electromagnet

length of path in core l
length of gap x
fields in core H_c ; B_c
fields in gap H_g ; B_g

"4 equations"

1. Ampère's law
$$H_c l + H_g x = nI$$
2. $B^{\perp}$ continuous
$$B_c = B_g$$
3. constitutive relation in gap
$$B_g = \mu_0 H_g$$
4. constitutive relation in material
$$B = f(H)$$
e.g. $B = \mu\mu_0 H$ if linear material
hysteresis loop if ferromagnet

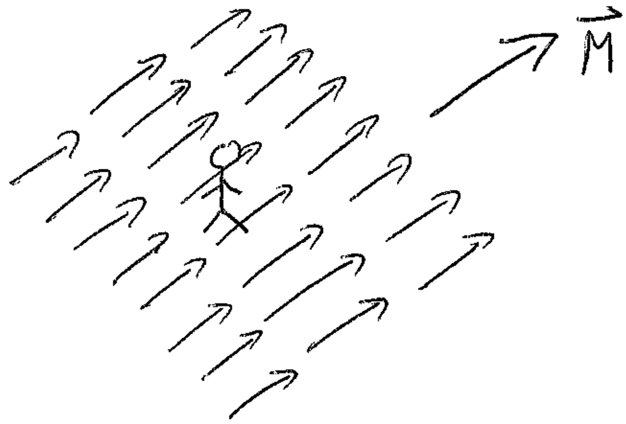
'Hidden' Symmetry (often called 'spontaneous symmetry breaking')

Ferromagnetic materials are a very good example of an exceptionally important phenomenon

– spontaneous symmetry breaking (SSB)

Although Maxwell's Eqns are rotationally symmetric (no preferred direction) inside a ferromagnetic domain this rotational symmetry is hidden – and indeed appears to be 'broken' (ie, violated)

Consider an observer inside a ferromagnet

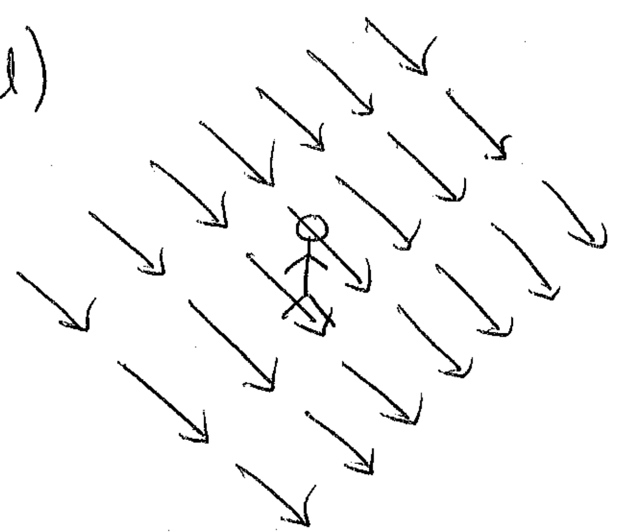


– the observer sees all the magnetic dipoles aligned in a particular direction, as is the associated $\vec{M}$. She sees no remnant of the full 3D rotation symmetry, (only a 2D symm around the axis of $\vec{M}$)

Experiments performed by the observer will be sensitive to this preferred direction, and she'll have a hard (but not impossible!) time deducing that the underlying eqns of EM are rotationally symm.

What has happened is that the <u>solution</u> of Maxwell's eqns in the presence of a ferromagnet has less symmetry than the underlying equations

If we explored another solution by heating the ferromagnet above the Curie temperature (so the mag. dipoles were disordered) and then cooling it down we would find the dipoles aligned again - but now in some <u>other</u> random direction (assuming no external biasing field)



Once again the observer does not see full 3D rotation symm. The fact that which exact dir'n the dipoles align in is random is a sign that the underlying eqns are symmetric.

Such spontaneous symmetry breaking phenomena are ubiquitous in the world!

Indeed our entire universe is really inside some type of SSB substance as the symmetry between $W^{\pm}$, Z and photons is broken (they have different masses) but the underlying eqns are symmetric. This SSB substance is the 'Higgs condensate'

In the words of Sidney Coleman all of us are 'the man in the magnet'!