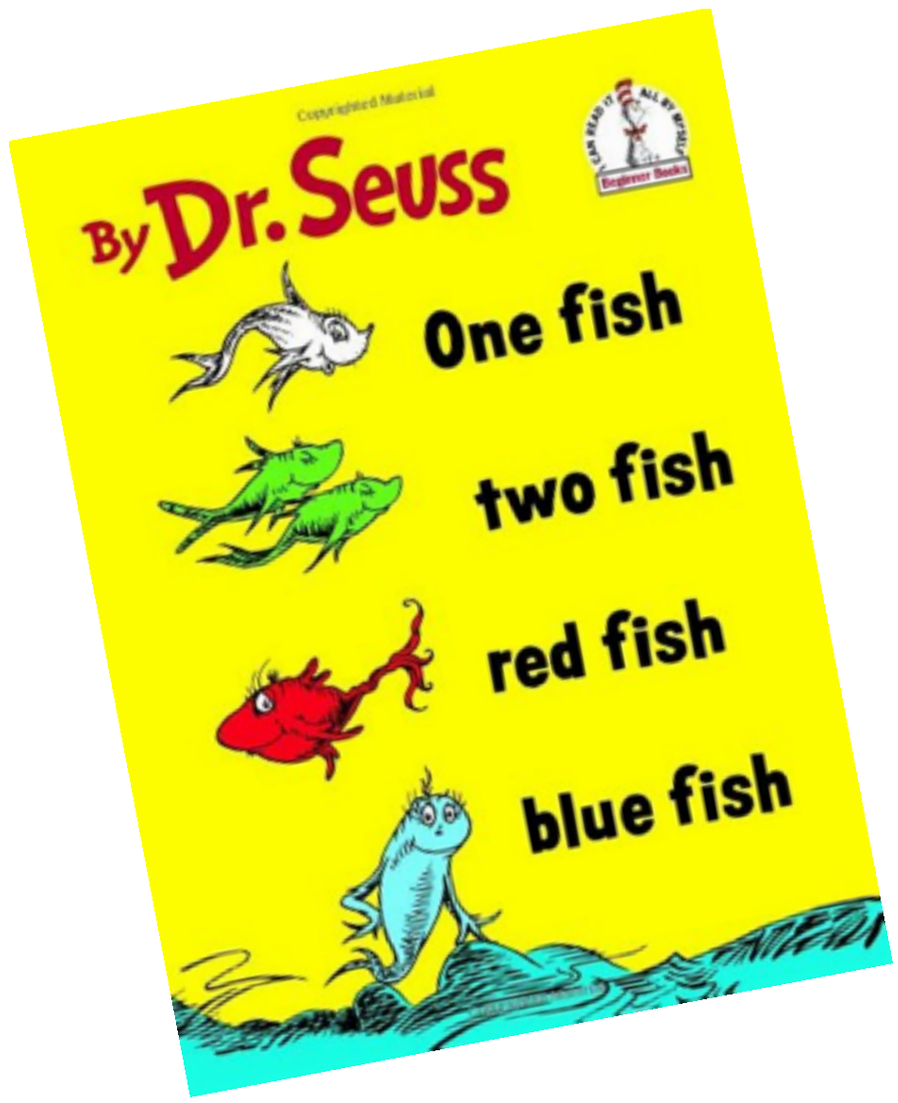
One fish, two fish, red fish, blue fish



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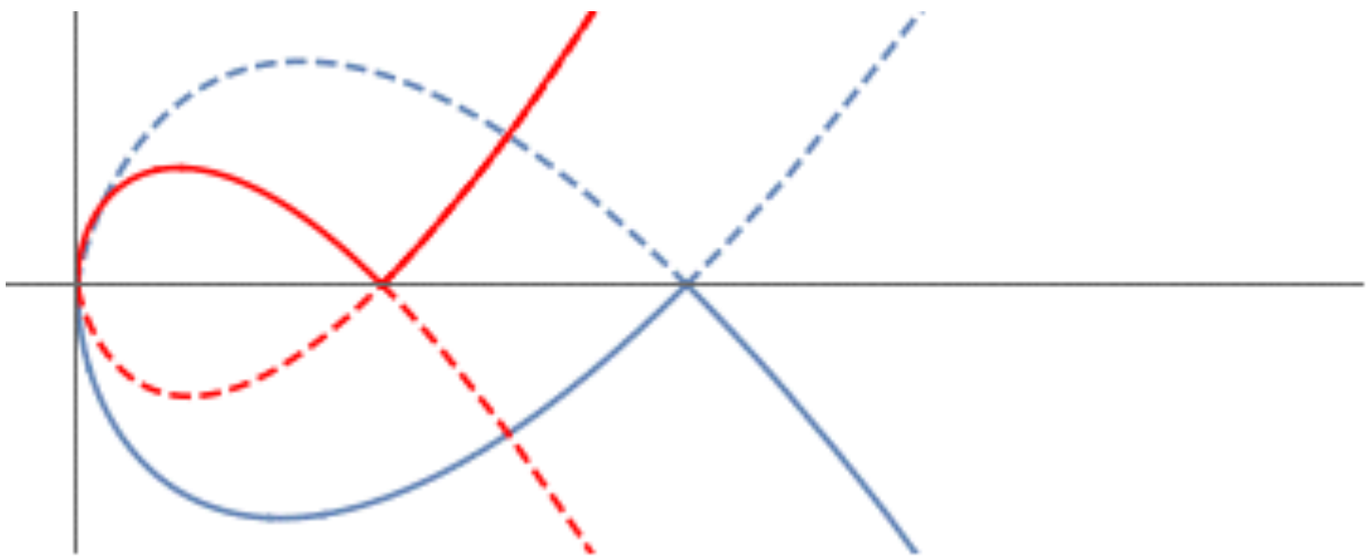
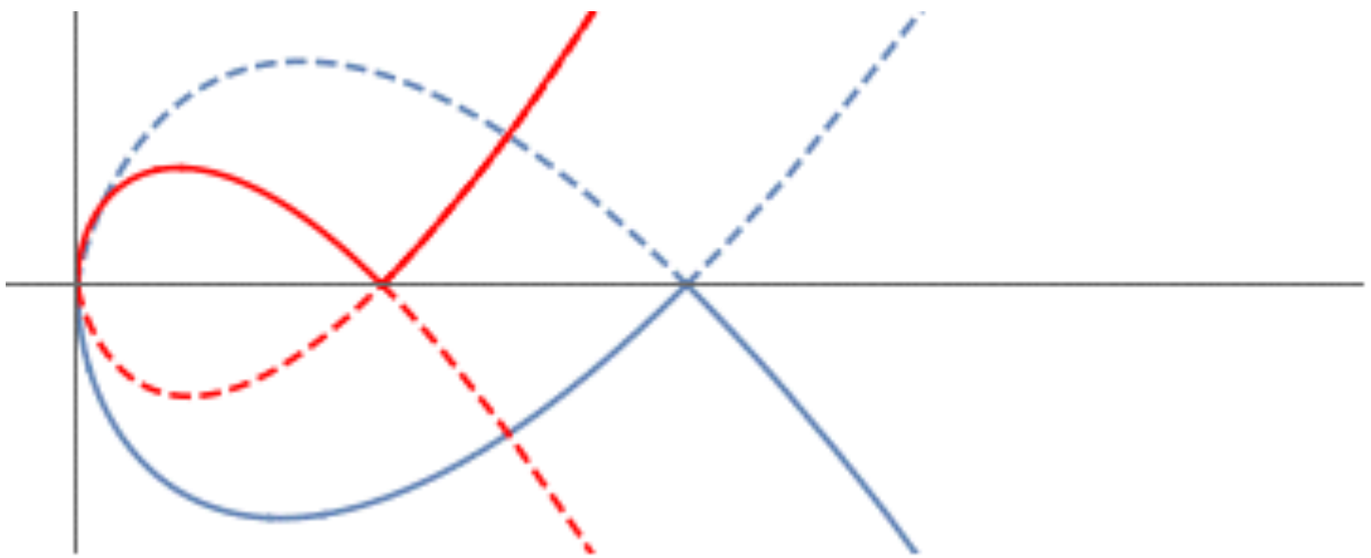
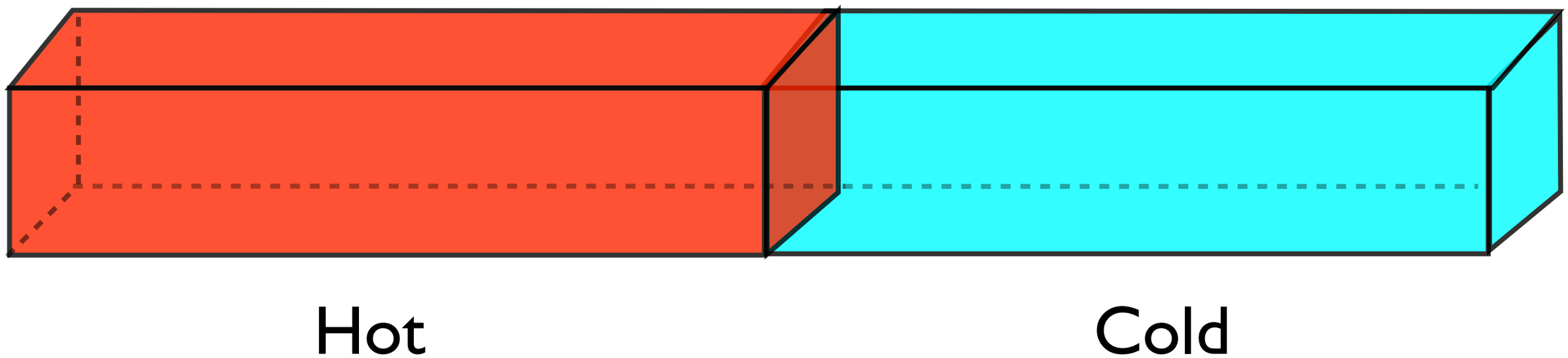


The Riemann problem at infinite d.



A. Karch, H. C. Chang, I. Amado, C. Herzog. M. Spillane

The problem I want to consider is as follows:



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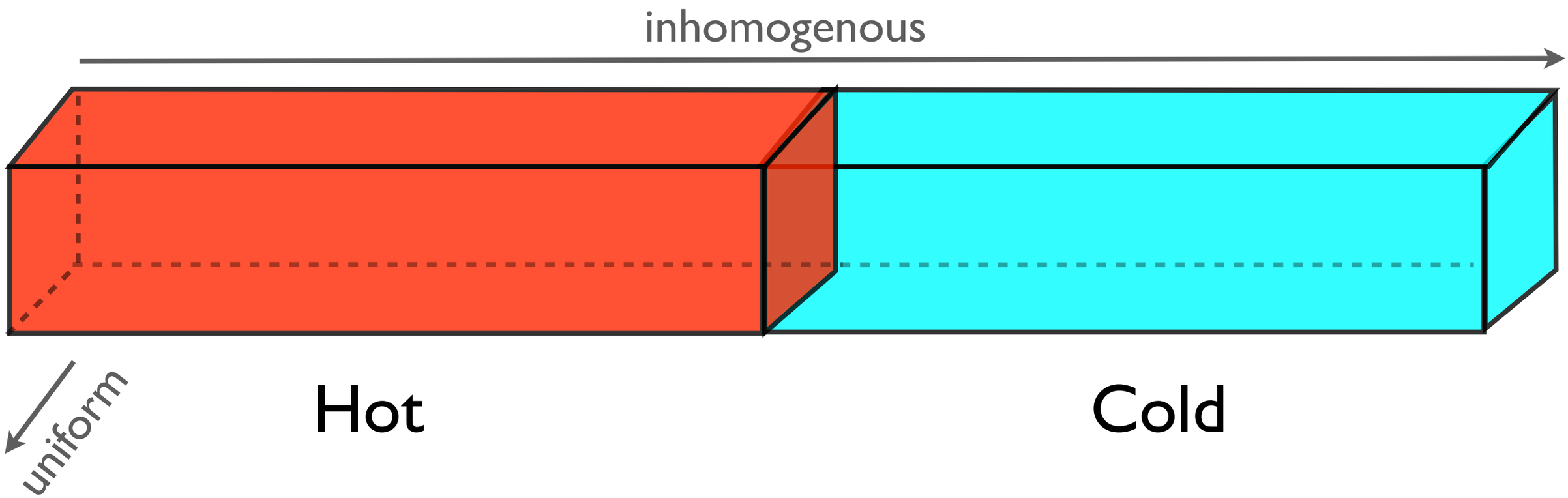

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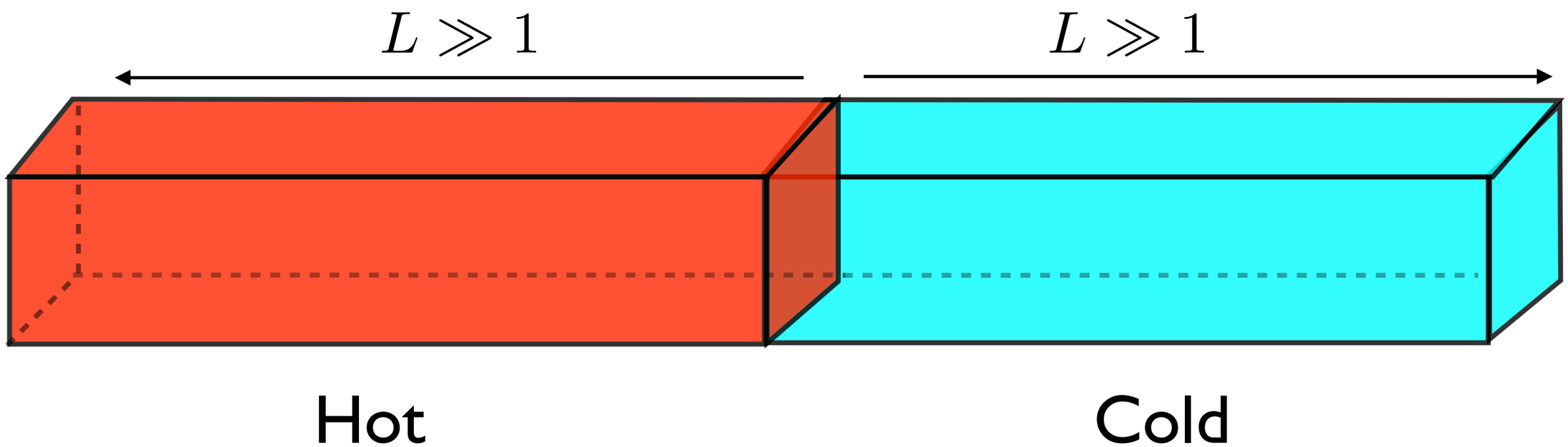

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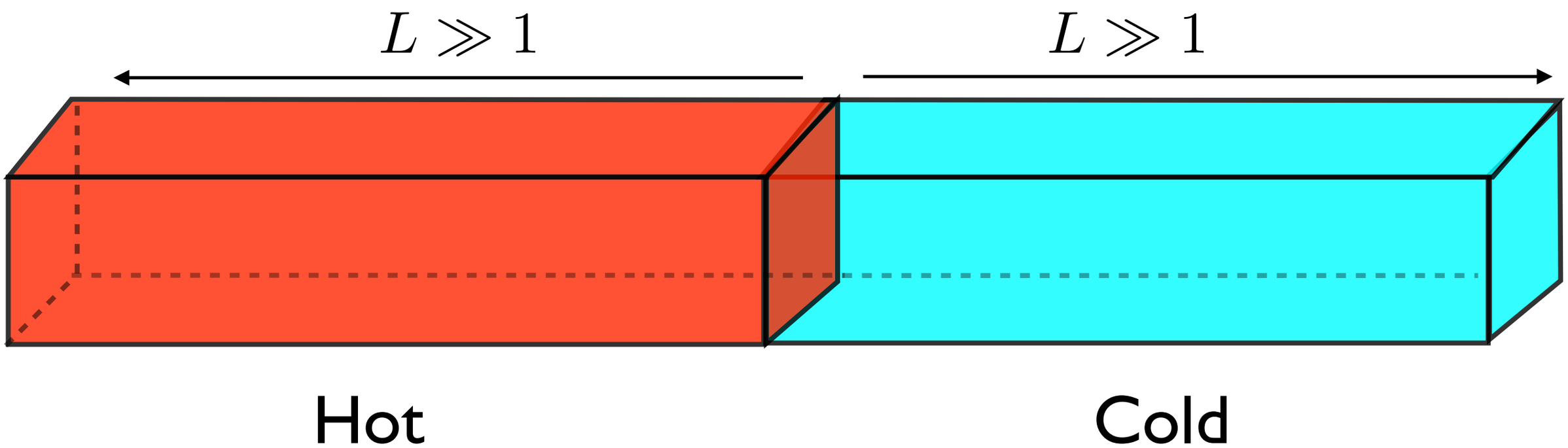

The evolution is governed by the dynamics of a conformal field theory:

$$\partial_\mu T^{\mu\nu} = 0 \qquad\qquad T^{\mu}{}_{\mu} = 0$$

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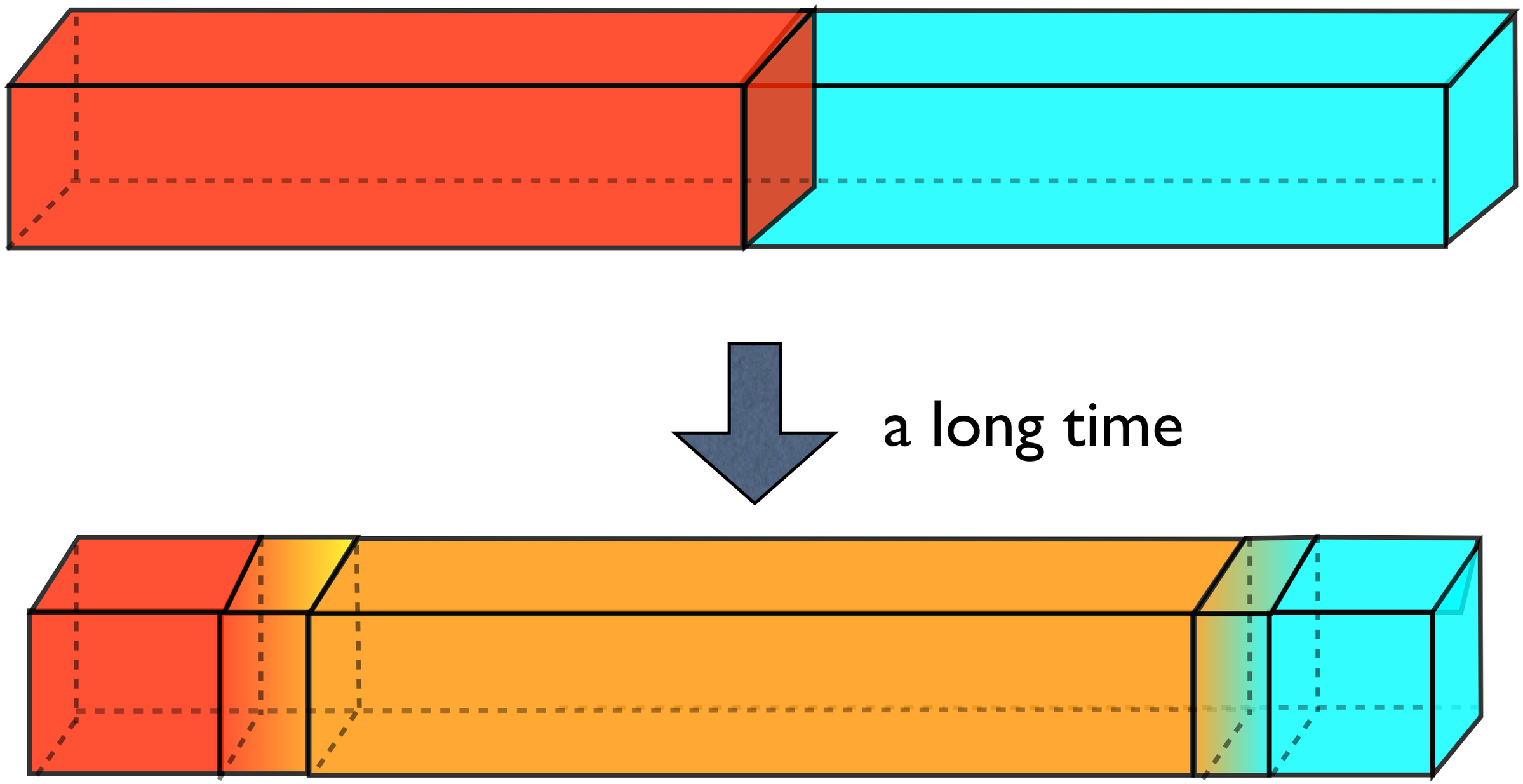

This configuration is an example of a non equilibrium steady state.

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Bernard & Doyon, 2012

Bhaseen et. al. 2013

H. -C. Chang, Karch & AY et. al. 2013

Amado & AY, 2015

Megias, 2015

PourHasan, 2015

Bachas, Skenderis & Withers, 2015

Lucas et. al., 2015

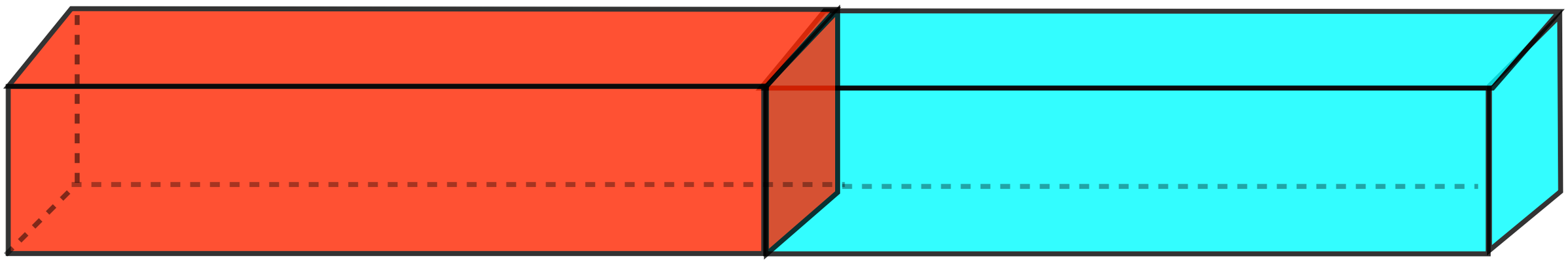
Herzog & Spillane, 2015

Herzog, Spillane, AY, 2016

Plan:

- Two dimensional systems
- Discussion of d>2
- Large d.

2d CFT's

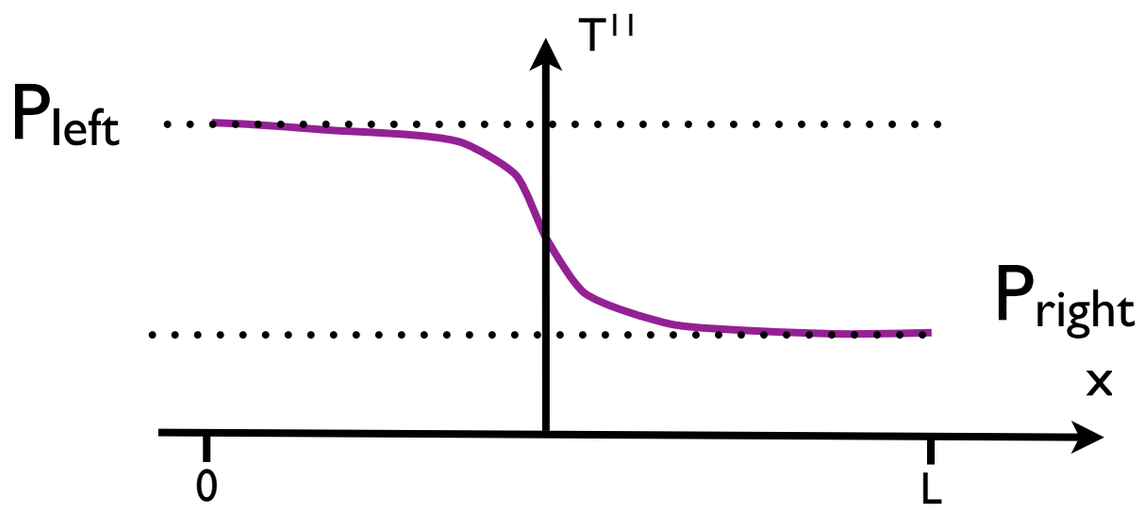


2d CFT's

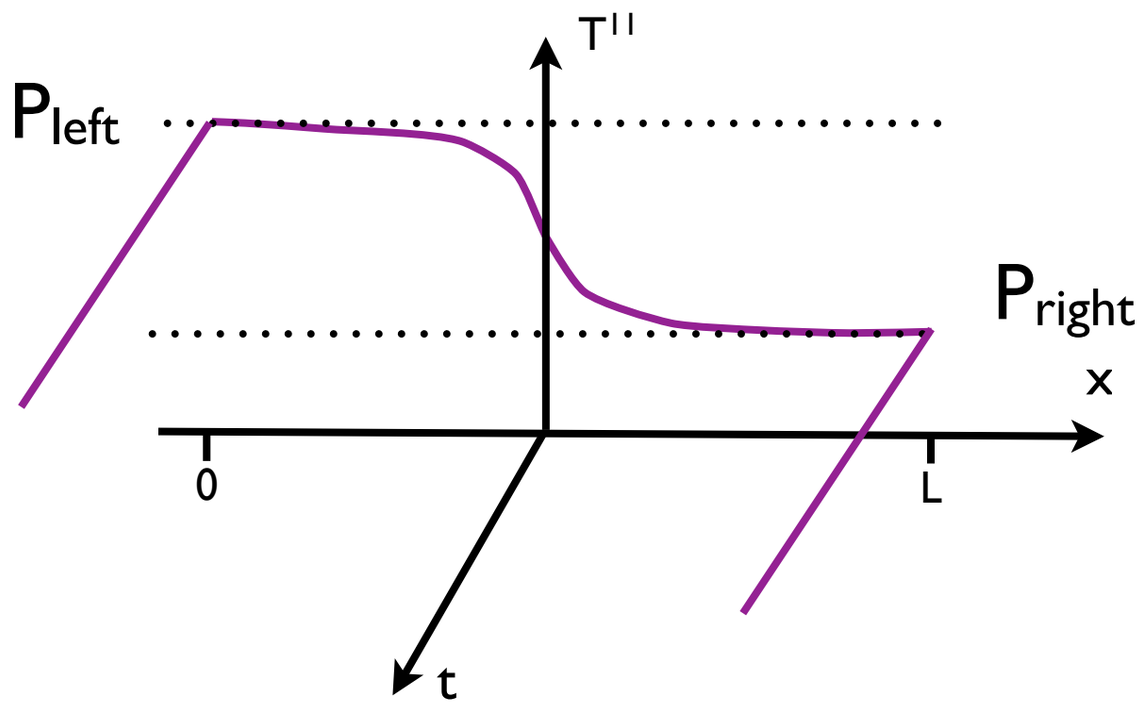
2d CFT's

2d CFT's

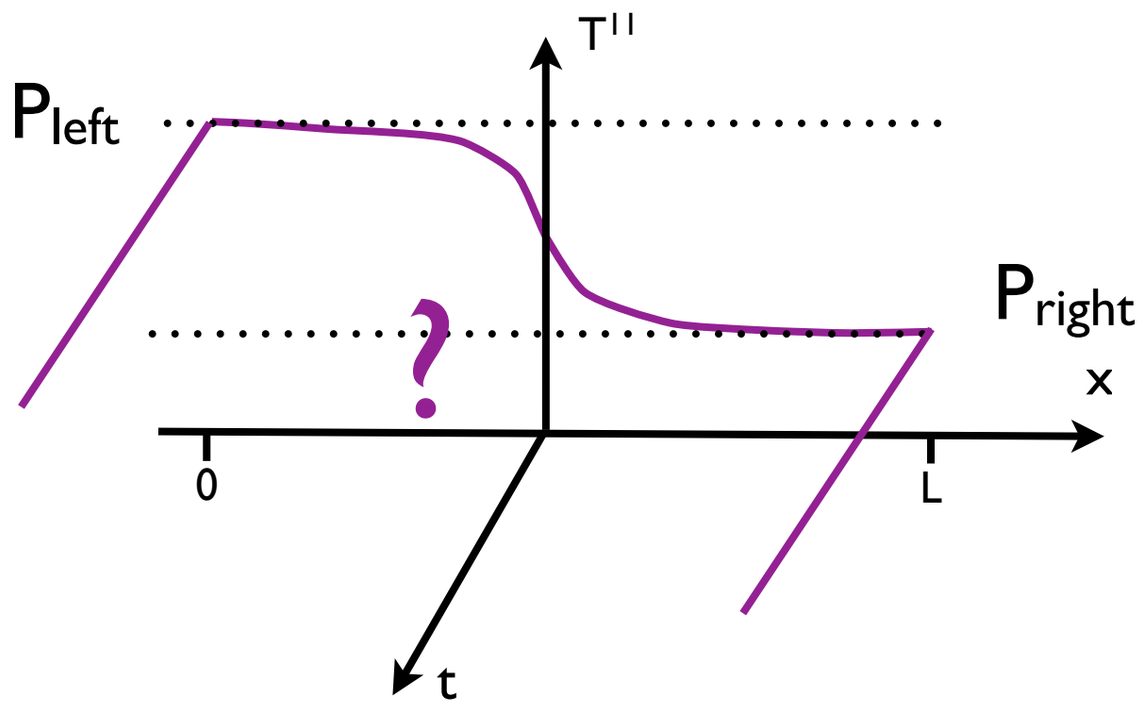
2d CFT's



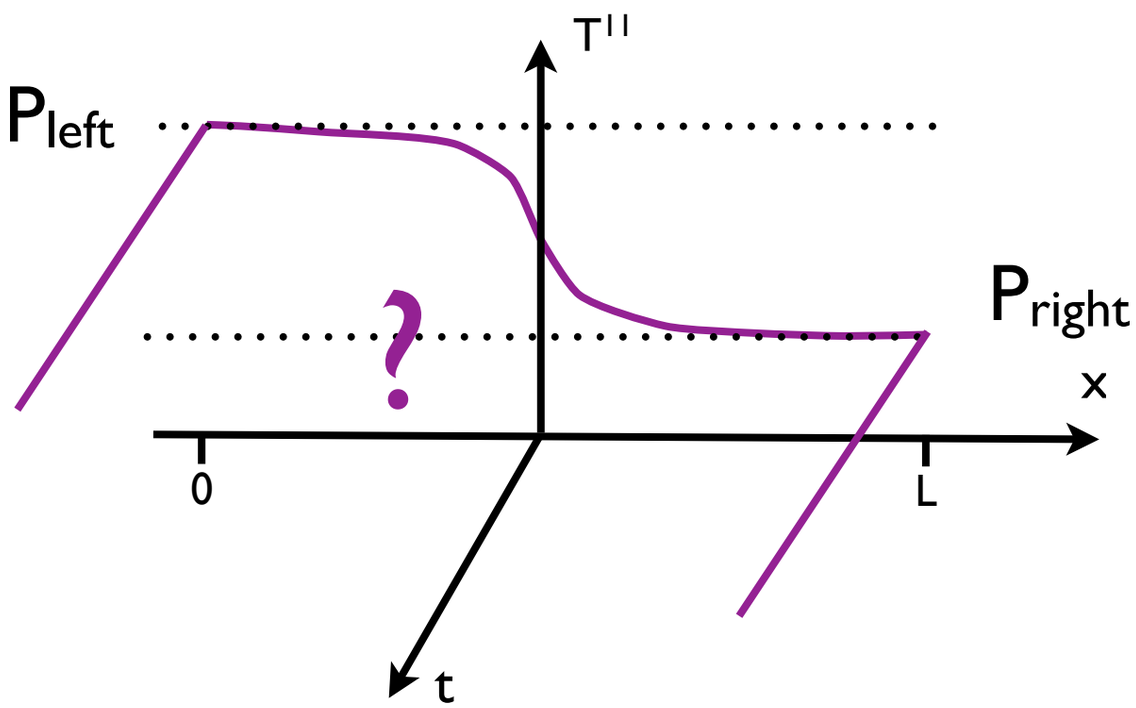
2d CFT's

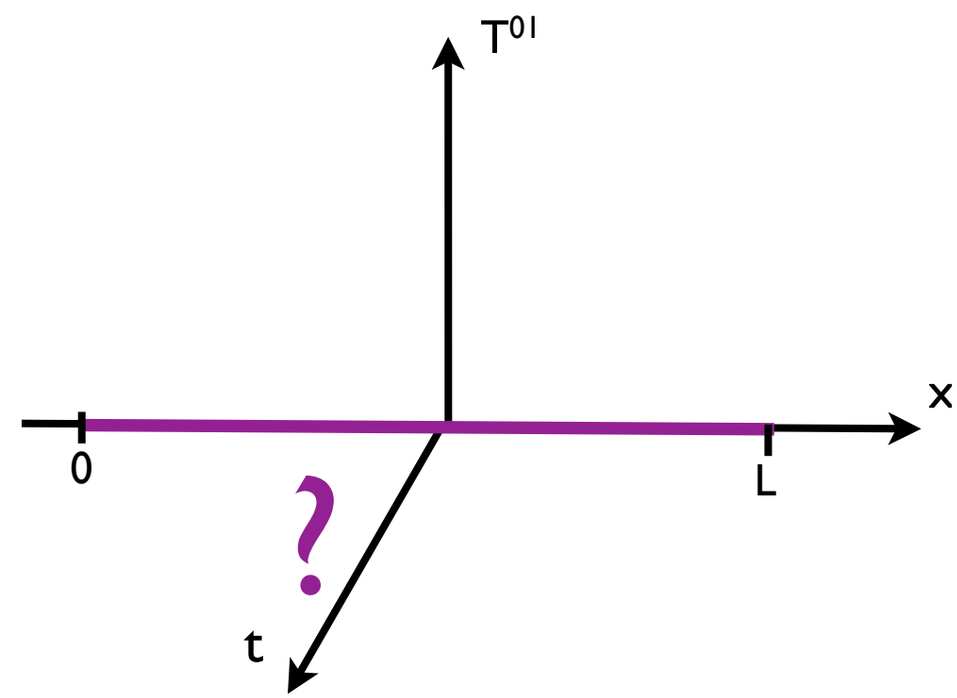


2d CFT's

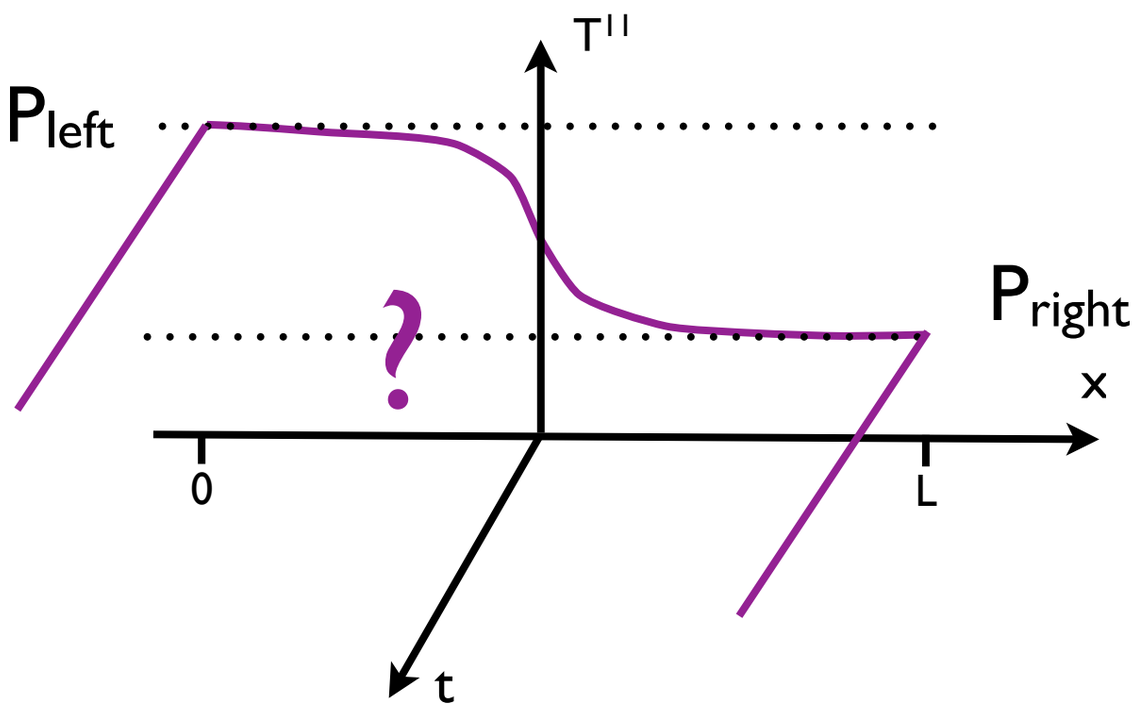


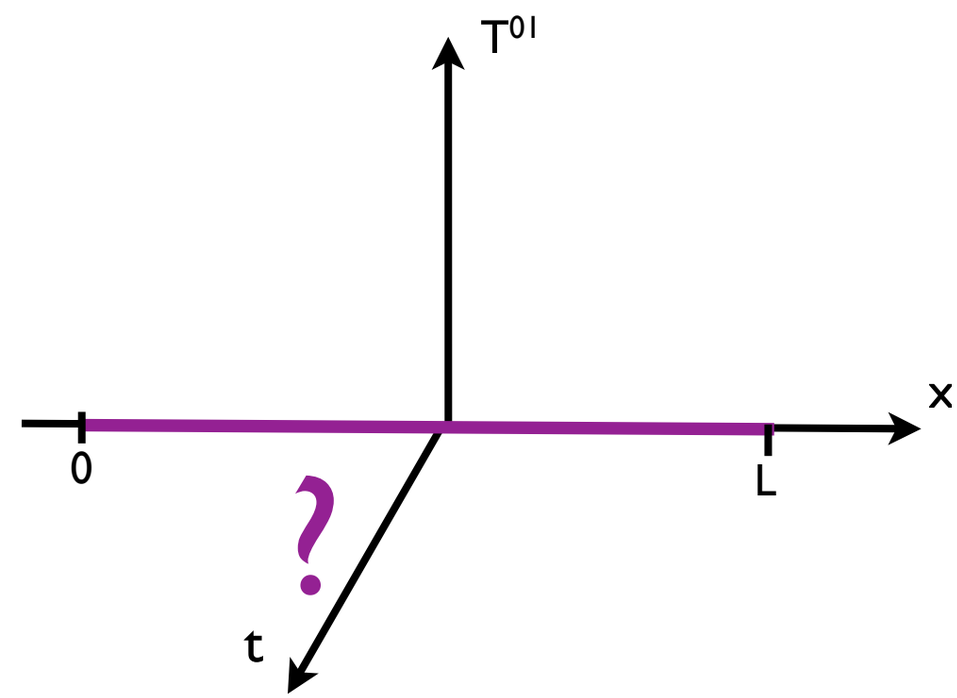
2d CFT's





2d CFT's

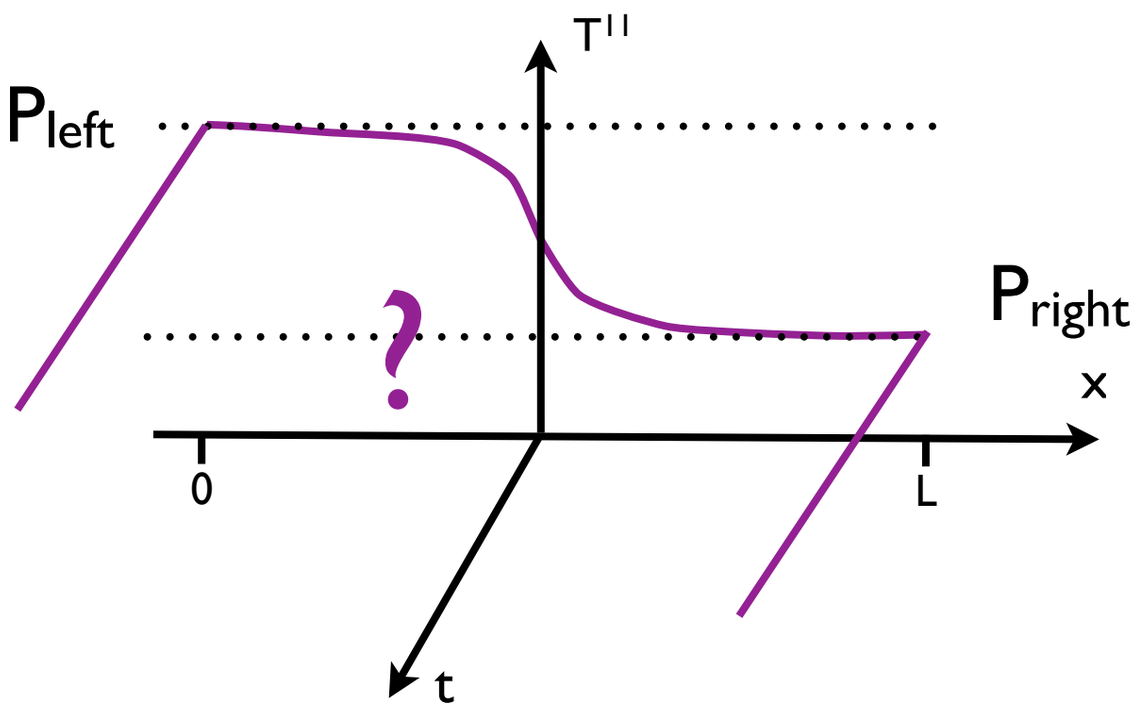


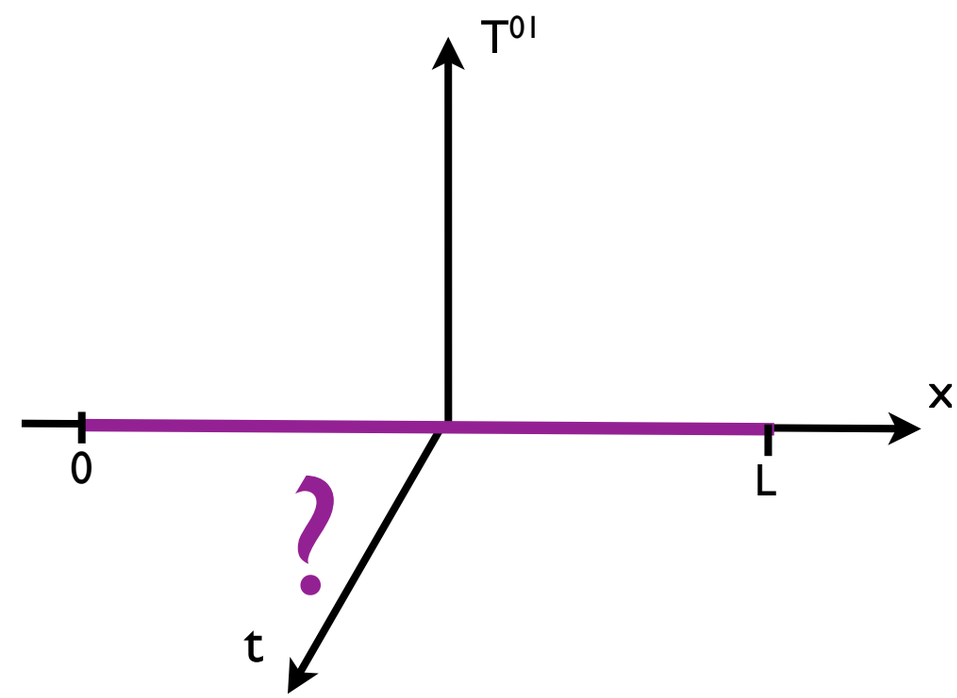


In a conformal theory

$$\partial_\mu T^{\mu\nu} = 0 \qquad\qquad T^\mu{}_\mu = 0$$

2d CFT's





In a conformal theory

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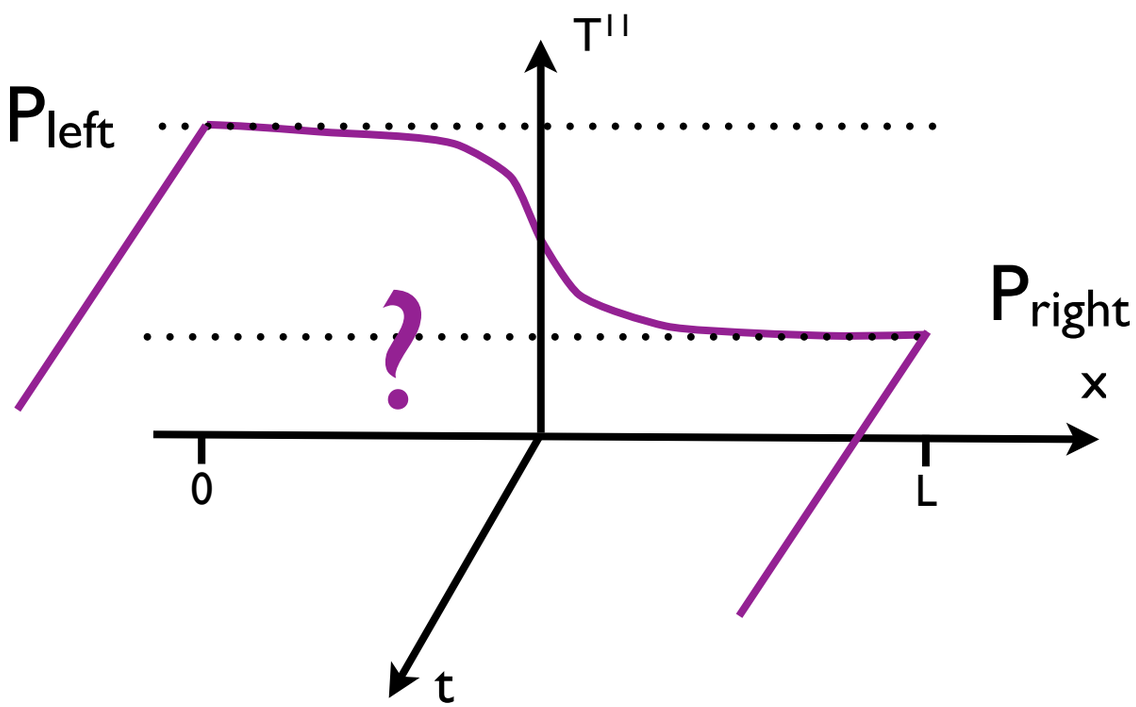
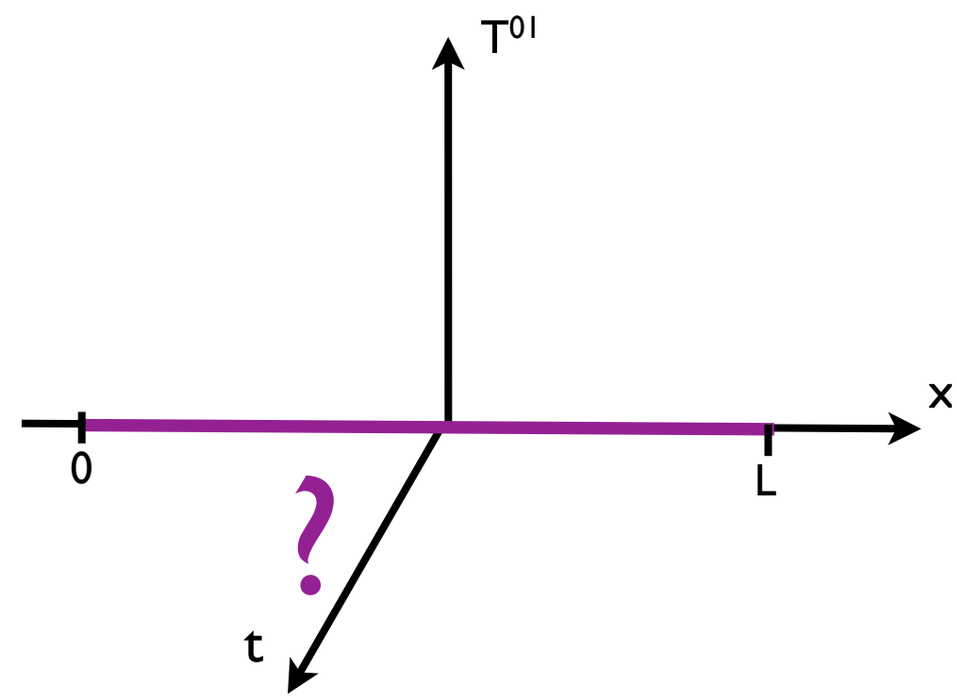
Equivalently

$$\partial_t T^{tt} = -\partial_x T^{tx} \qquad \partial_t T^{tx} = -\partial_x T^{xx} \qquad -T^{tt} + T^{xx} = 0$$

2d CFT's





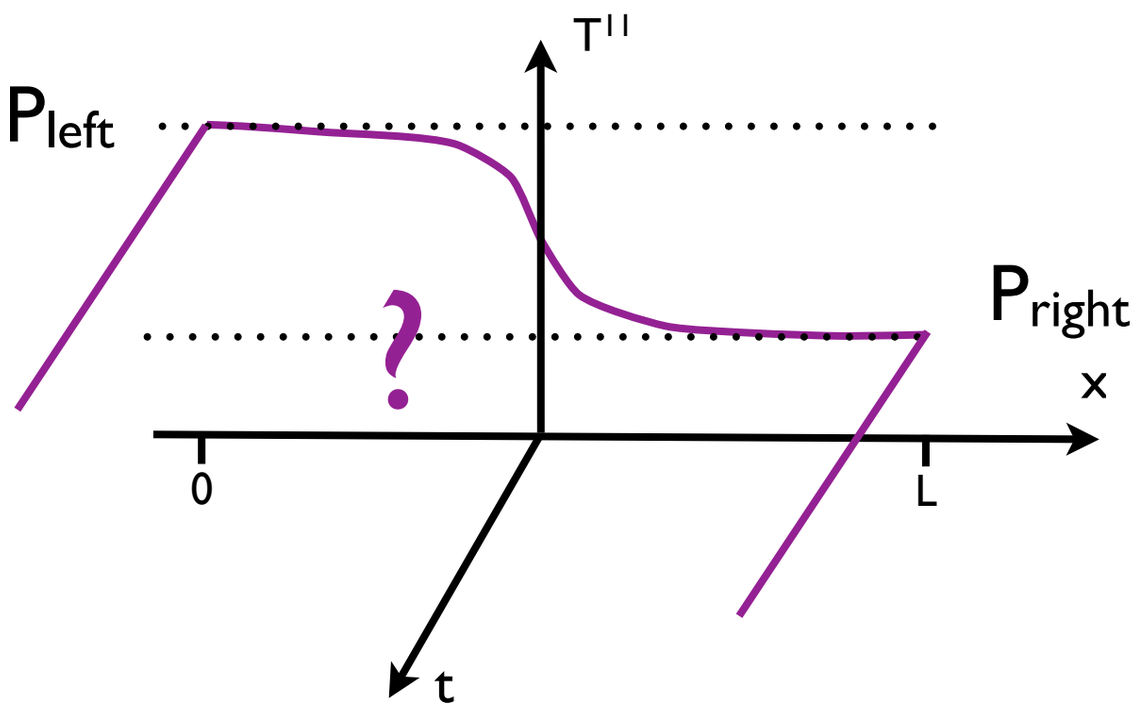
In a conformal theory

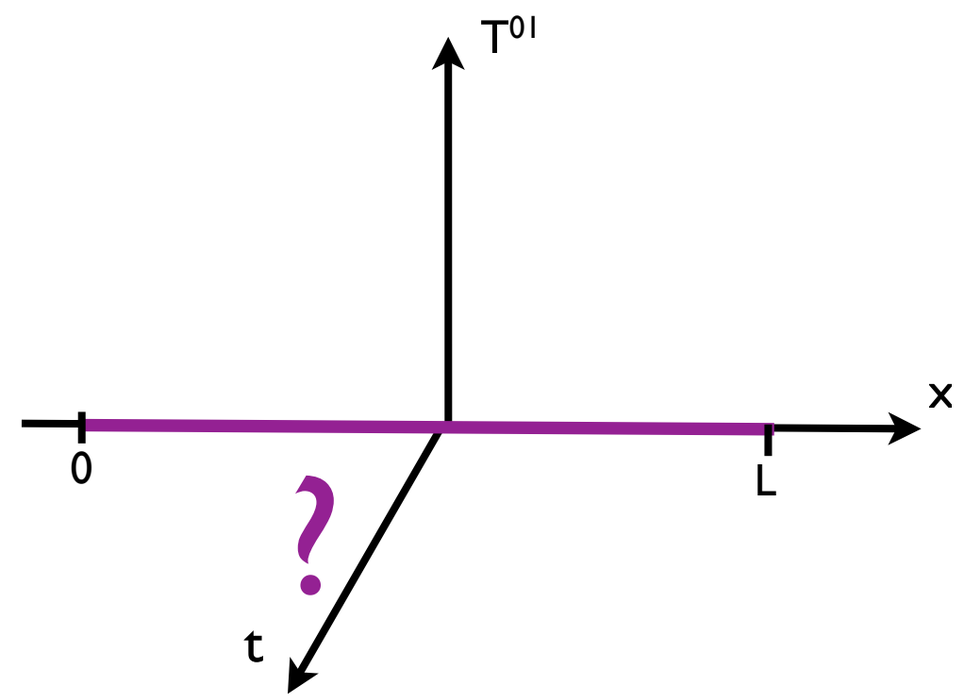
$$\partial_t T^{tt} = -\partial_x T^{tx} \qquad \partial_t T^{tx} = -\partial_x T^{xx} \qquad -T^{tt} + T^{xx} = 0$$

whose solution is

$$T^{\mu\nu} = \begin{pmatrix} T_+(t+x) + T_-(-t+x) & T_-(-t+x) - T_+(t+x) \\ T_-(-t+x) - T_+(t+x) & T_+(t+x) + T_-(-t+x) \end{pmatrix}$$

2d CFT's





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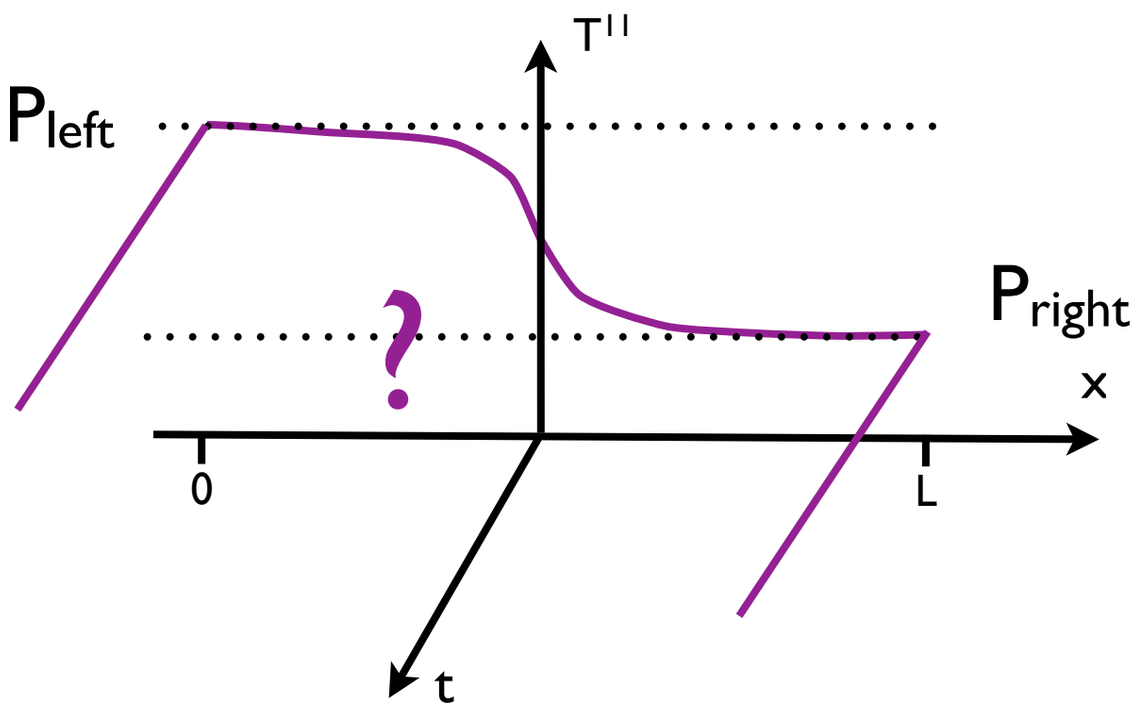
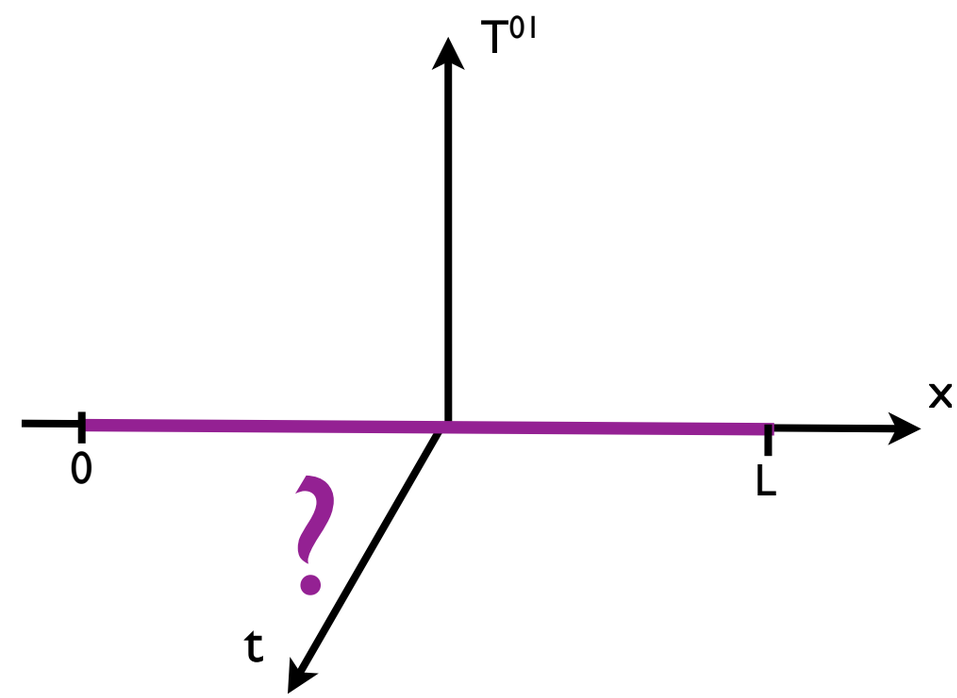
The initial conditions imply that:

$$T_-(u) = T_+(u)\,, \quad 2T_+(u) = T^{11}(t=0, x=u)\,, \qquad 0 < u < L$$

2d CFT's





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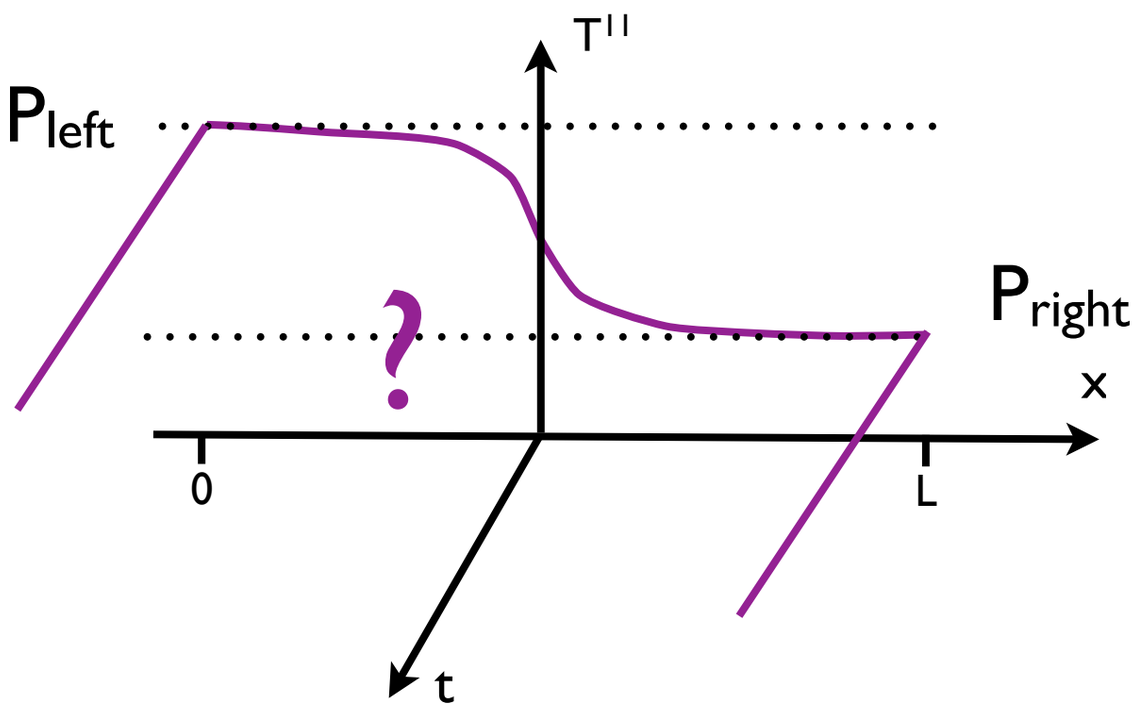

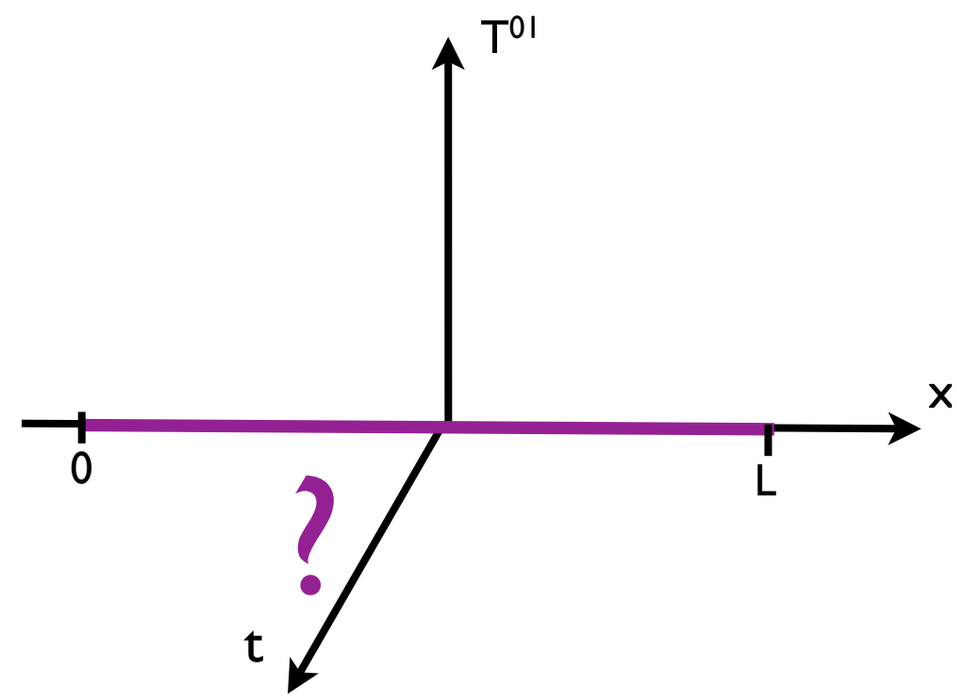

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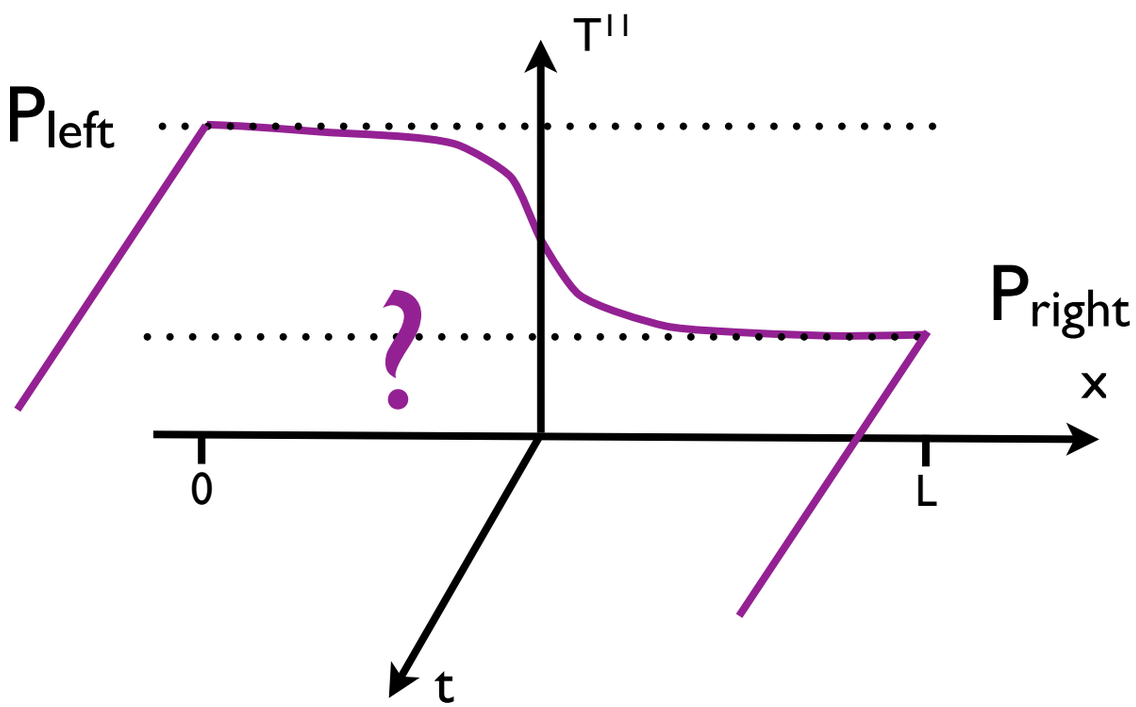
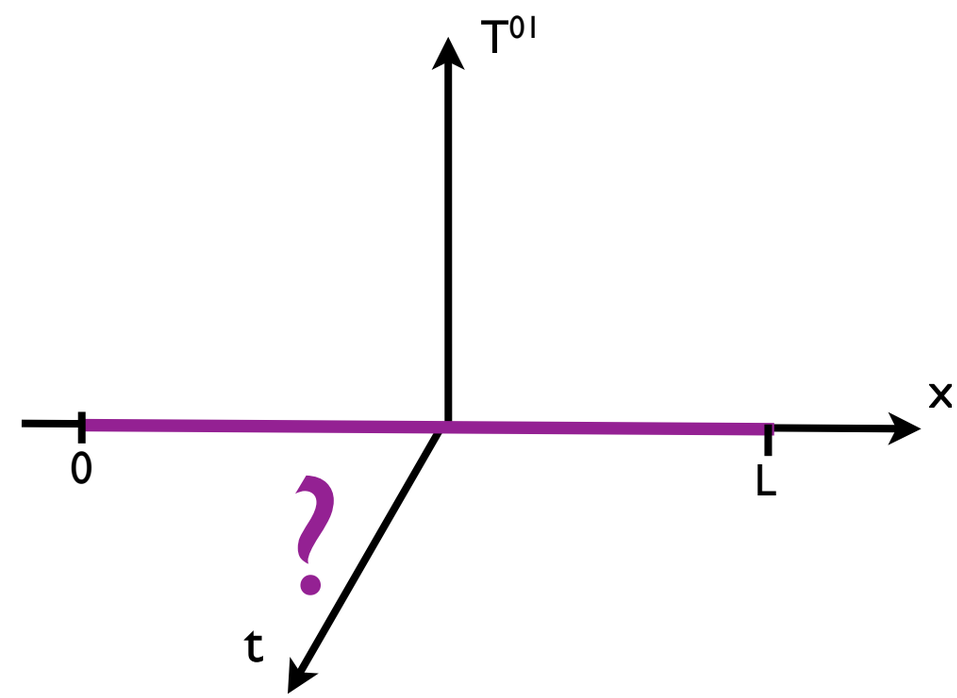
The initial and boundary conditions imply that:

$$T_-(u) = T_+(u)\,, \quad 2T_+(u) = T^{11}(t=0, x=u)\,, \qquad 0 < u < L$$
$$T_-(-u) + T_+(u) = P_{\text{left}}\,, \quad T_-(-u+L) + T_+(u+L) = P_{\text{right}}\,, \qquad \forall\, u$$

2d CFT's





We find:

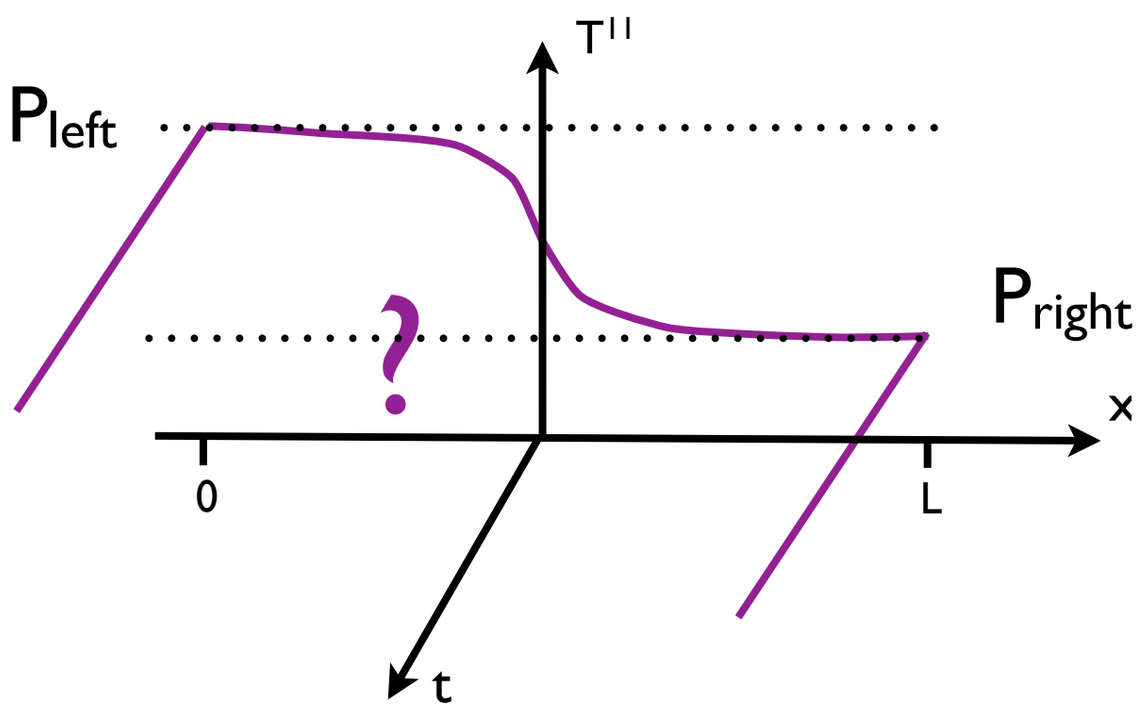
$$T_+(u) = \begin{cases} -n(P_{\text{left}} - P_{\text{right}}) + T_+(u_0) & u_0 > 0 \\ -(n-1)(P_{\text{left}} - P_{\text{right}}) + (P_{\text{right}} - T_-(-u_0)) & u_0 < 0 \end{cases}$$

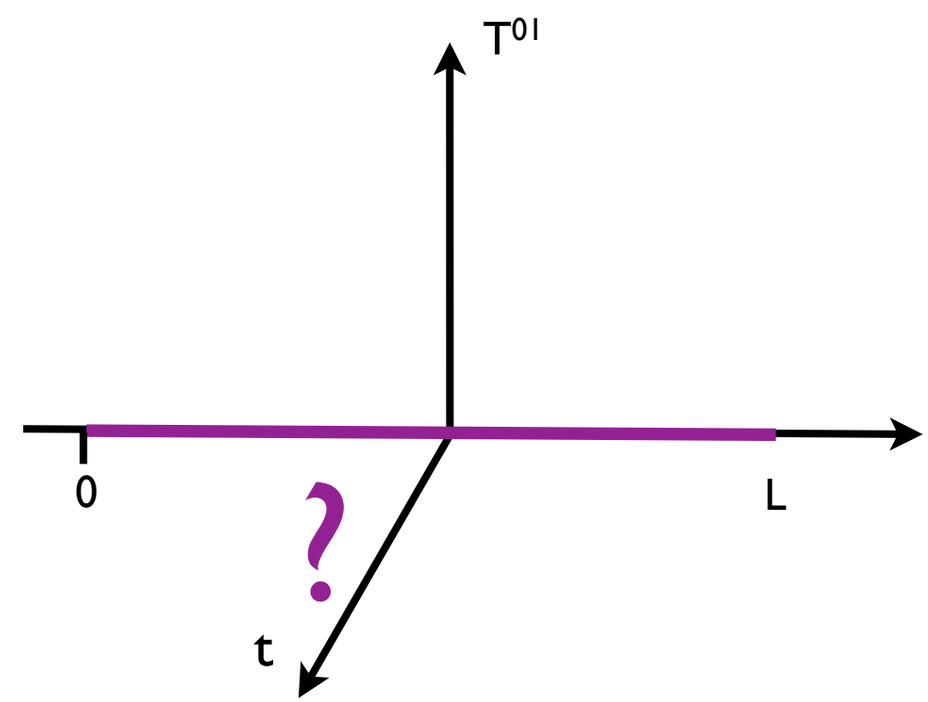
$$T_-(u) = \begin{cases} -n(P_{\text{left}} - P_{\text{right}}) + T_-(u_0) & u_0 > 0 \\ -(n-1)(P_{\text{left}} - P_{\text{right}}) + (Pn_{\text{right}} - T_+(-u_0)) & u_0 < 0 \end{cases}$$

where

$$u = u_0 + 2nL \qquad -L < u_0 < L \qquad n \in \mathbb{Z}$$

2d CFT's





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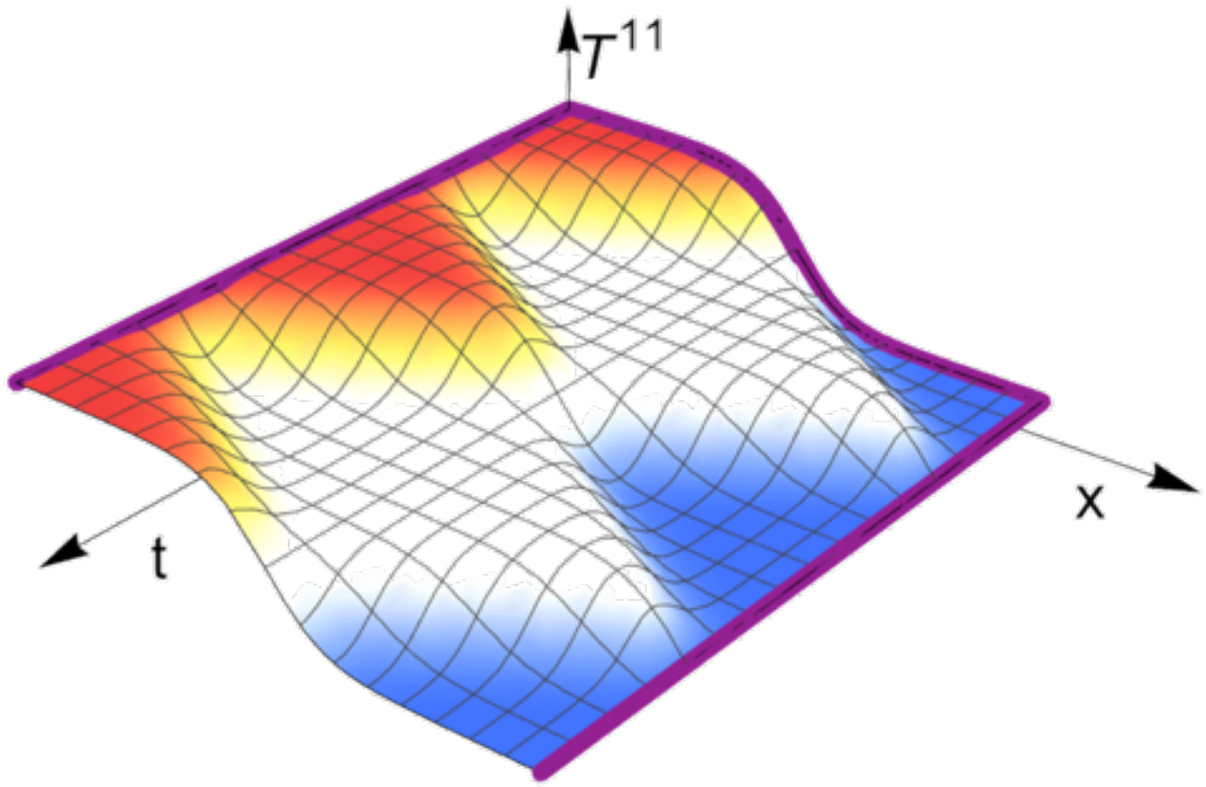
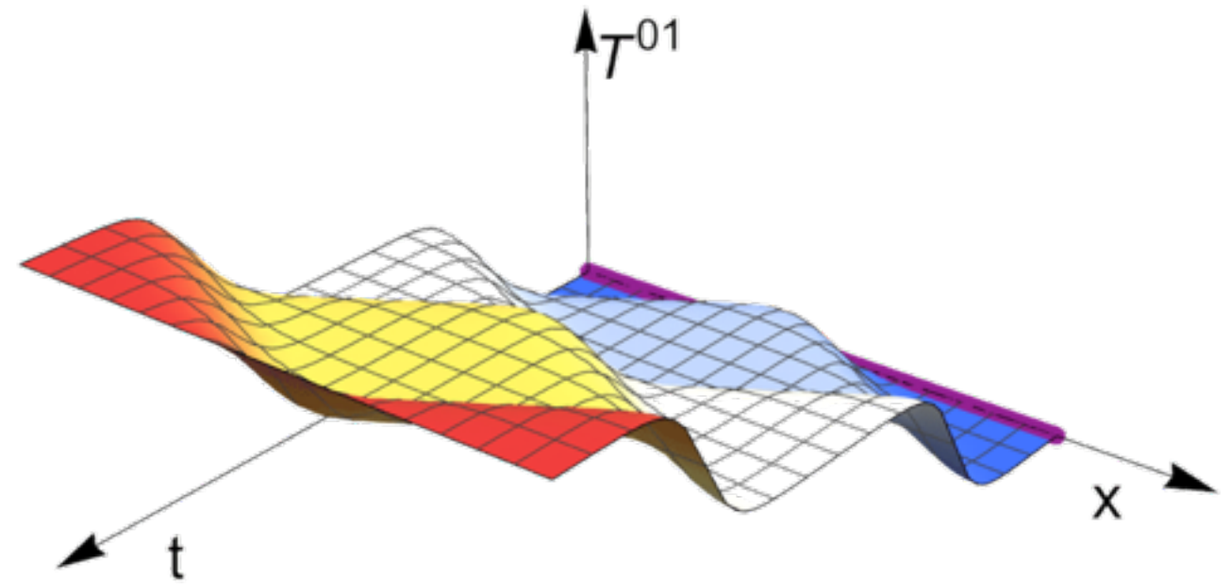
2d CFT's





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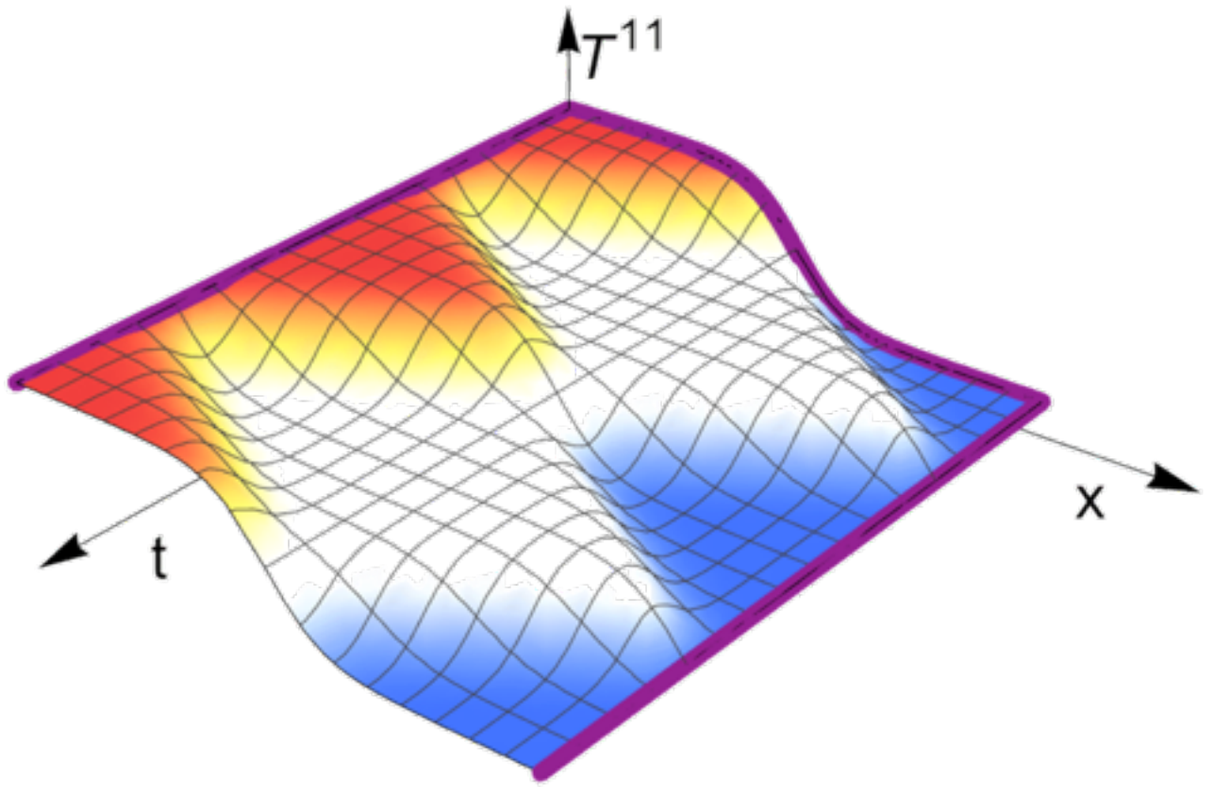
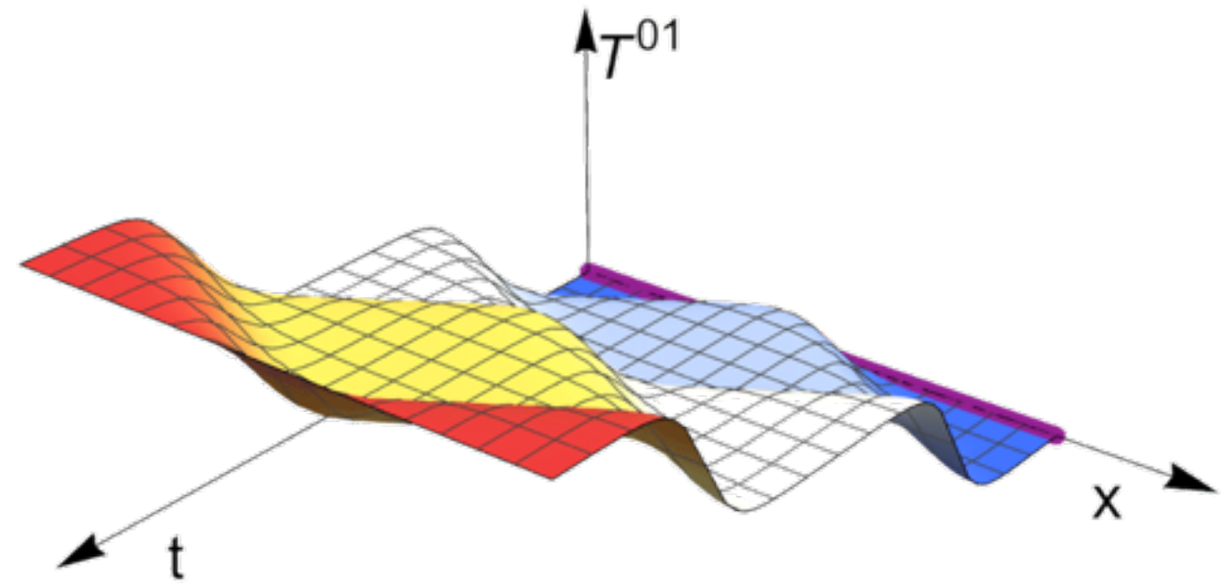
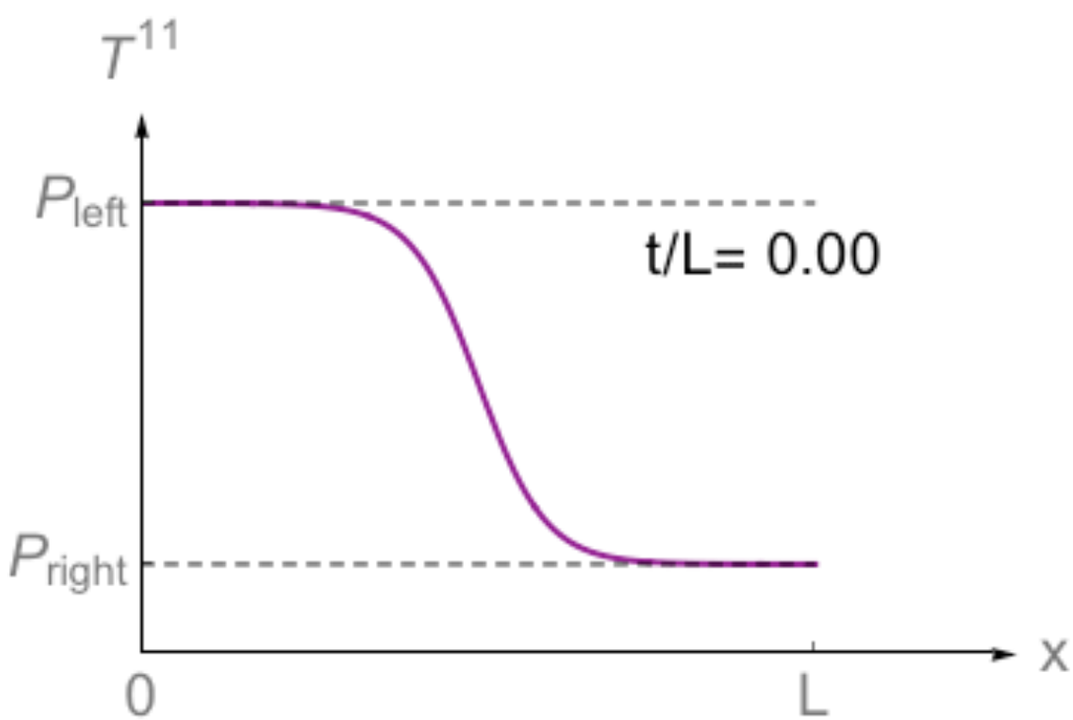
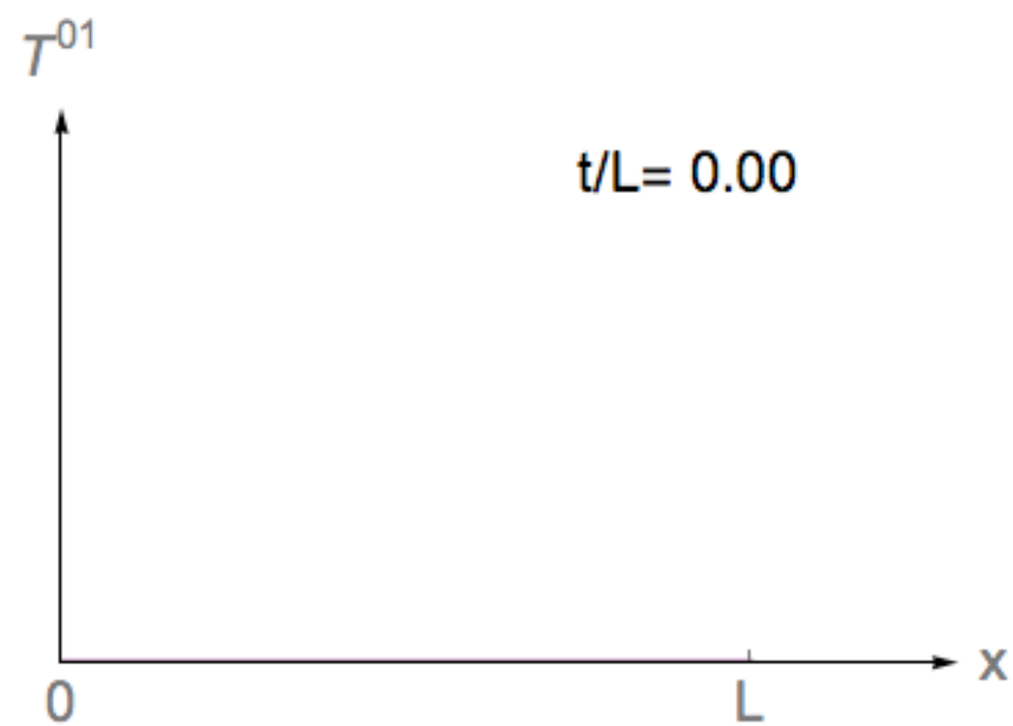
2d CFT's


T^{11}
t
x


T^{01}
t
x


T^{11}
P_{left}
P_{right}
t/L= 0.00
0
L
x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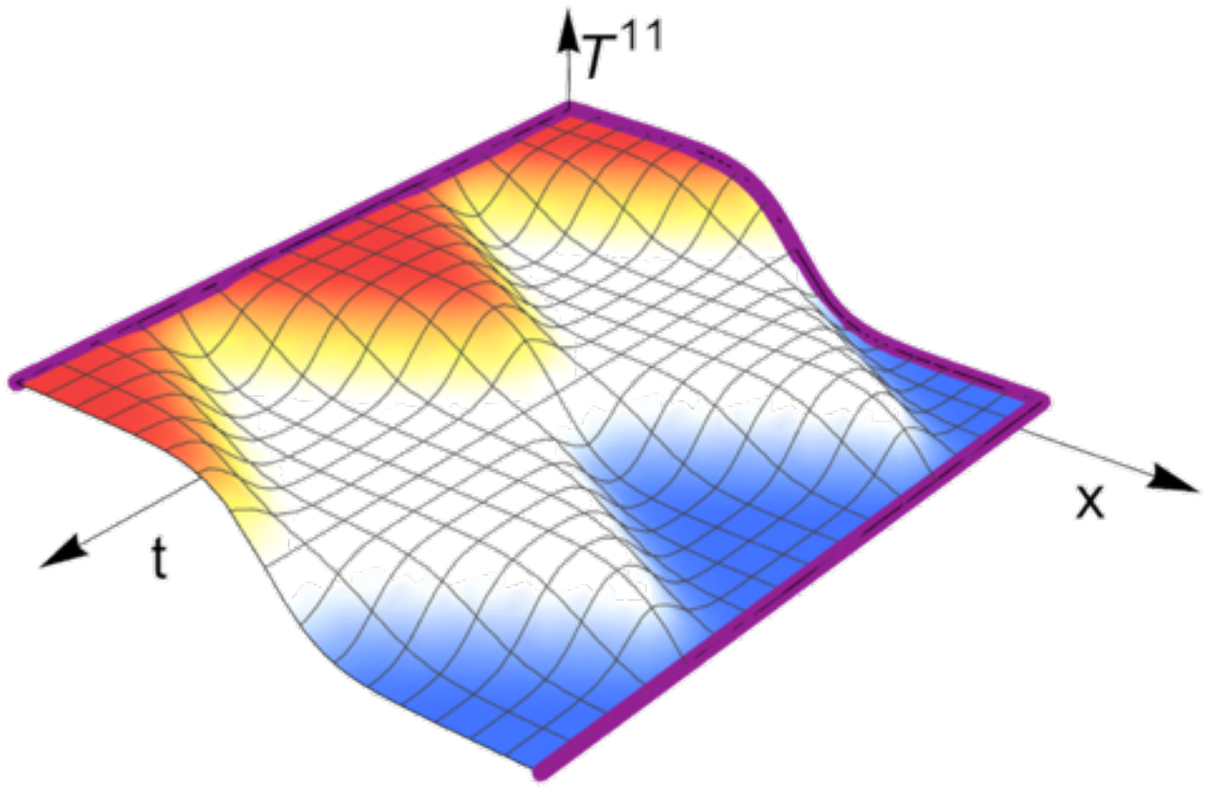
t/L= 0.00
0
L
x

2d CFT's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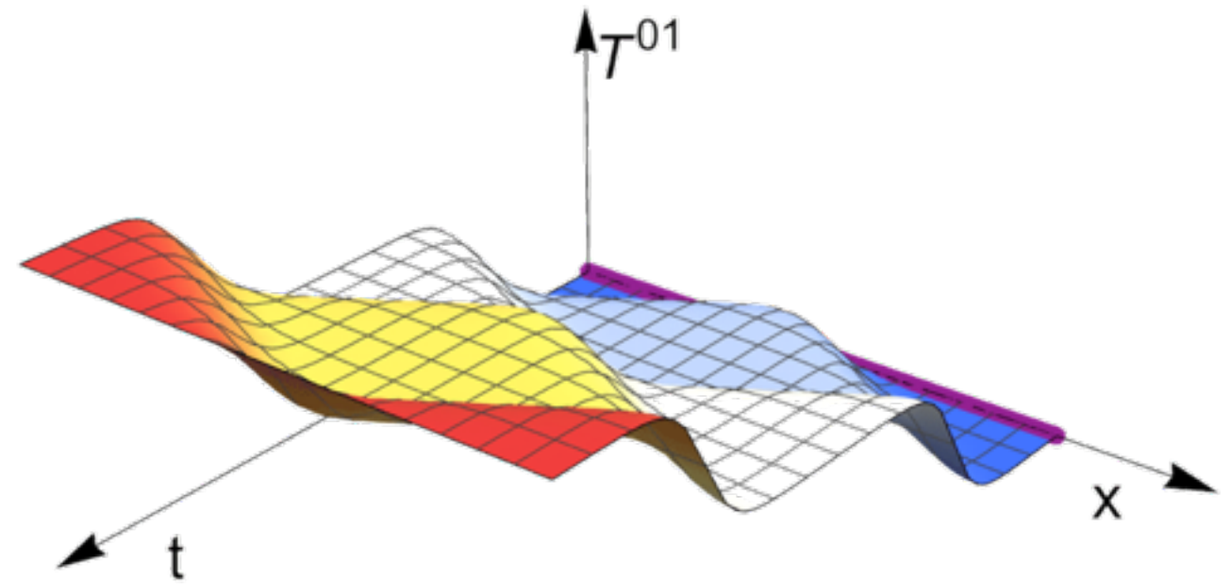
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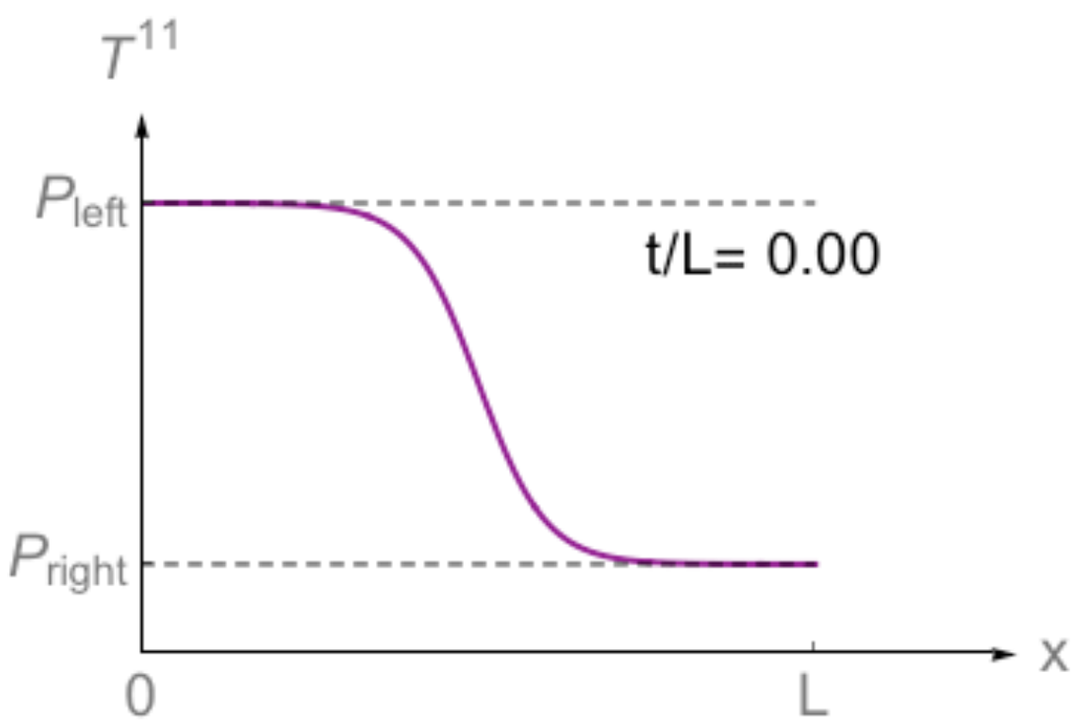
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P_{left}
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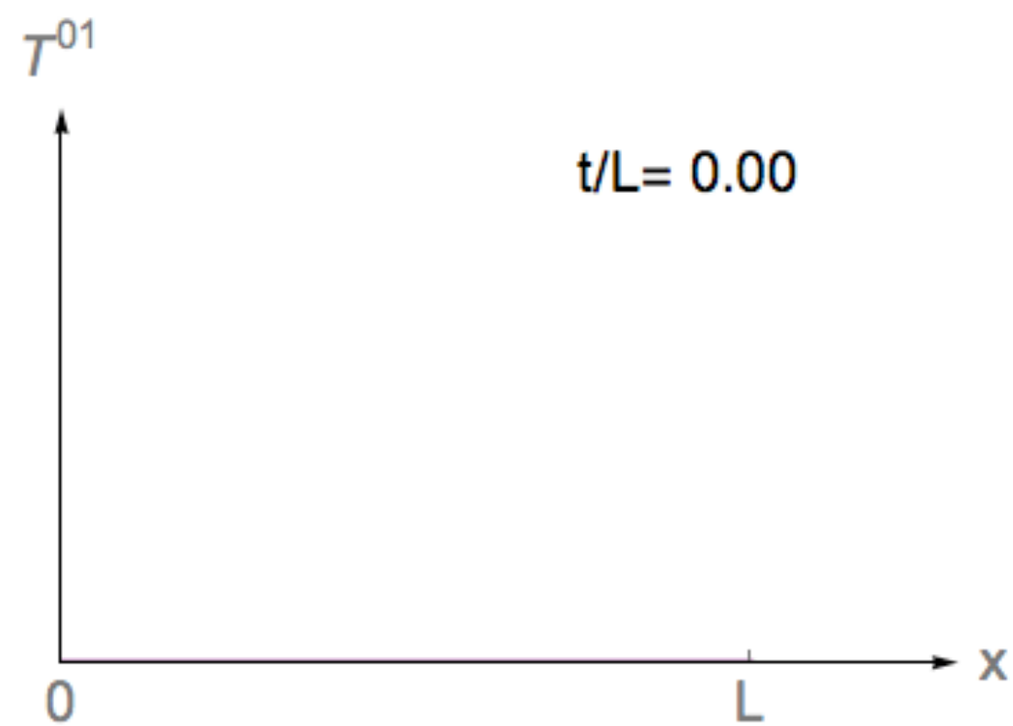
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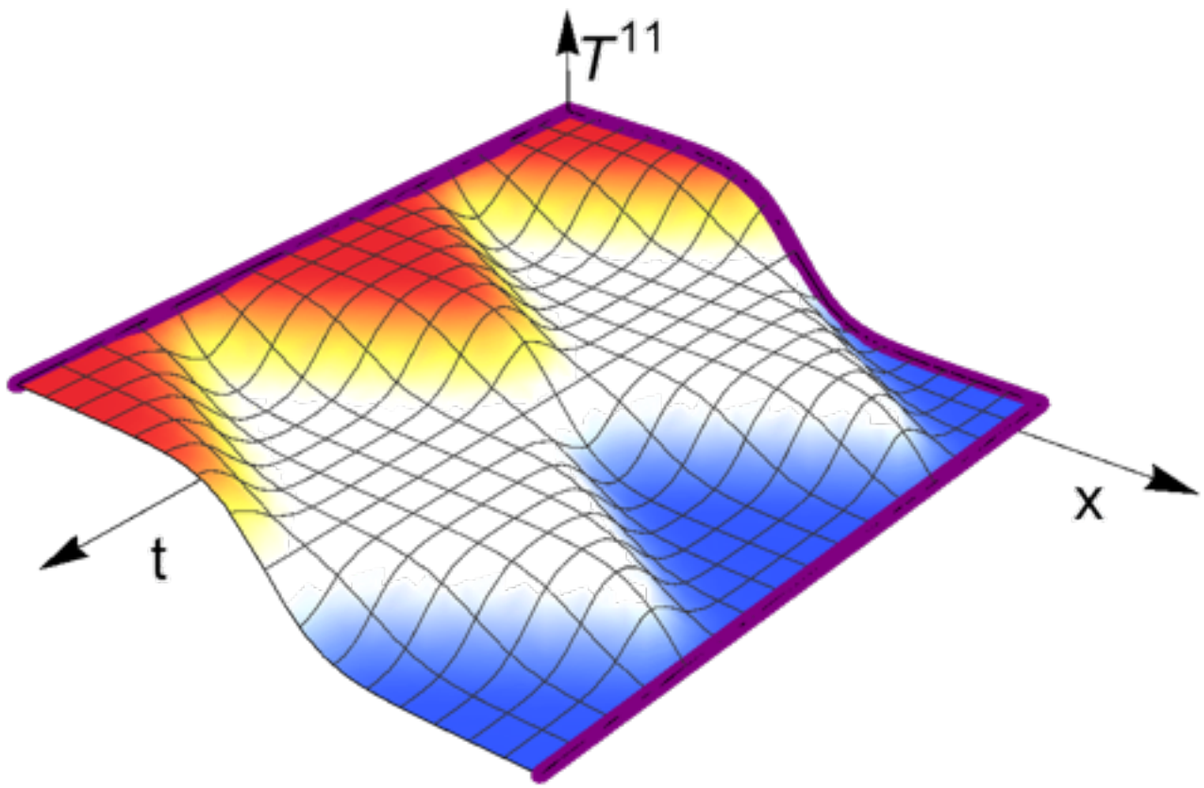
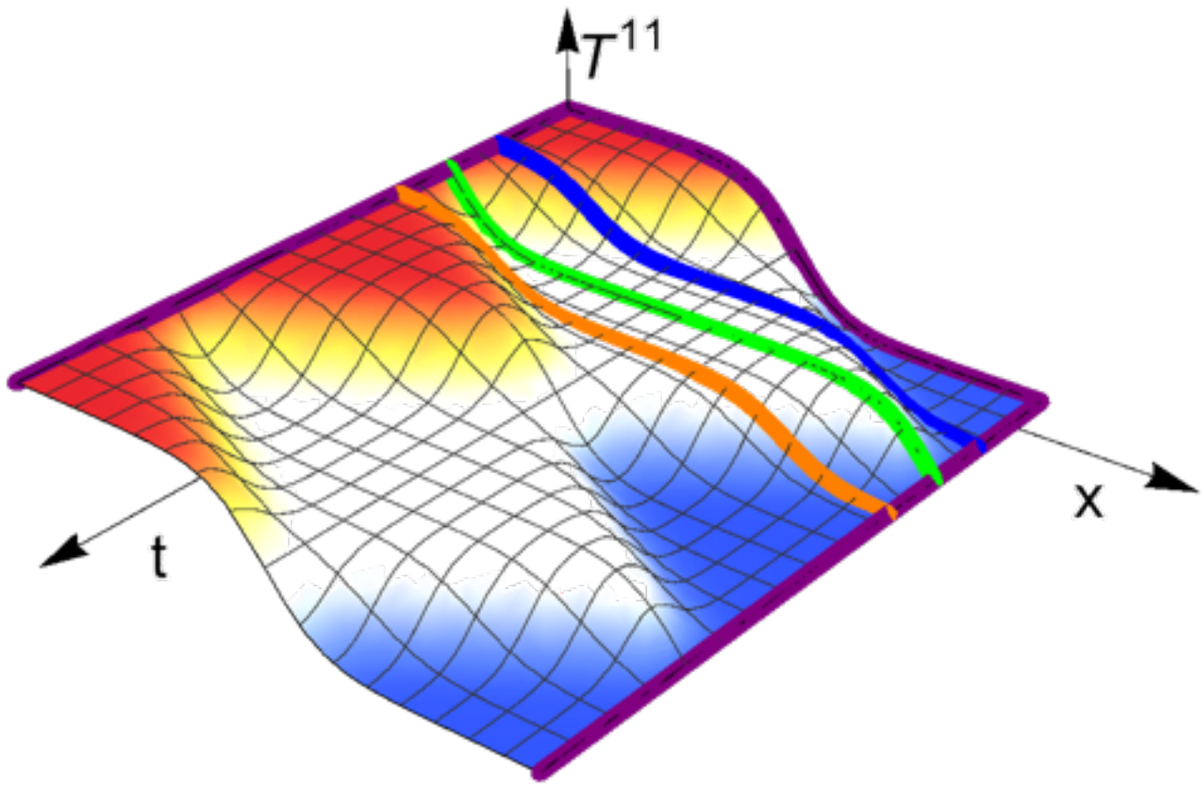
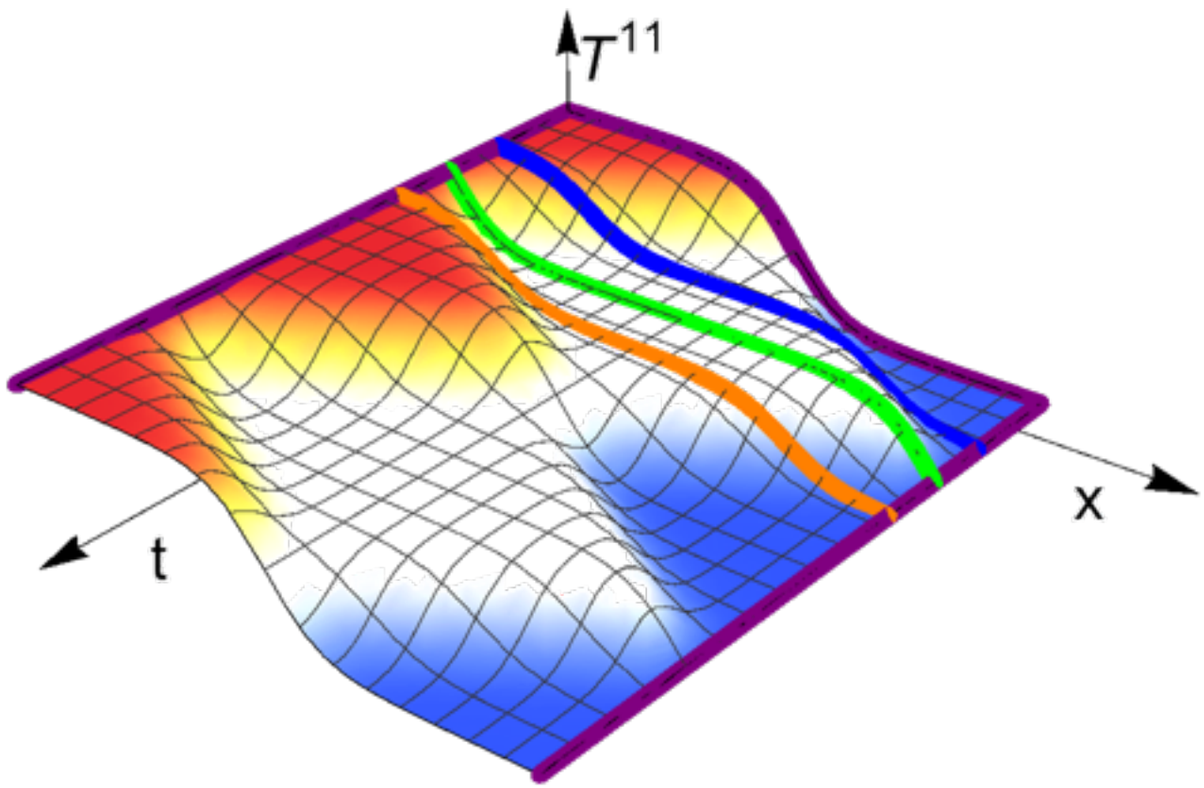
2d CFT's

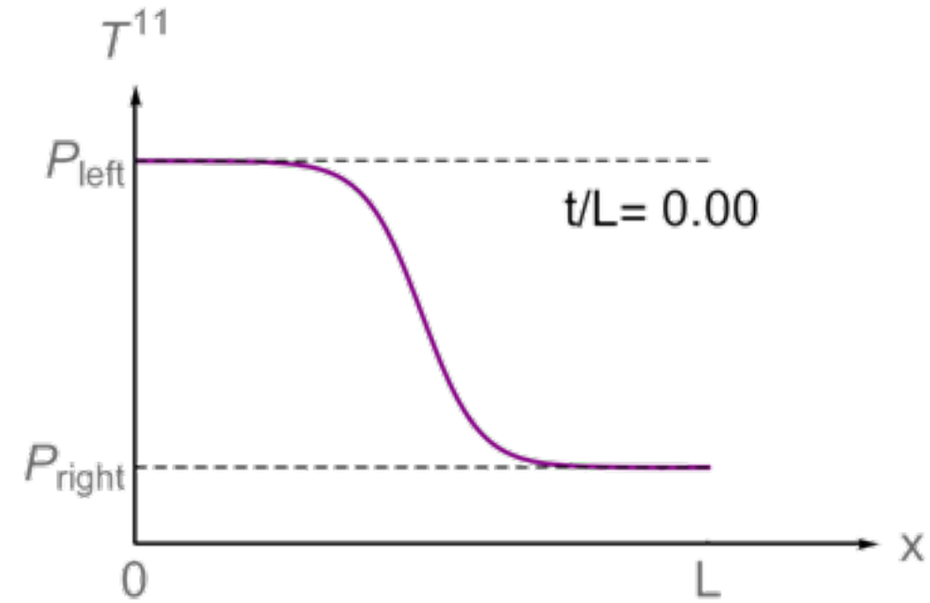


2d CFT's

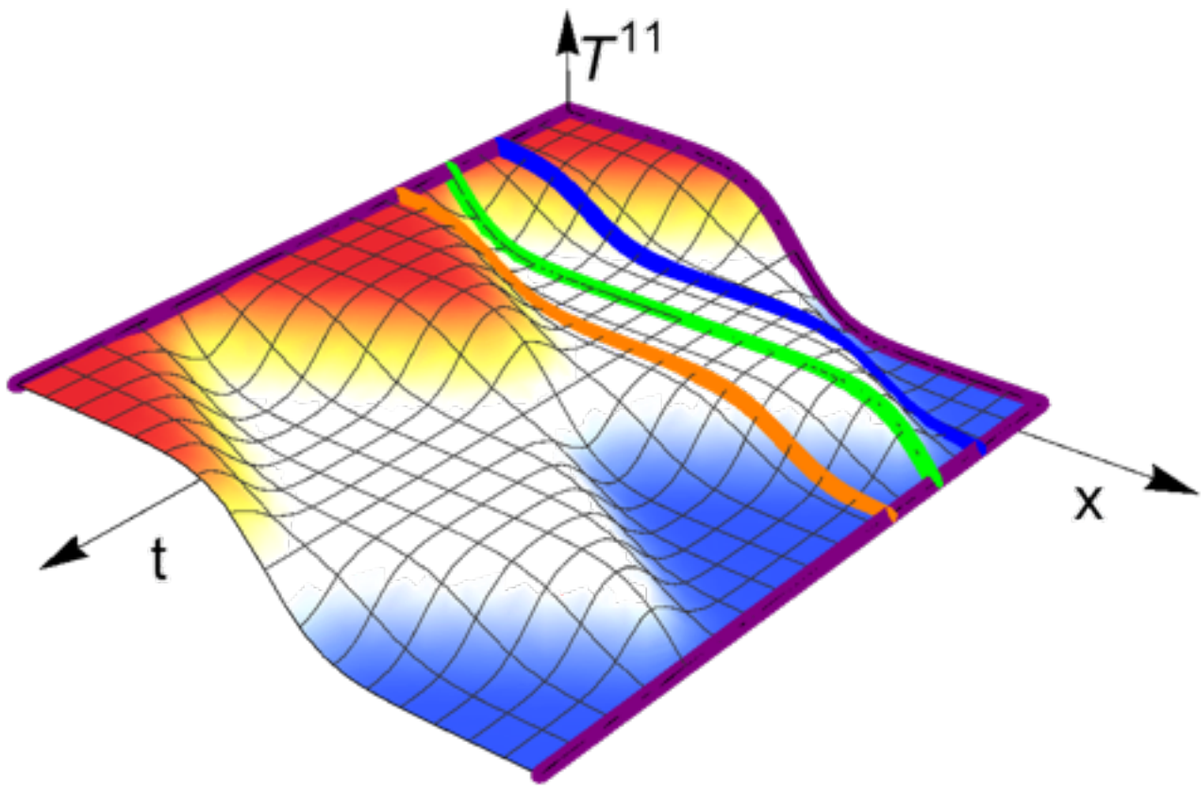


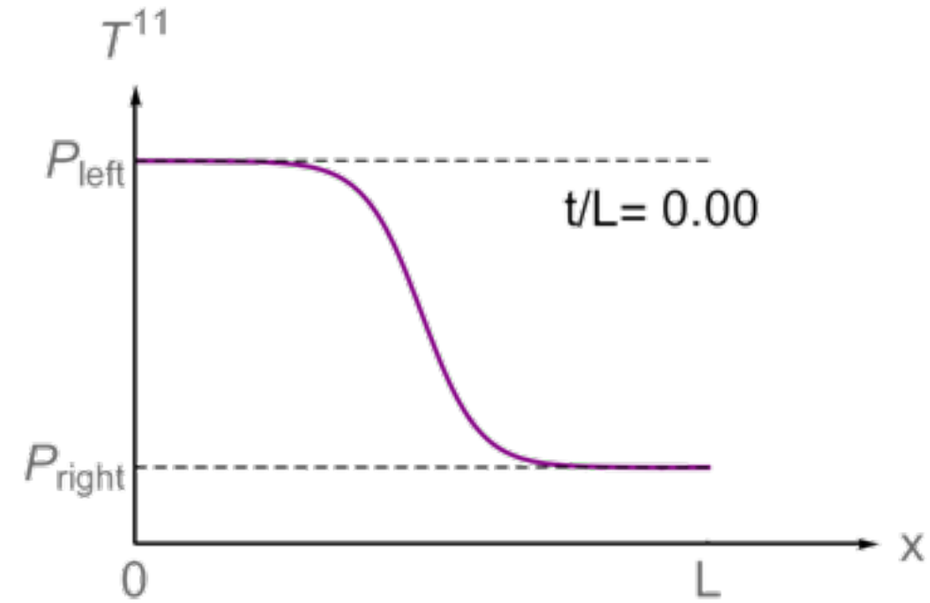
2d CFT's

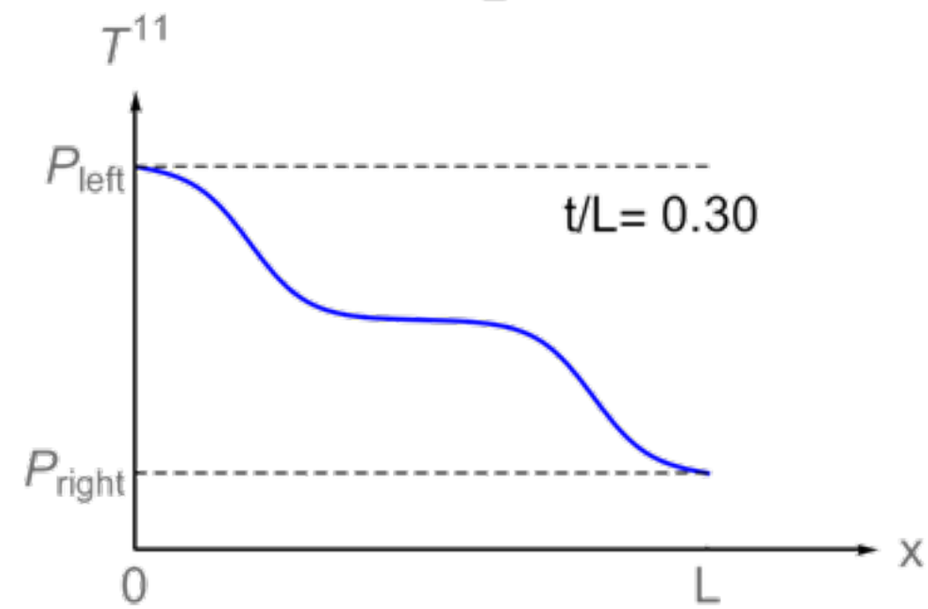




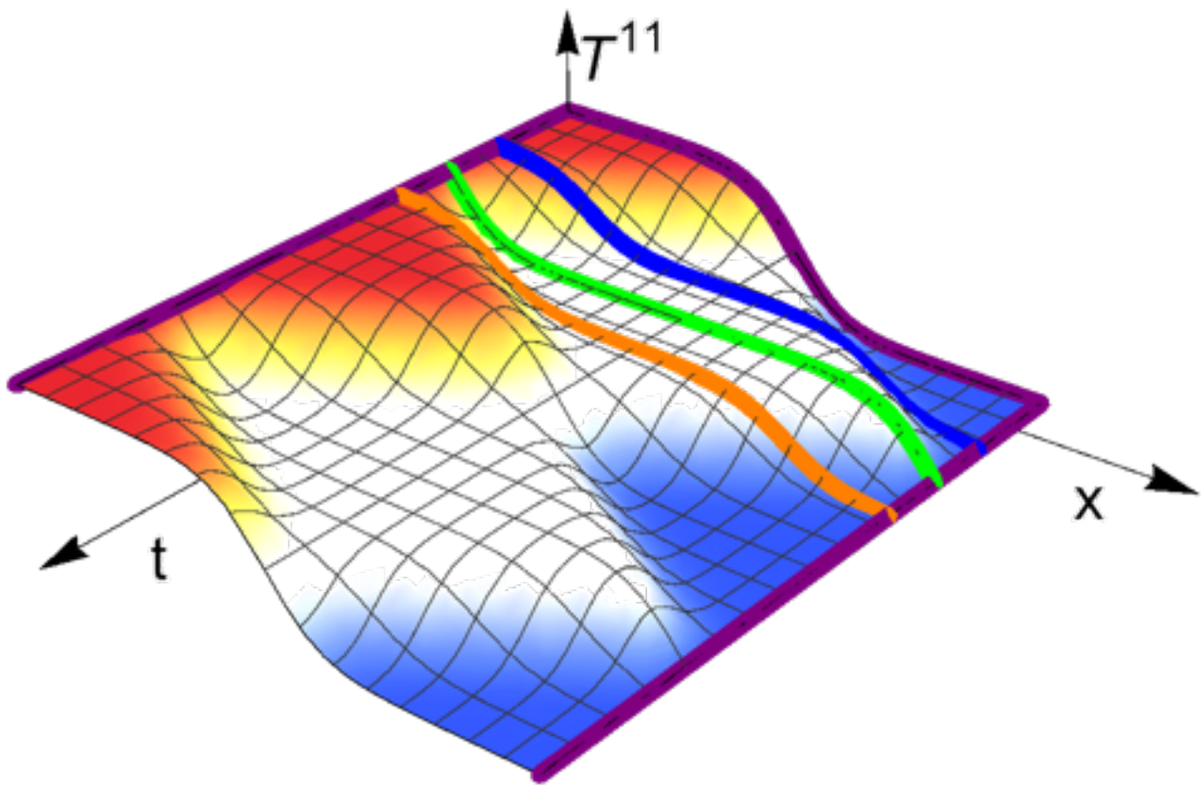
2d CFT's

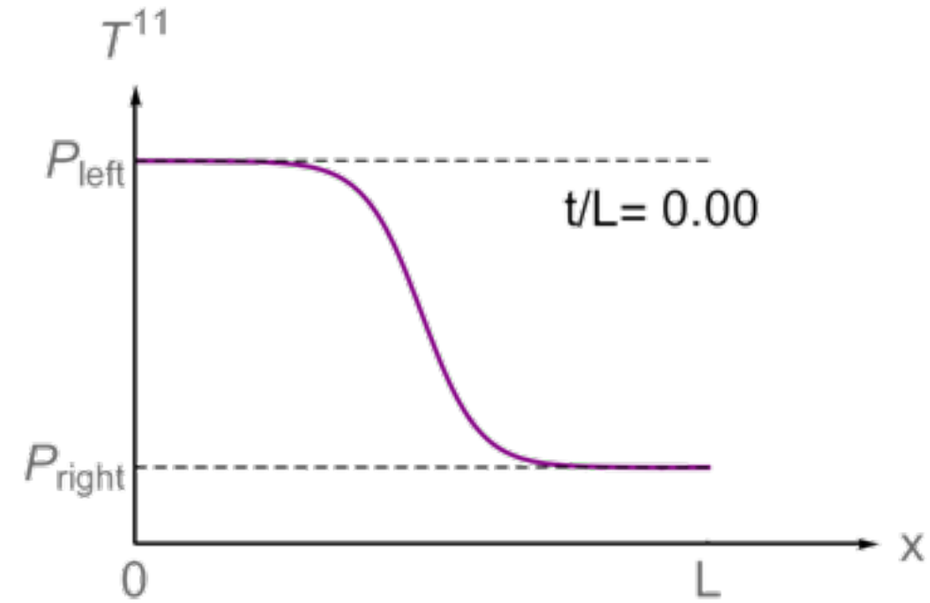


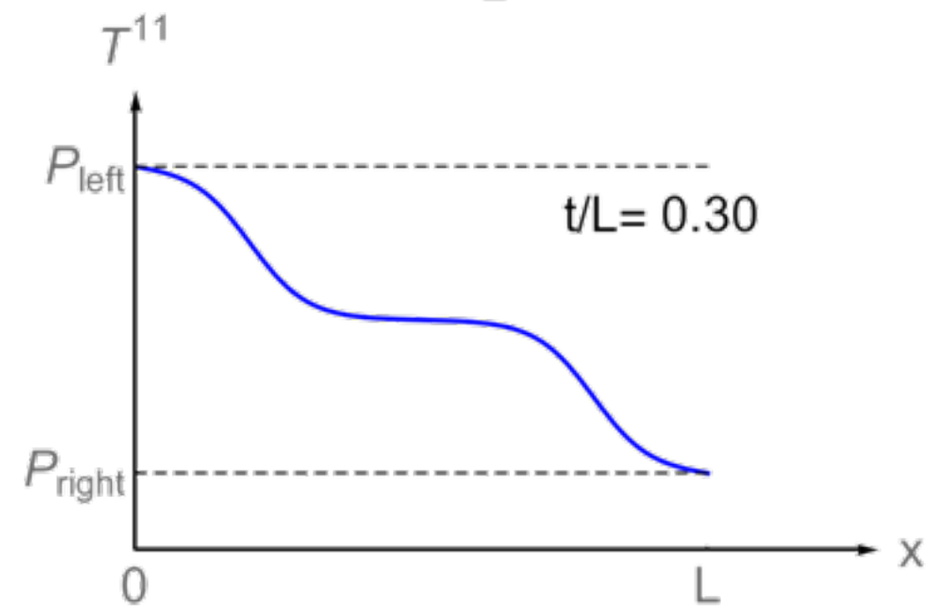


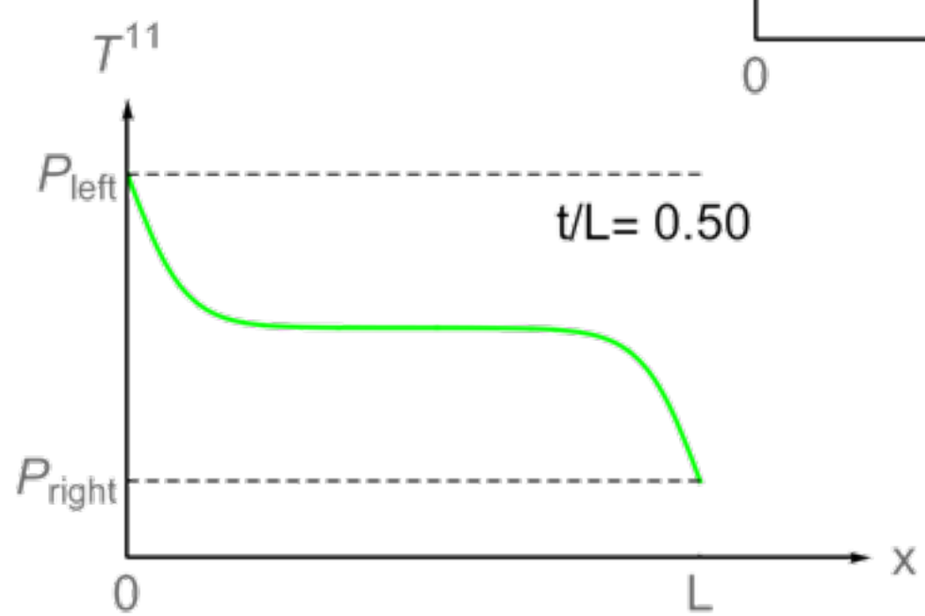


2d CFT's

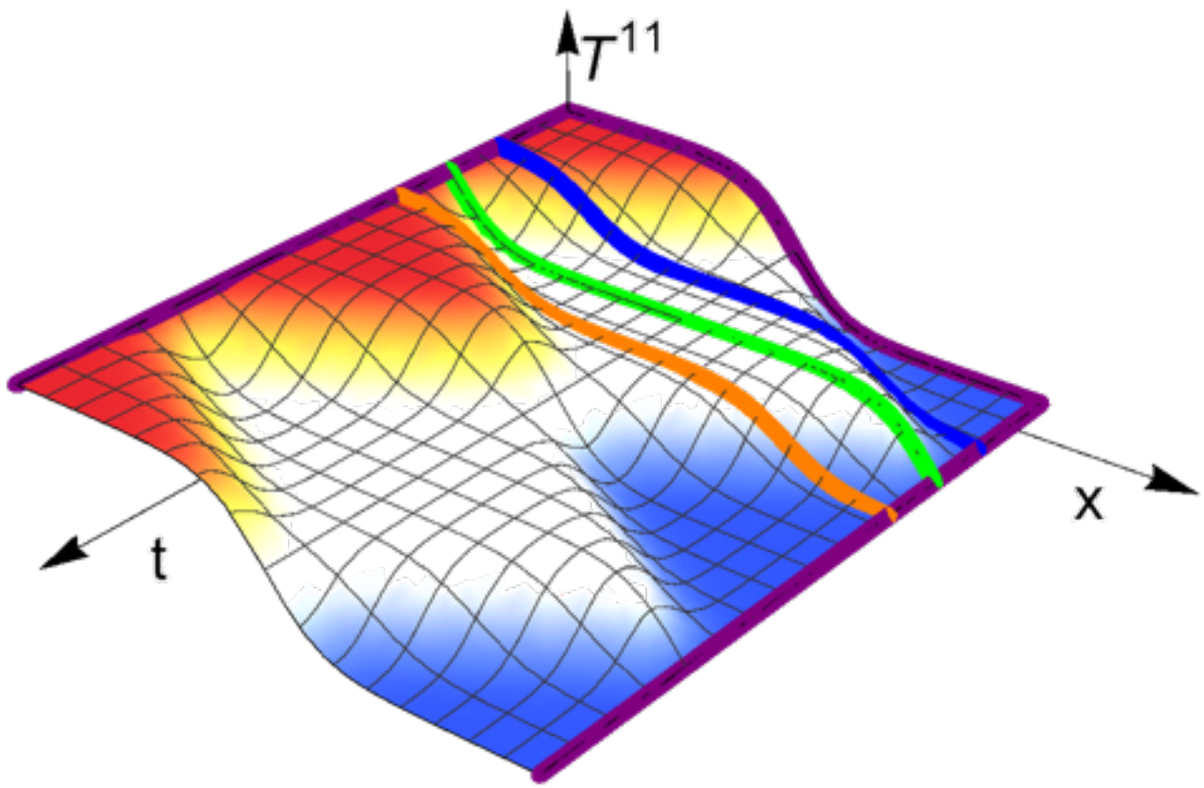

T^{11}
t
x

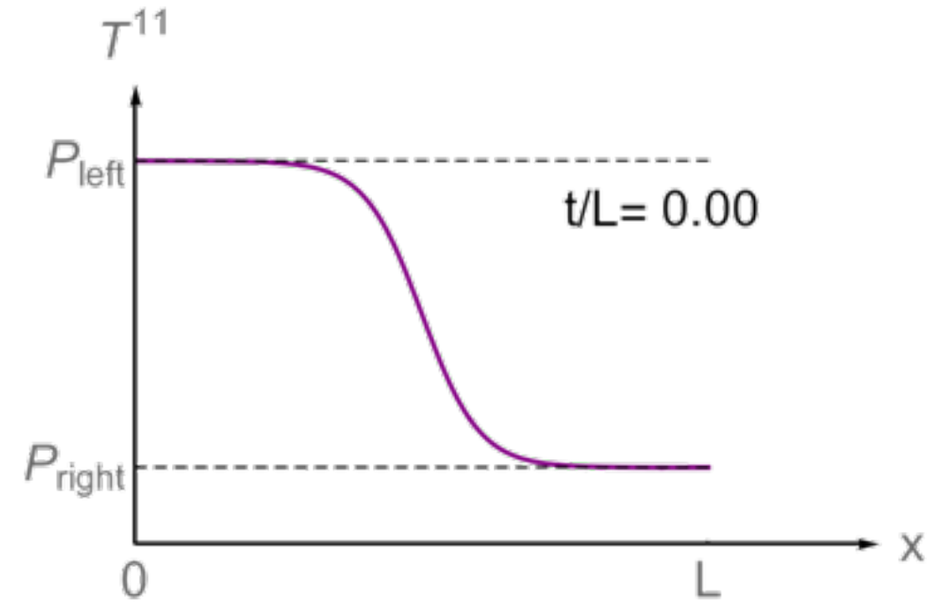

T^{11}
P_{left}
t/L= 0.00
P_{right}
0
L
x

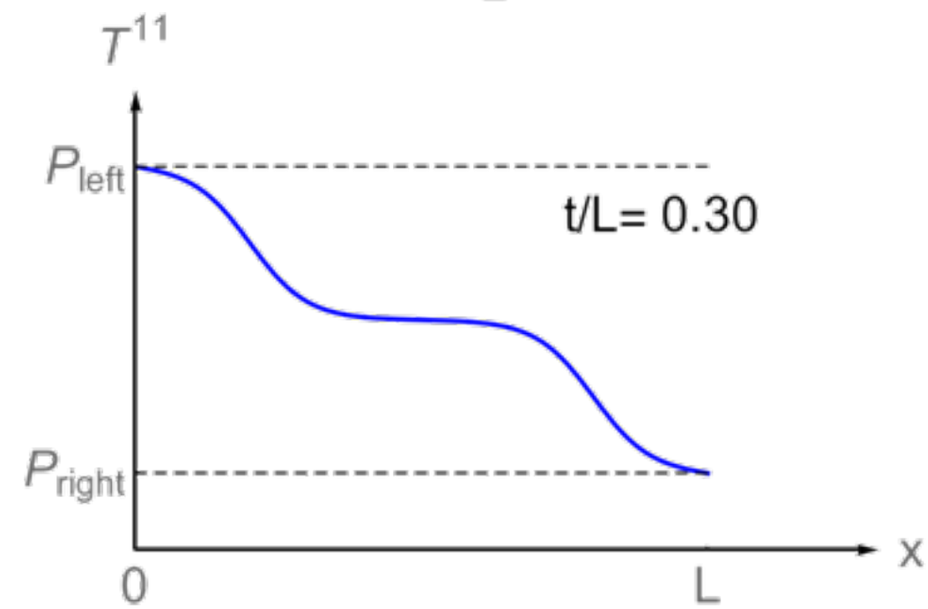

T^{11}
P_{left}
t/L= 0.30
P_{right}
0
L
x

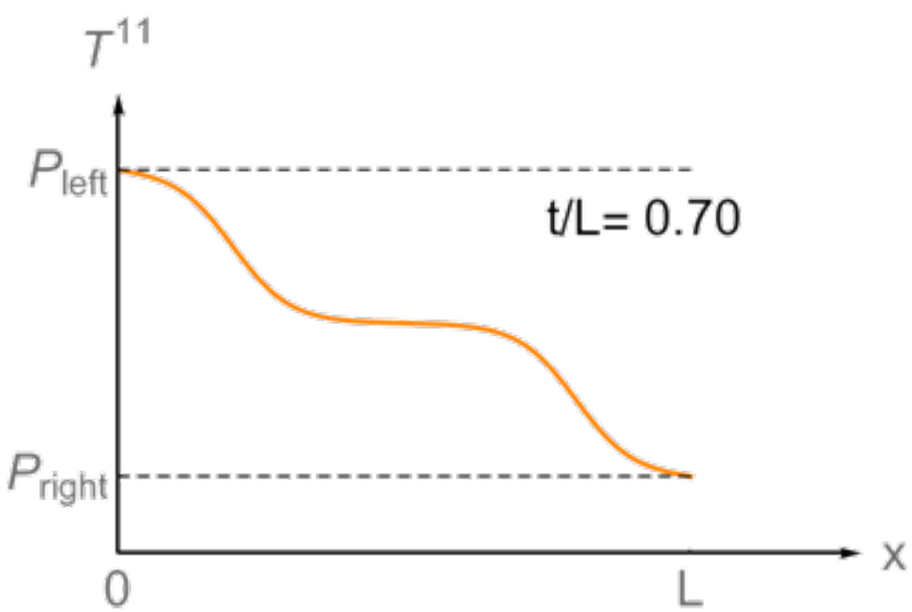

T^{11}
P_{left}
t/L= 0.50
P_{right}
0
L
x

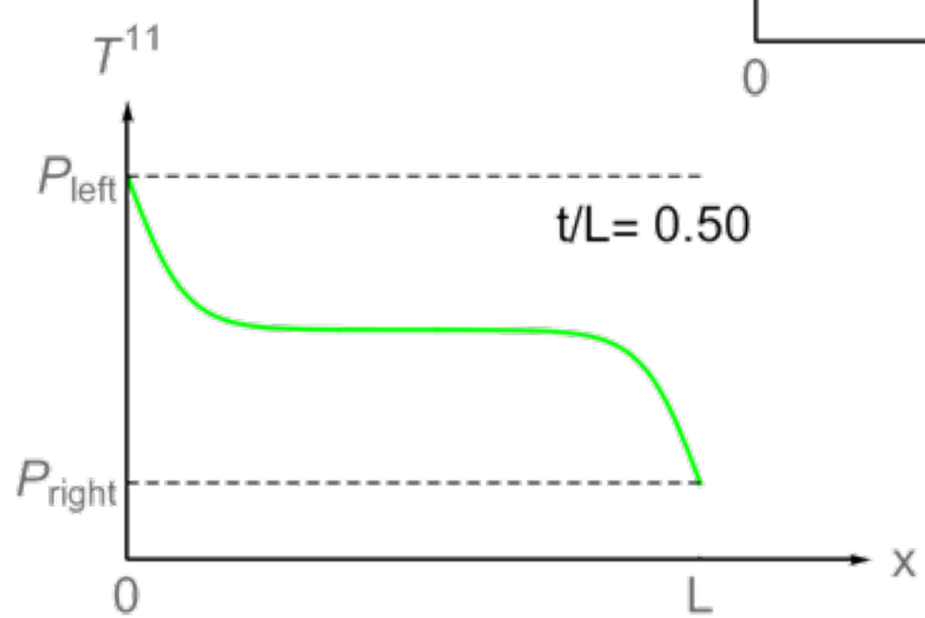
2d CFT's


T^{11}
t
x

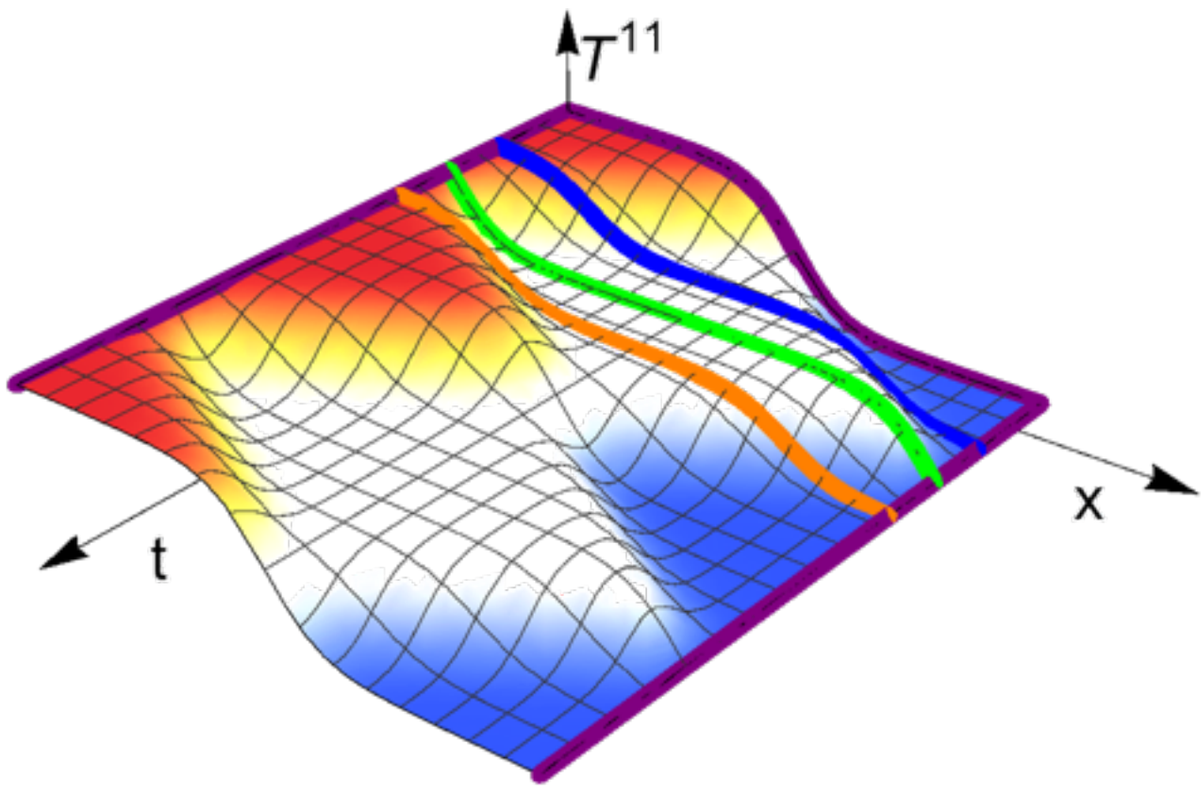

T^{11}
P_{left}
t/L= 0.00
P_{right}
0
L
x

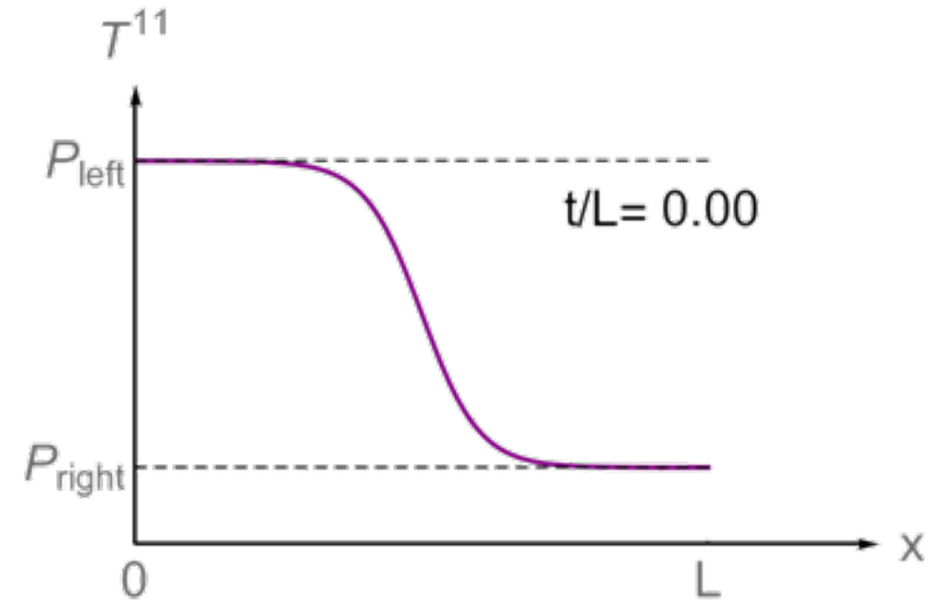

T^{11}
P_{left}
t/L= 0.30
P_{right}
0
L
x


T^{11}
P_{left}
t/L= 0.70
P_{right}
0
L
x


T^{11}
P_{left}
t/L= 0.50
P_{right}
0
L
x

2d CFT's





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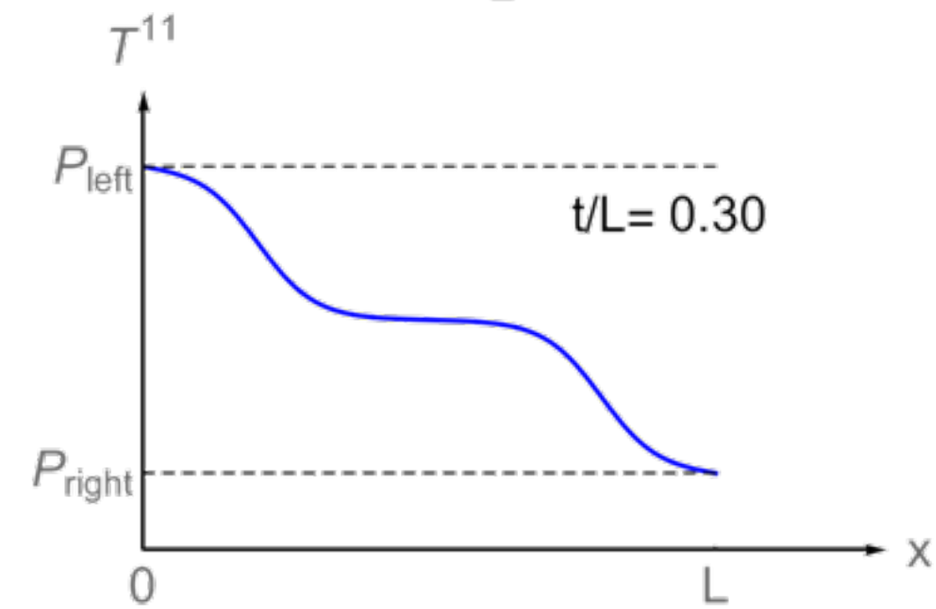
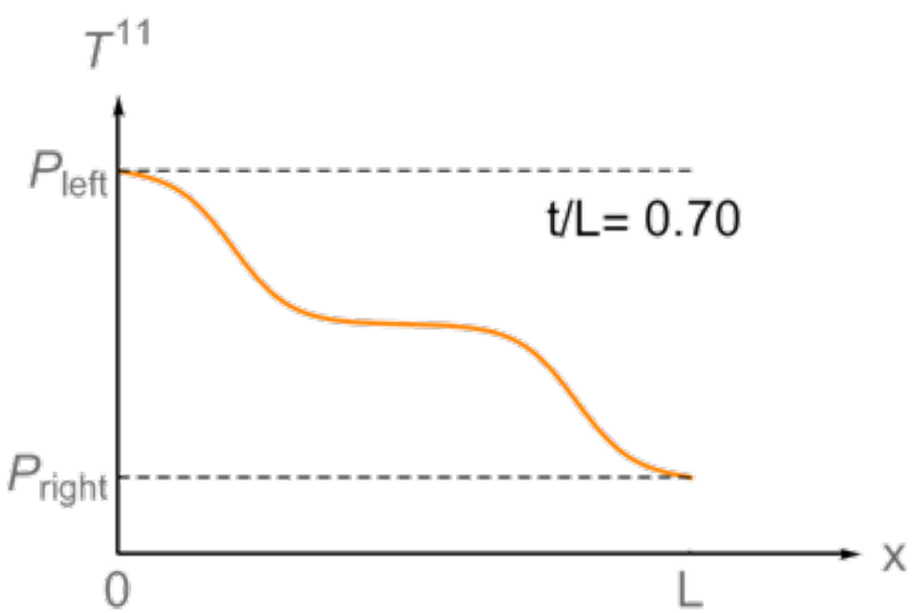
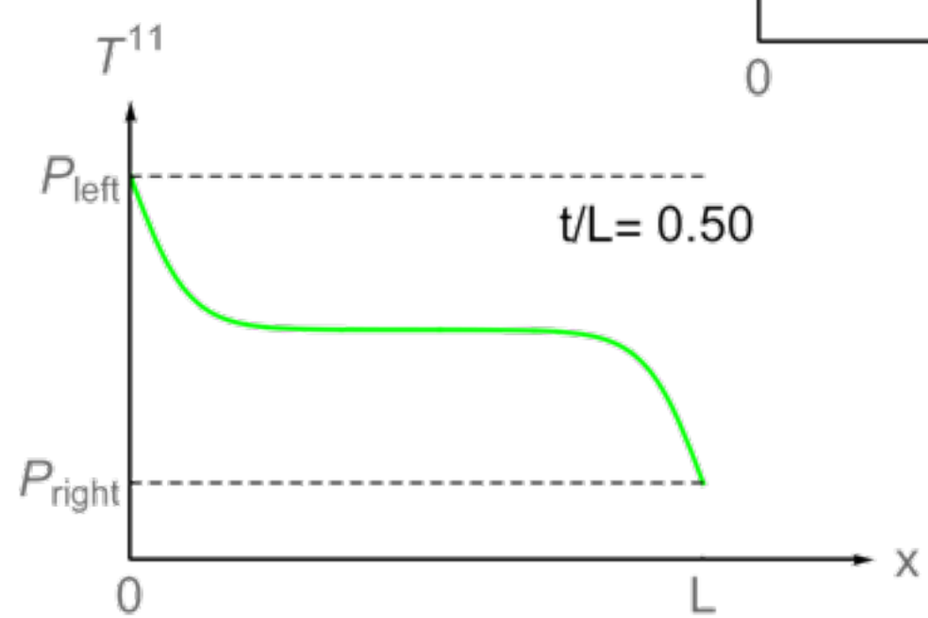
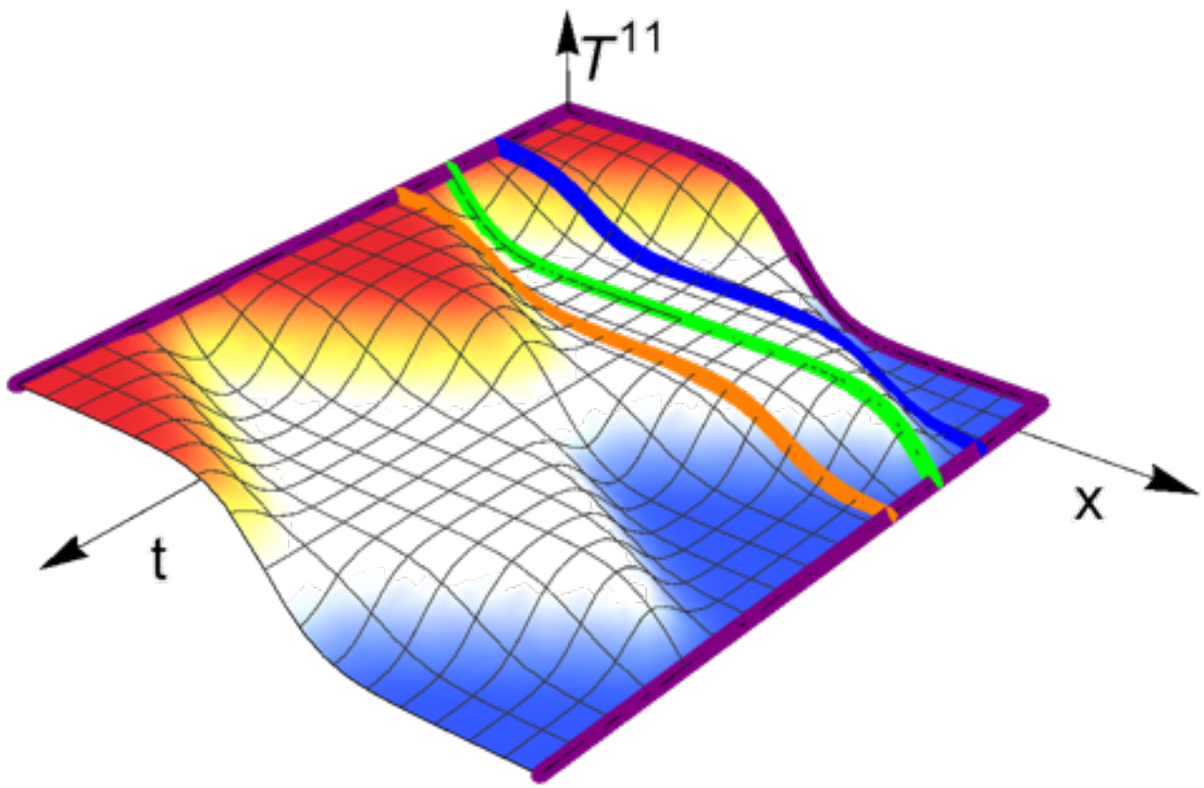
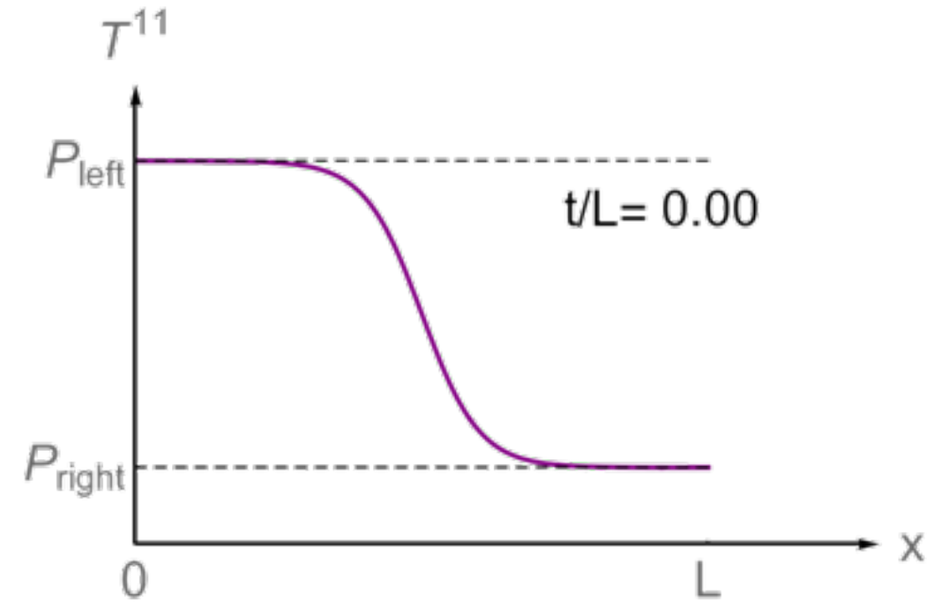






2d CFT's





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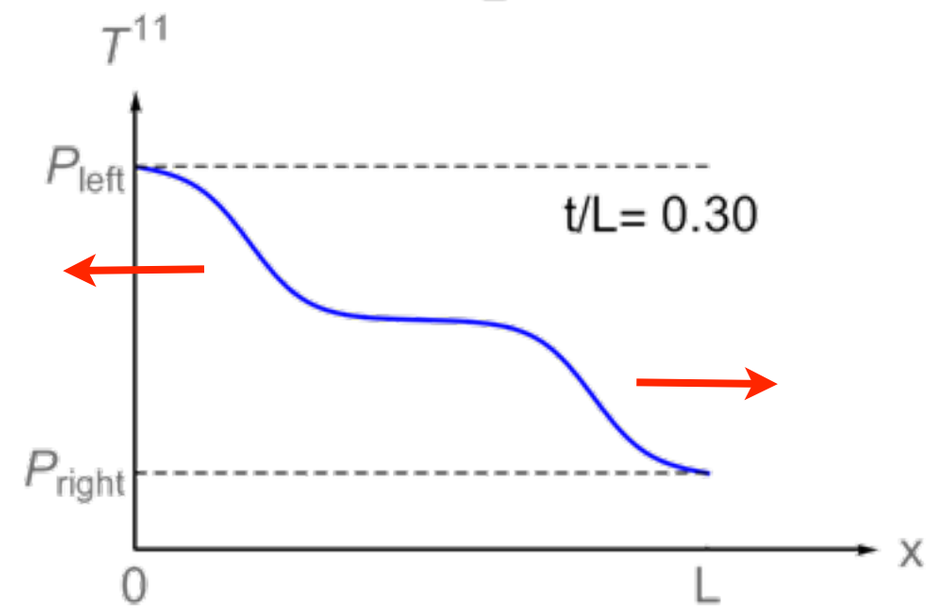
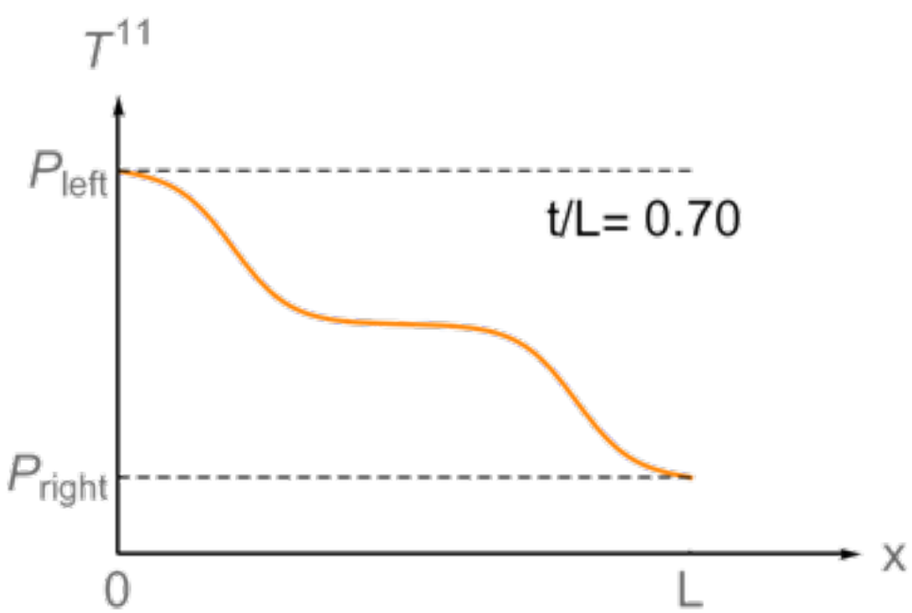
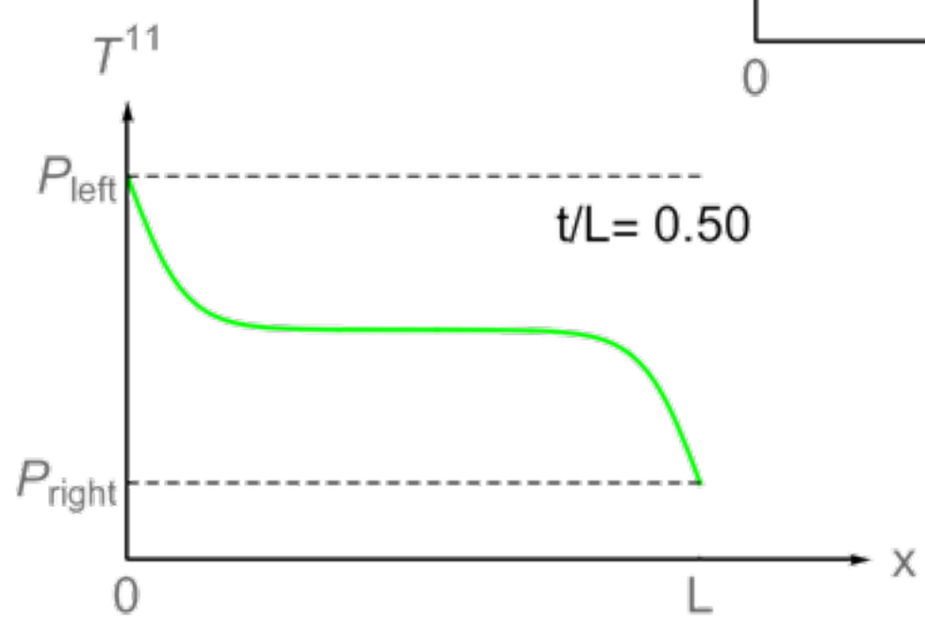
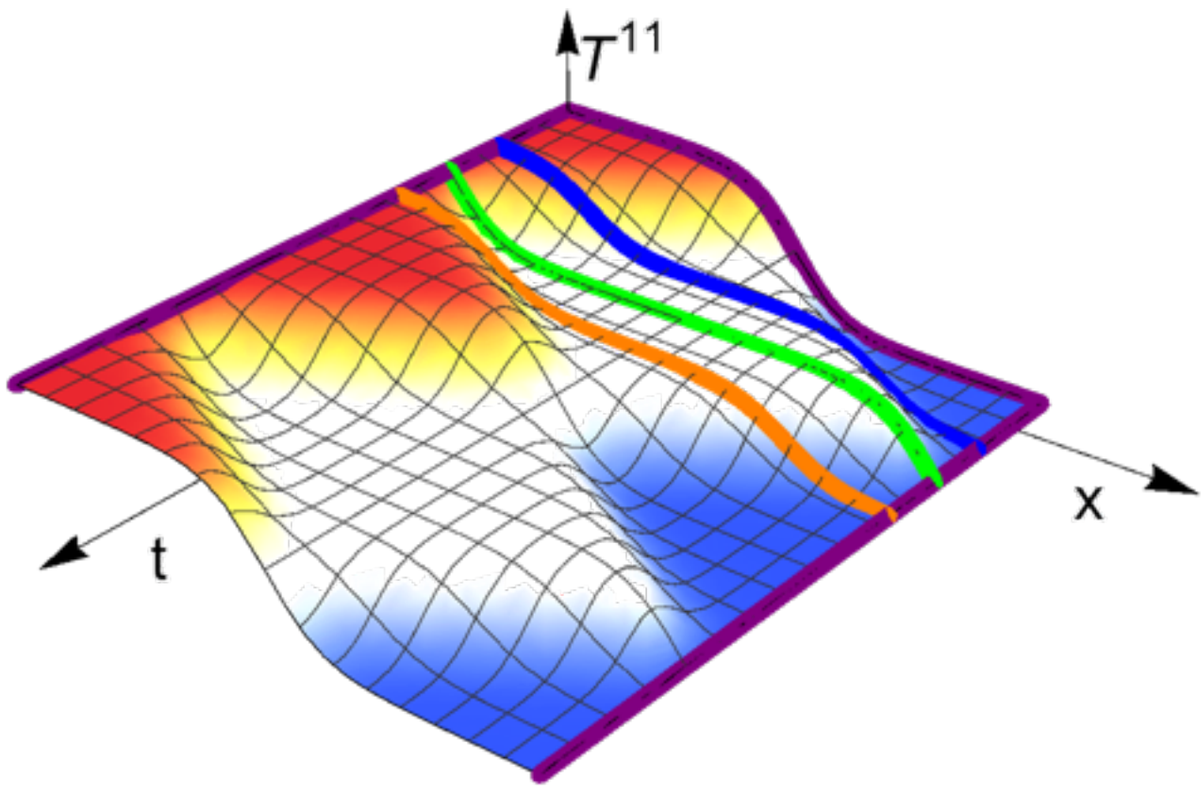
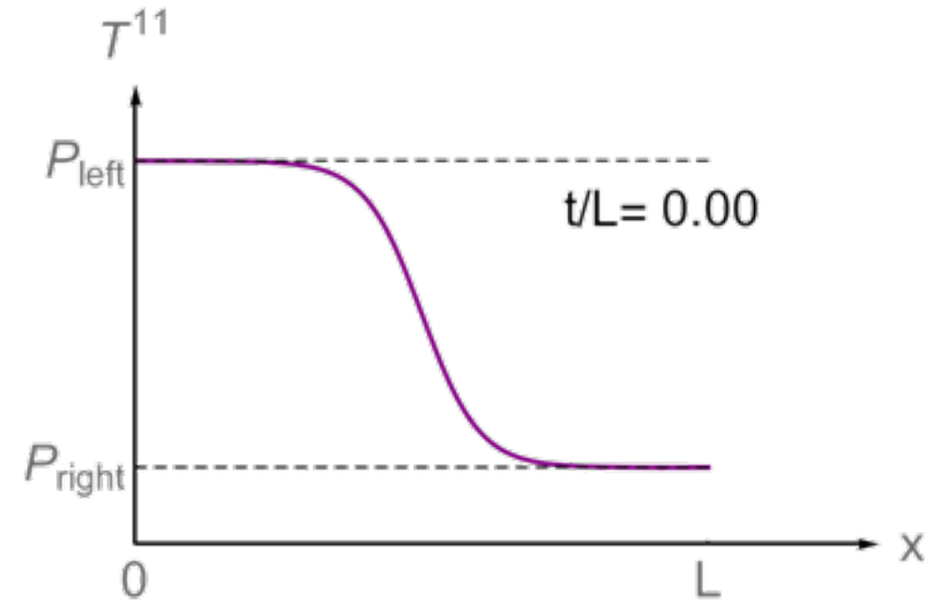






2d CFT's





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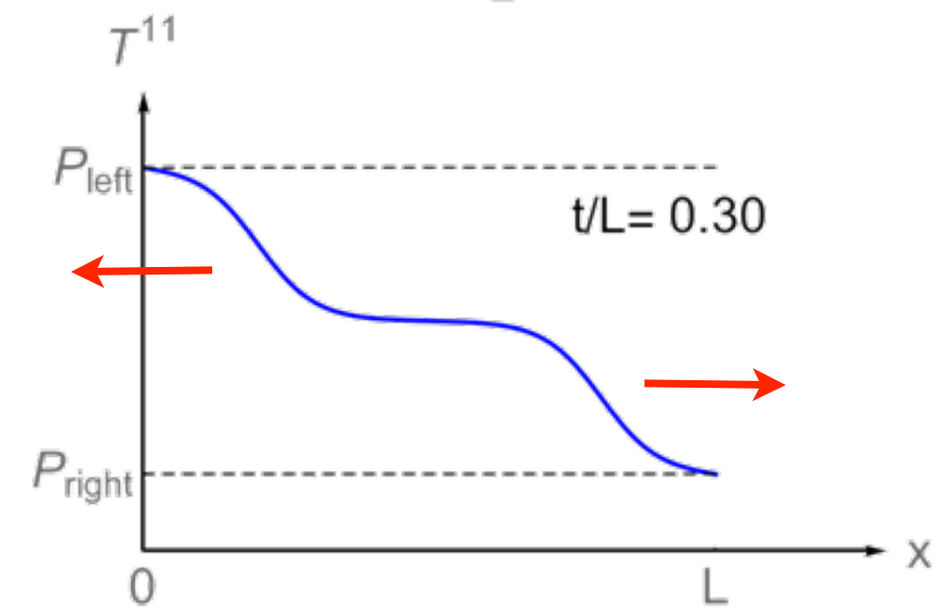

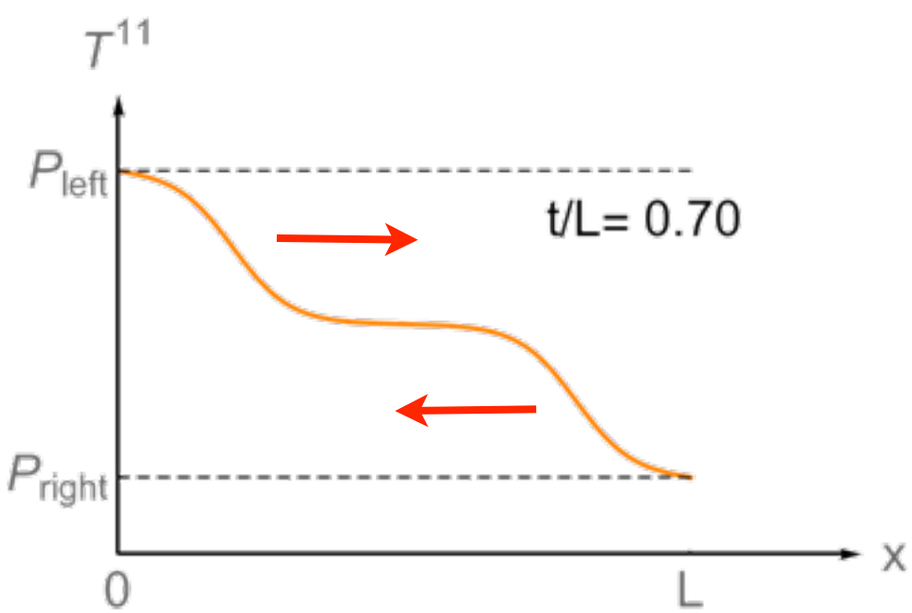

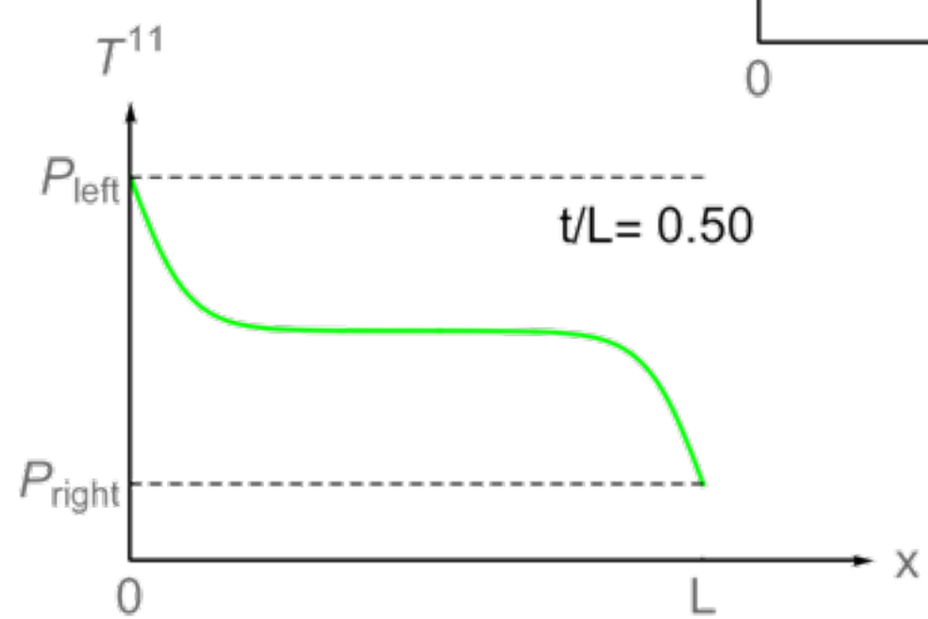

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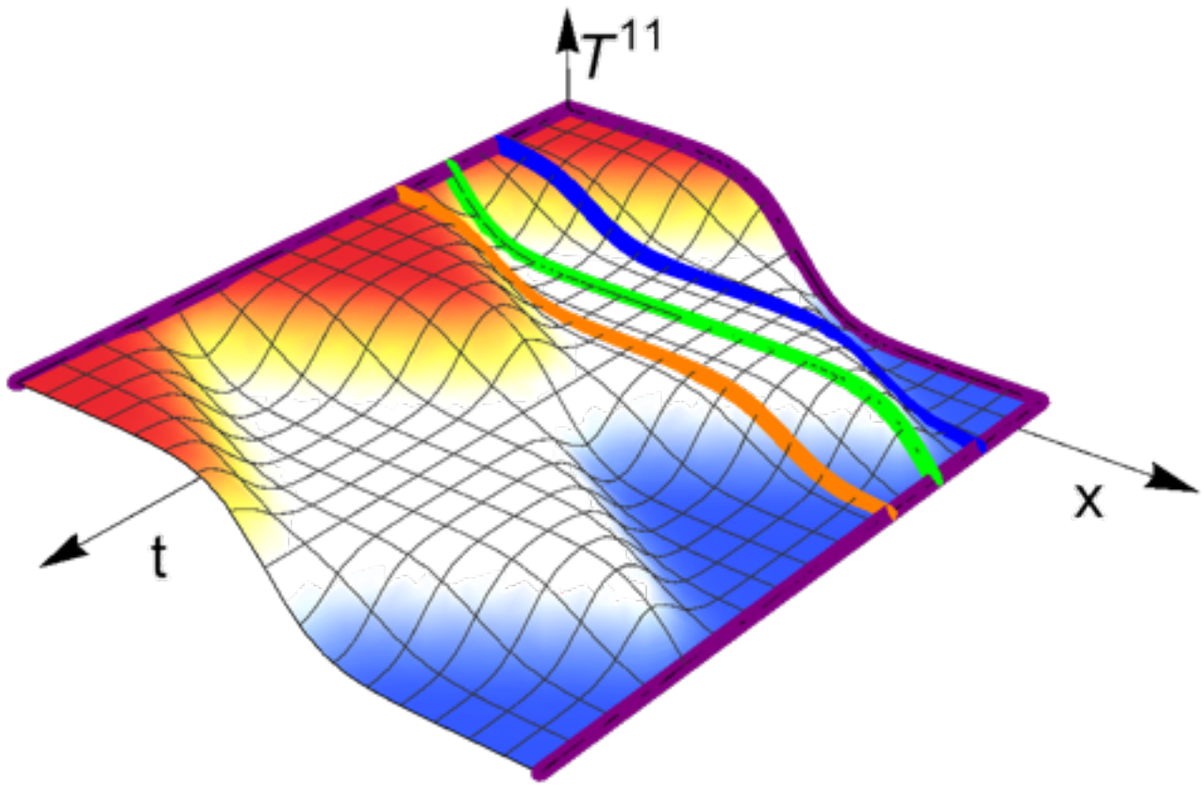

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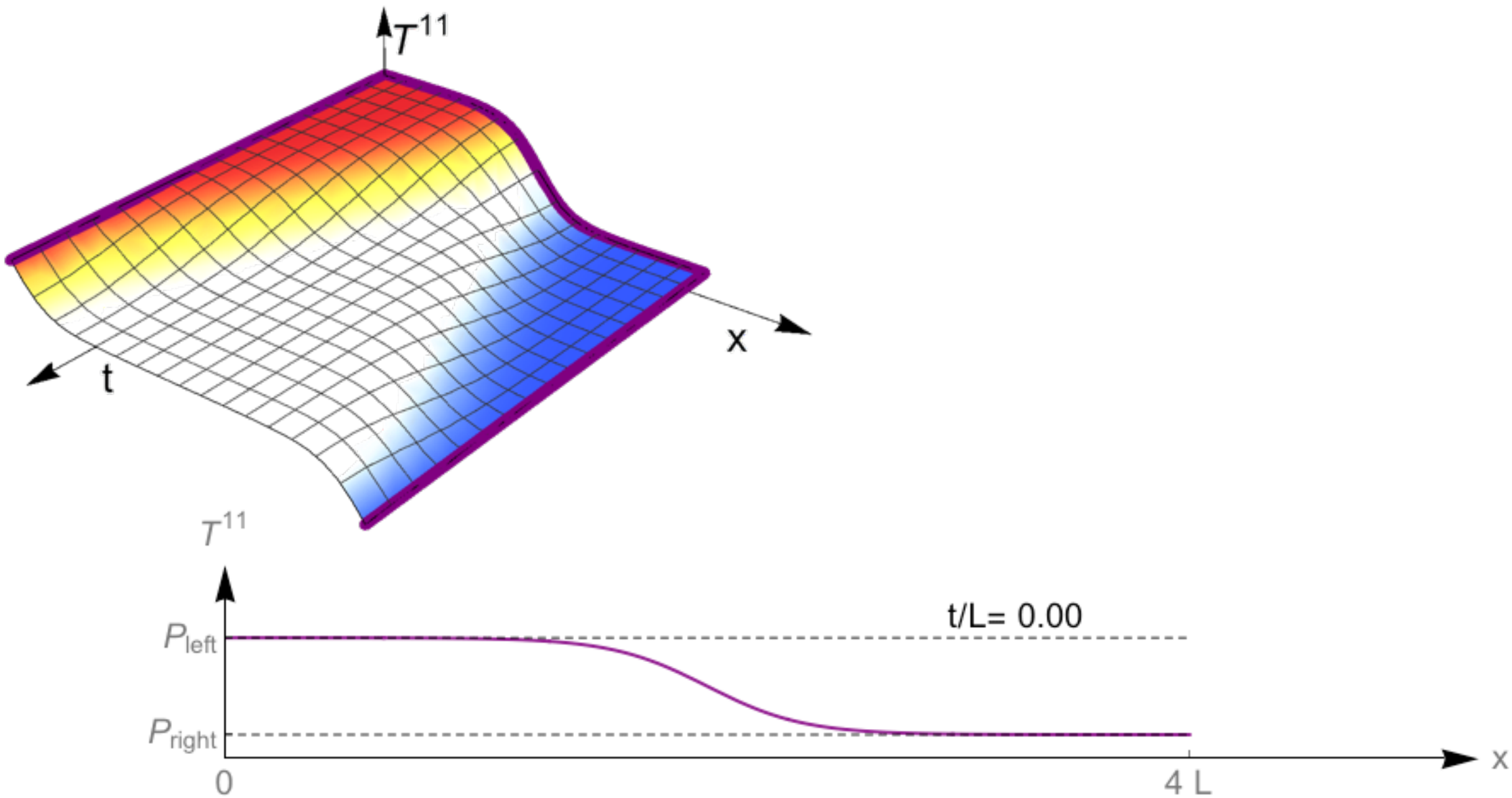

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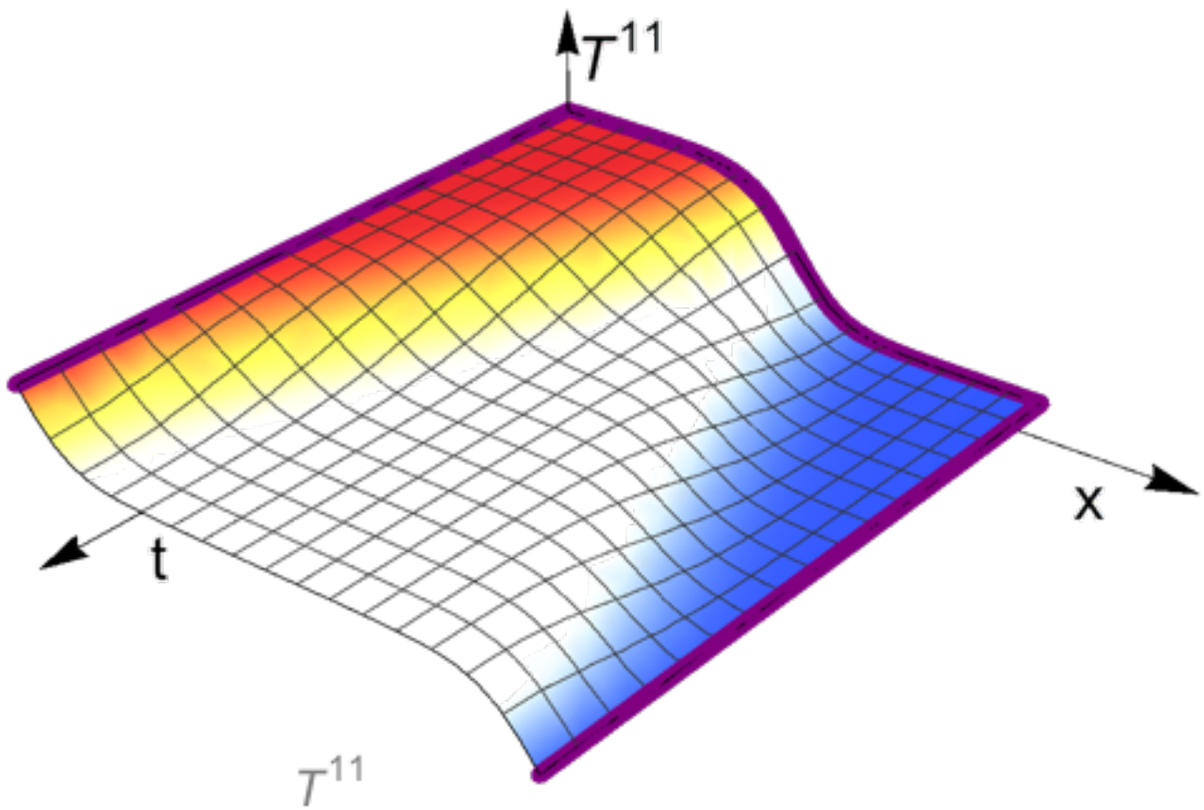

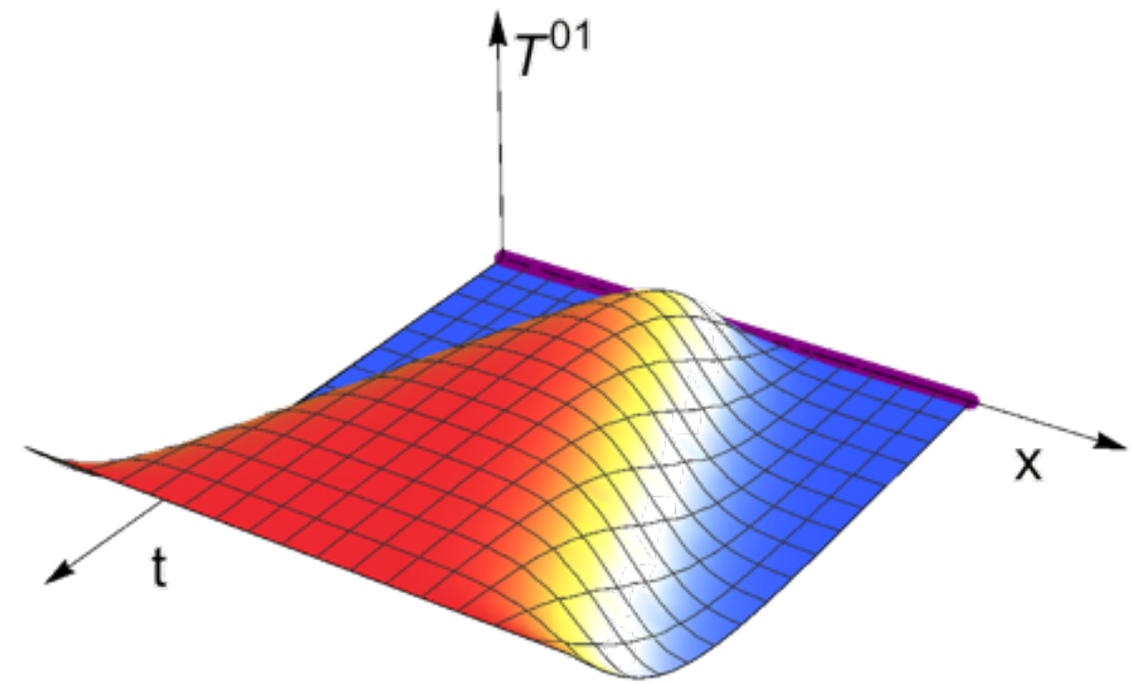

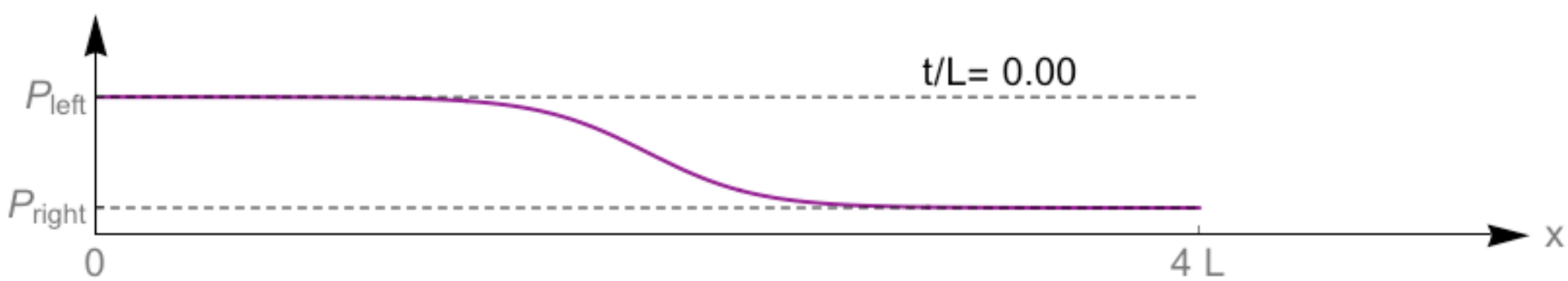

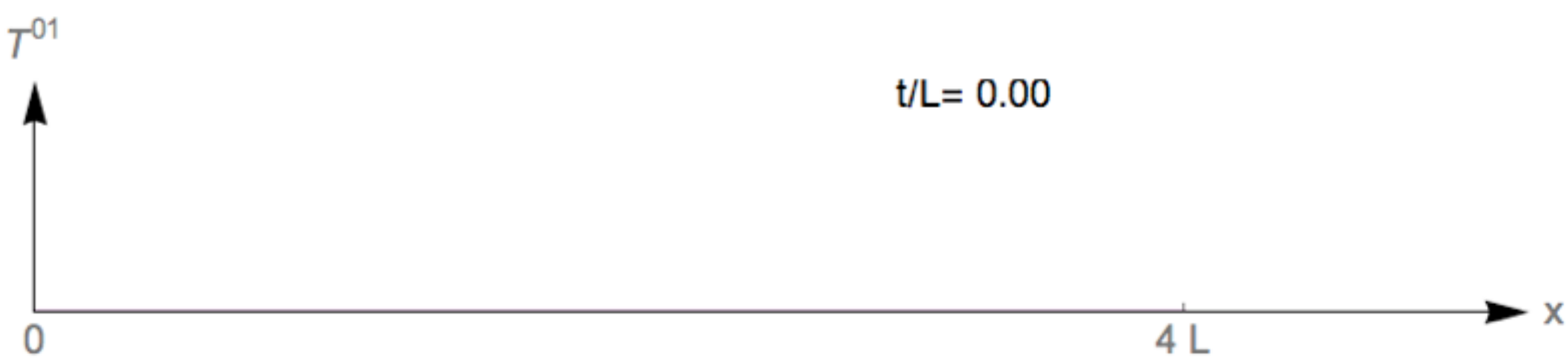

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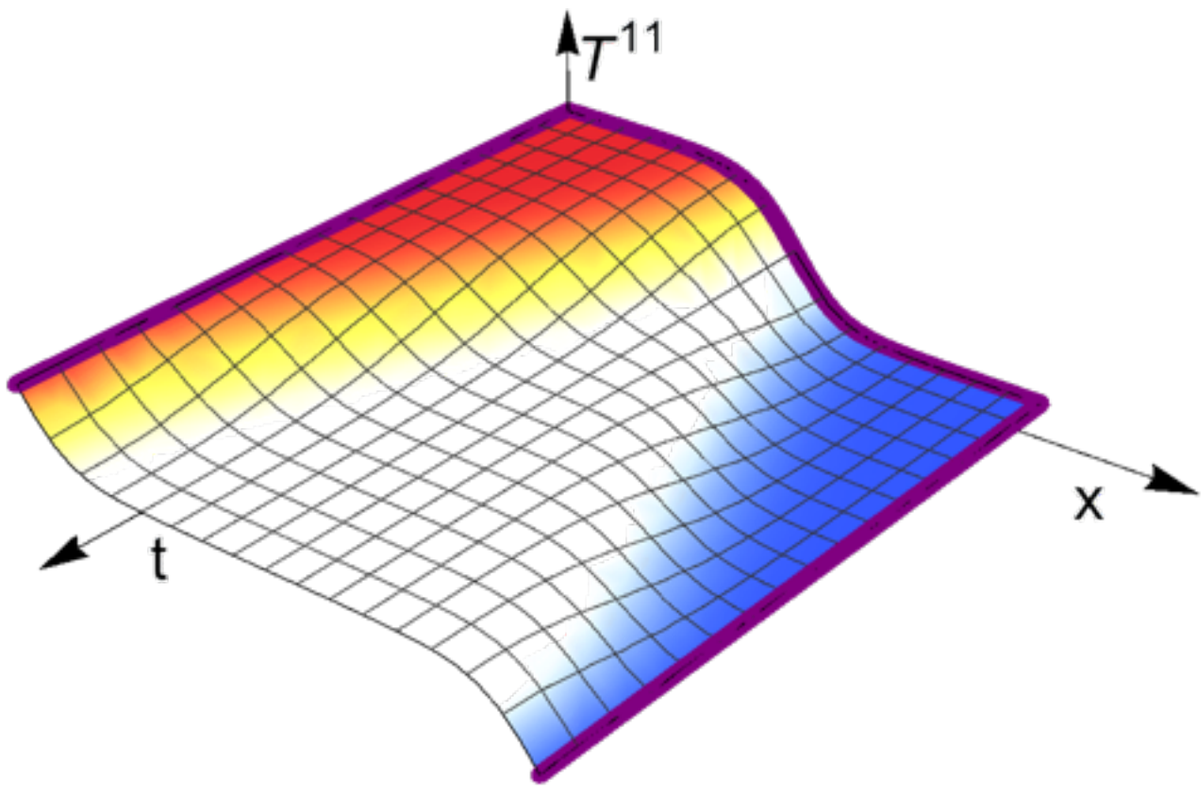

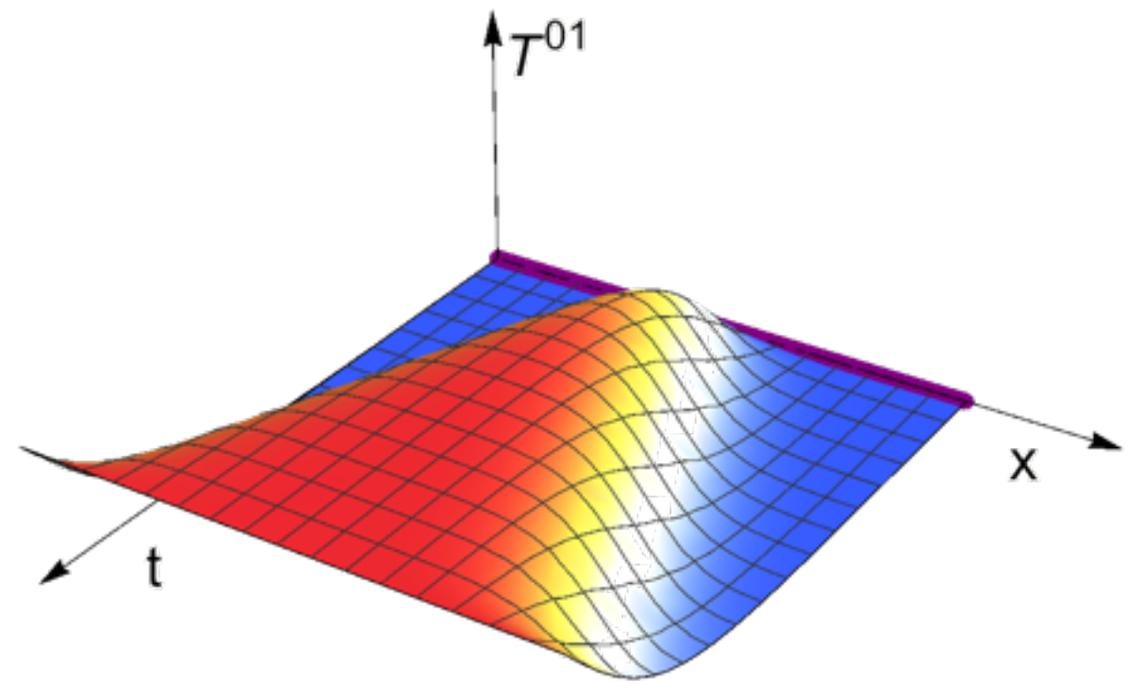

If we set $L \to \infty$, then in the finite x, $t \to \infty$ limit we expect to see a time independent steady state.

2d CFT's

Recall that in a conformal theory:

$$T^{\mu\nu} = \begin{pmatrix} T_+(t+x) + T_-(-t+x) & T_-(-t+x) - T_+(t+x) \\ T_-(-t+x) - T_+(t+x) & T_+(t+x) + T_-(-t+x) \end{pmatrix}$$

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At x=∞ we have the right heat bath

$$T_+(\infty) + T_-(\infty) = P_{\text{right}}\,, \quad T_-(\infty) - T_+(\infty) = 0$$

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At x=-∞ we have the left heat bath

$$T_+(-\infty) + T_-(-\infty) = P_{\text{left}}\,, \quad T_-(-\infty) - T_+(-\infty) = 0$$

2d CFT's

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Therefore, at t=∞ we have

$$T^{11} = T_+(\infty) + T_-(-\infty) = \frac{1}{2}\left(P_{\text{left}} + P_{\text{right}}\right)\,,$$

$$T^{01} = T_-(-\infty) - T_+(\infty) = \frac{1}{2}\left(P_{\text{left}} - P_{\text{right}}\right)$$

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Therefore, at t=∞ we have (See also, Bernard and Doyon, 2012)

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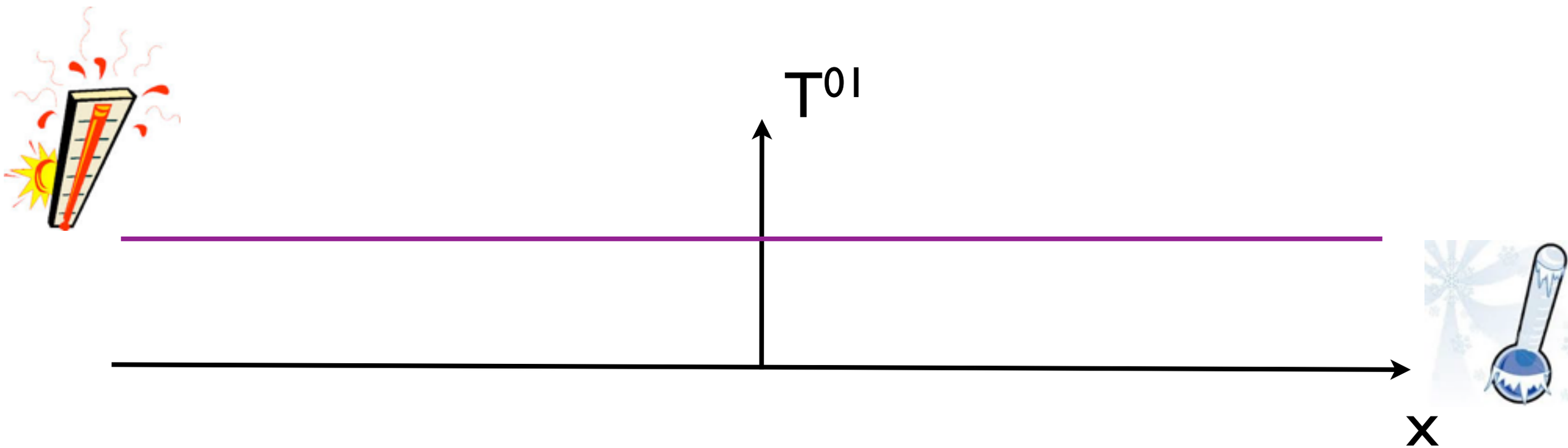
2d CFT's

The exact same analysis can be used to consider more complicated configurations:



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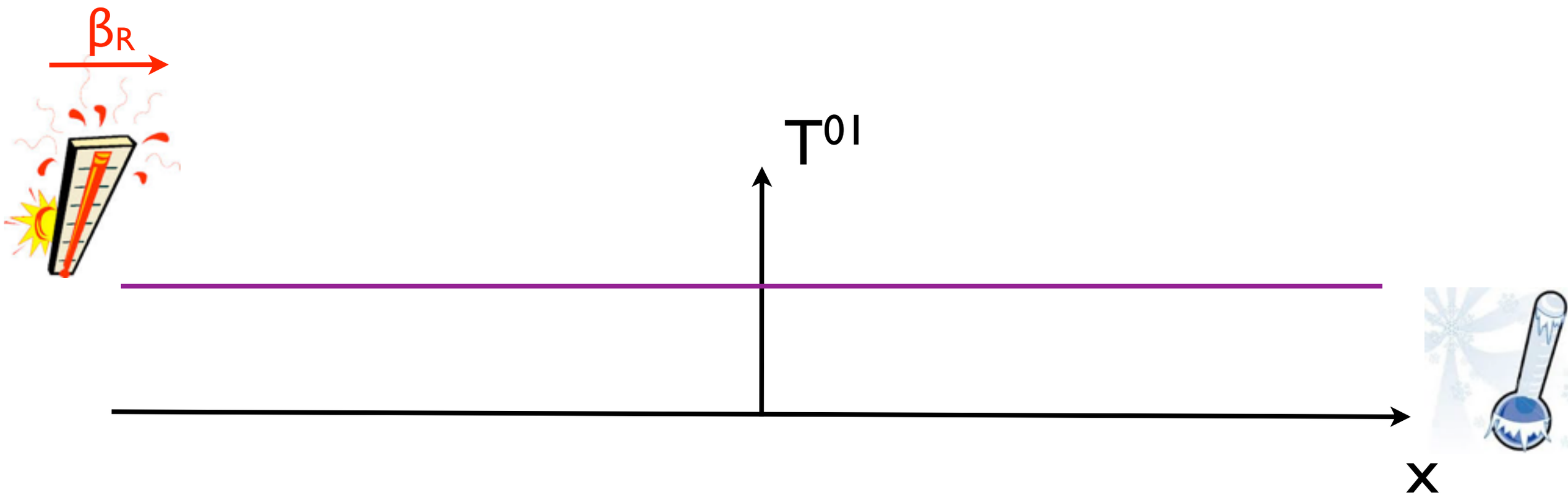

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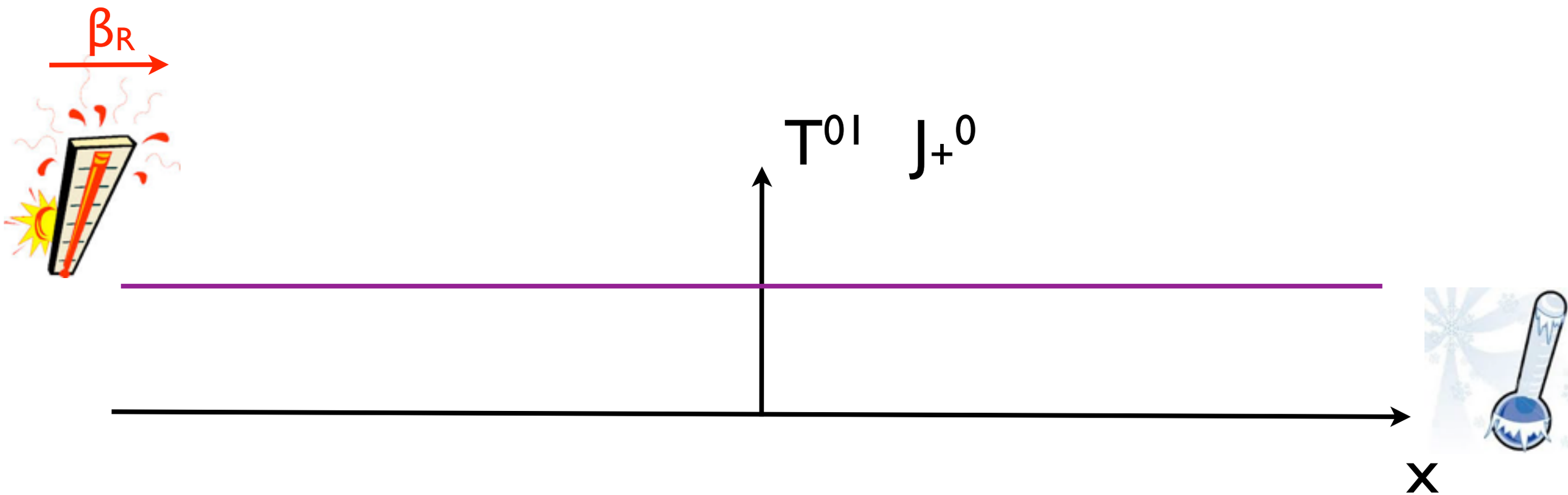

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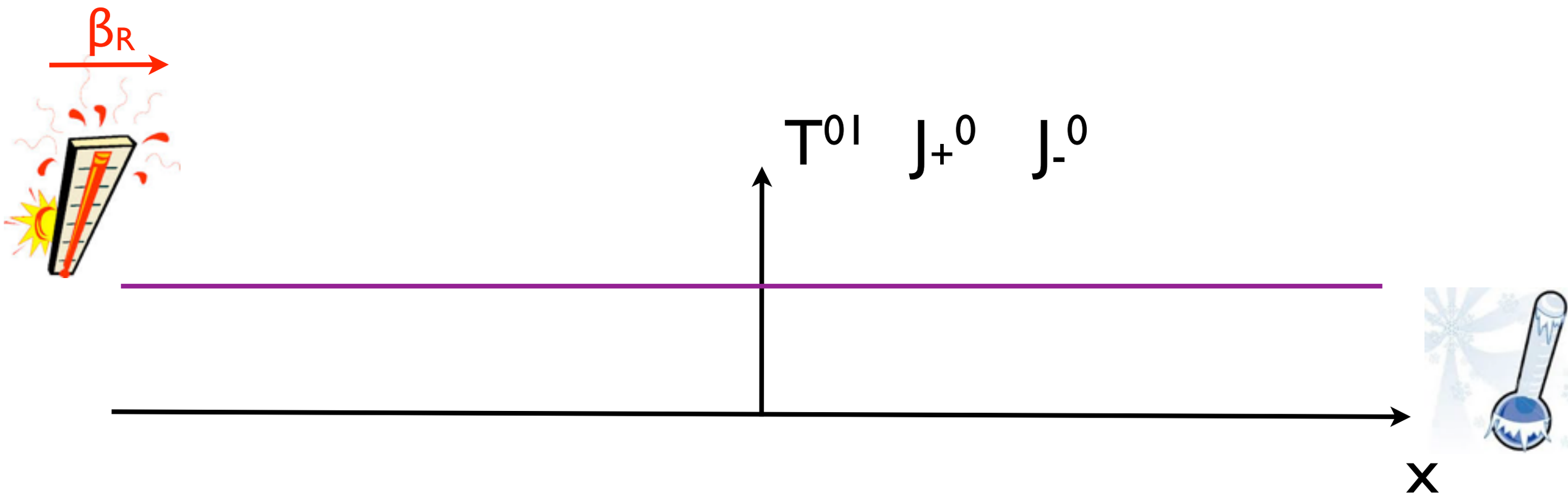

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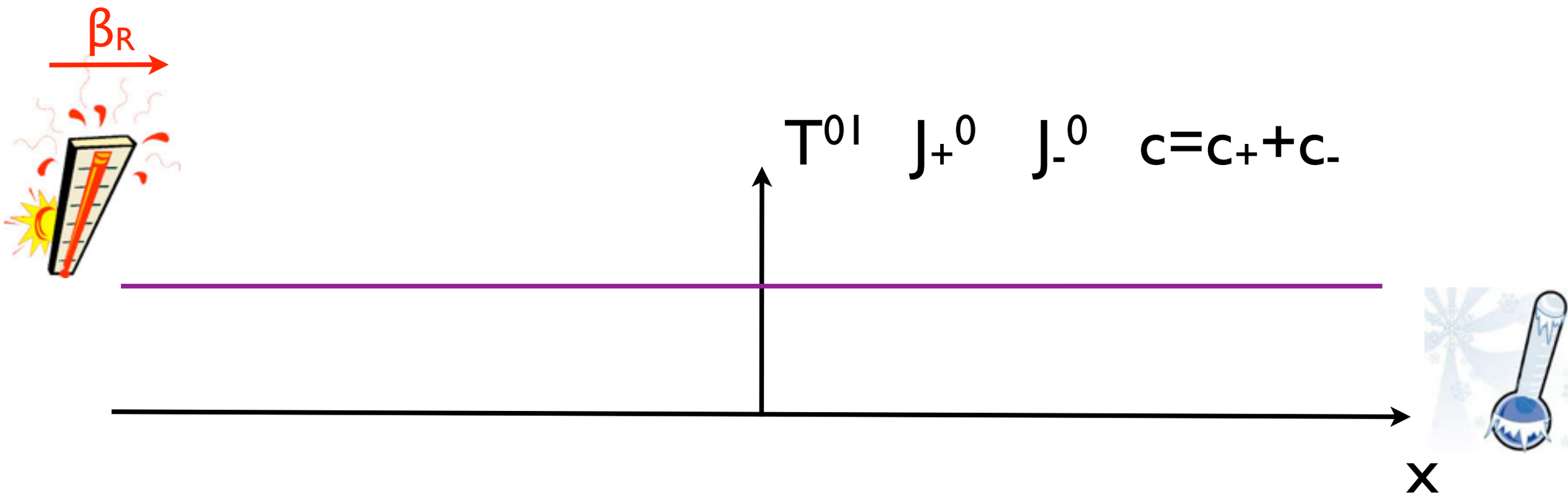

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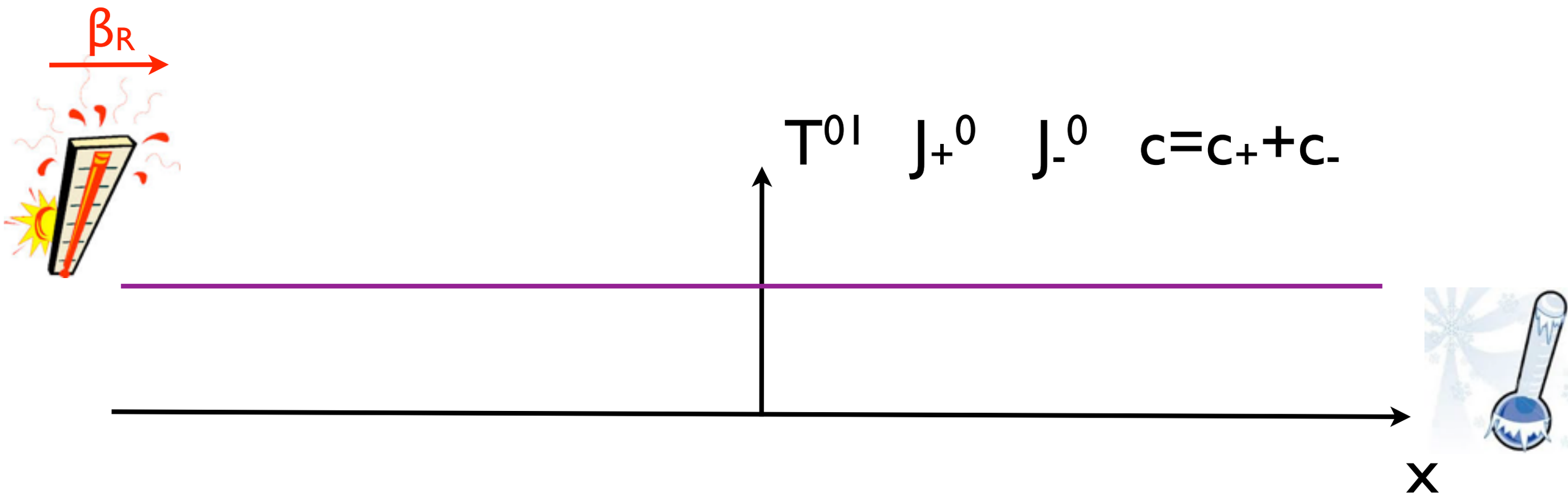

$$T^{01}(t\to\infty)=\frac{\pi}{12}\left(c_-T_L^2-c_+T_R^2\frac{1-\beta_R}{1+\beta_R}\right)+\frac{1}{2\pi}\left(k_-\mu_L^--k_+\mu_R^+\frac{1-\beta_R}{1+\beta_R}\right)$$

d>2

Main ingredient:

$$T^{\mu\nu} = \begin{pmatrix} T_+(t+x) + T_-(-t+x) & T_-(-t+x) - T_+(t+x) \\ T_-(-t+x) - T_+(t+x) & T_+(t+x) + T_-(-t+x) \end{pmatrix}$$

It follows from:

$$\partial_\mu T^{\mu\nu} = 0\,, \quad T^{\mu}{}_{\mu} = 0$$

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Within our ansatz

$$T^{\mu\nu}(t,x) = \begin{pmatrix} T^{00} & T^{01} & 0 \\ T^{01} & T^{11} & 0 \\ 0 & 0 & T_\perp \end{pmatrix}$$

d>2

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So for d>2 we have 4 components of the stress tensor but only three non trivial equations.

d>2

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So for d>2 we have 4 components of the stress tensor but only three non trivial equations.

We need more input.

d>2

Main ingredient:

It follows from:

$$\partial_\mu T^{\mu\nu} = 0\,, \quad T^\mu{}_\mu = 0$$

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d>2

Energy momentum conservation and conformal invariance imply:

$$\partial_\mu T^{\mu\nu} = 0\,, \quad T^\mu{}_\mu = 0$$

Within our ansatz

$$T^{\mu\nu}(t,x) = \begin{pmatrix} T^{00} & T^{01} & 0 \\ T^{01} & T^{11} & 0 \\ 0 & 0 & T_\perp \end{pmatrix}$$

Let us assume, in addition, that the system is described by a perfect inviscid fluid:

$$T^{\mu\nu} = \epsilon(P) u^\mu u^\nu + (\eta^{\mu\nu} + u^\mu u^\nu) P$$

d>2

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energy density

d>2

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energy density

4-velocity

d>2

Energy momentum conservation and conformal invariance imply:

$$\partial_\mu T^{\mu\nu} = 0\,, \quad T^\mu{}_\mu = 0$$

Within our ansatz

$$T^{\mu\nu}(t,x) = \begin{pmatrix} T^{00} & T^{01} & 0 \\ T^{01} & T^{11} & 0 \\ 0 & 0 & T_\perp \end{pmatrix}$$

Let us assume, in addition, that the system is described by a perfect inviscid fluid:

$$T^{\mu\nu} = \epsilon(P) u^\mu u^\nu + (\eta^{\mu\nu} + u^\mu u^\nu) P$$

energy density 4-velocity Pressure

d>2

Energy momentum conservation and conformal invariance imply:

$$\partial_\mu T^{\mu\nu} = 0\,, \quad T^\mu{}_\mu = 0$$

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For a conformal field theory:

$$\epsilon = (d-1)P$$

d>2

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d>2

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d>2

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Consider an ansatz

$$T^{00}(x,t) = (T_r^{00} - T_l^{00})\theta(x - st) + T_l^{00}$$

$$T^{01}(x,t) = (T_r^{01} - T_l^{01})\theta(x - st) + T_l^{01}$$

d>2

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d>2

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d>2

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d>2

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$$T^{01}(x,t) = (\overset{4}{T_r^{01}} - \overset{5}{T_l^{01}})\theta(x - st) + T_l^{01}$$

We have a three parameter family of solutions.

d>2

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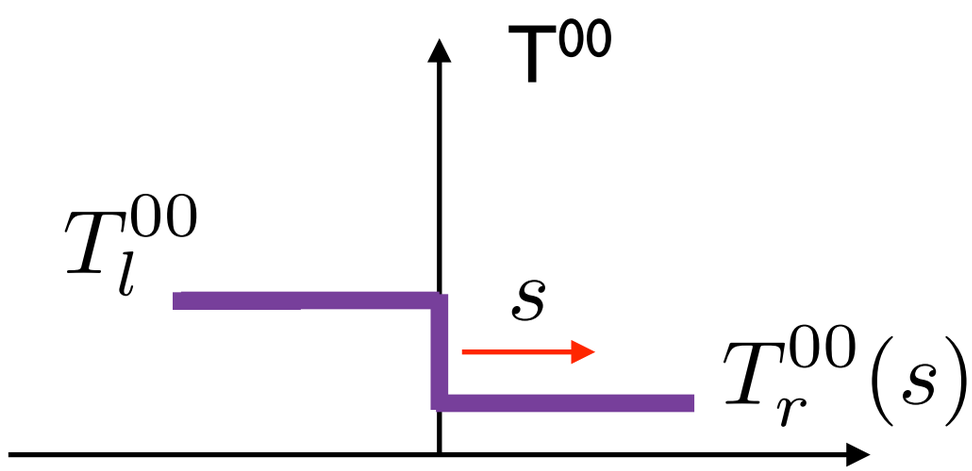
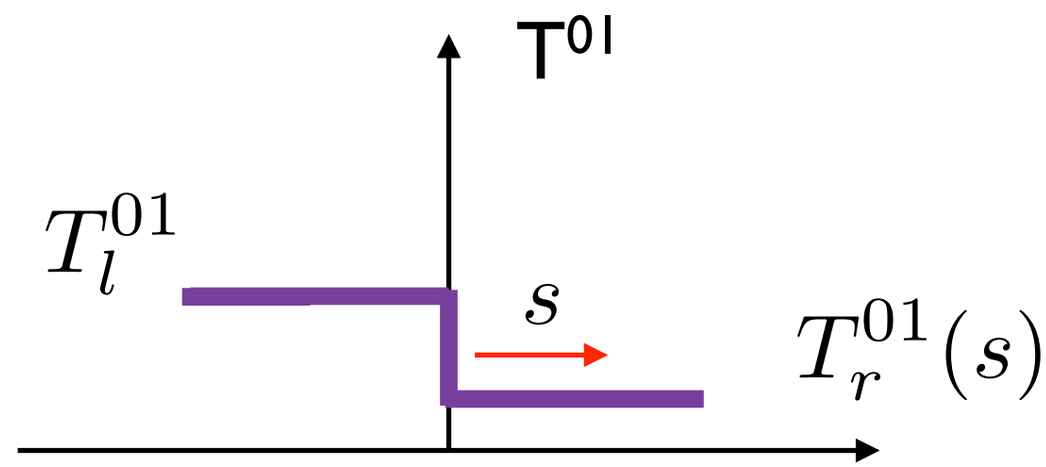
For instance, let us fix $T^{00}{}_l$ and $T^{01}{}_l$,





then the solution to the equations of motion will tell us $T^{00}{}_r(s)$ and $T^{01}{}_r(s)$ as a function of s.

d>2

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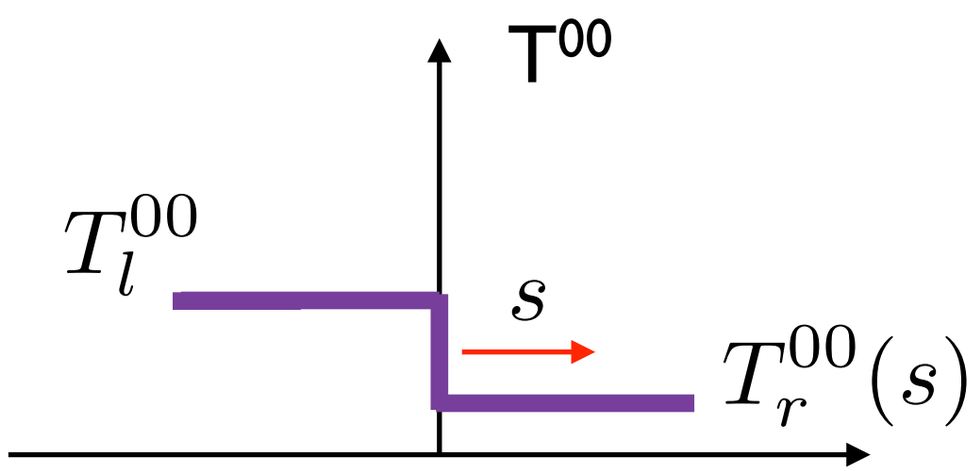
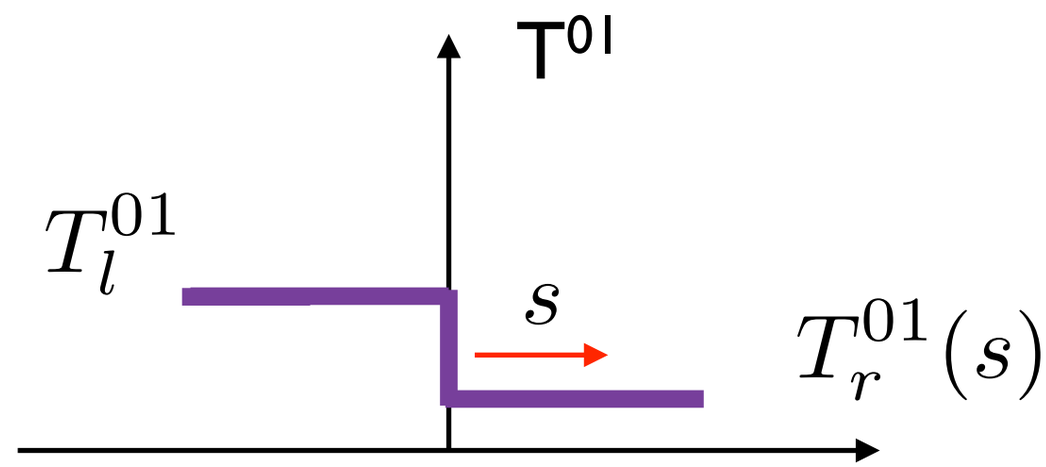




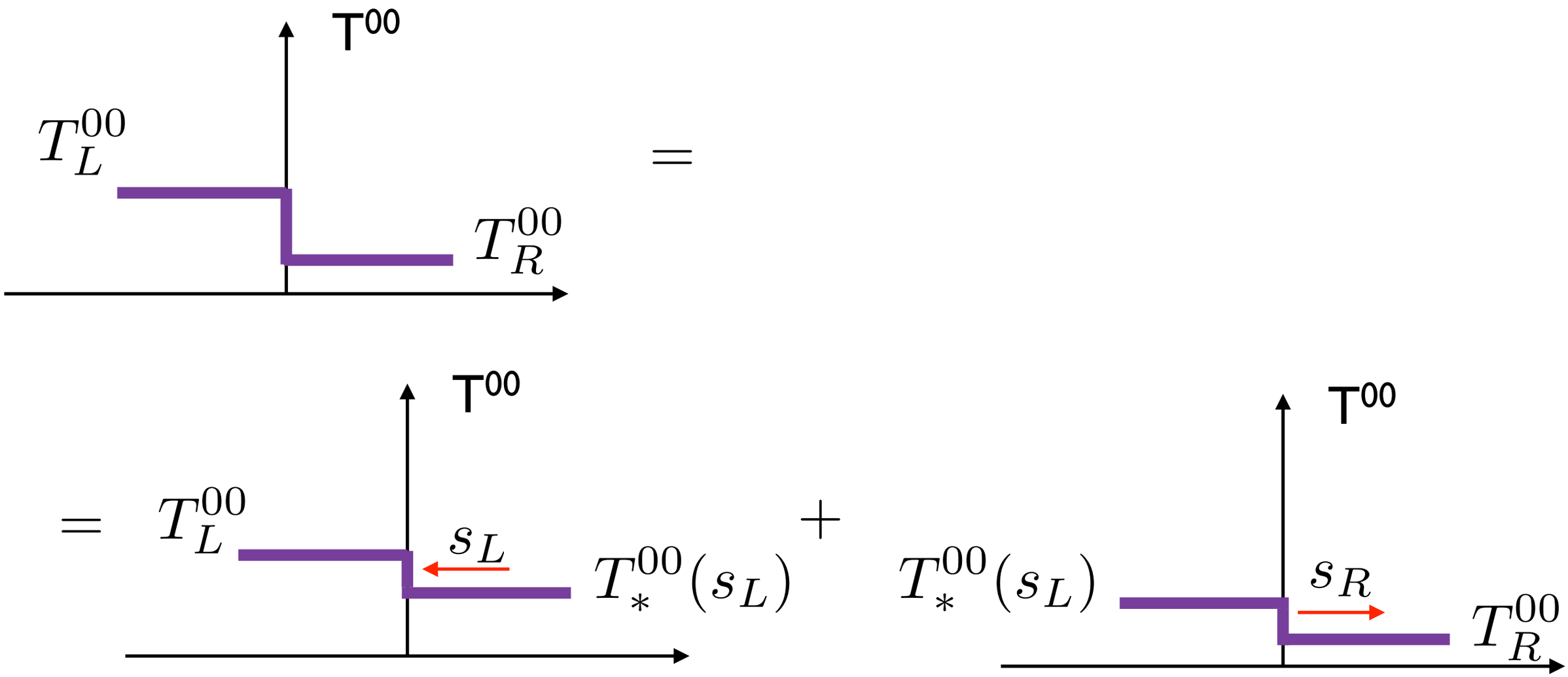
then the solution to the equations of motion will tell us $T^{00}{}_r(s)$ and $T^{01}{}_r(s)$ as a function of s.

Thus, if we specify $T^{00}{}_l$, $T^{01}{}_l$, $T^{00}{}_r$ and $T^{01}{}_r$ the problem will be overdetermined.

d>2

Thus, if we specify $T^{00}{}_l$, $T^{01}{}_l$, $T^{00}{}_r$ and $T^{01}{}_r$ the problem will be overdetermined.

But we can glue two solutions, e.g.,



d>2

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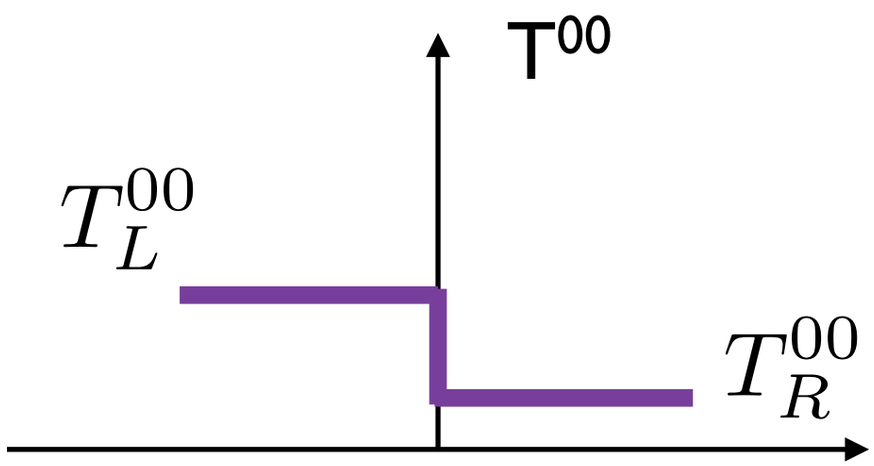
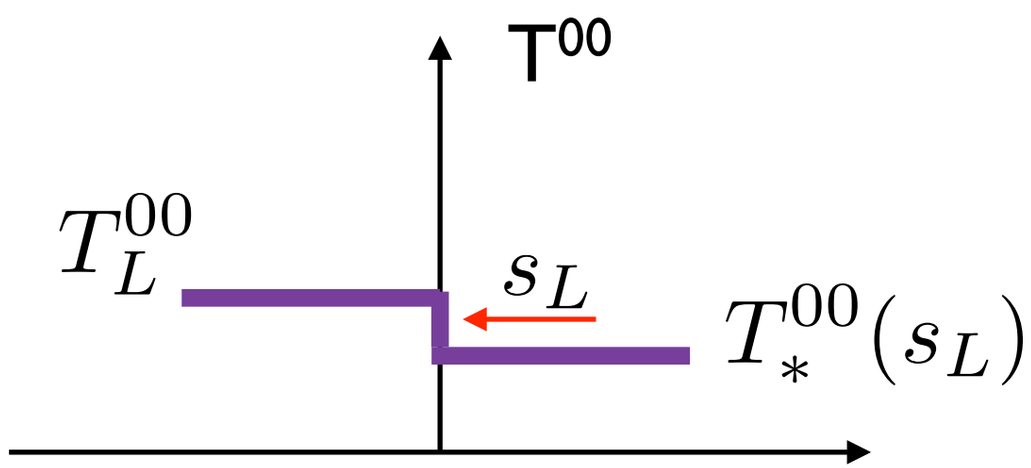
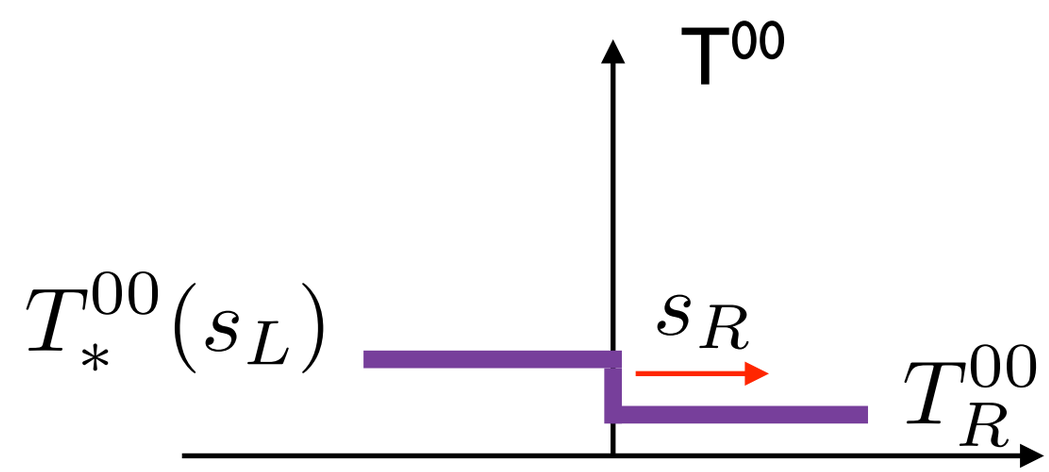


$=$

$=$



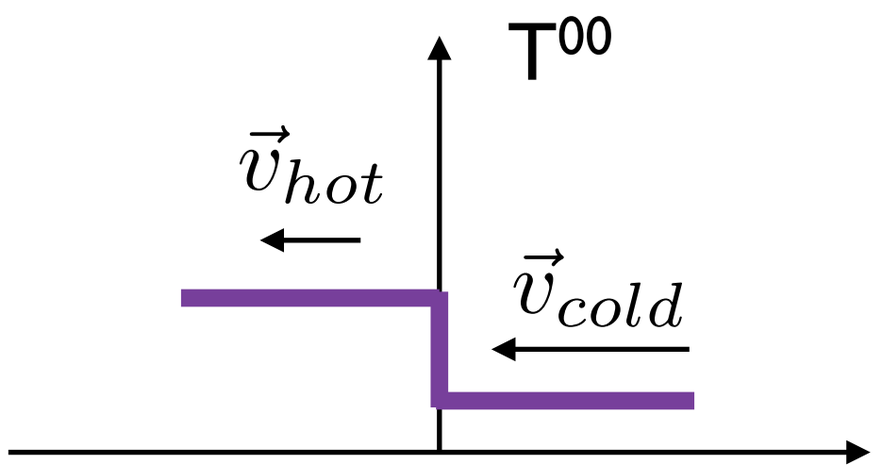
$+$

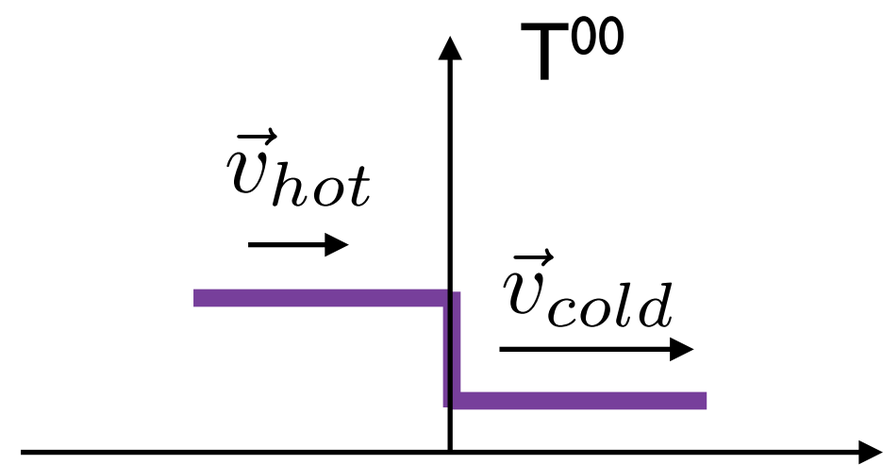


The resulting steady state will be characterised by: T_*^{00} T_*^{01}

d>2

Let us move to a frame where s=0, we obtain, e.g.,





d>2

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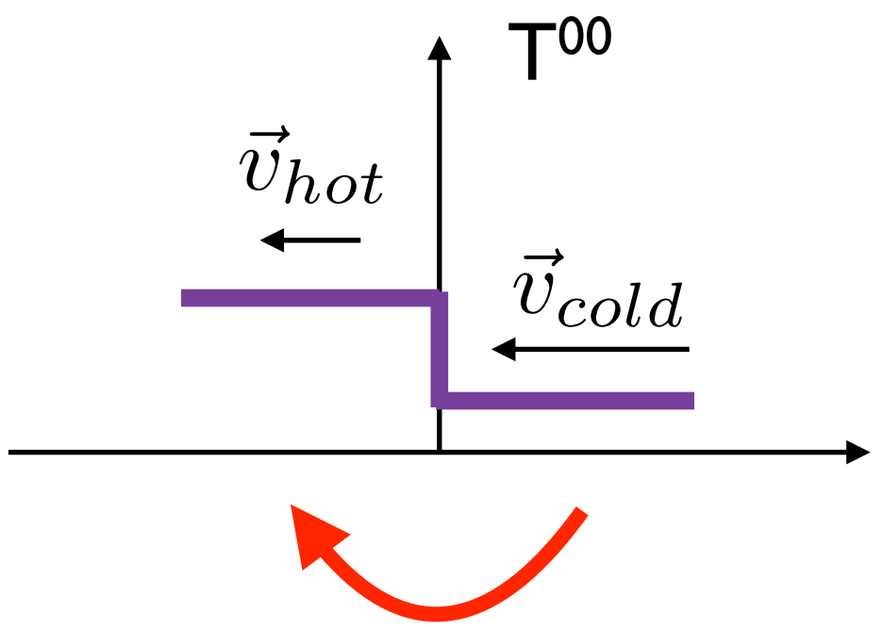

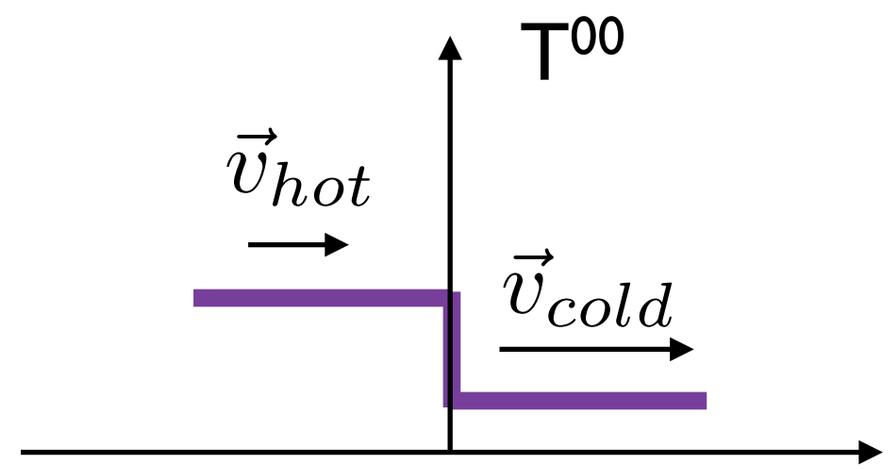

Kinetic energy
converted to heat

d>2

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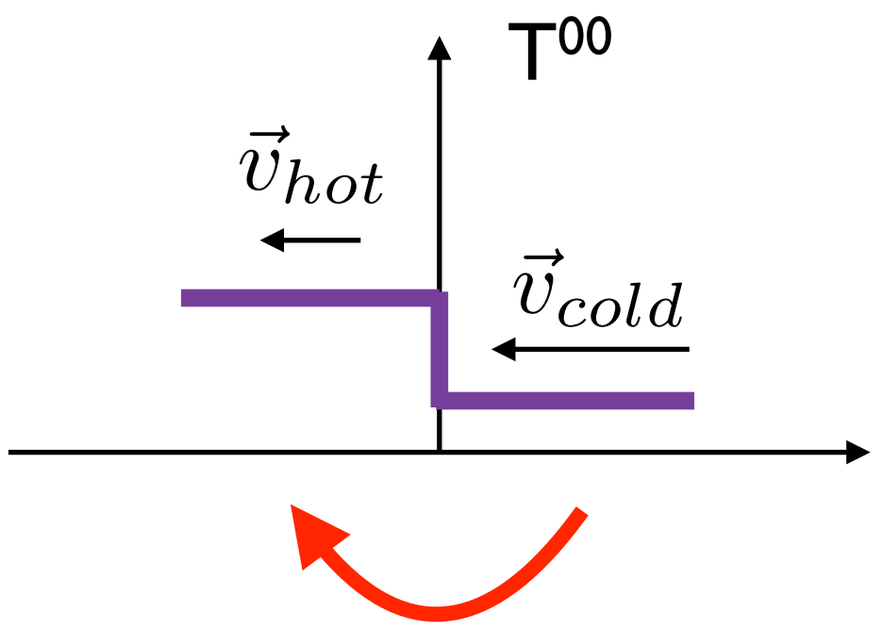

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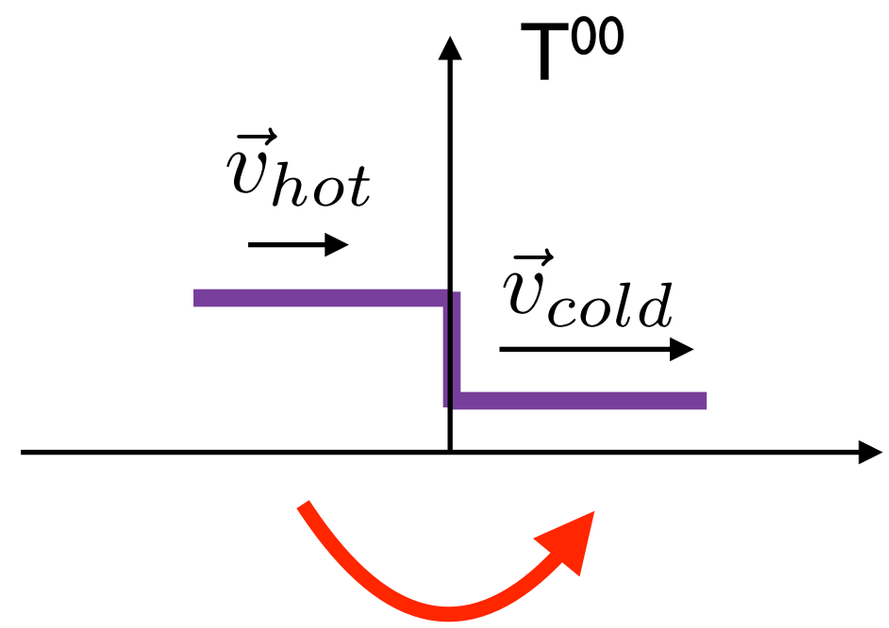
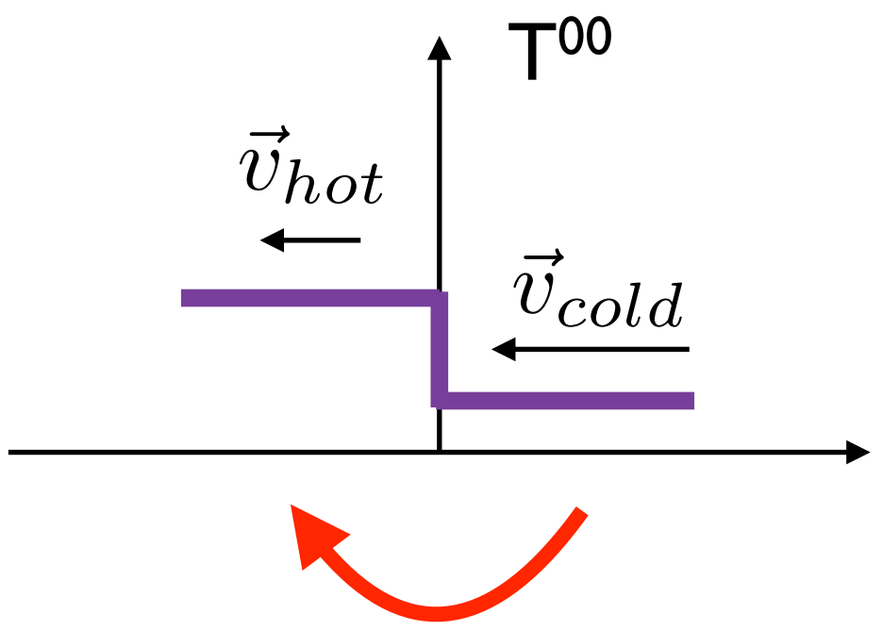


Heat converted to kinetic energy (?)

d>2

Let us move to a frame where s=0, we obtain, e.g.,

"Good" shocks



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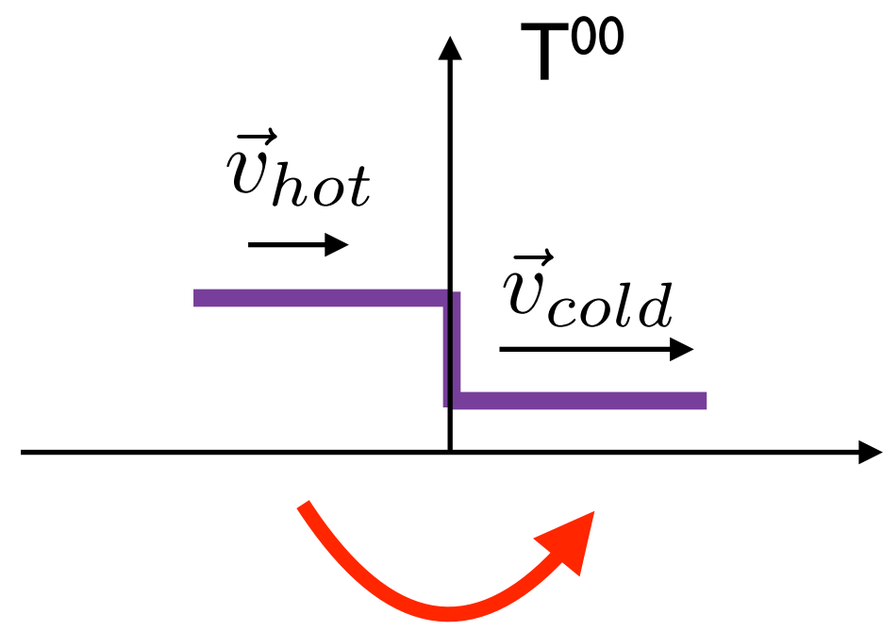

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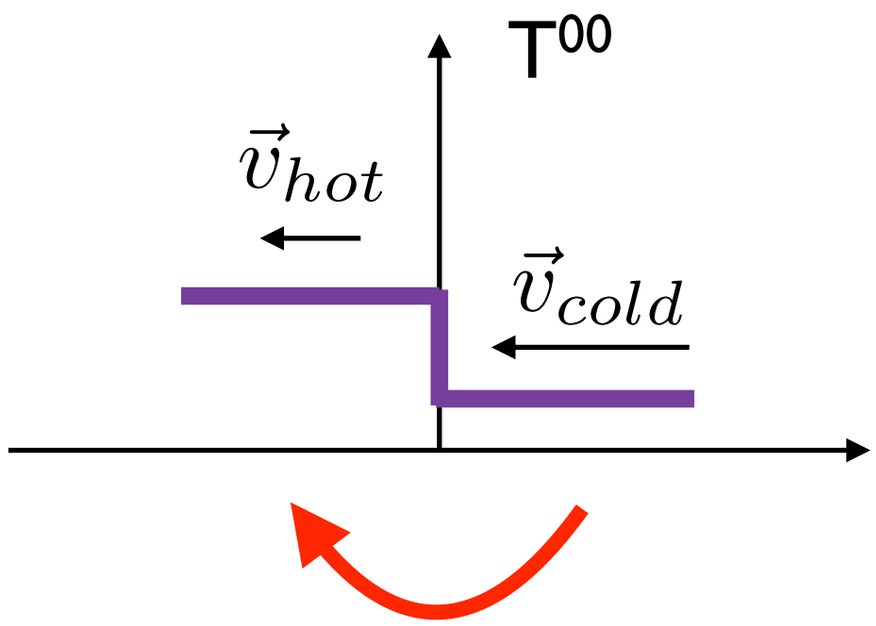
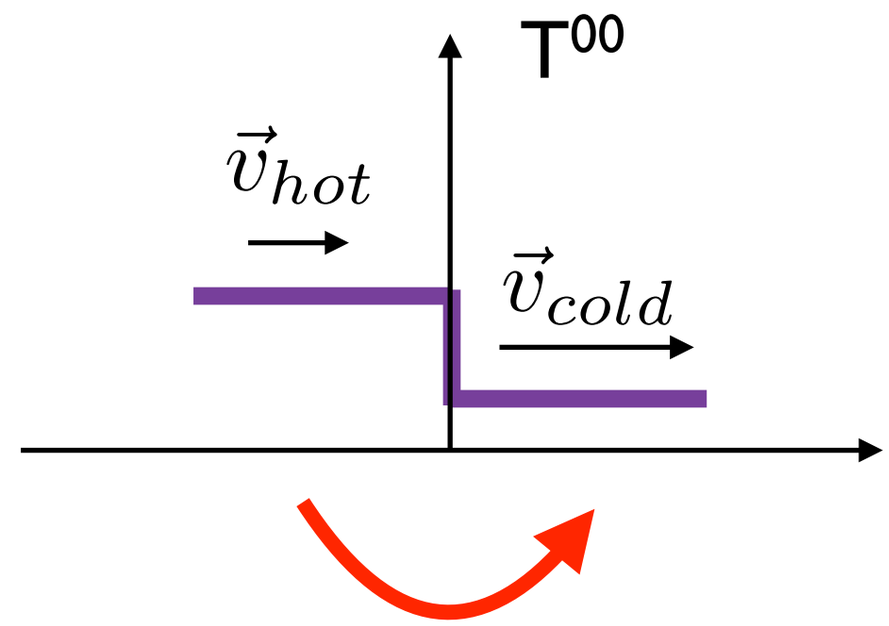


Kinetic energy converted to heat

"Bad" shocks



Heat converted to kinetic energy (?)

d>2

Energy momentum conservation and conformal invariance imply:

$$\partial_\mu T^{\mu\nu} = 0\,, \quad T^{\mu}{}_{\mu} = 0$$

The resulting system of non linear equations is still difficult to solve.

Consider another ansatz (rarefaction wave)

$$T^{00}(x,t) = T^{00}(x/t)$$

$$T^{01}(x,t) = T^{01}(x/t)$$

d>2

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Up to an overall rescaling, we have a three parameter family of solutions.

d>2

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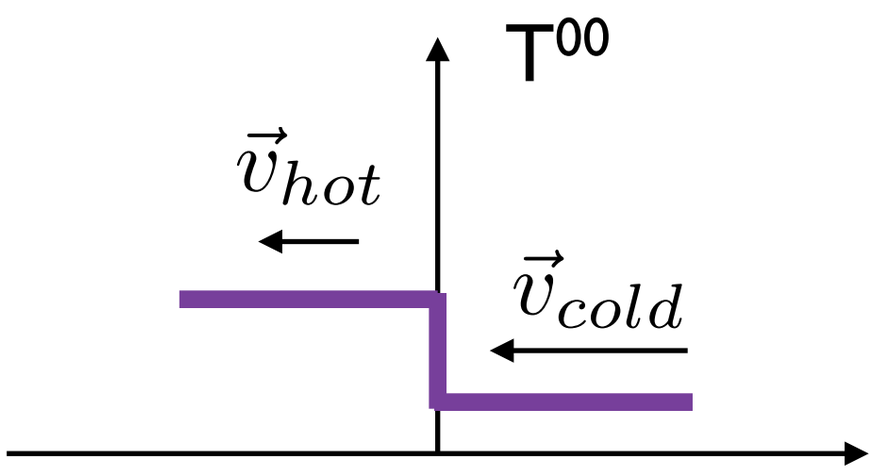
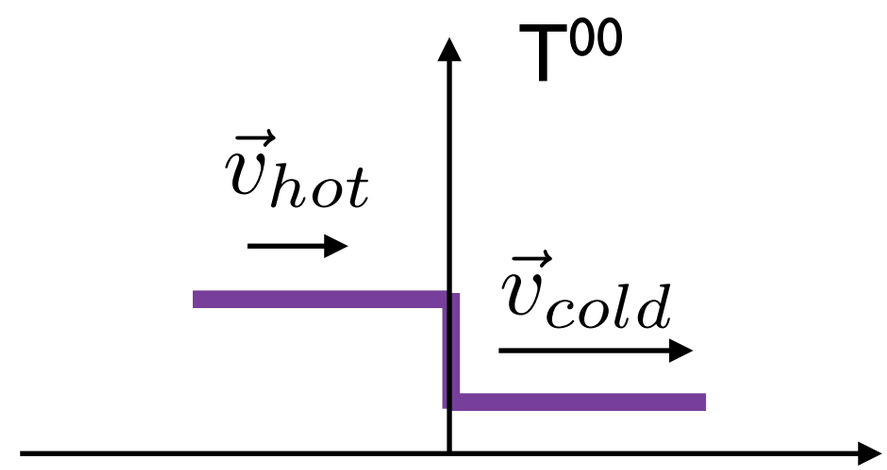
"Good" shocks



"Bad" shocks



d>2

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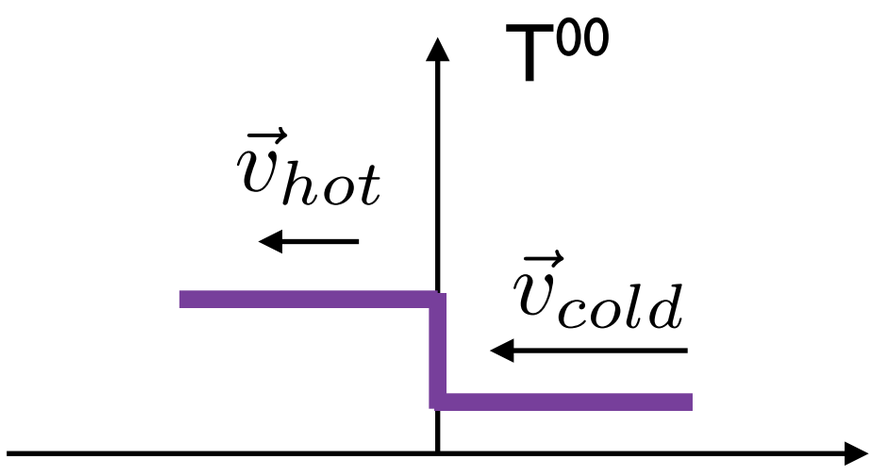
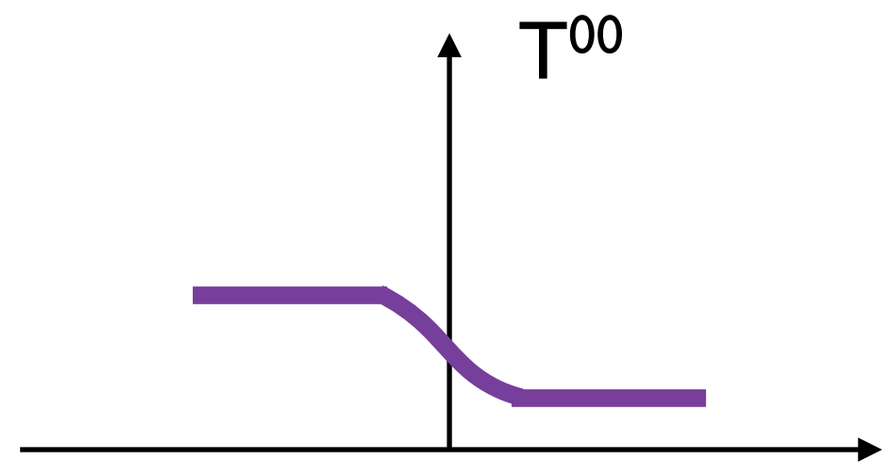


Rarefaction waves



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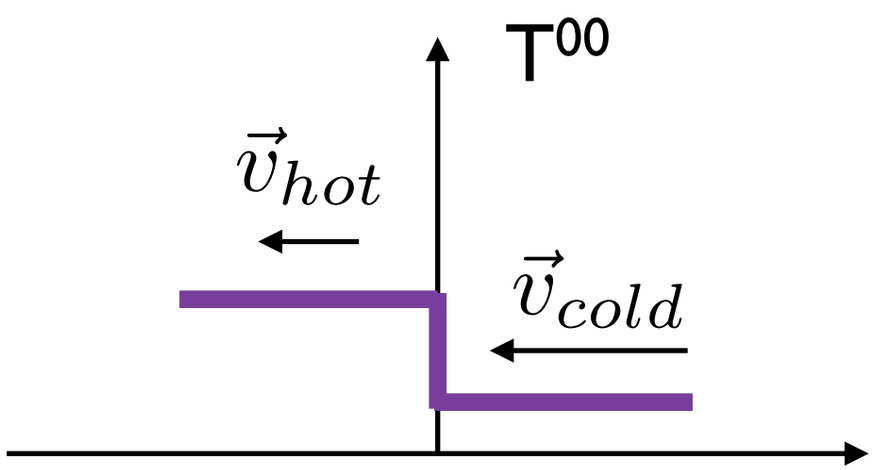

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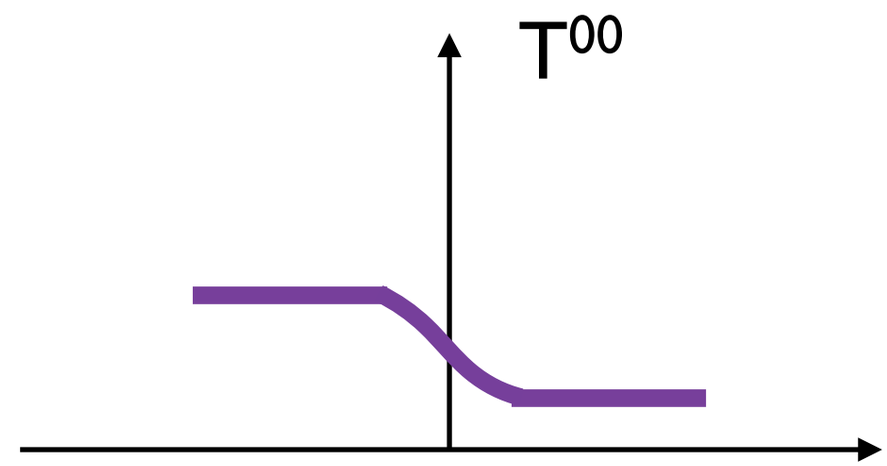

So now, we have 4 possibilities:

d>2

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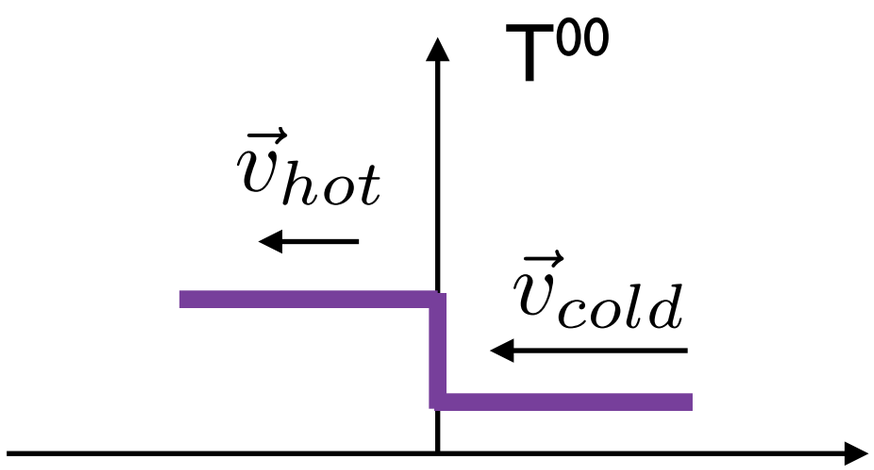

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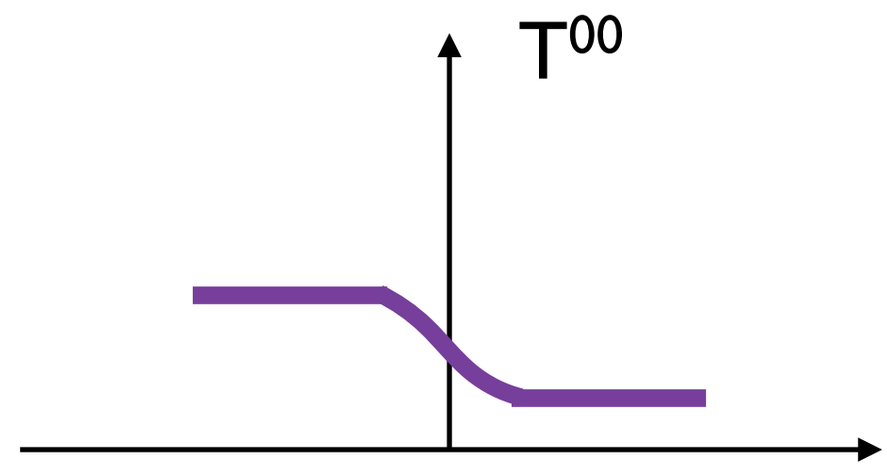

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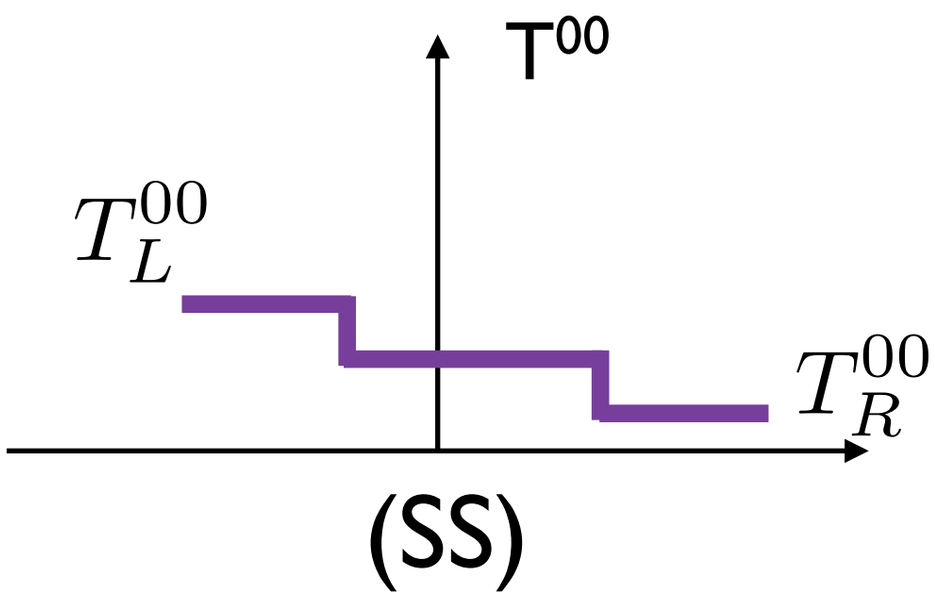

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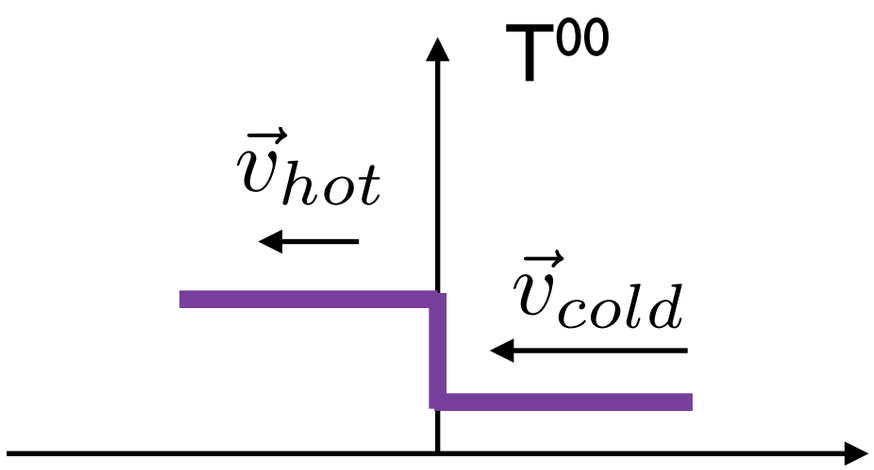
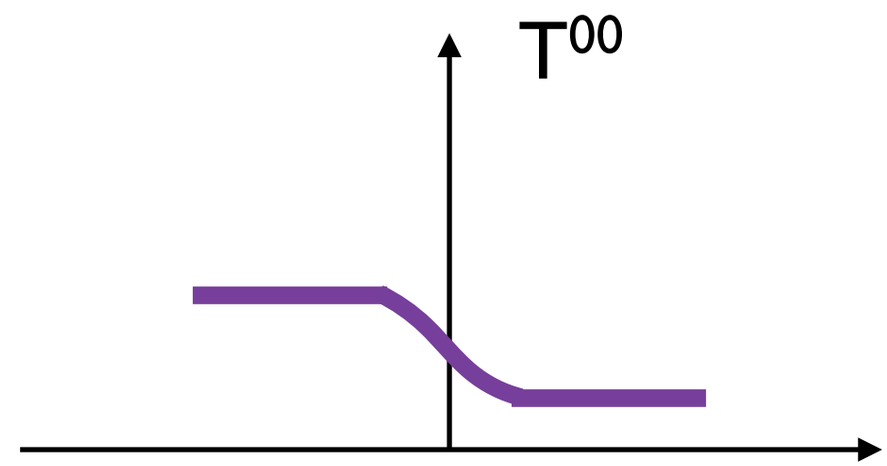


Rarefaction waves



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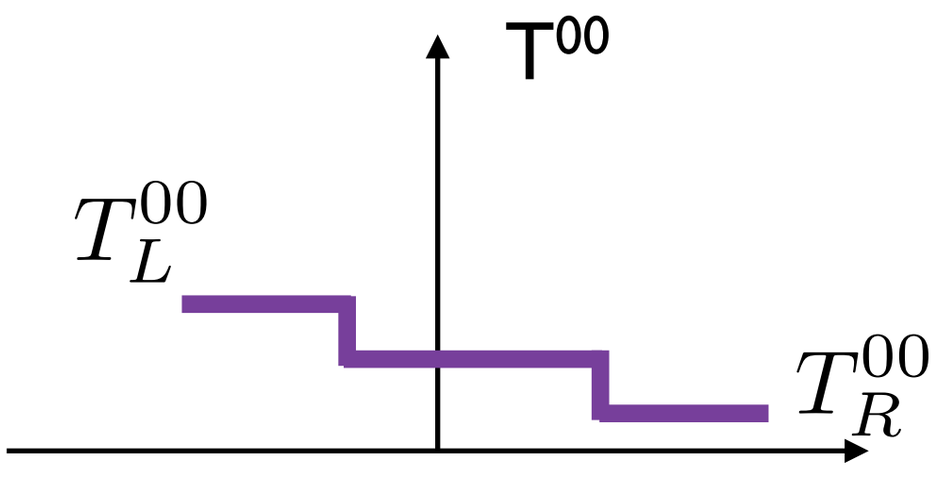
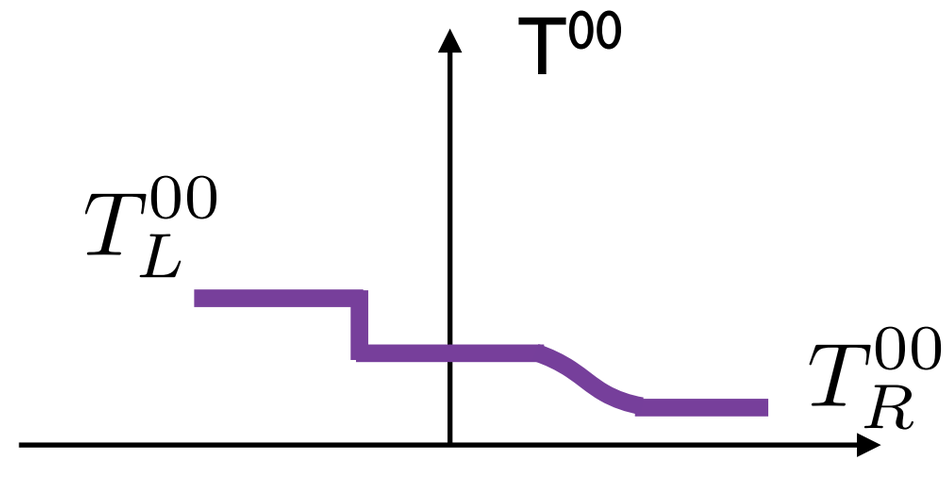


(SS)



(SR)

d>2

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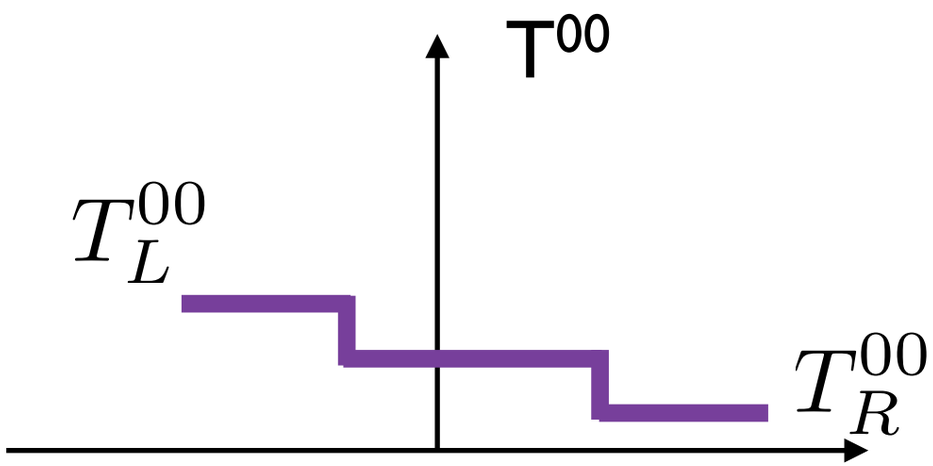
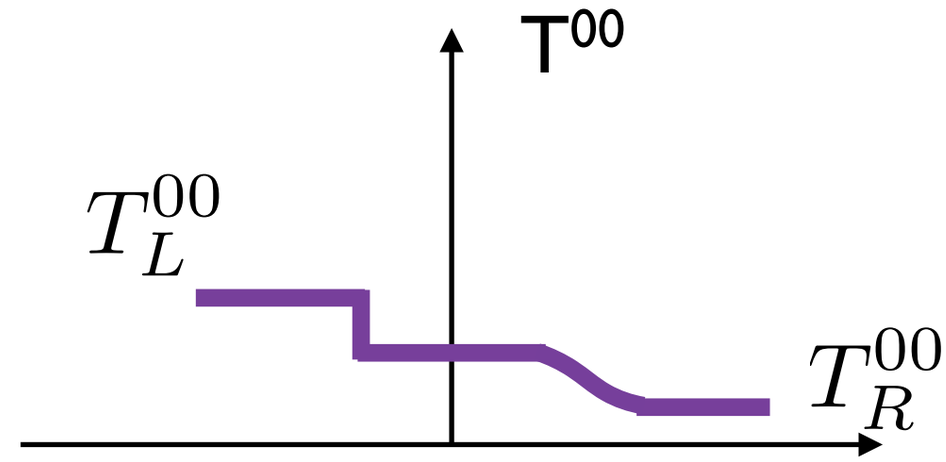
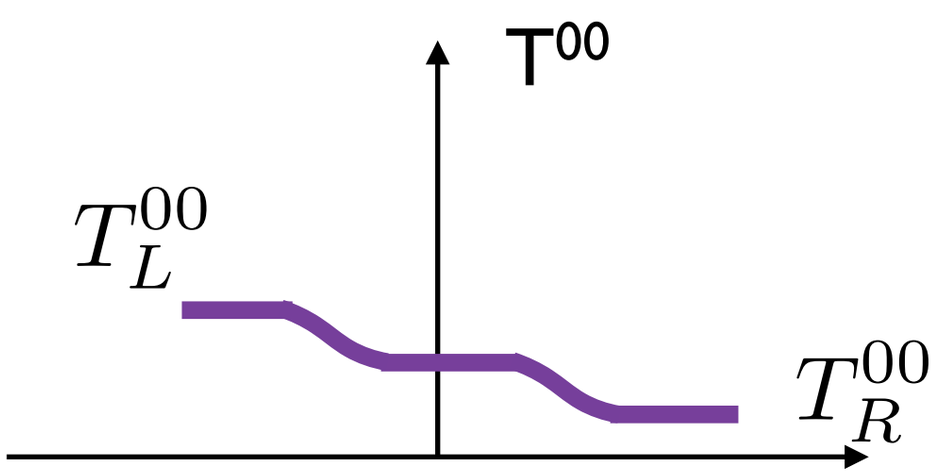


(SS)



(SR)



(RR)

d>2

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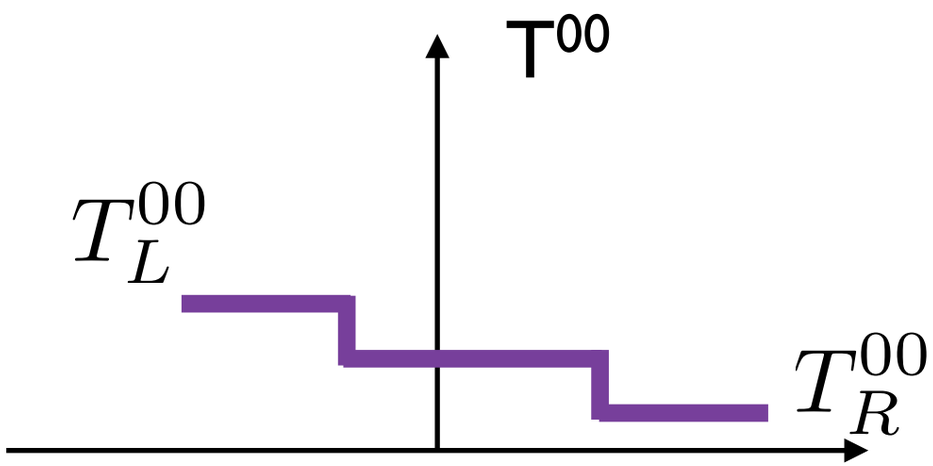

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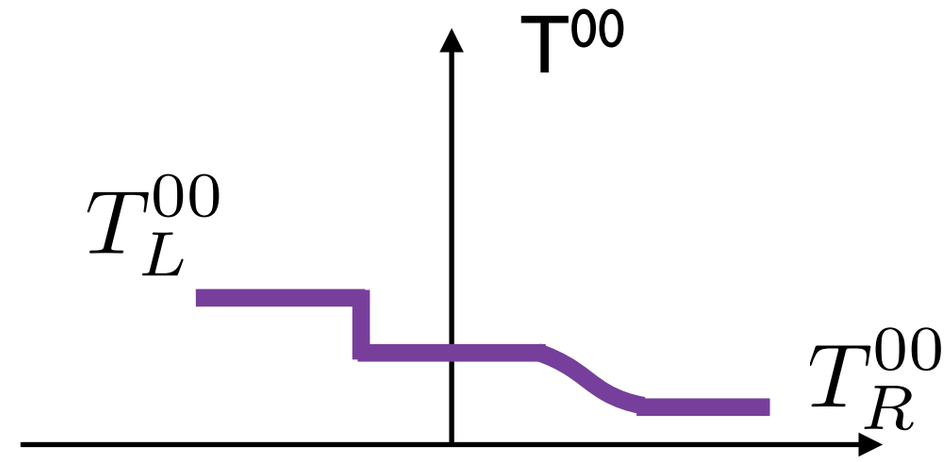

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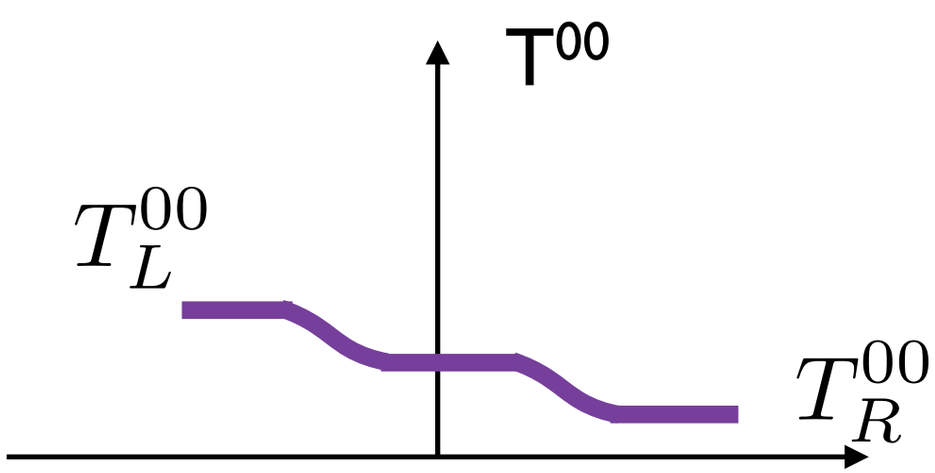

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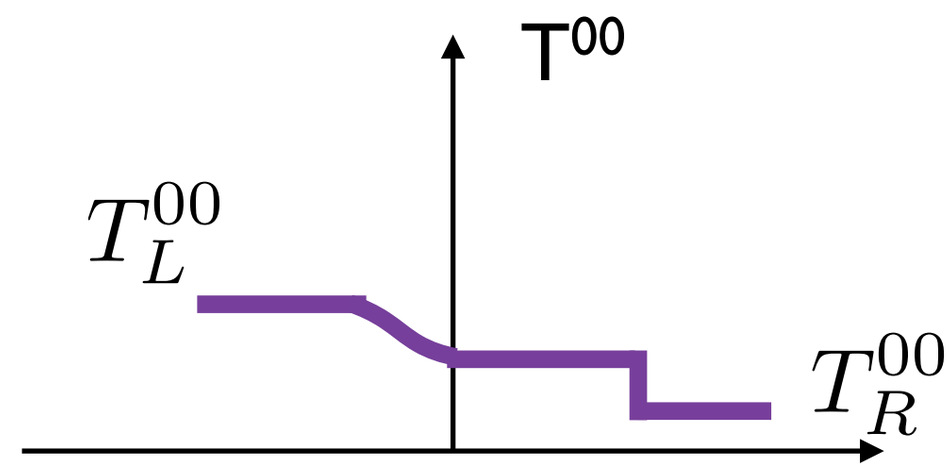
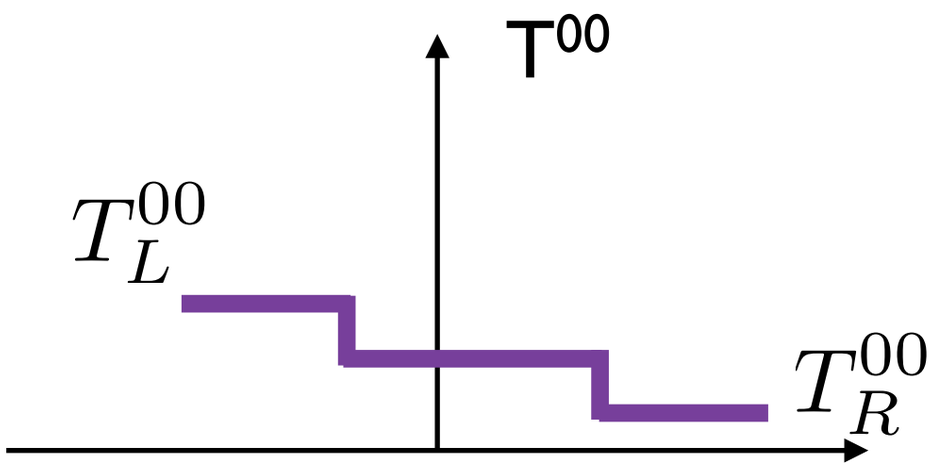


(RS)

d>2



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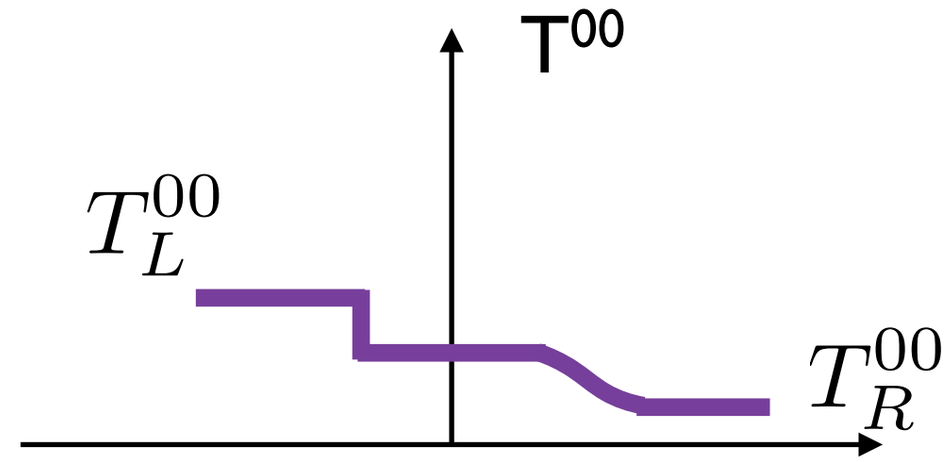

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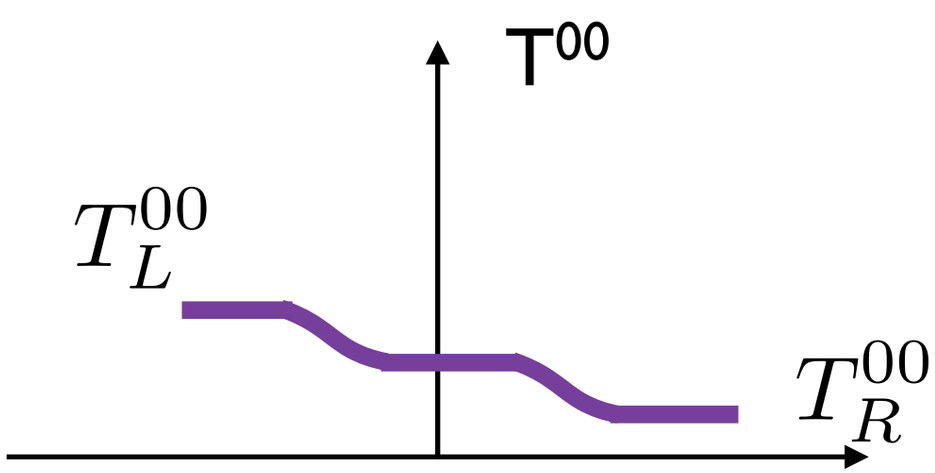

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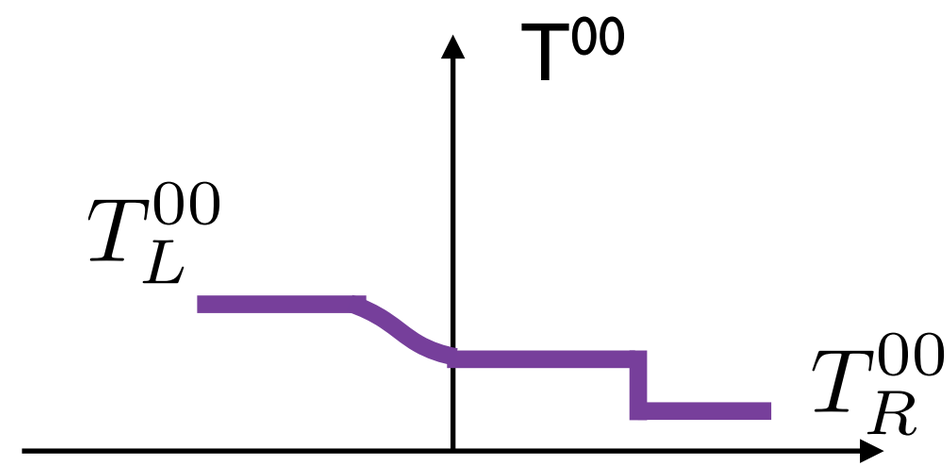

(RS)

d>2

Energy momentum conservation and conformal invariance imply:

$$\partial_\mu T^{\mu\nu} = 0\,, \quad T^\mu{}_\mu = 0$$

Let us assume, in addition, that the system is described by a perfect inviscid fluid:

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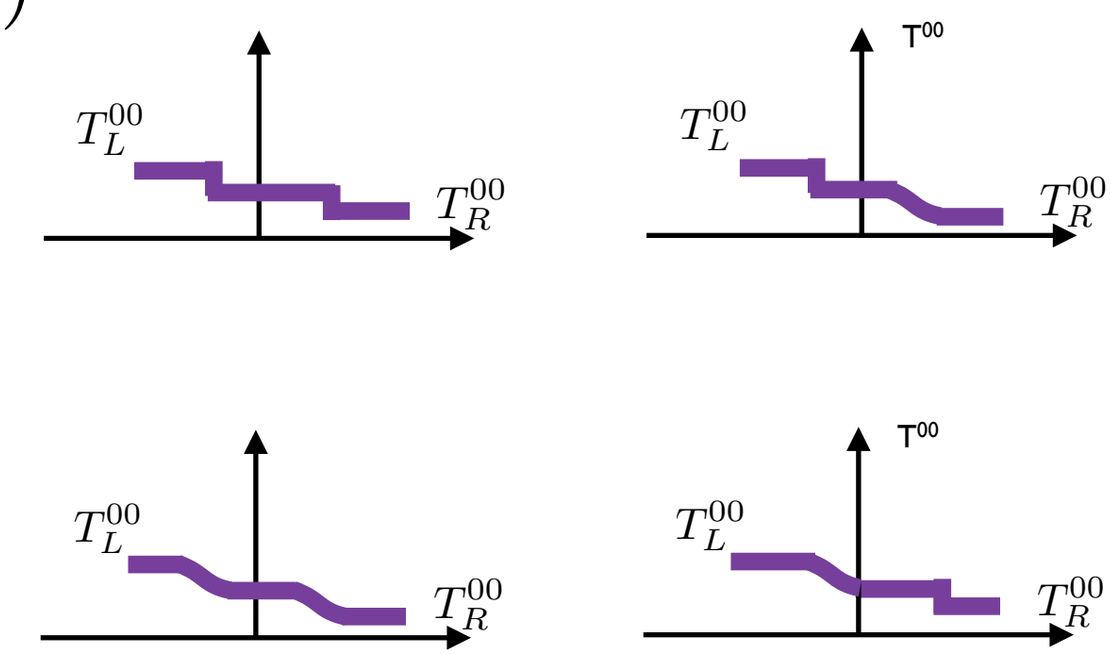

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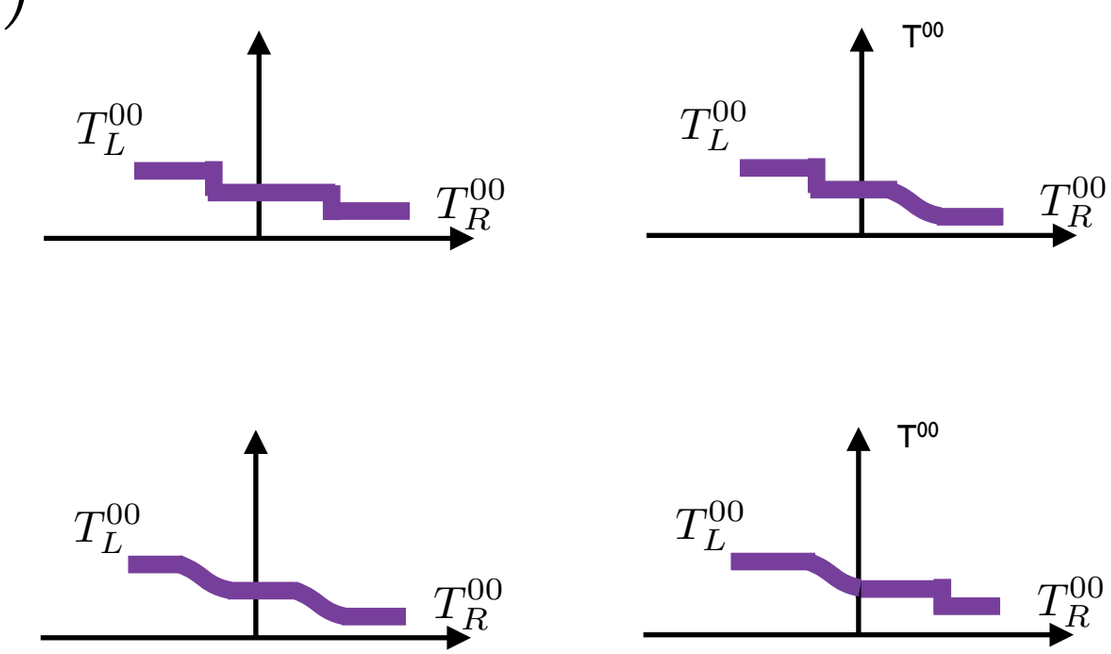

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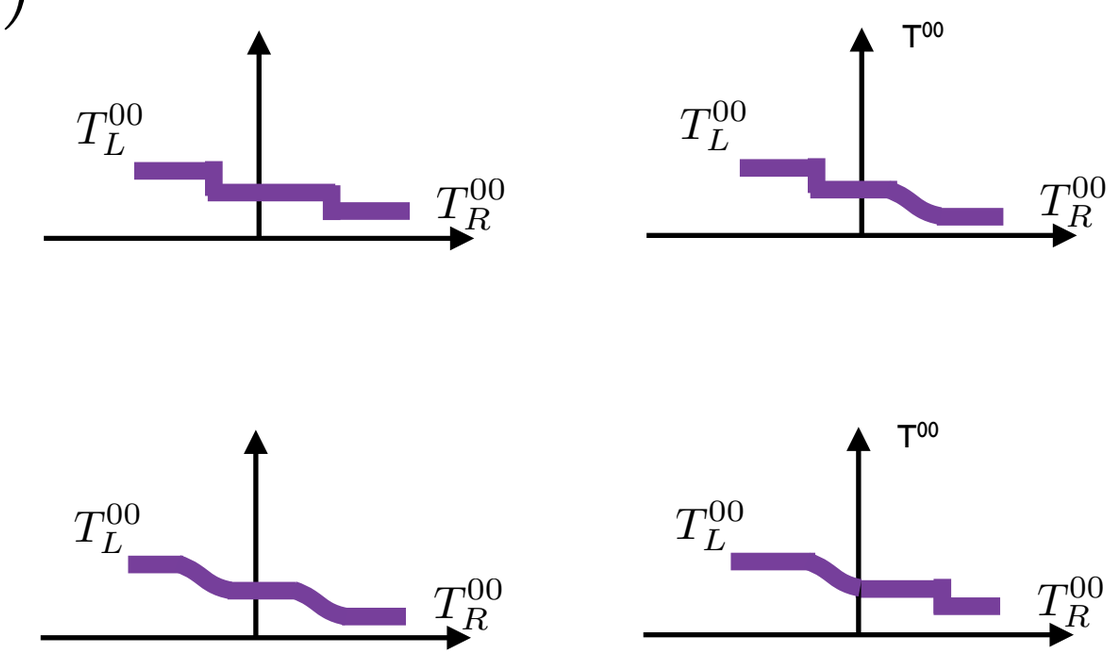

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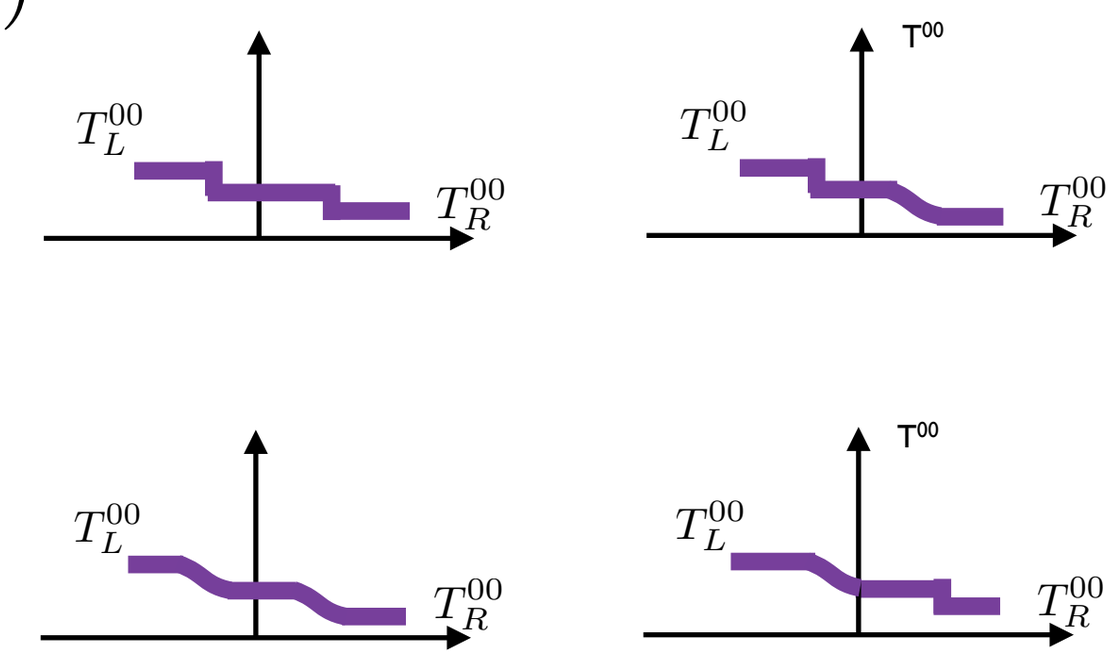

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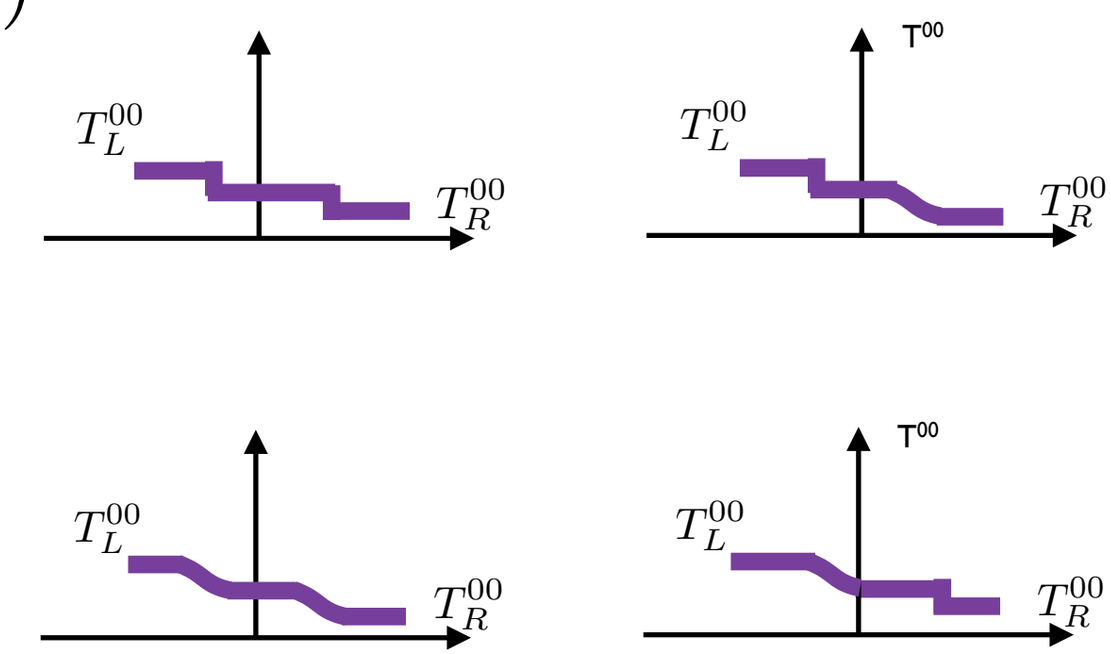
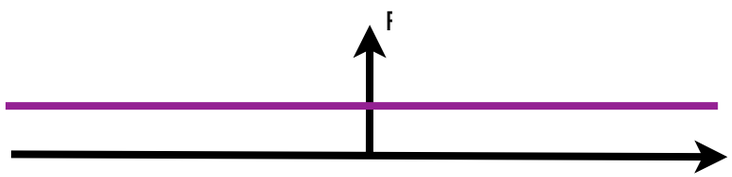


Holography

Holography

Let us start by considering an equilibrated configuration



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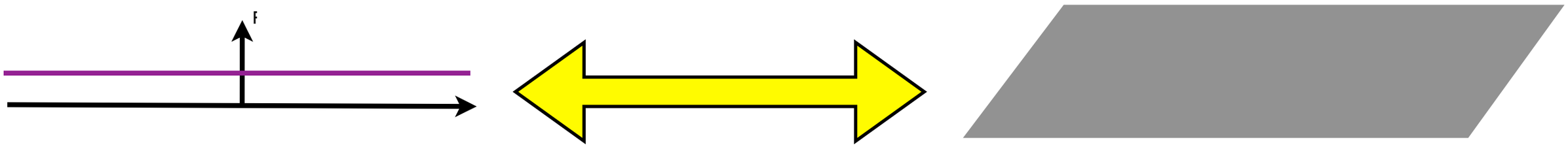

A planar event horizon:

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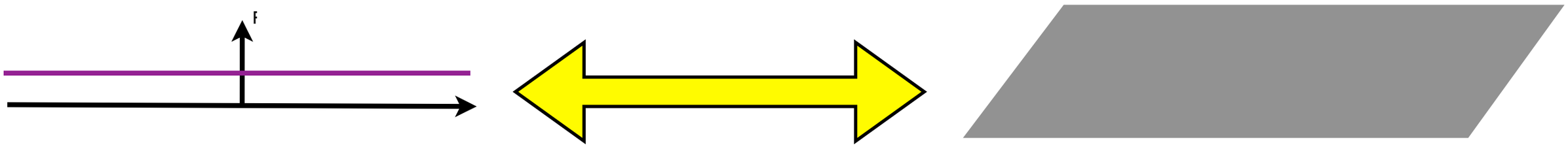

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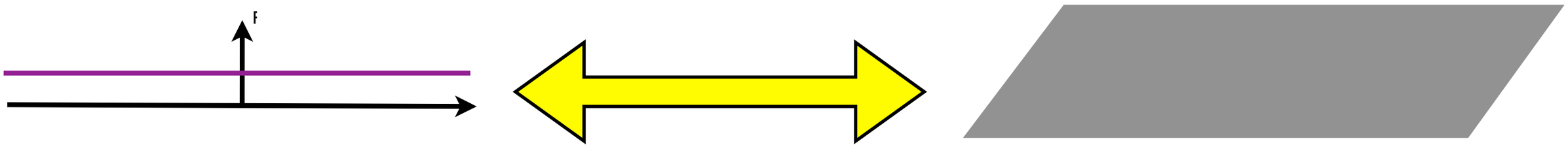

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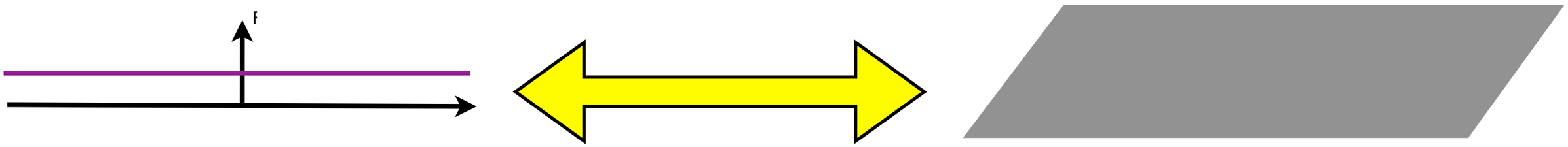

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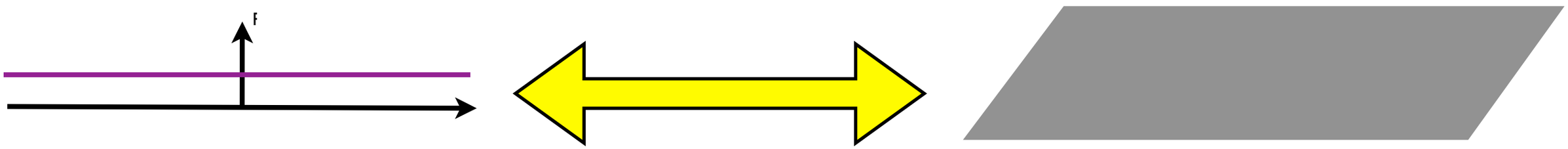

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e.g., in ABJM

$$p_0 = \frac{2N^2}{9\sqrt{2\lambda}} \qquad \lambda = \frac{N}{k}$$

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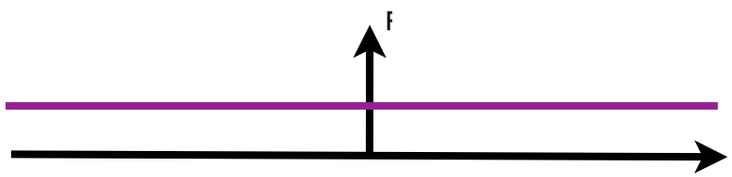

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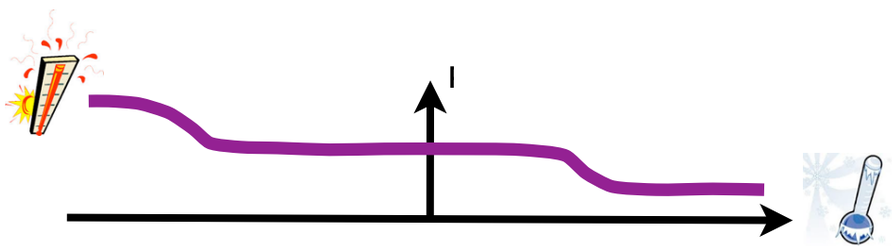

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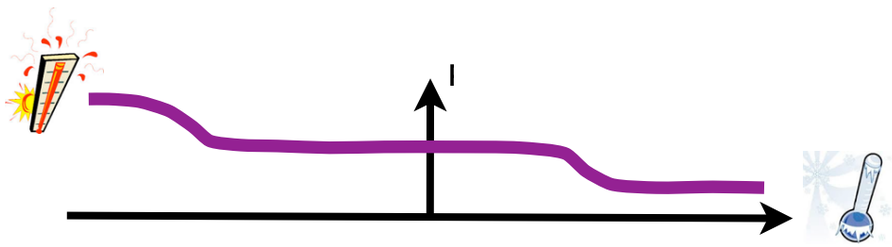

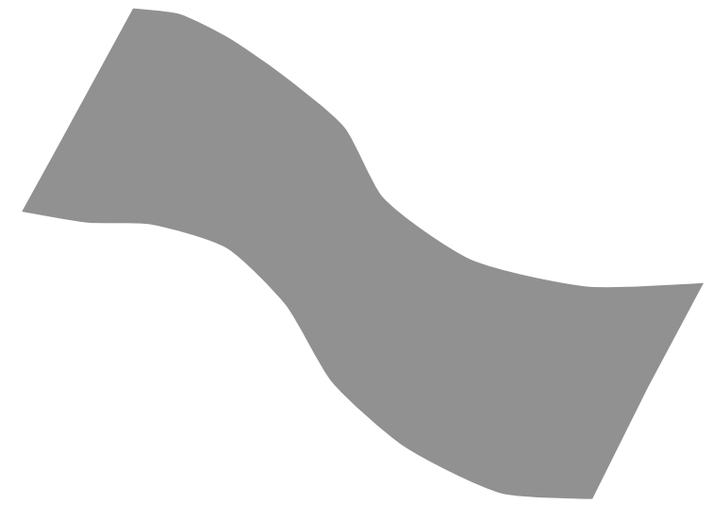

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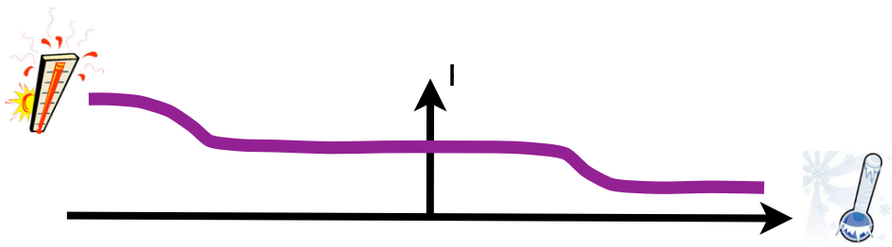

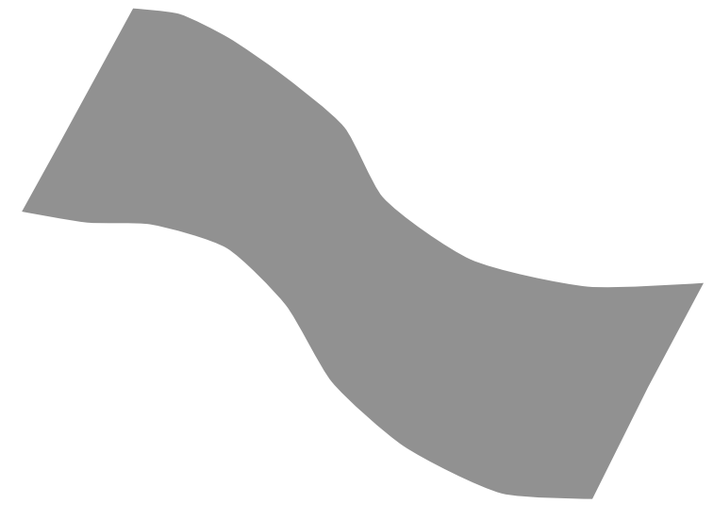

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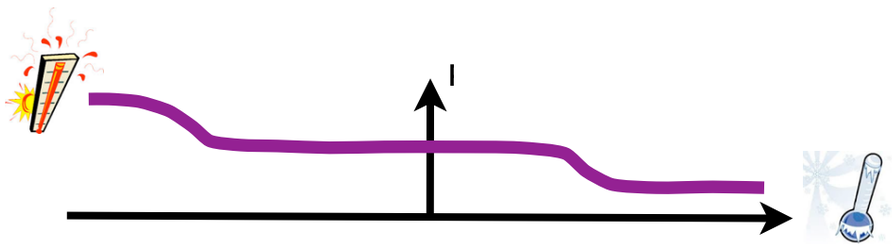

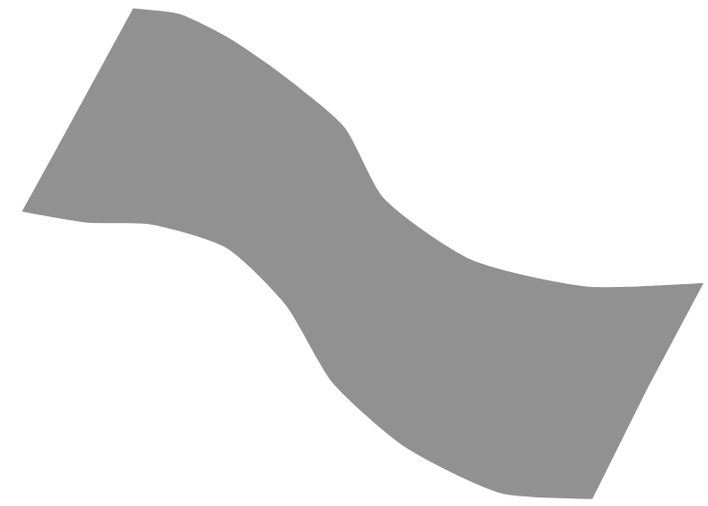

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the Einstein equations reduce to a set of nested linear differential equations in the radial coordinate 'r'. We have solved these equations numerically.

(Chesler, Yaffe, 2012)

Holography

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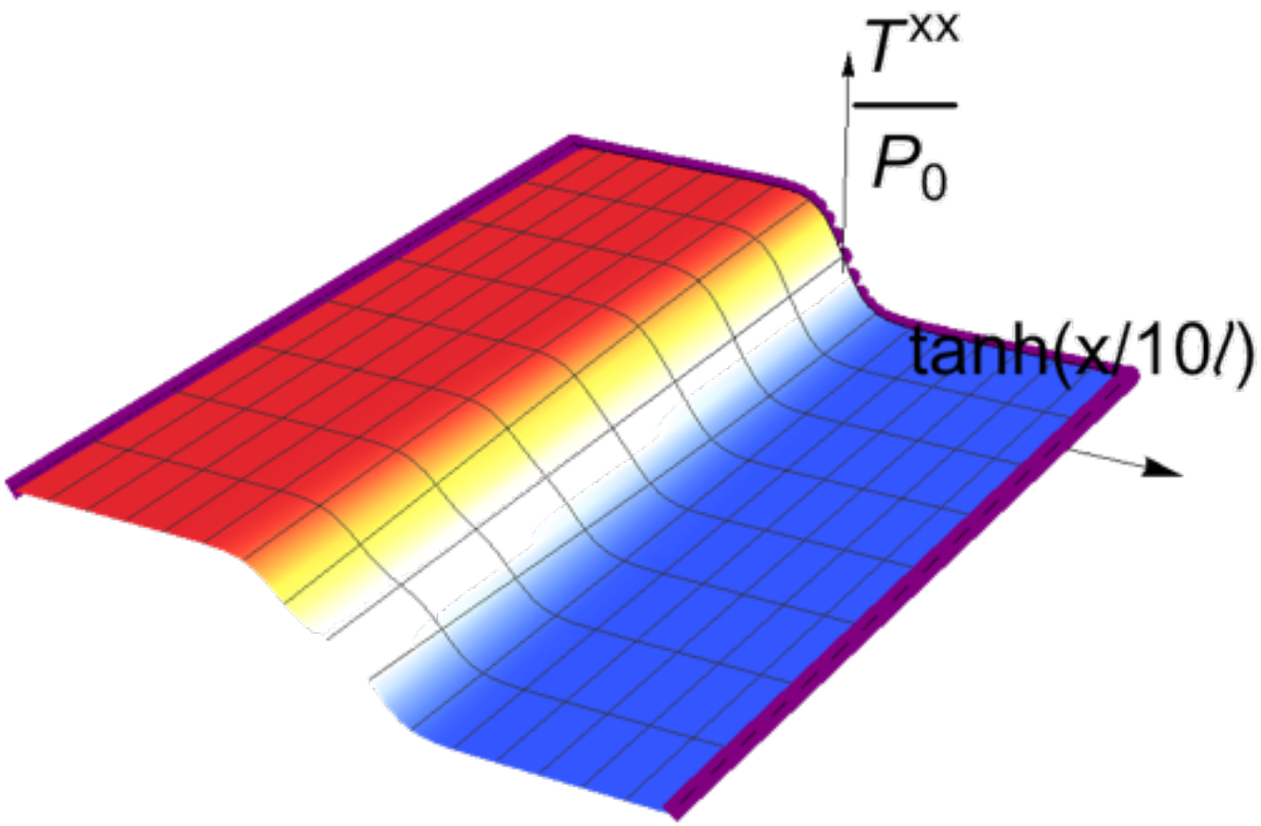

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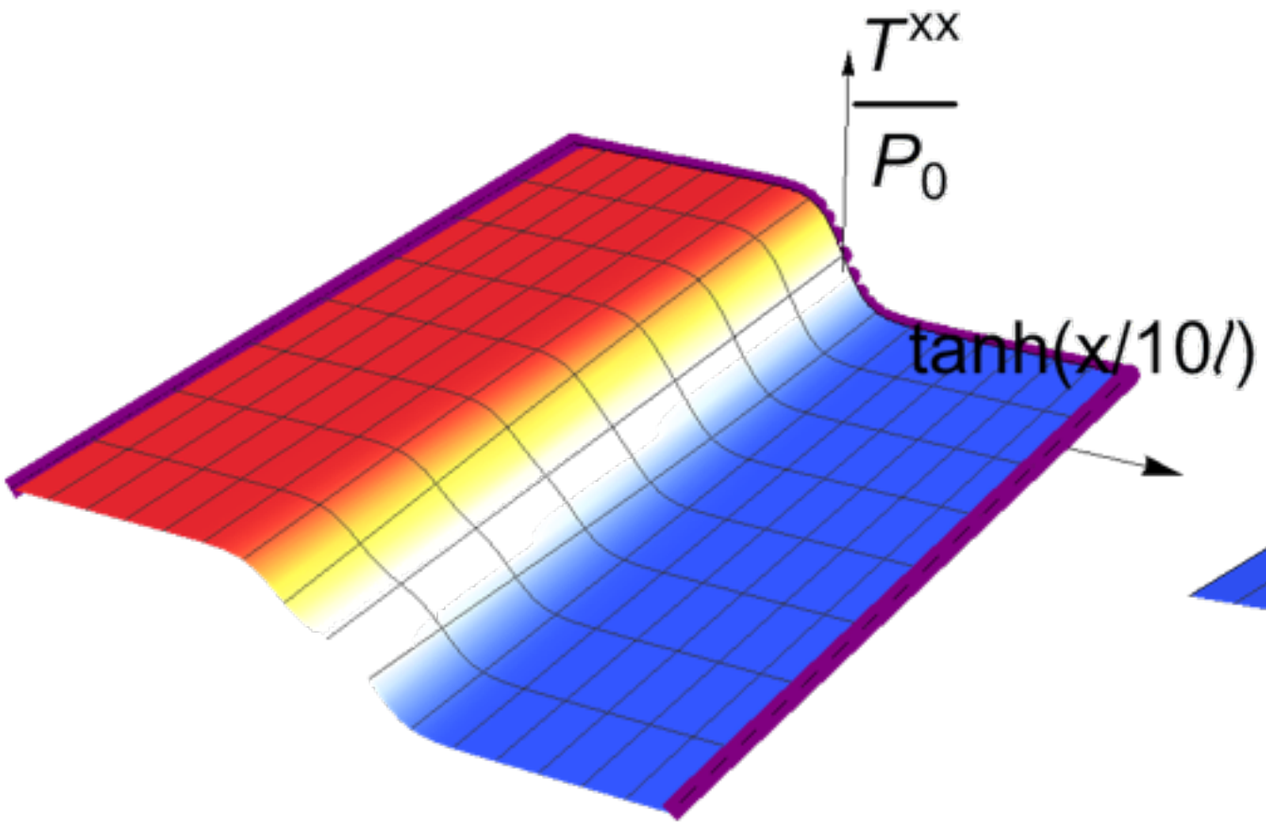

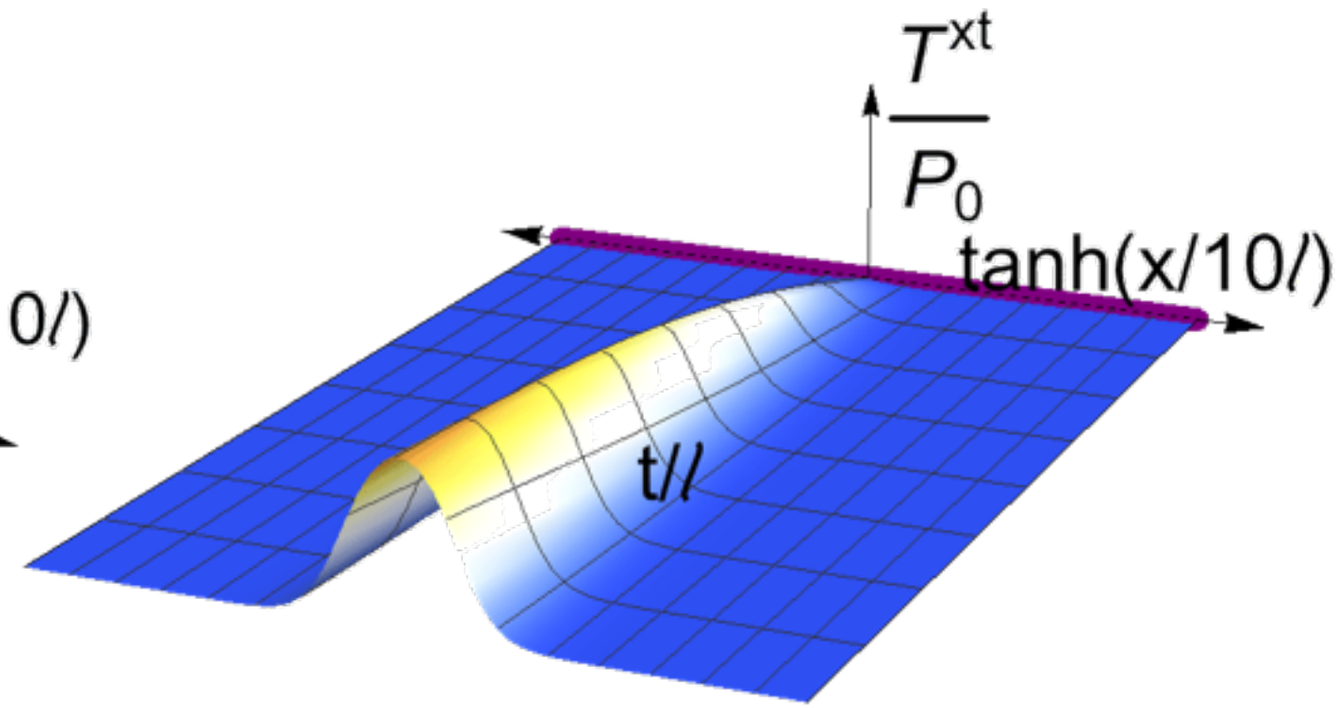

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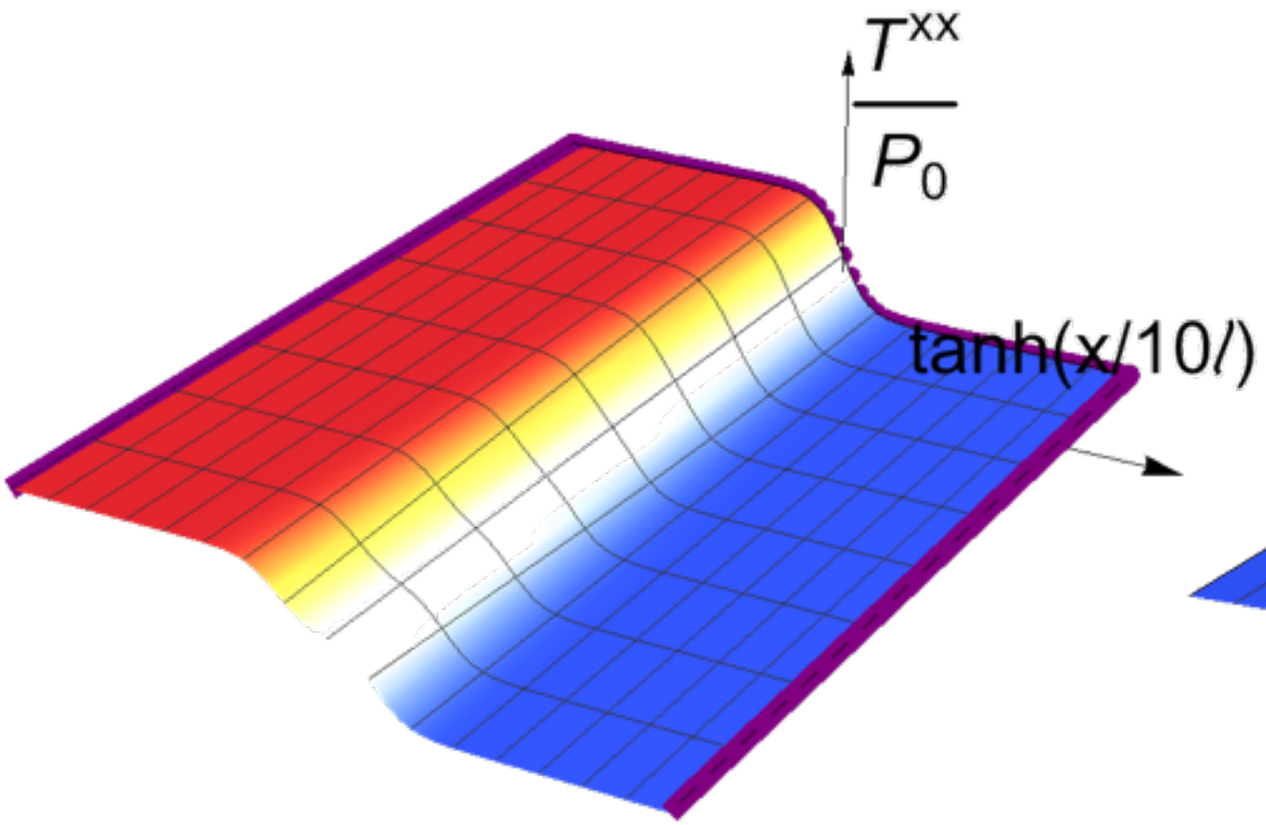

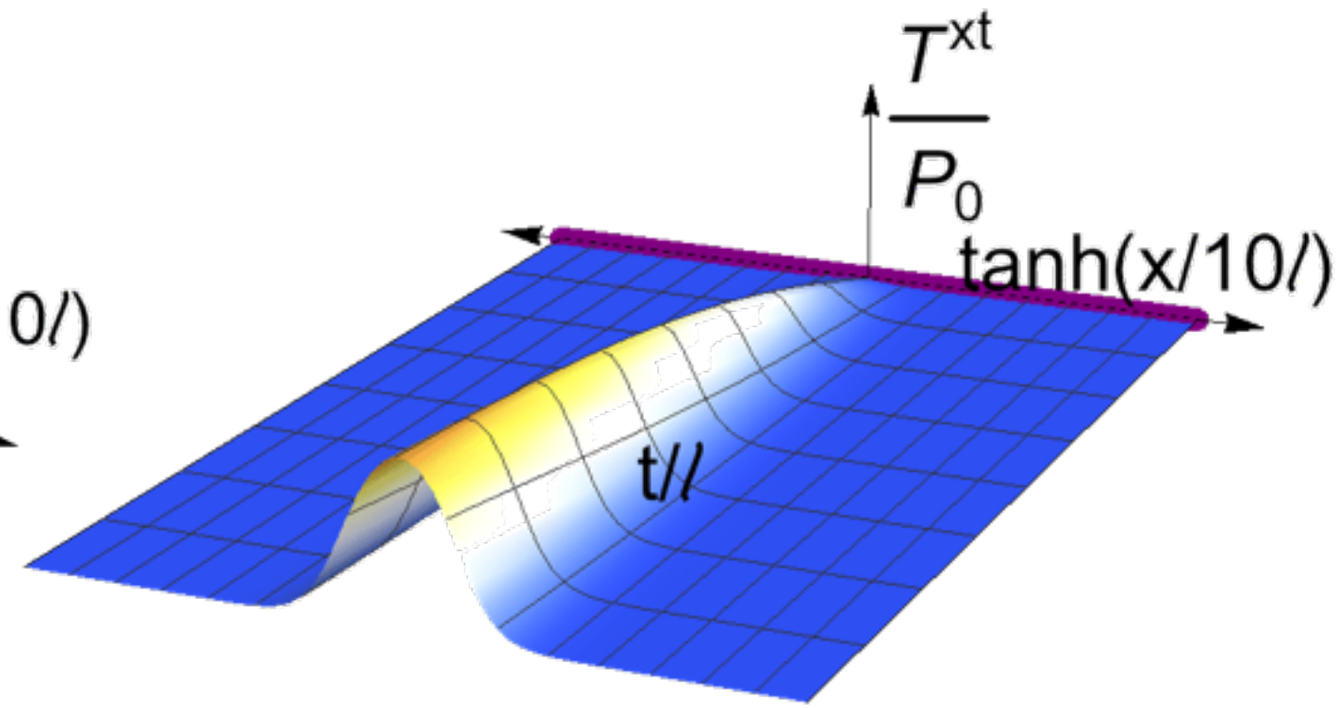

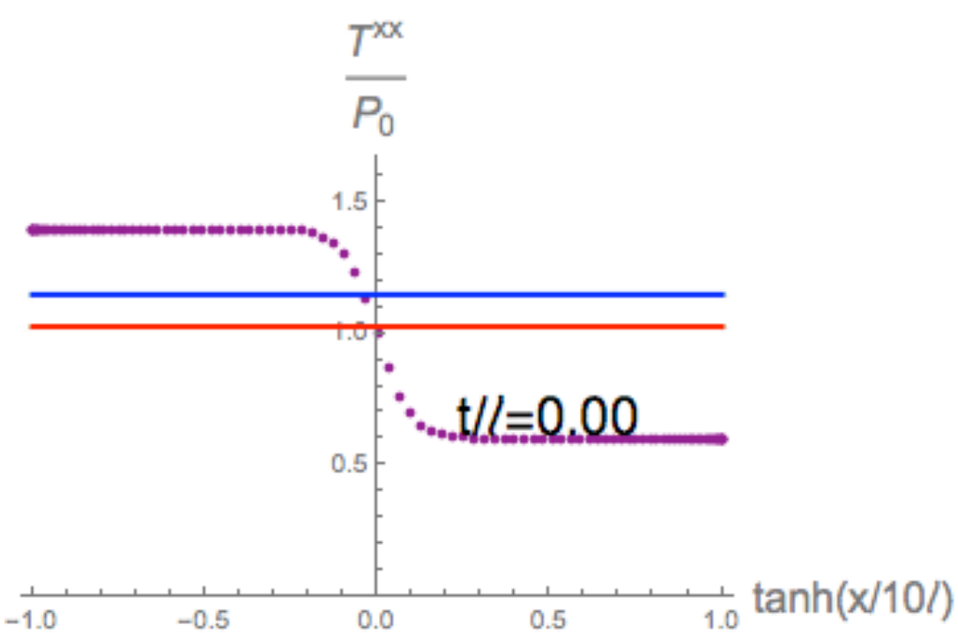

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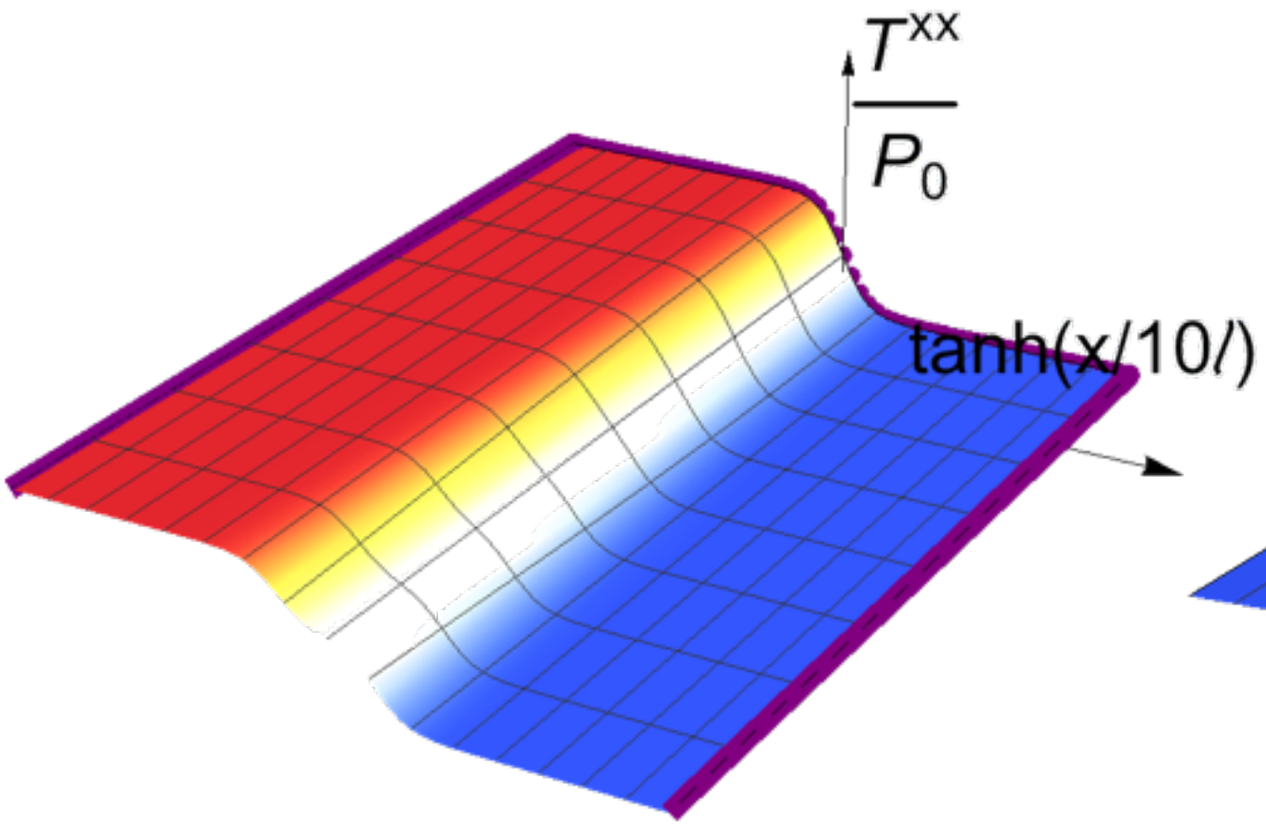

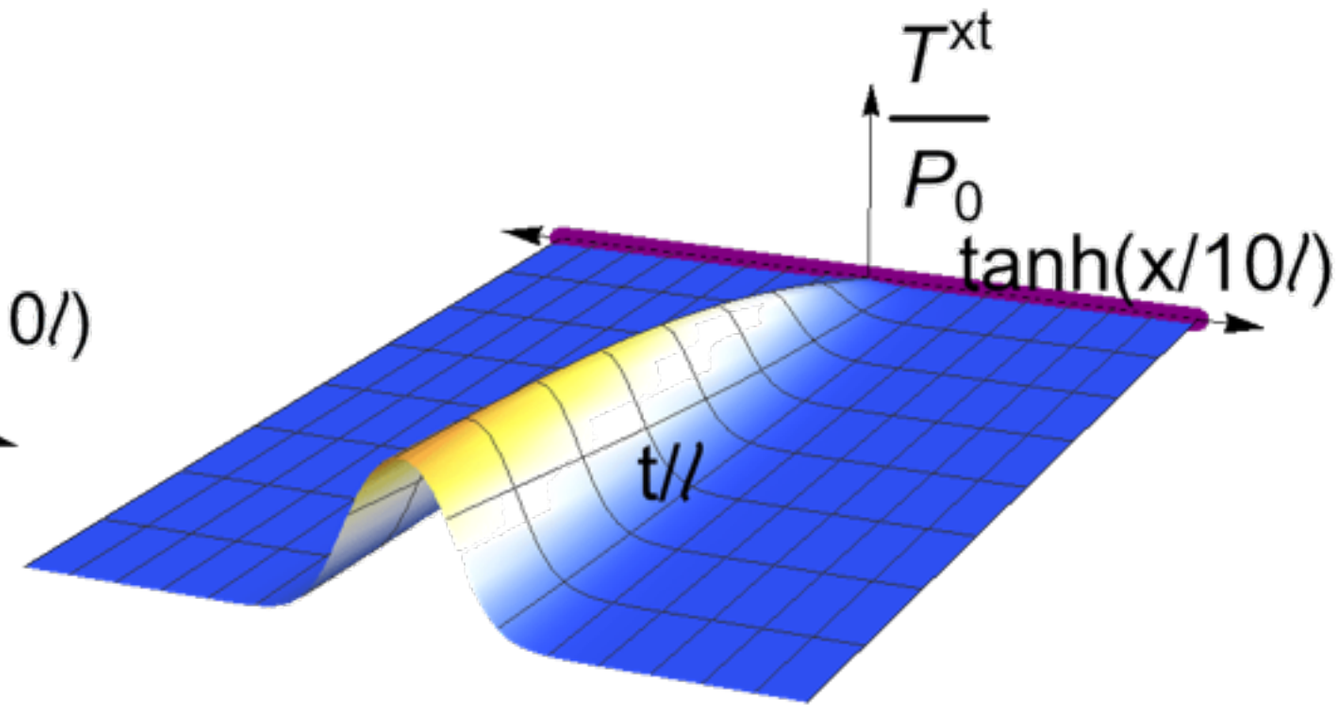

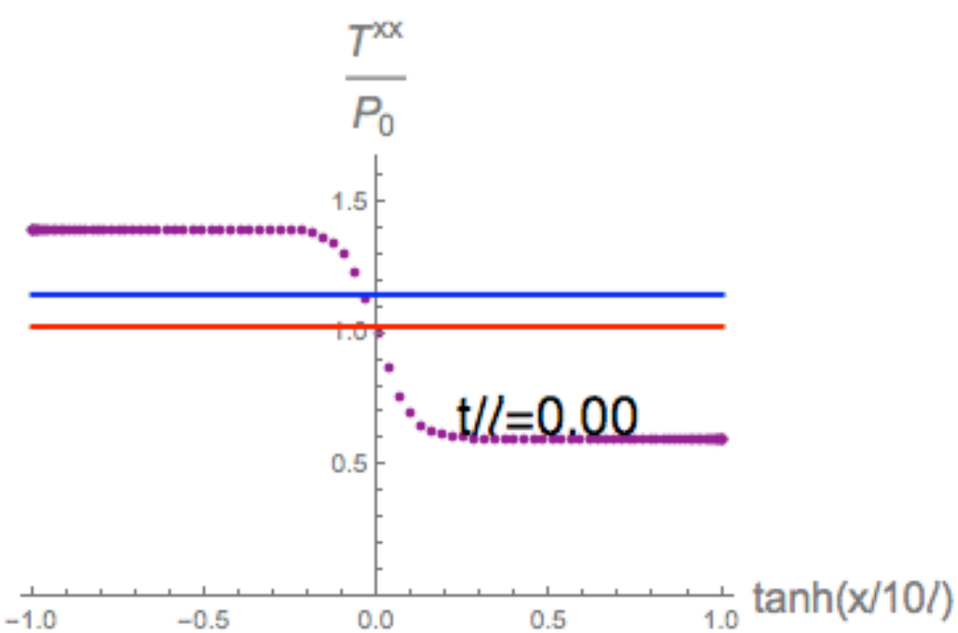

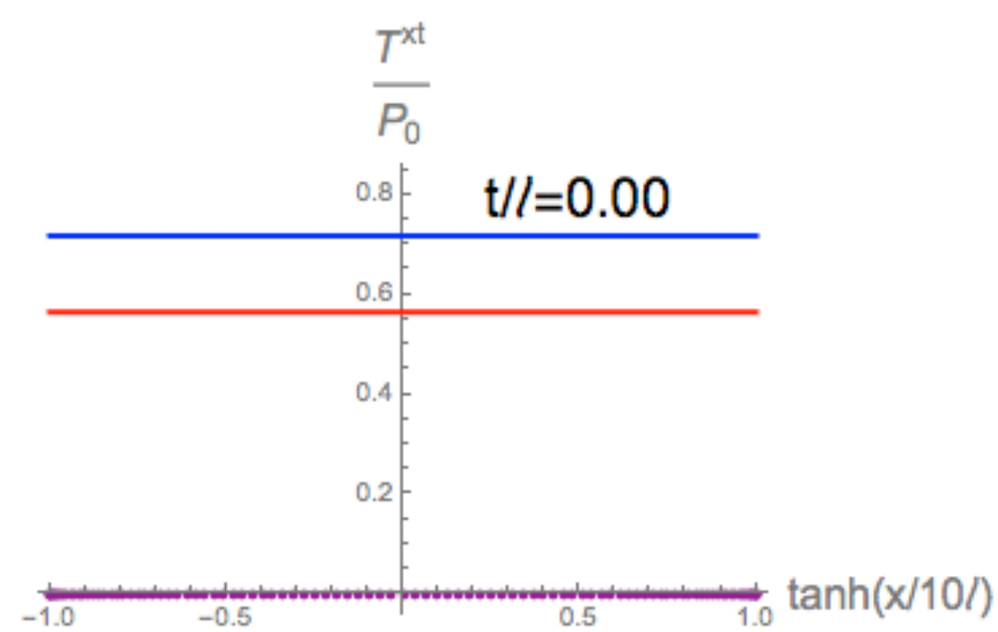

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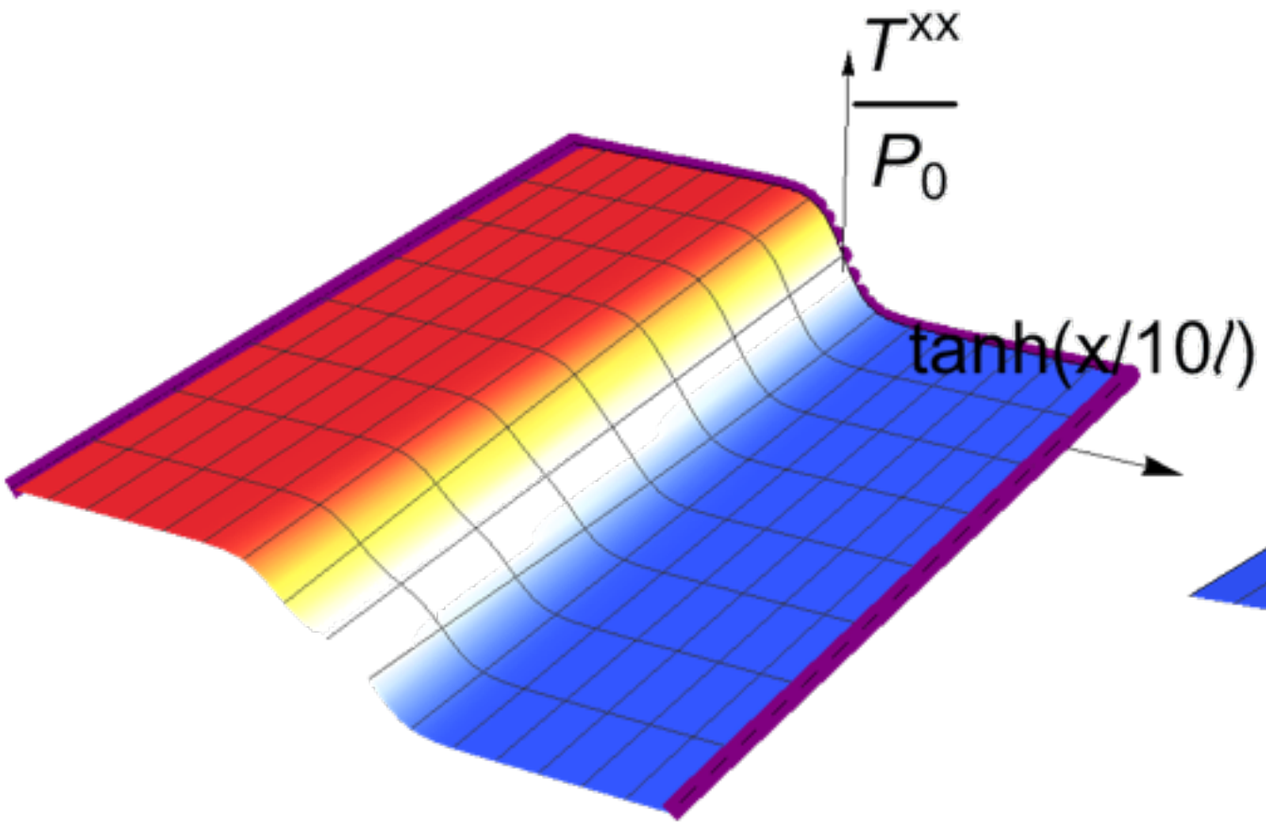
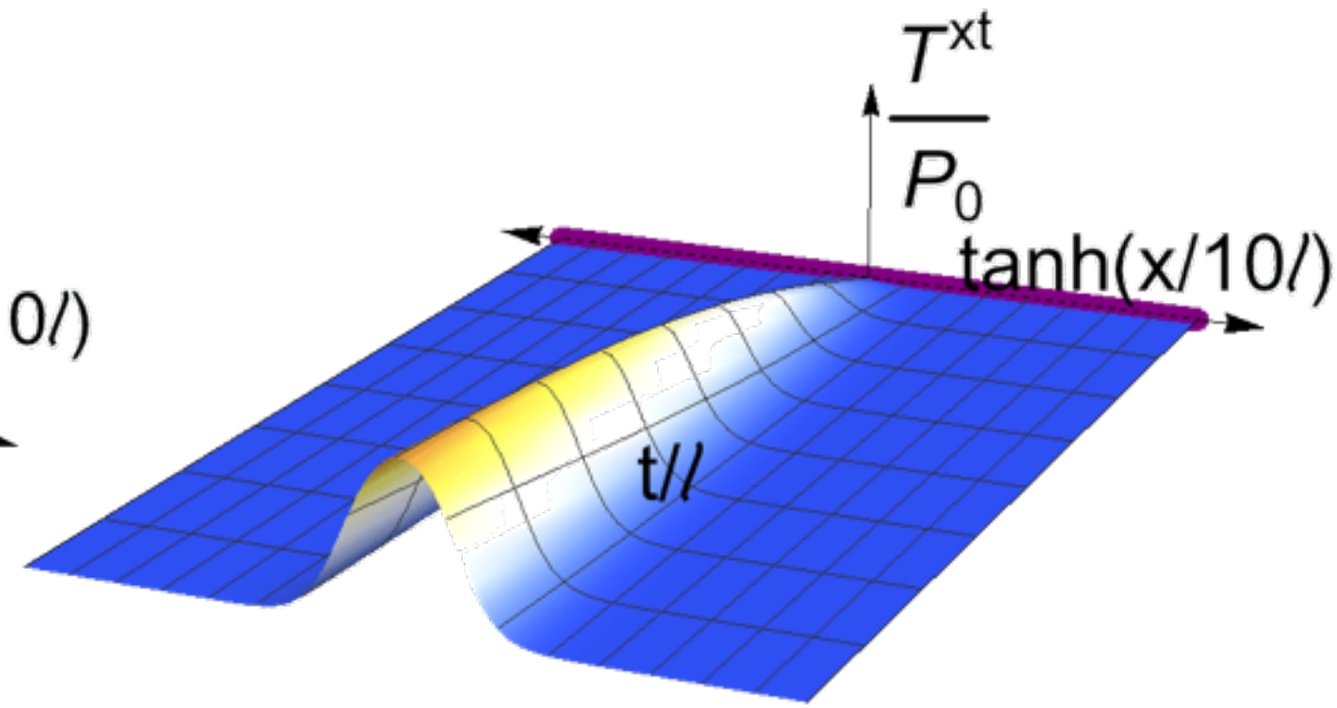
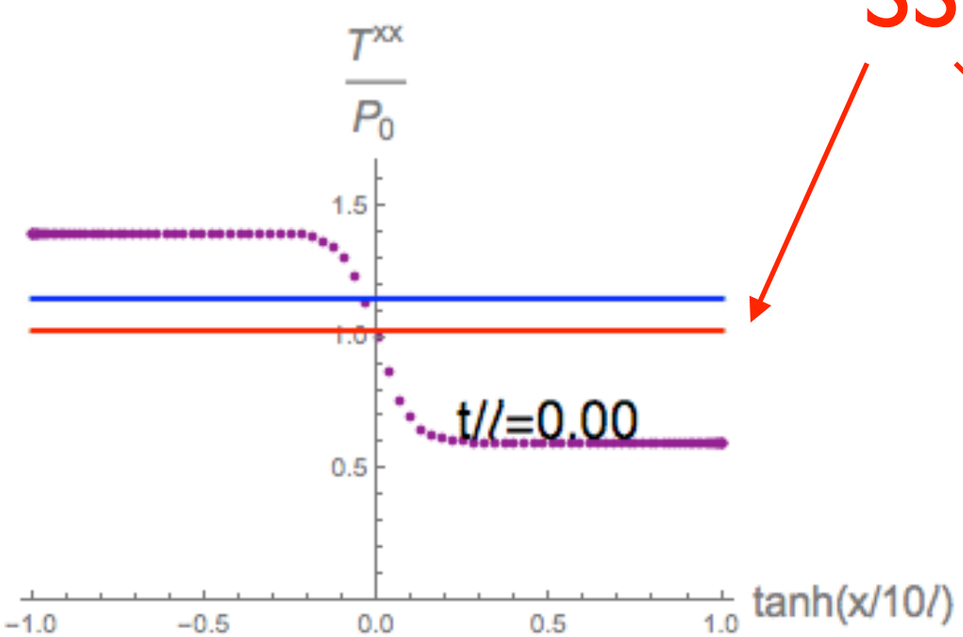
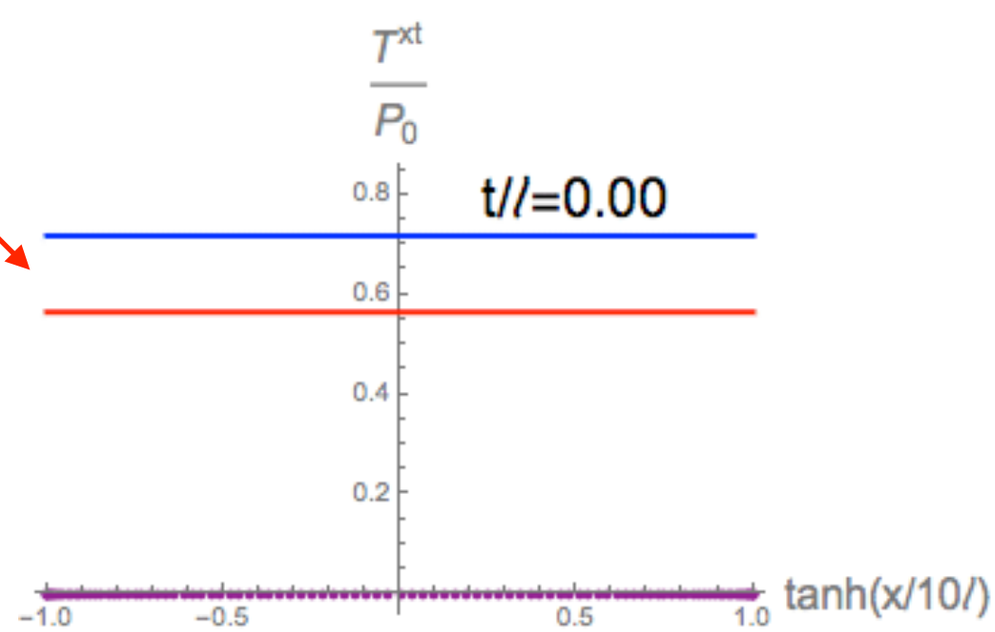




SS





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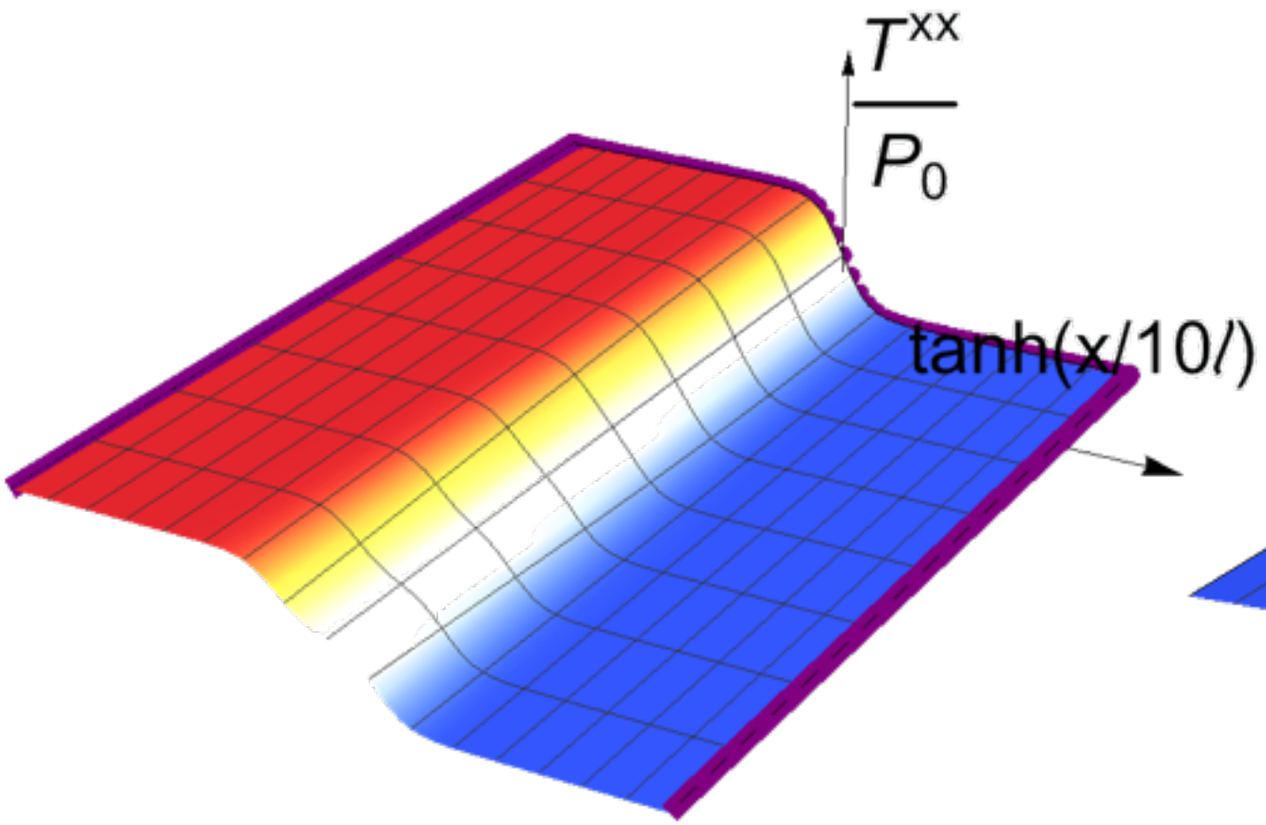
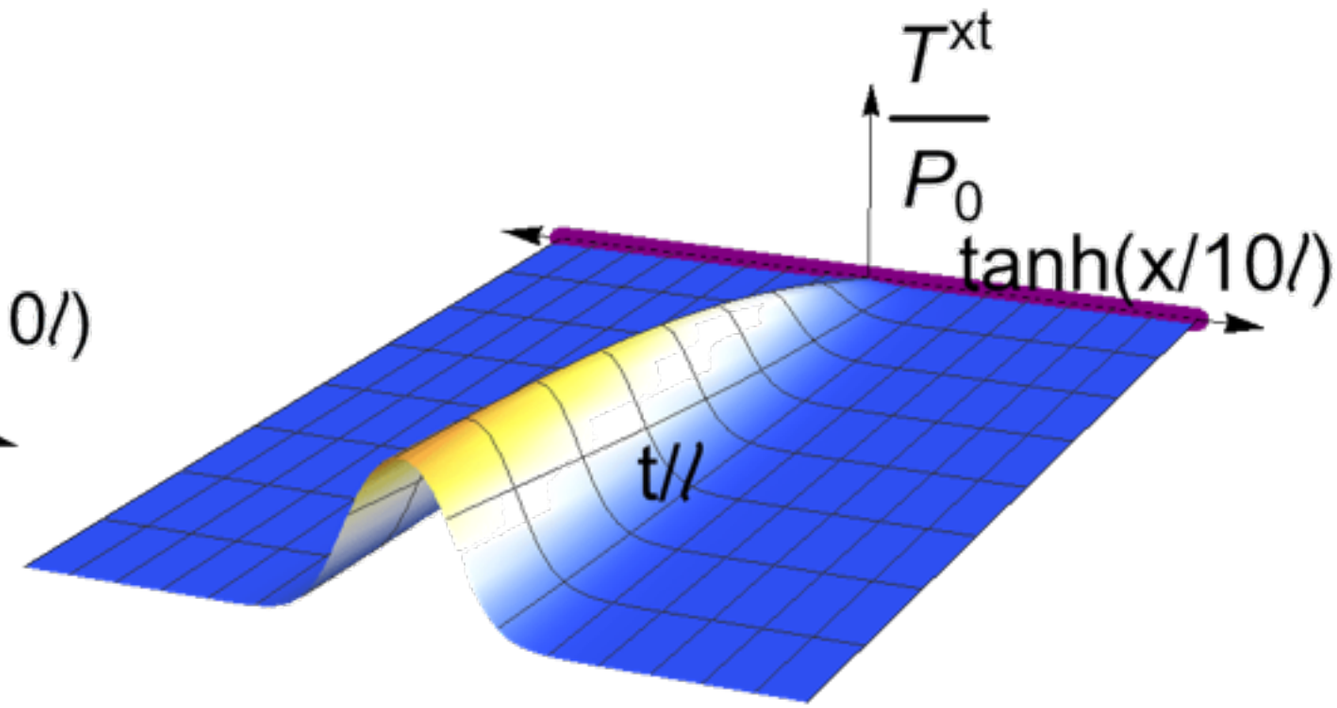
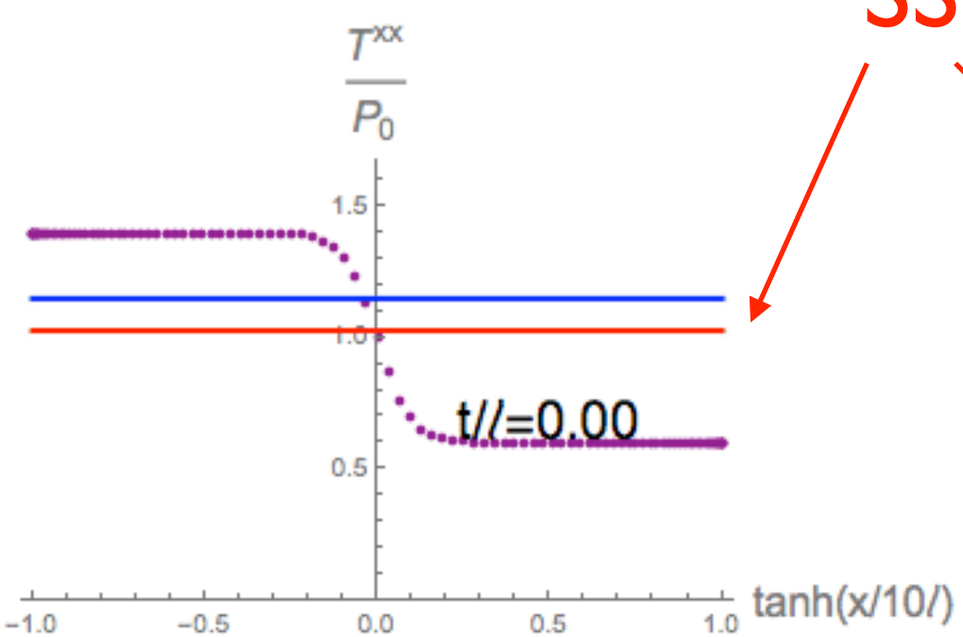
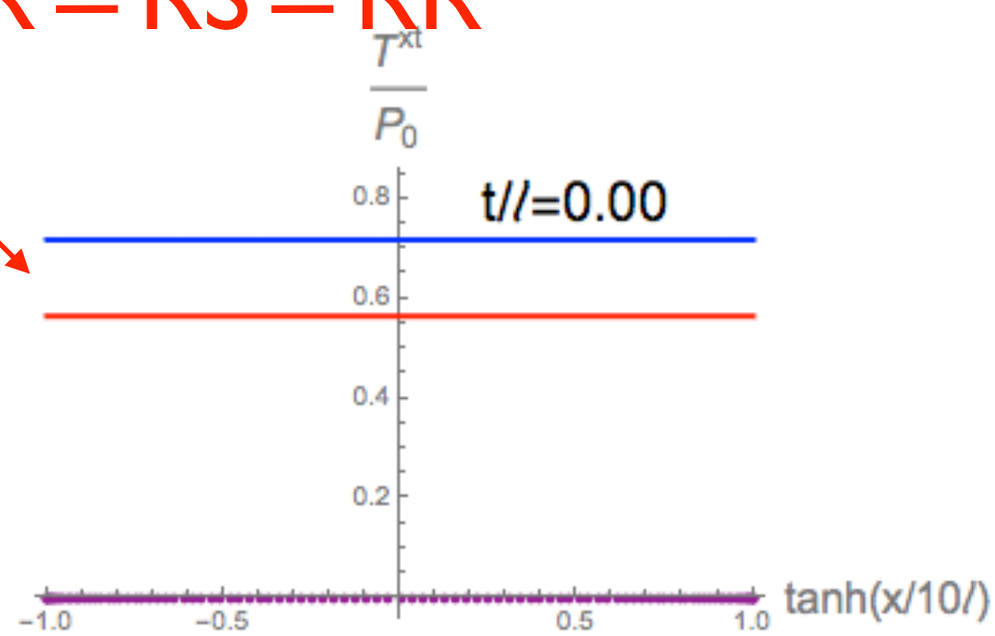




SS ≅ SR ≅ RS ≅ RR





Infinite d

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$$T^{\mu\nu} = \begin{pmatrix} e & j \\ j & e + \frac{j^2}{e} \end{pmatrix}$$

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Consider

$$e(\chi, t) = (e_r - e_l)\theta(x - st) + e_l$$

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then we have a one parameter family of solutions

$e_r(s) \quad j_r(s)$

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Consider

$$e(\chi, t) = (e_r - e_l)\theta(x - st) + e_l$$

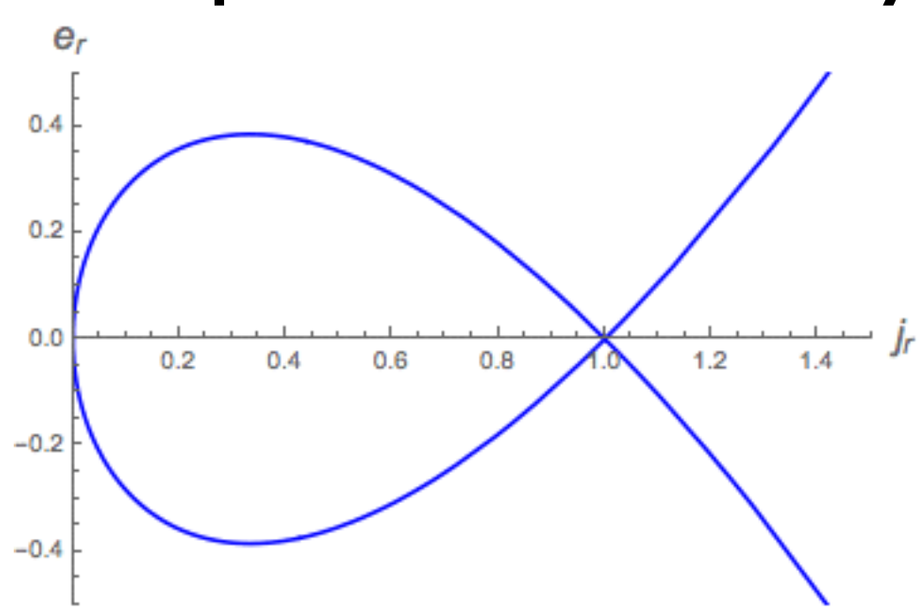
$$j(\chi, t) = (j_r - j_l)\theta(x - st) + j_l$$

If we choose a reference frame with $j_l = 0$

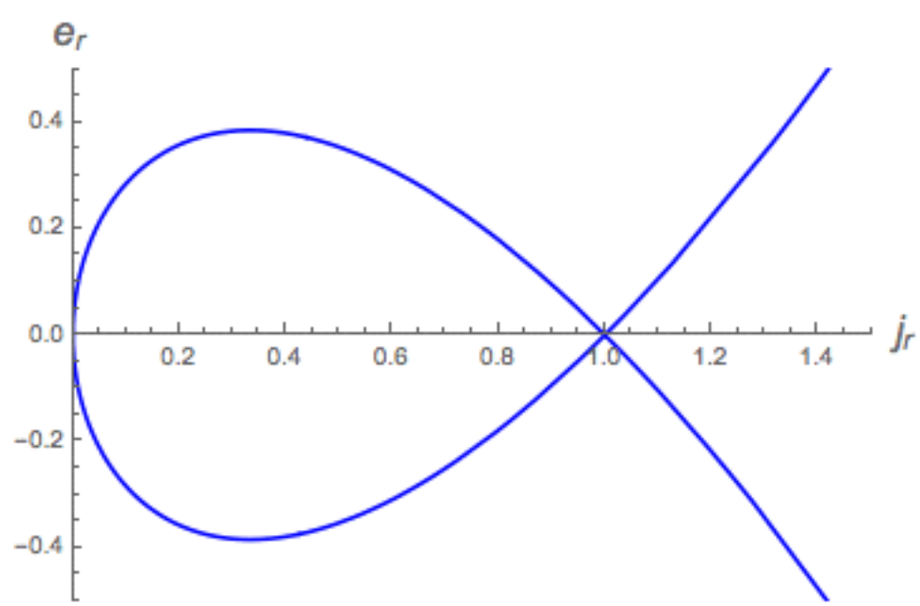
and scale the energy so that $e_l = 1$

then we have a one parameter family of solutions

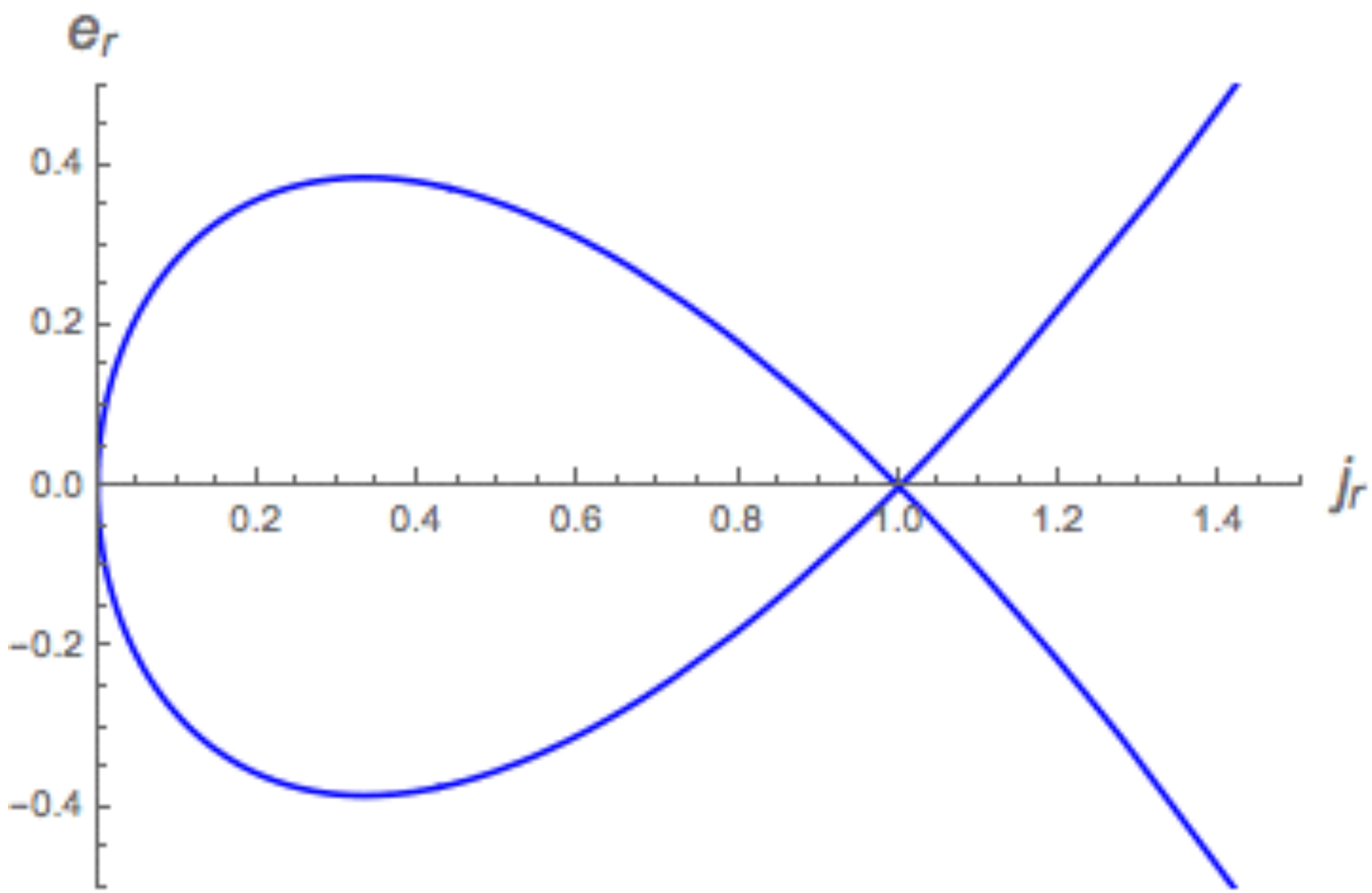
$e_r(s) \quad j_r(s)$



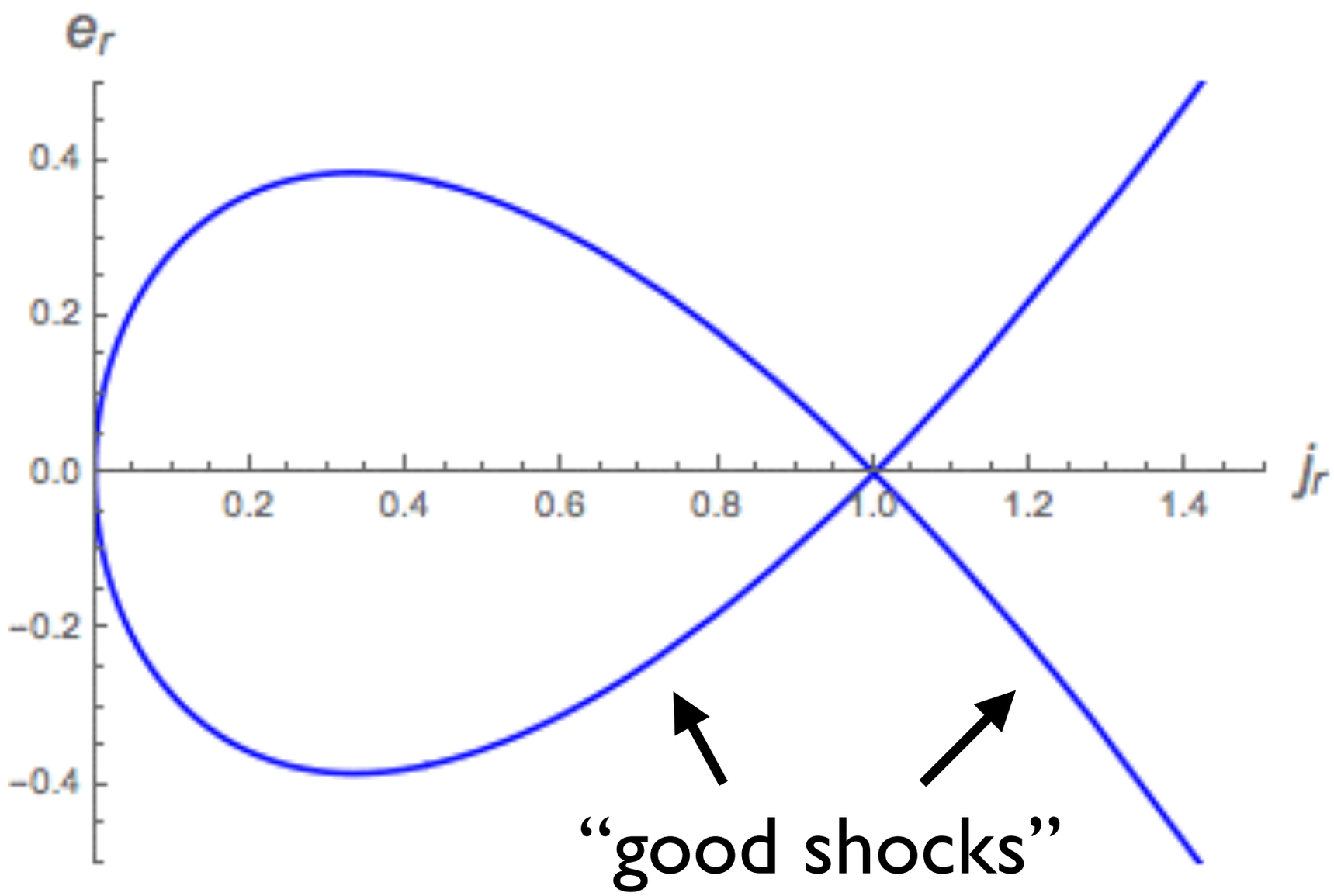
Infinite d



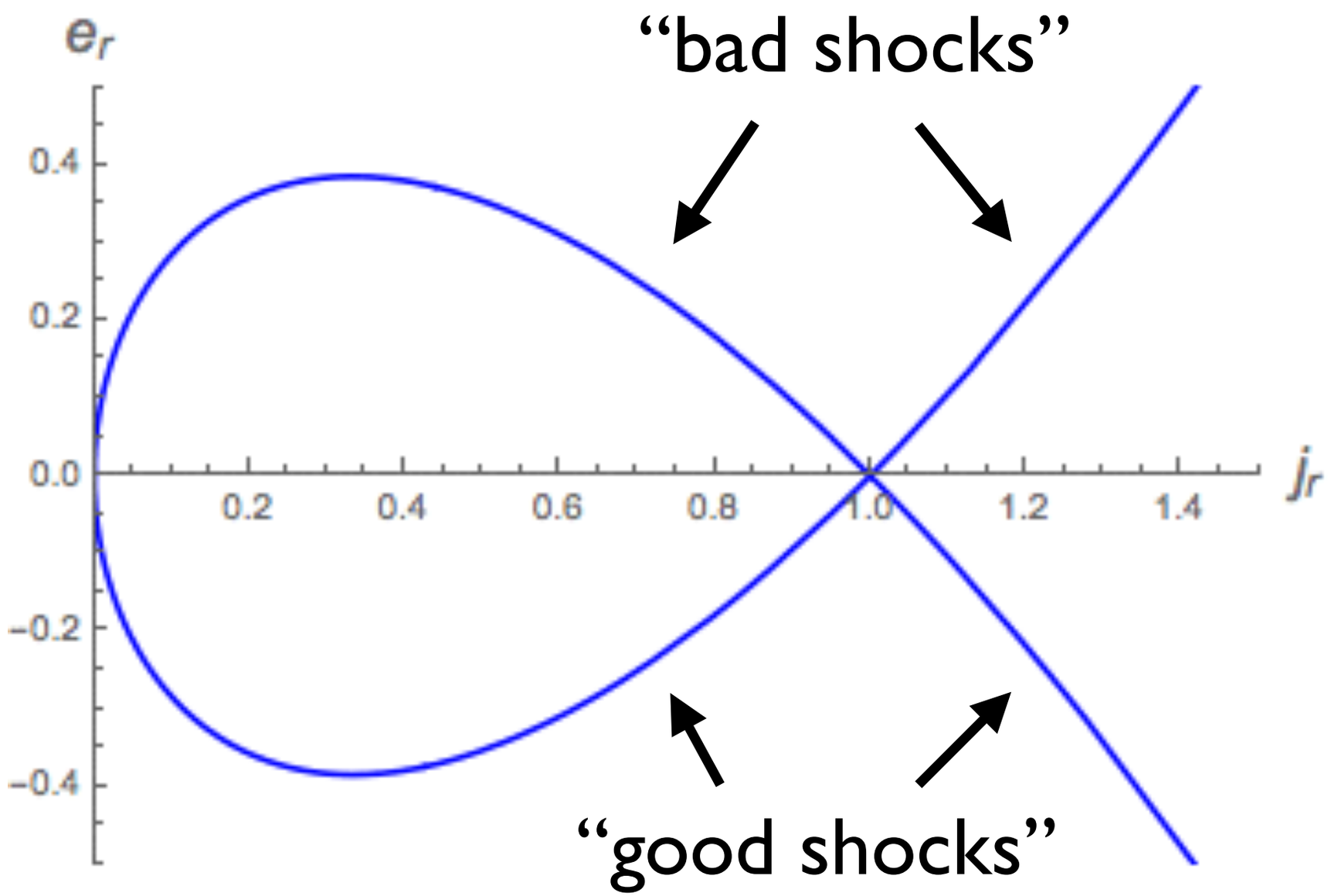
Infinite d



Infinite d



Infinite d



Infinite d

If we send d to infinity the difference between the solutions becomes prominent.

One finds

$$T^{\mu\nu} = \begin{pmatrix} e & j \\ j & e + \frac{j^2}{e} \end{pmatrix}$$

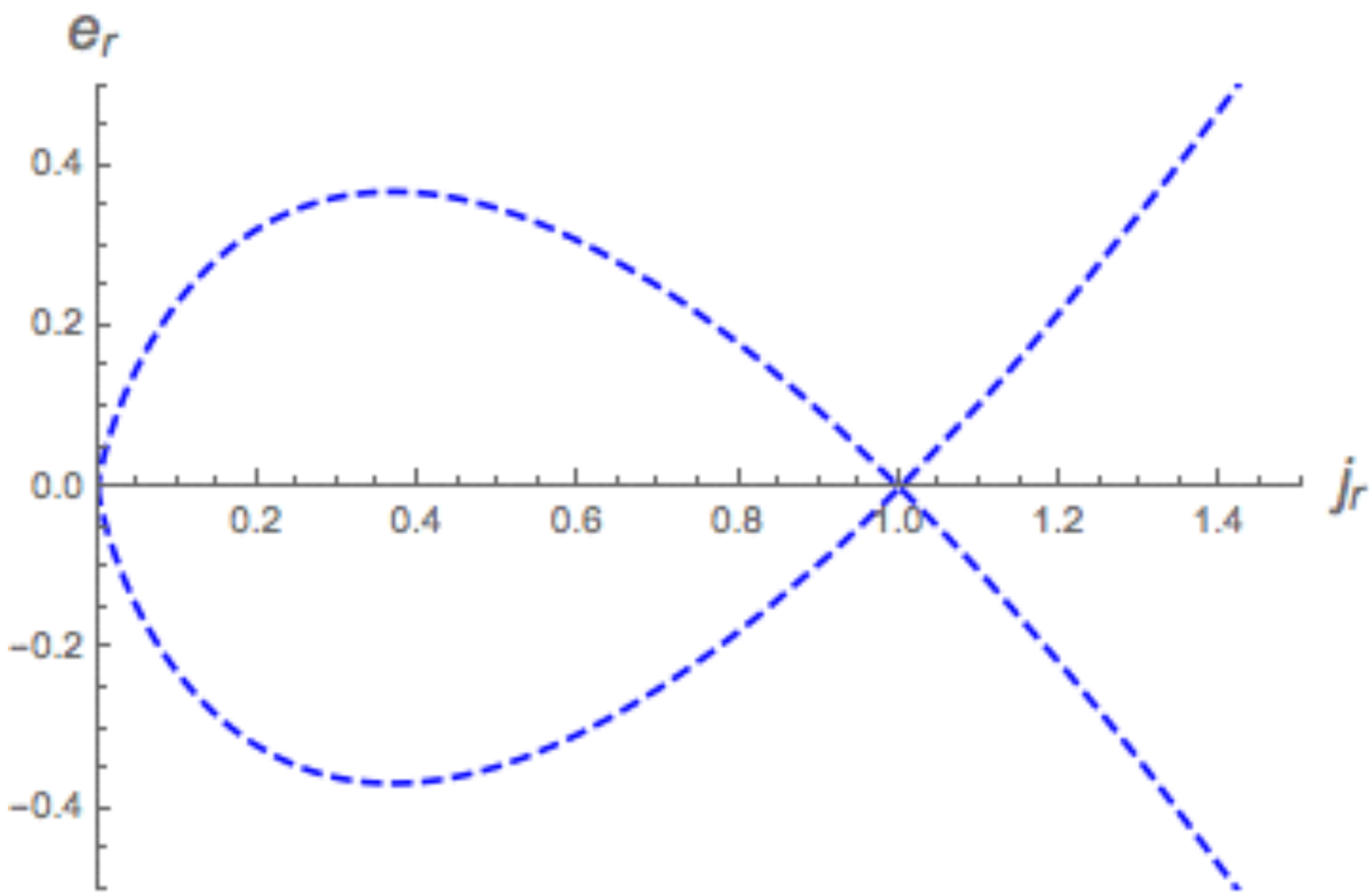
Consider

$$e(\chi, t) = e(\chi/t)$$

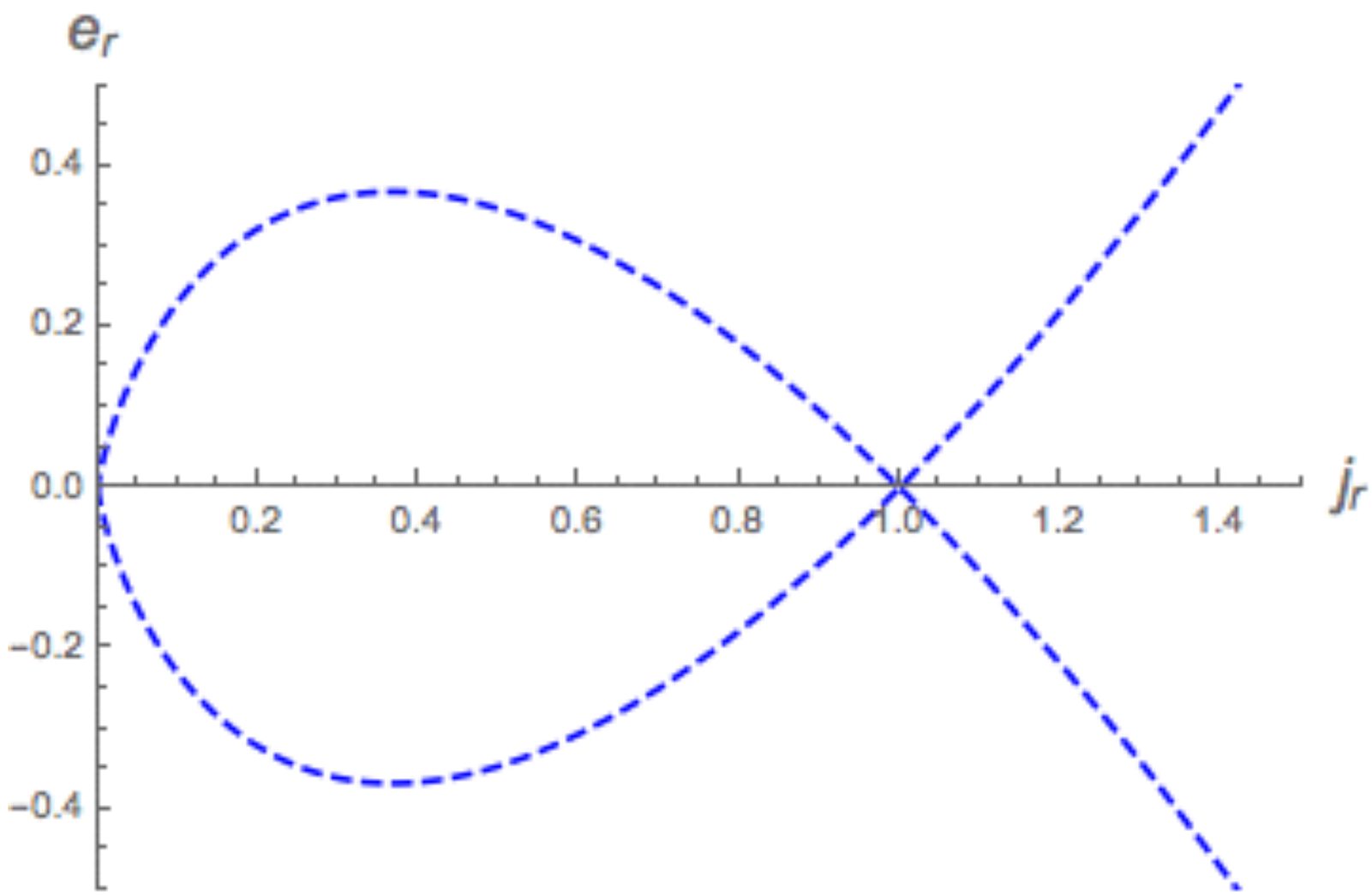
$$j(\chi, t) = j(\chi/t)$$

with $j_l = 0$ and $e_l = 1$

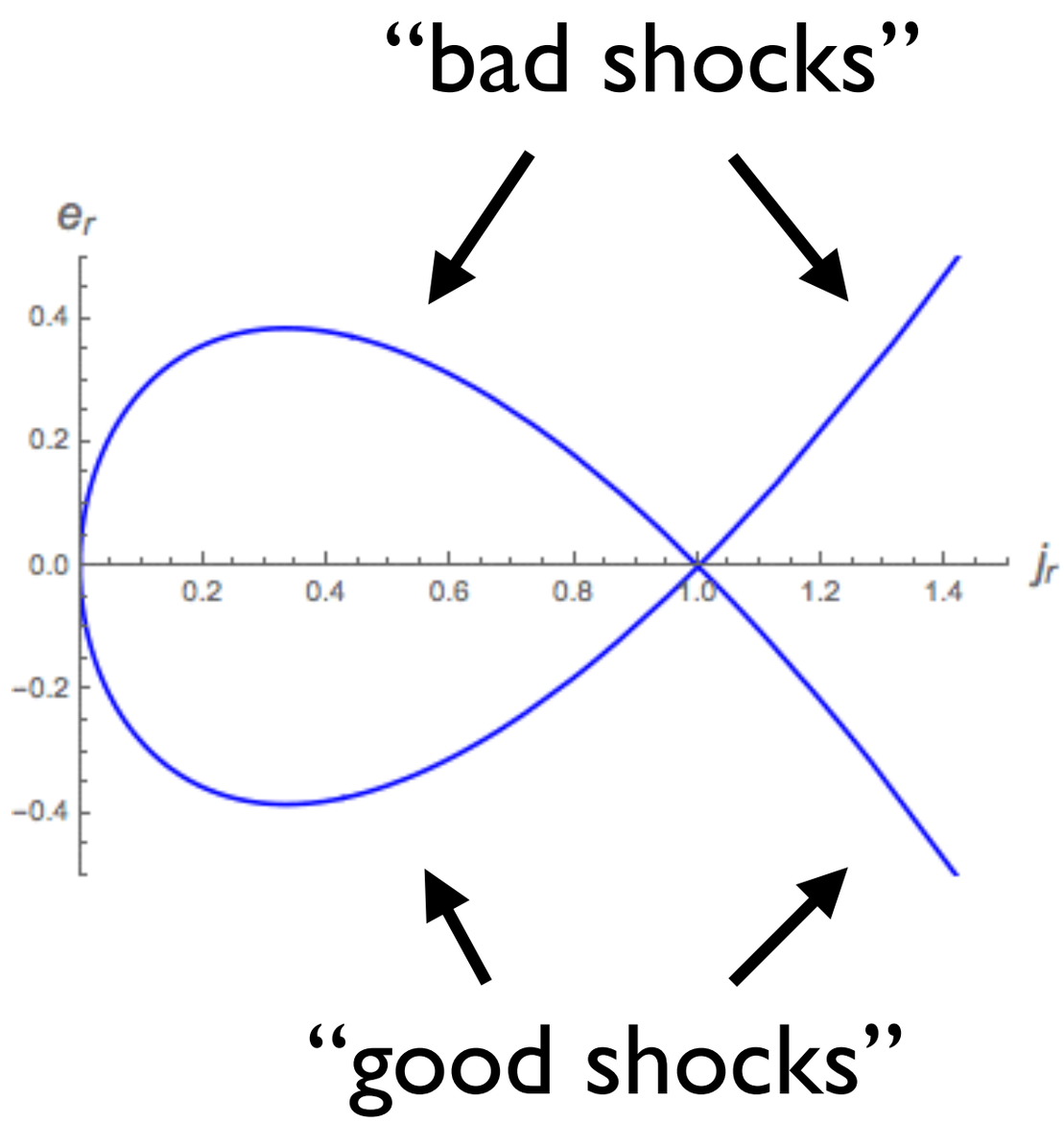
Infinite d

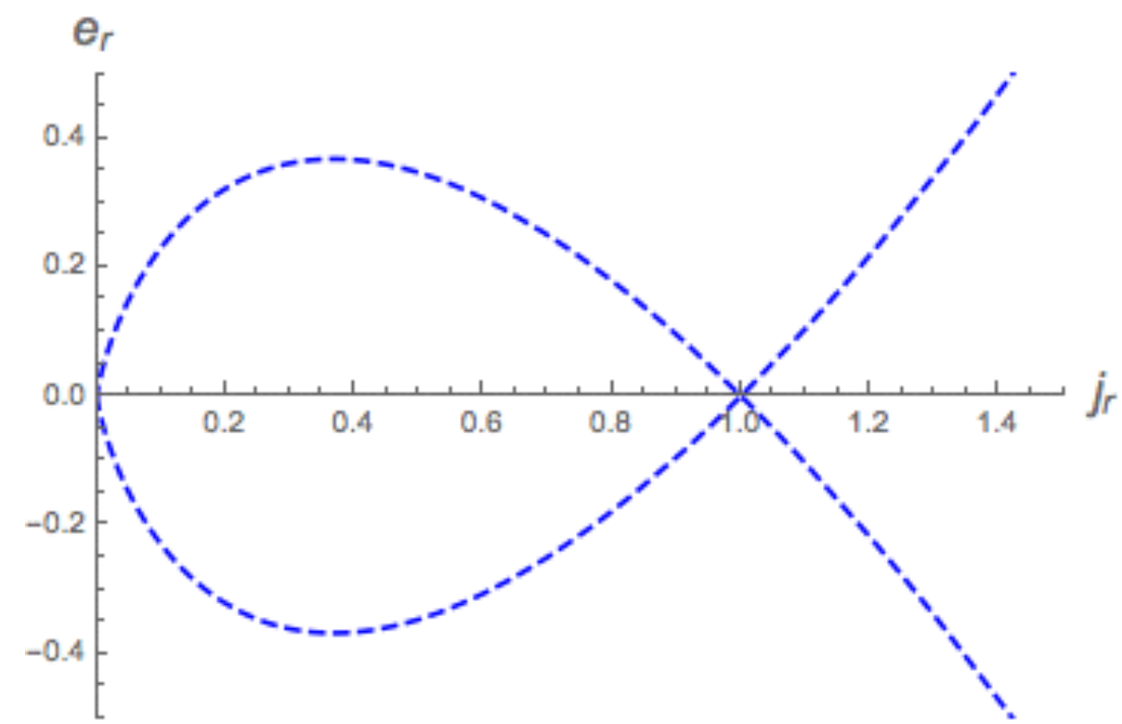


Infinite d

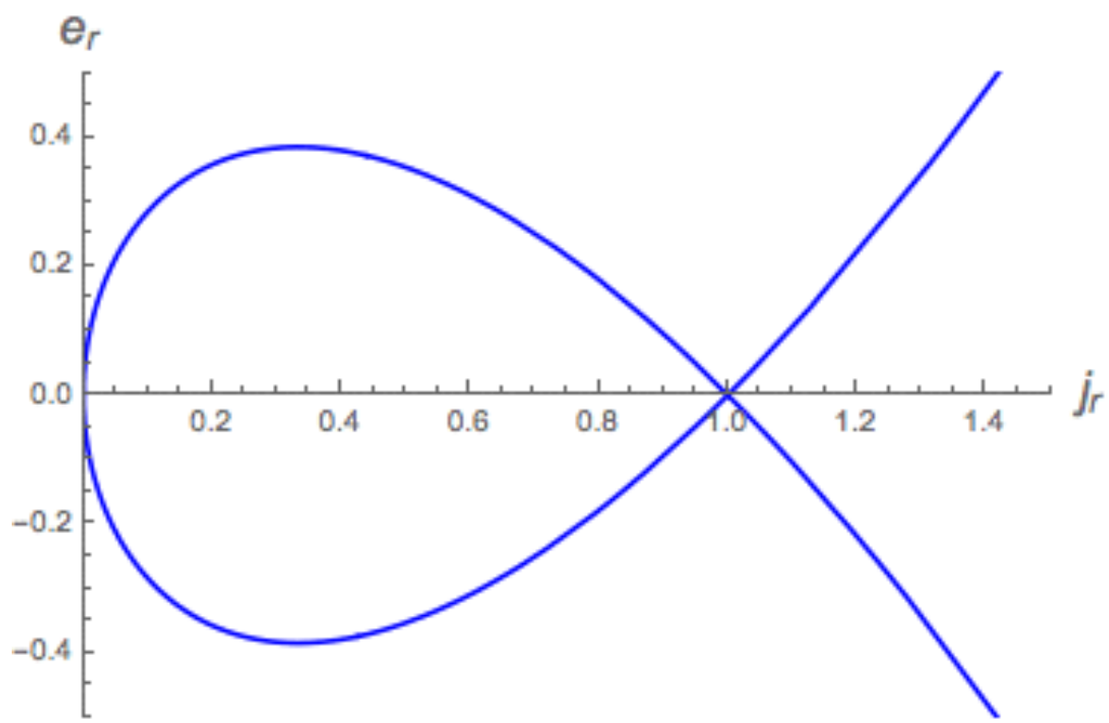


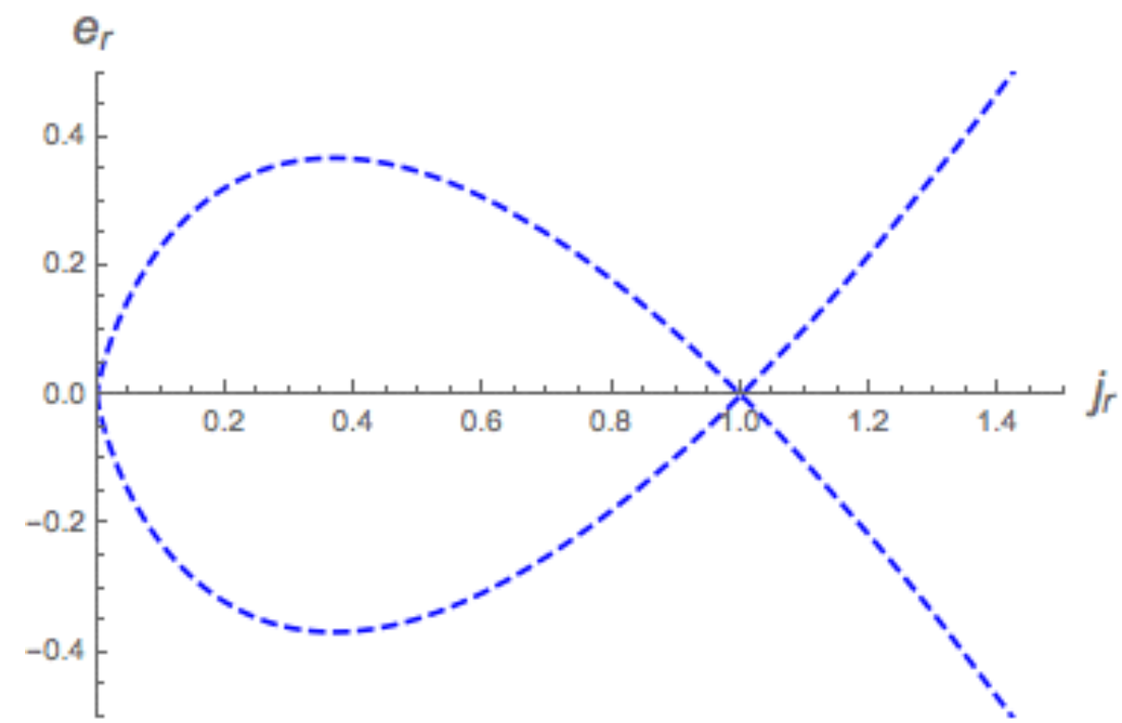
Infinite d


"bad shocks"
e_r
0.4
0.2
0.0
−0.2
−0.4
0.2
0.4
0.6
0.8
1.0
1.2
1.4
j_r
"good shocks"

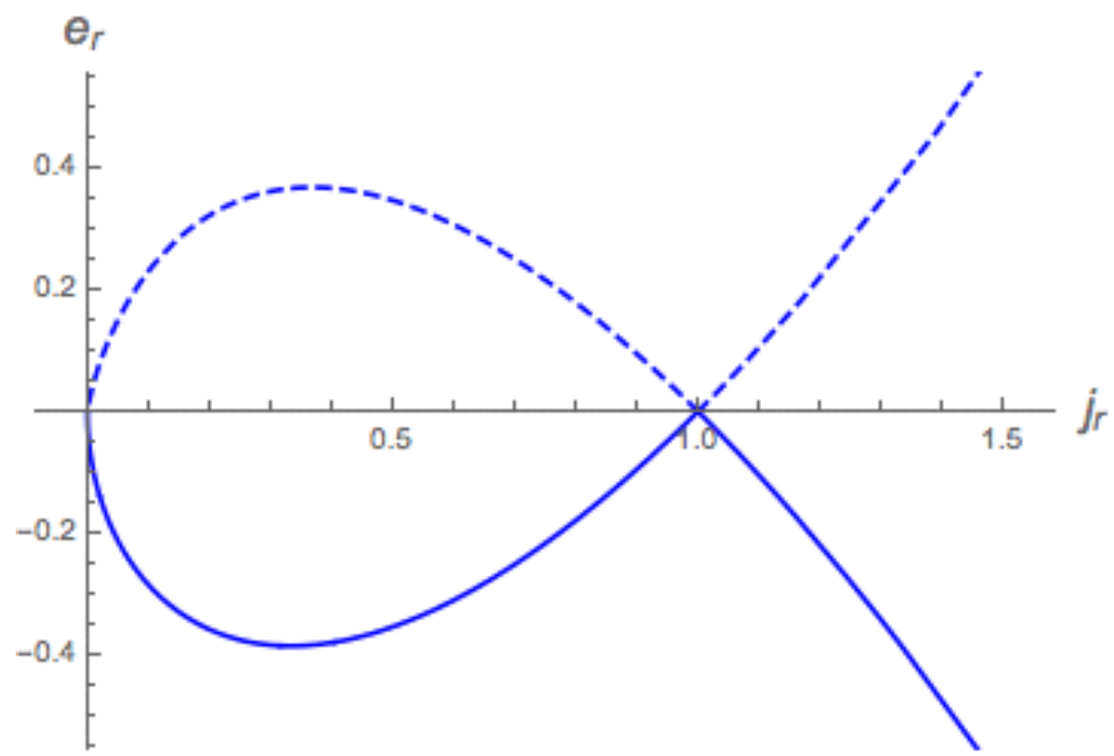

e_r
0.4
0.2
0.0
−0.2
−0.4
0.2
0.4
0.6
0.8
1.0
1.2
1.4
j_r

Infinite d


e_r
0.4
0.2
0.0
−0.2
−0.4
0.2
0.4
0.6
0.8
1.0
1.2
1.4
j_r

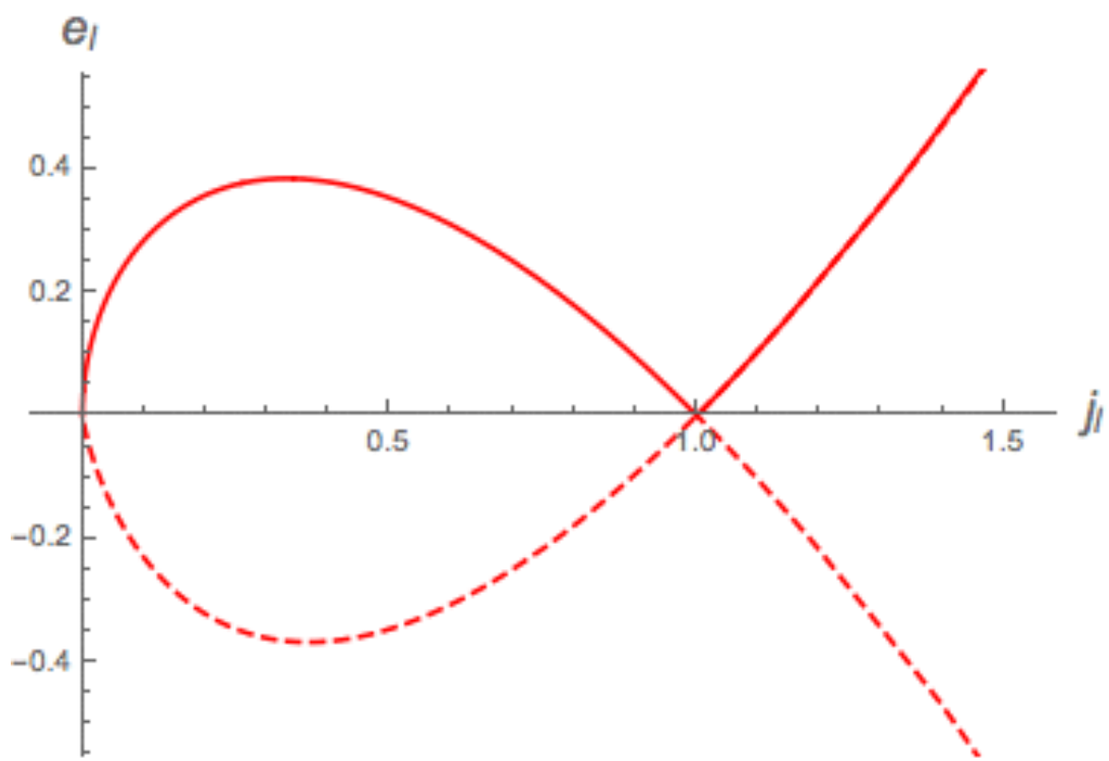

e_r
0.4
0.2
0.0
−0.2
−0.4
0.2
0.4
0.6
0.8
1.0
1.2
1.4
j_r

Infinite d



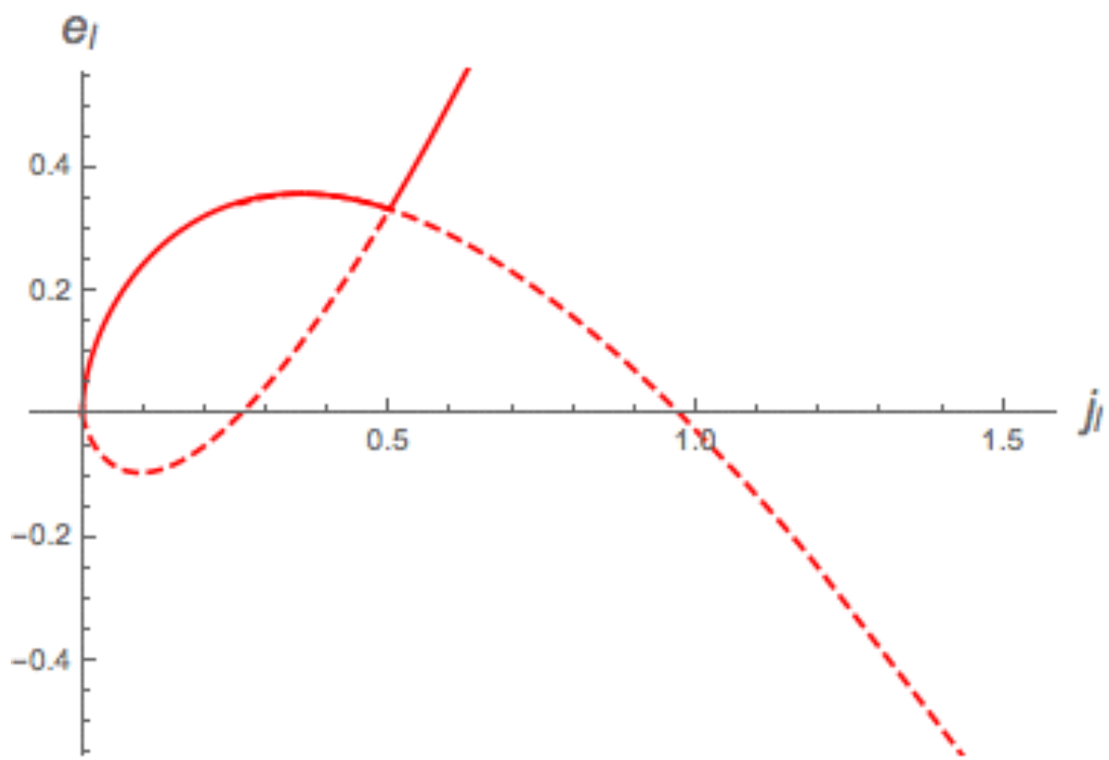
Infinite d

We can fix instead the right asymptotic: $e_r = 1 \qquad j_r = 0$



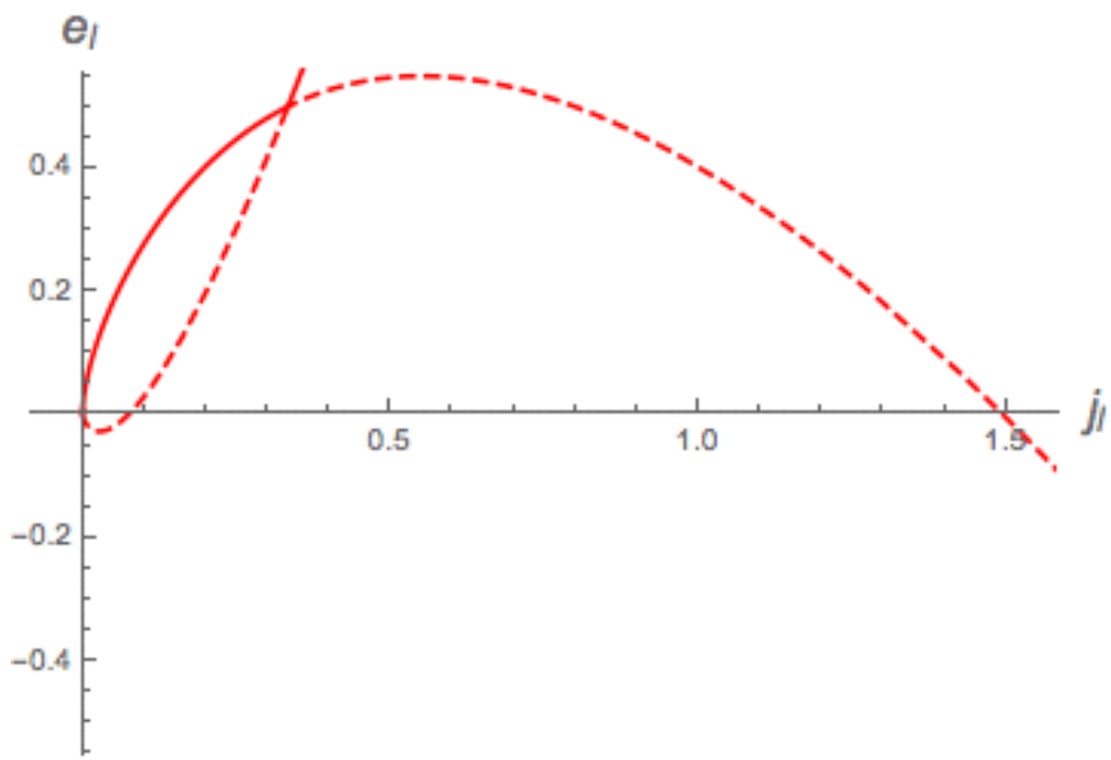
Infinite d

Or:



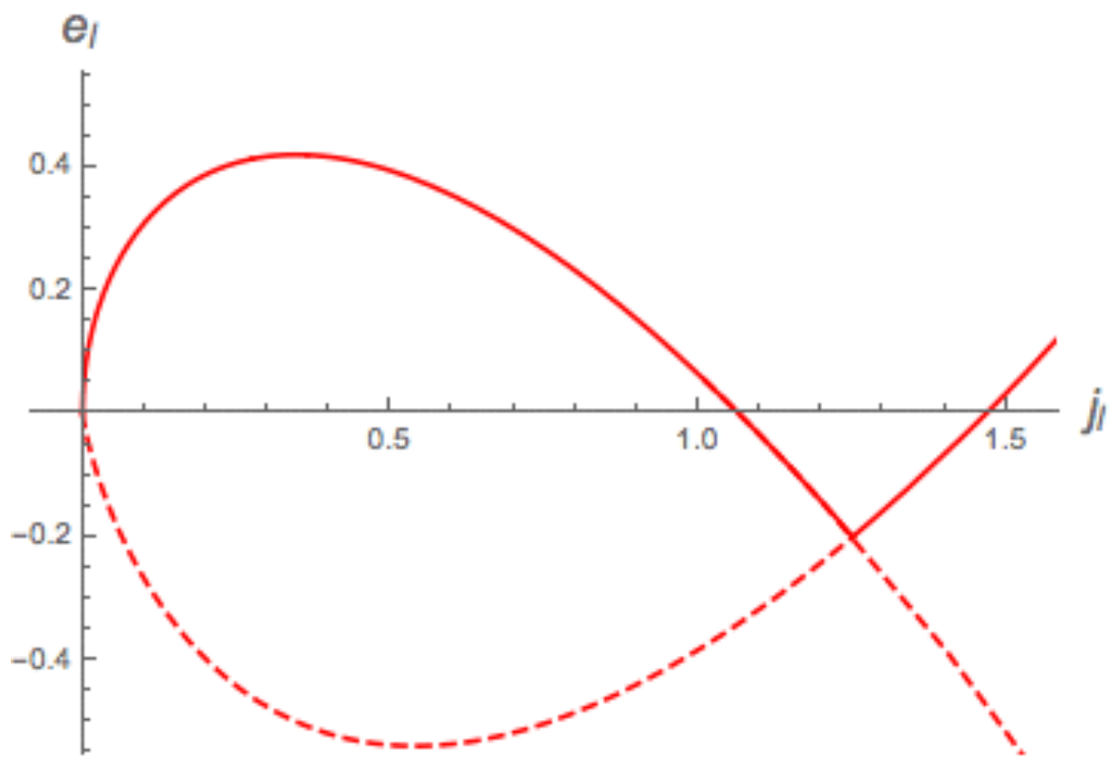
Infinite d

Or:



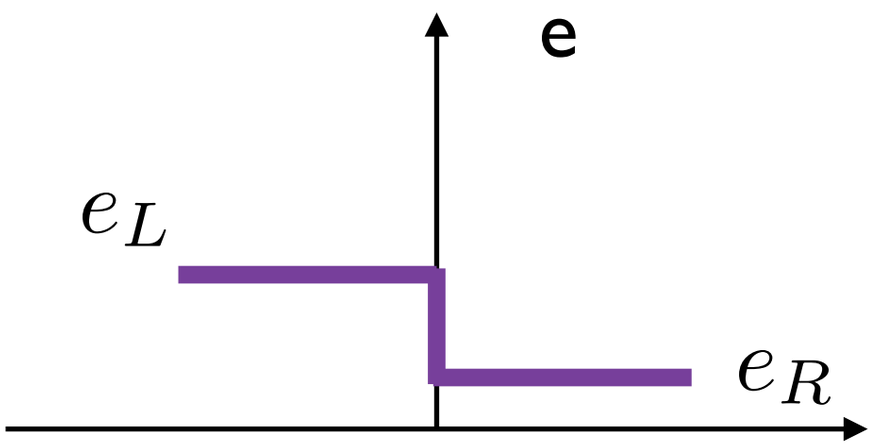
Infinite d

Or:



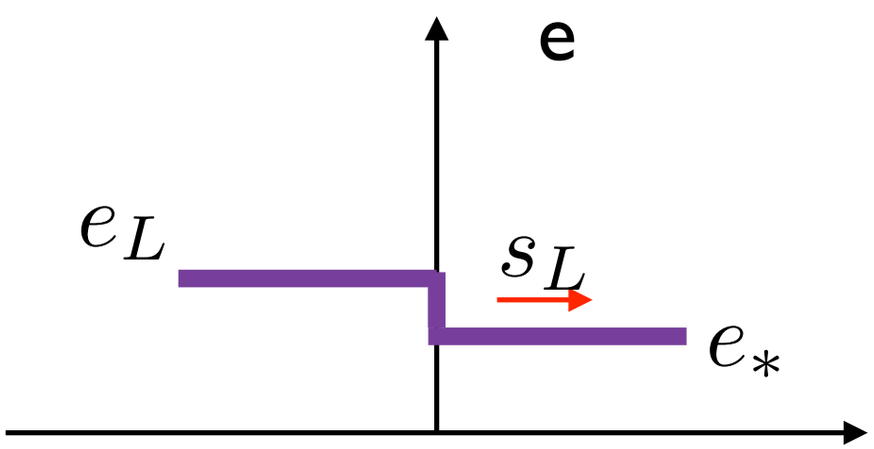
Infinite d

We can now glue solutions like we did before:

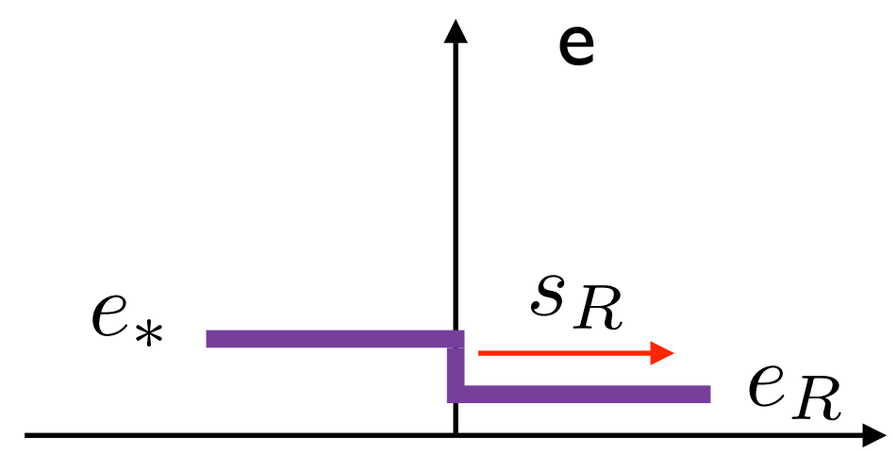


$=$

$=$



$+$



Infinite d

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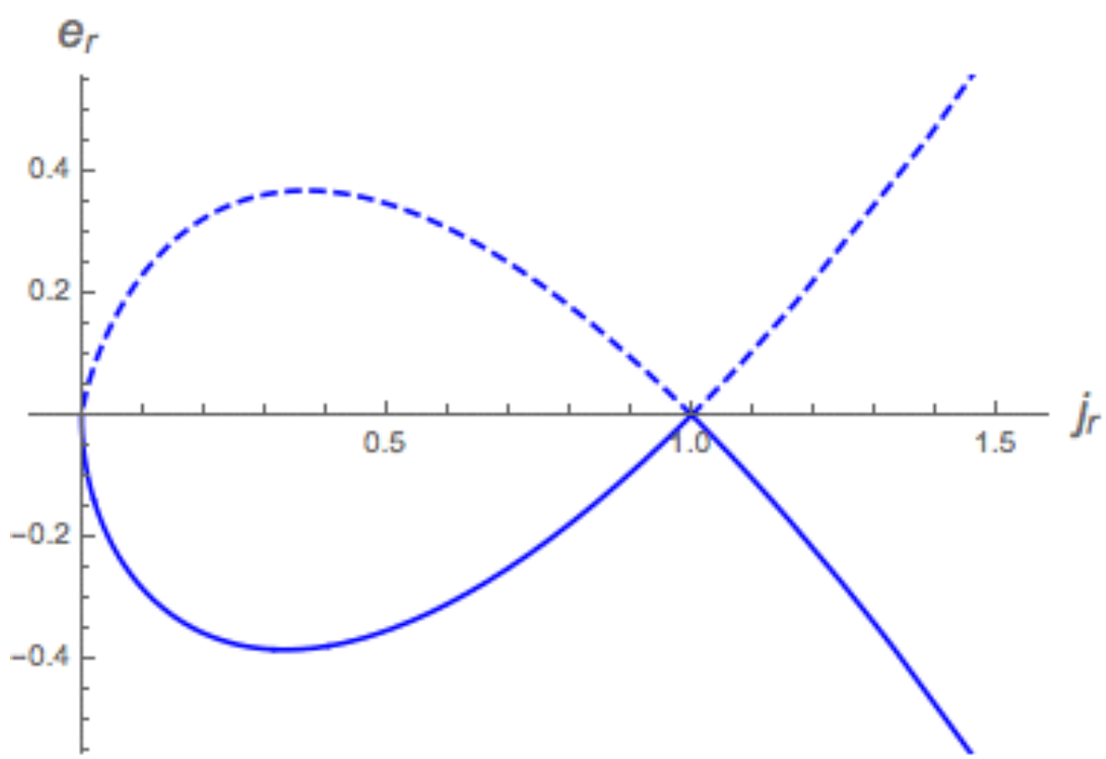

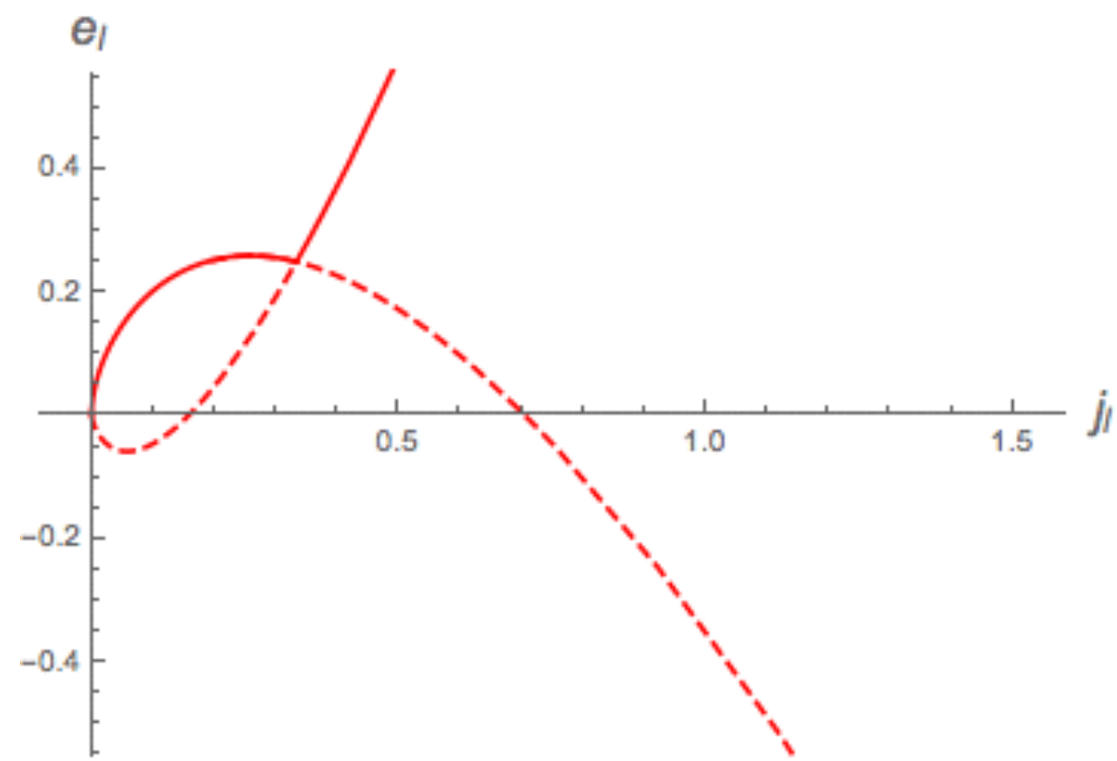

Infinite d

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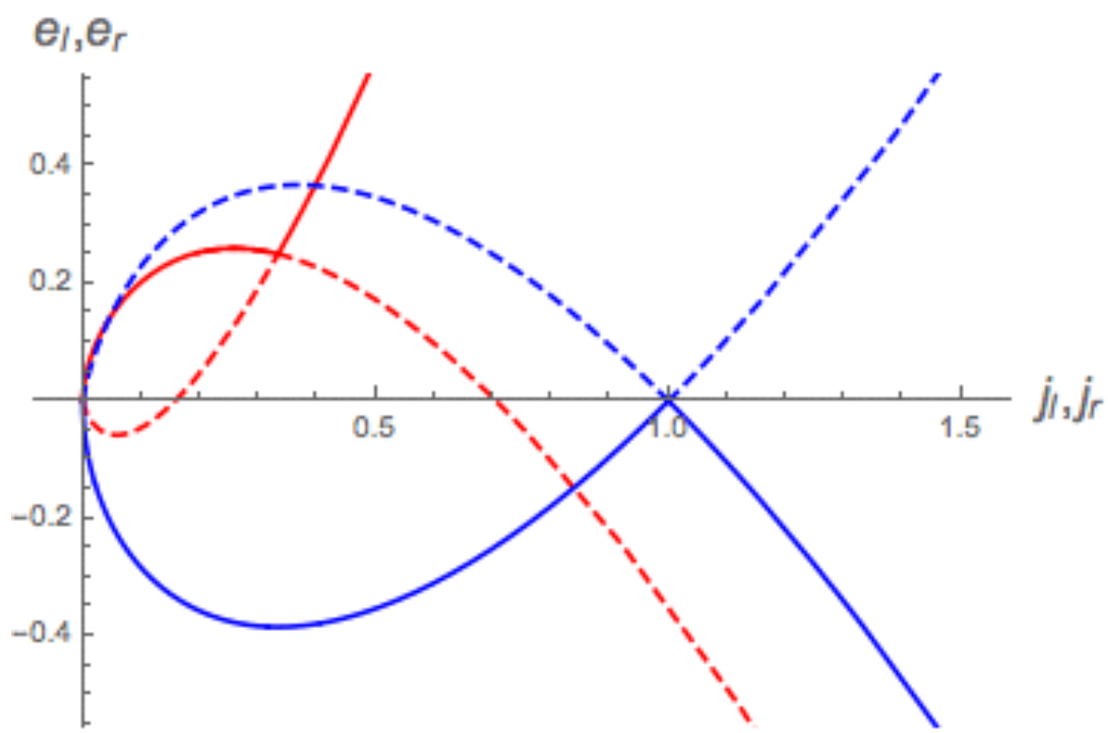
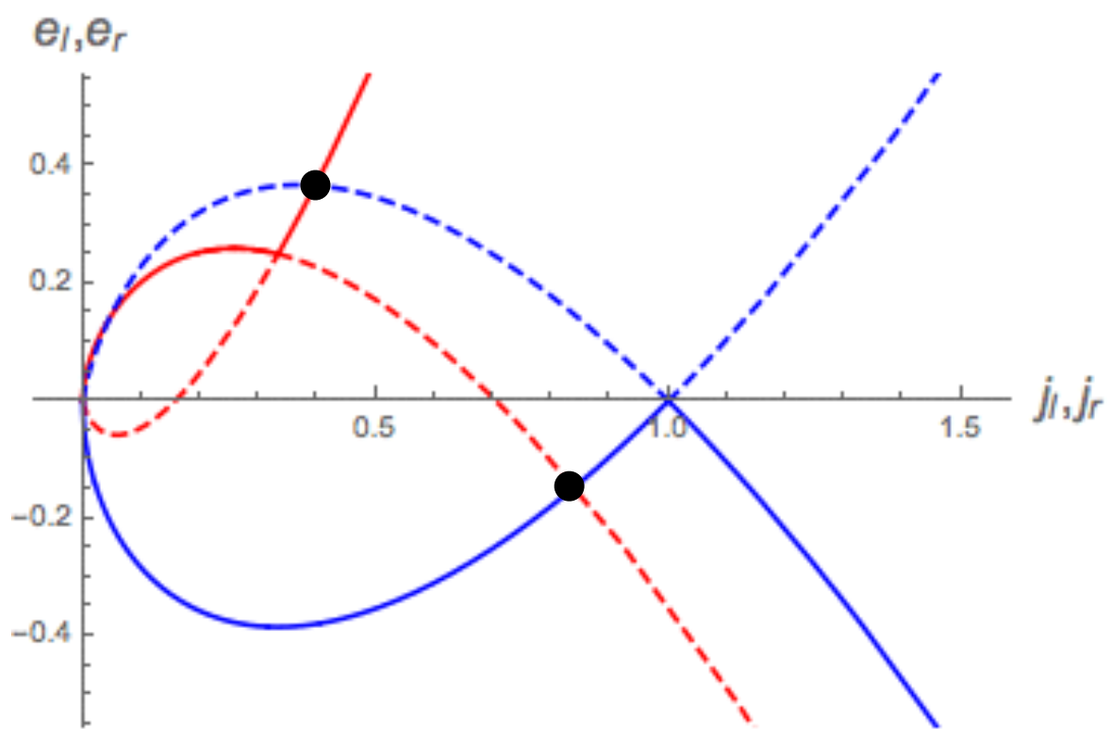


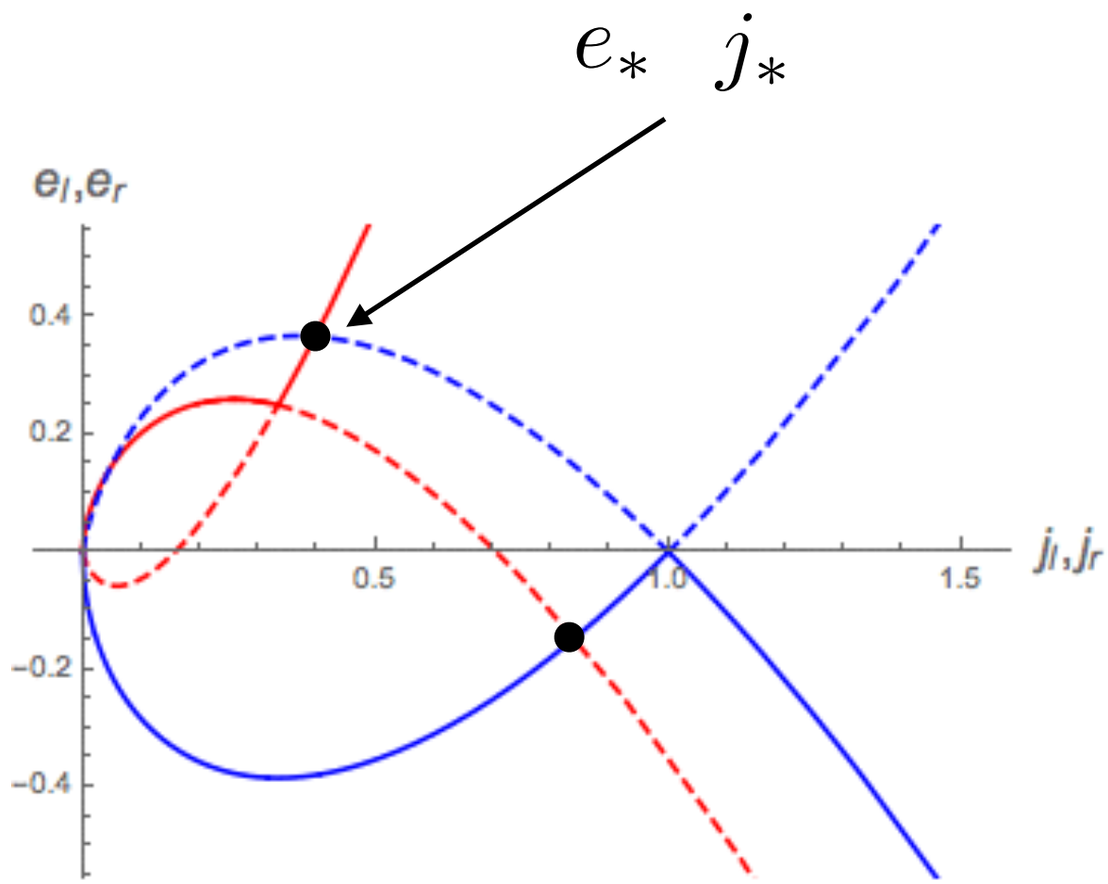
Infinite d

We can now glue solutions like we did before:



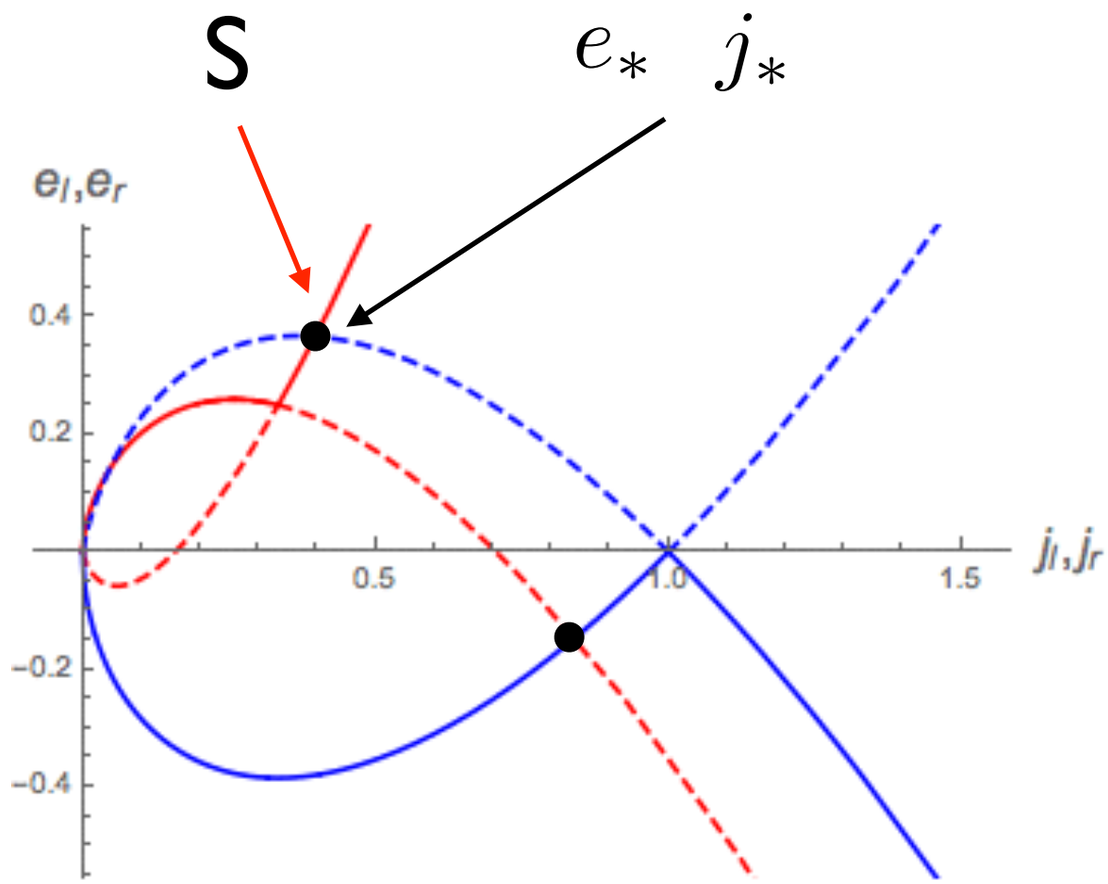
Infinite d

We can now glue solutions like we did before:



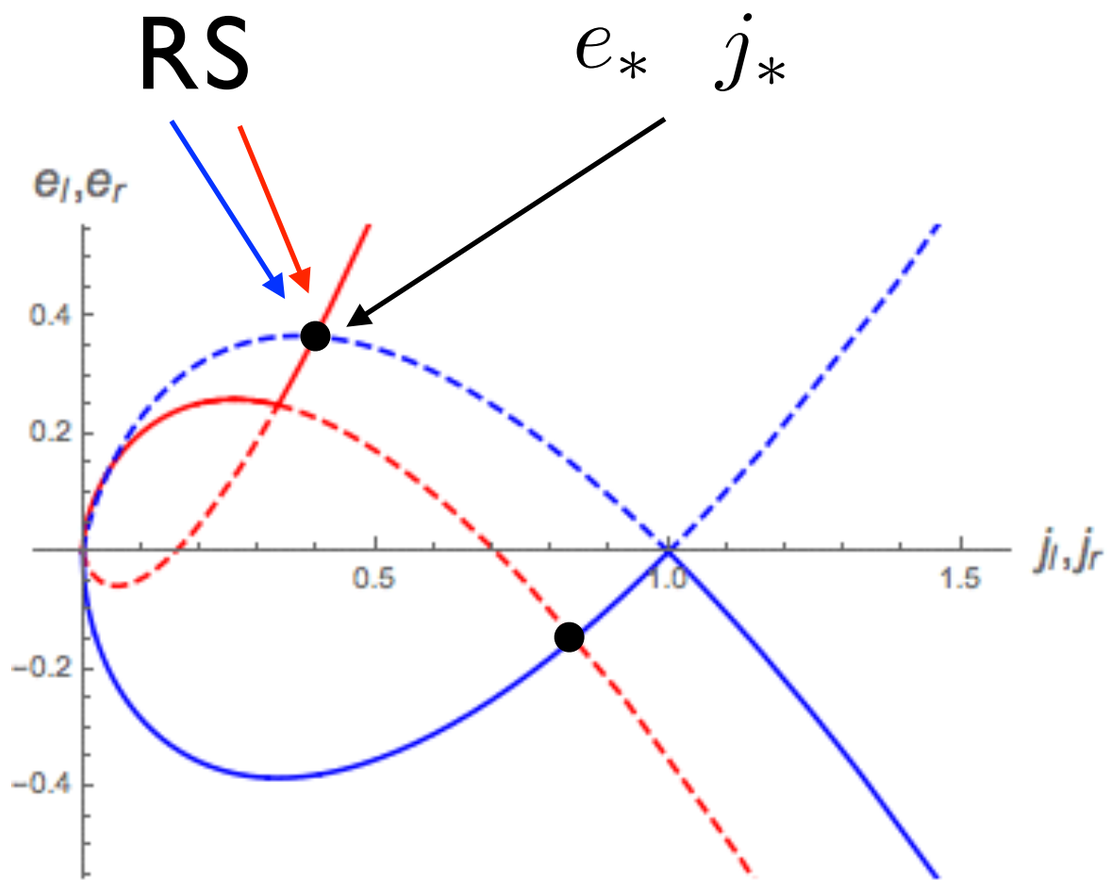
Infinite d

We can now glue solutions like we did before:



Infinite d

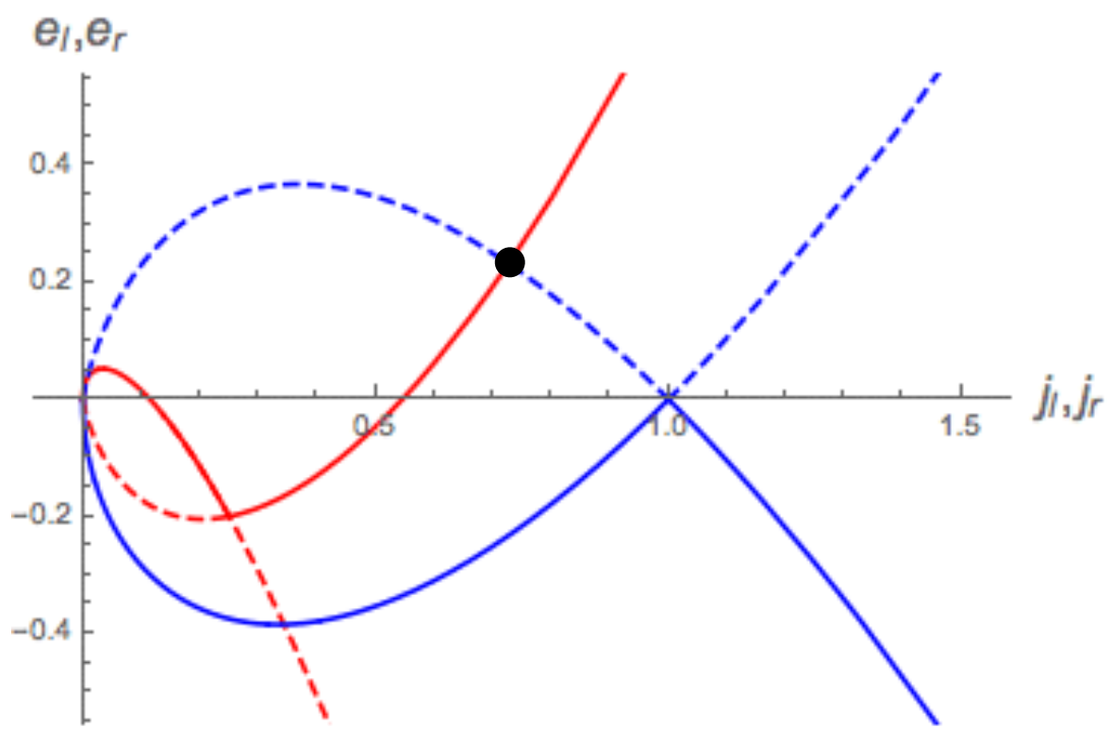
We can now glue solutions like we did before:



Infinite d

We can now glue solutions like we did before:

RS



Infinite d

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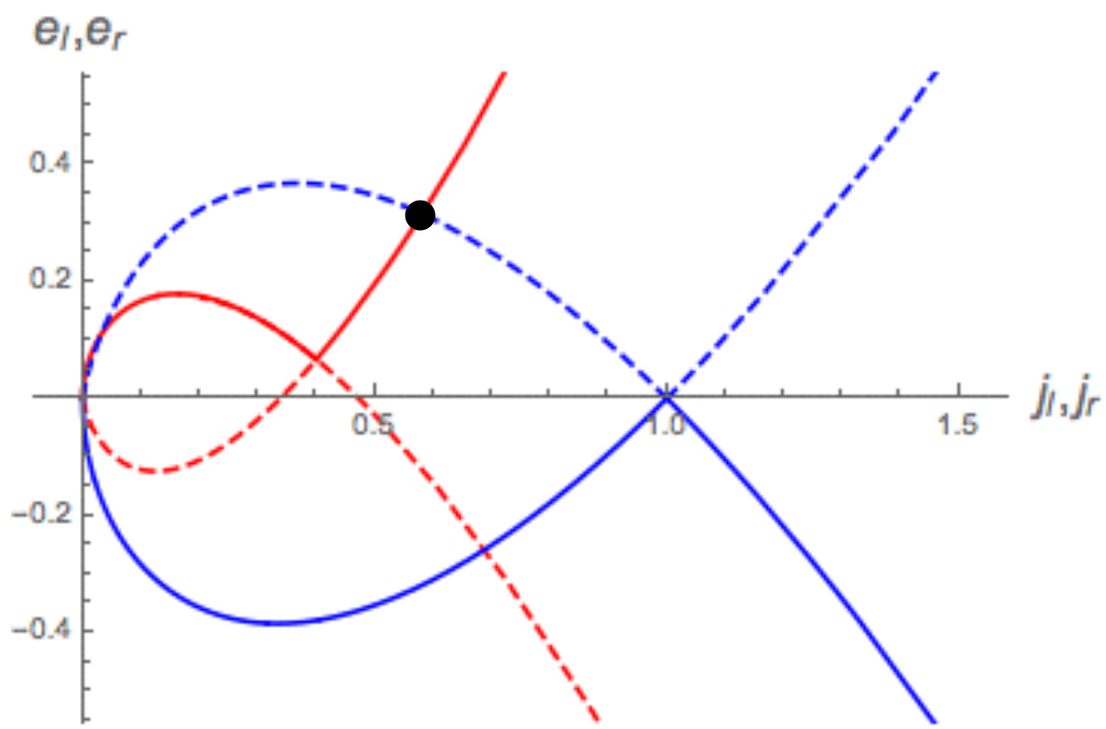
RS



Infinite d

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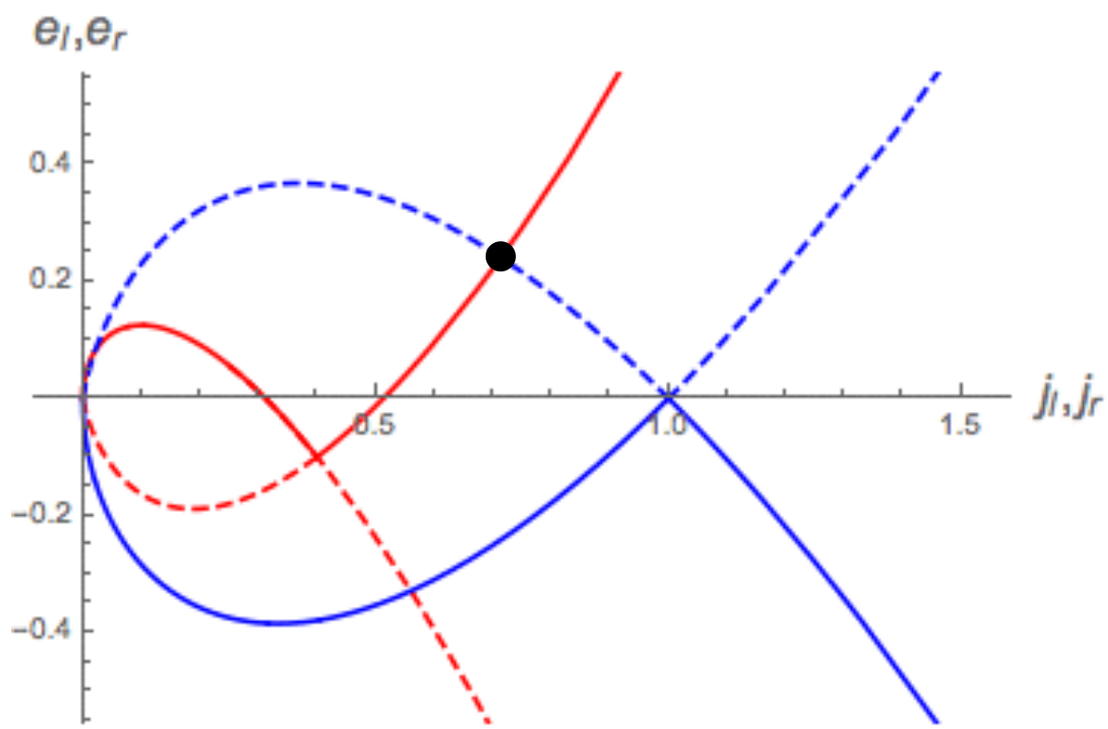
RS



Infinite d

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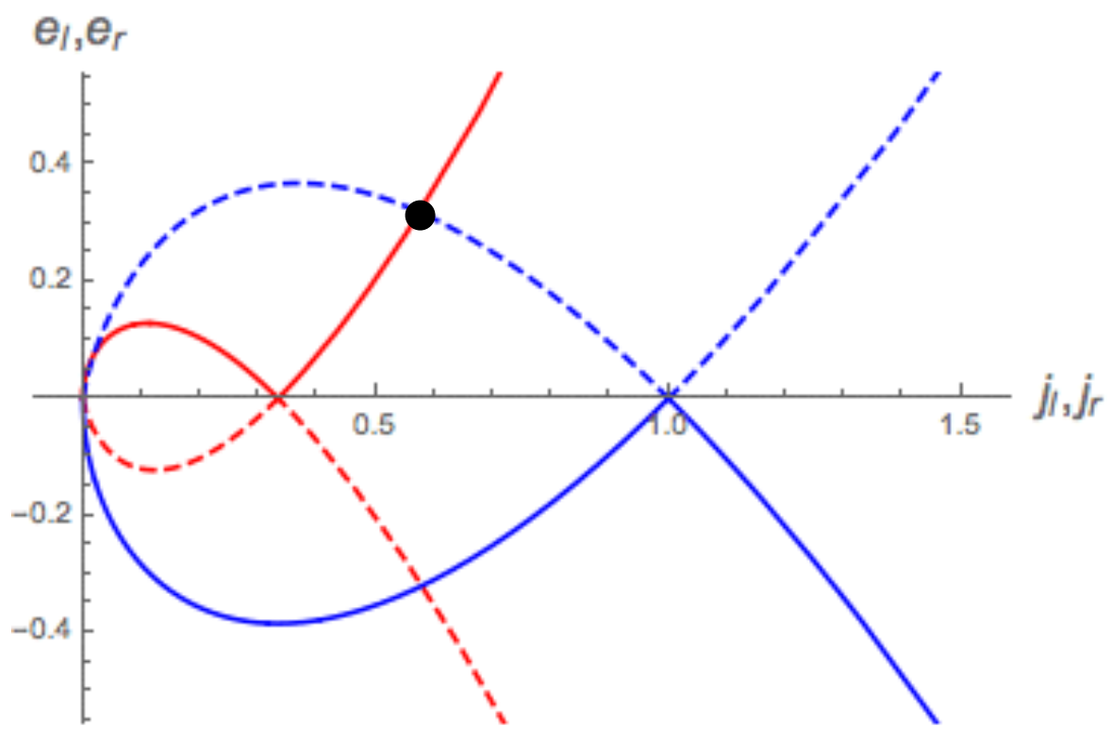
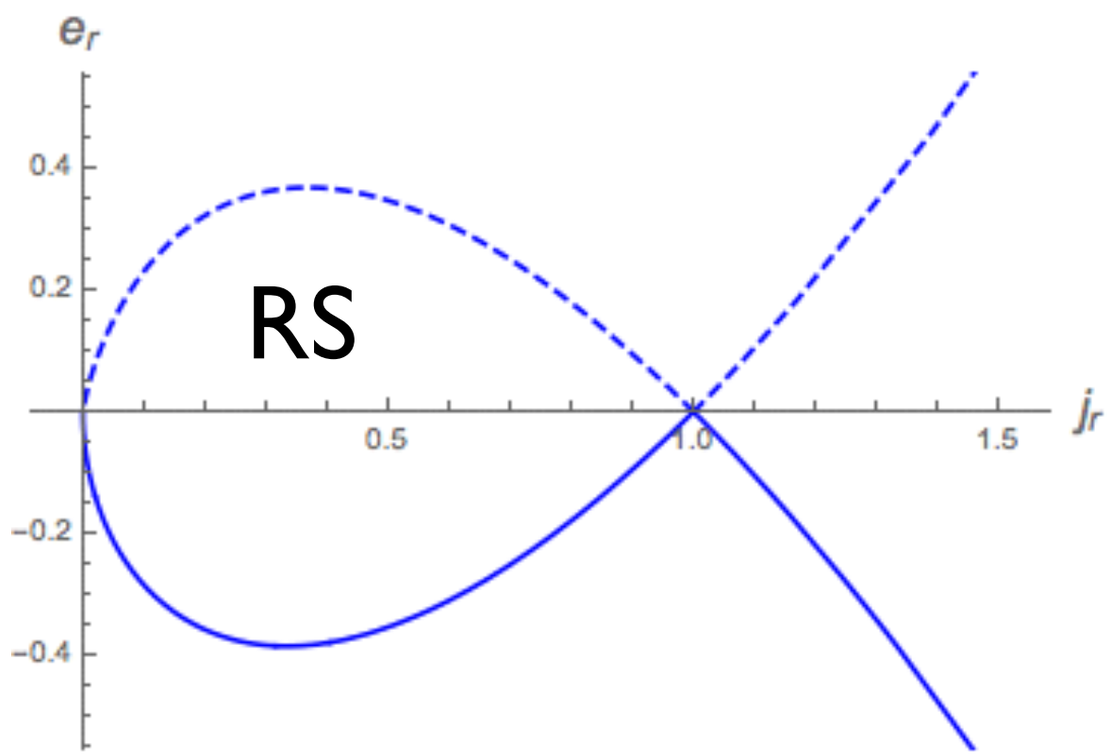
RS



Infinite d

We can now glue solutions like we did before:



Infinite d

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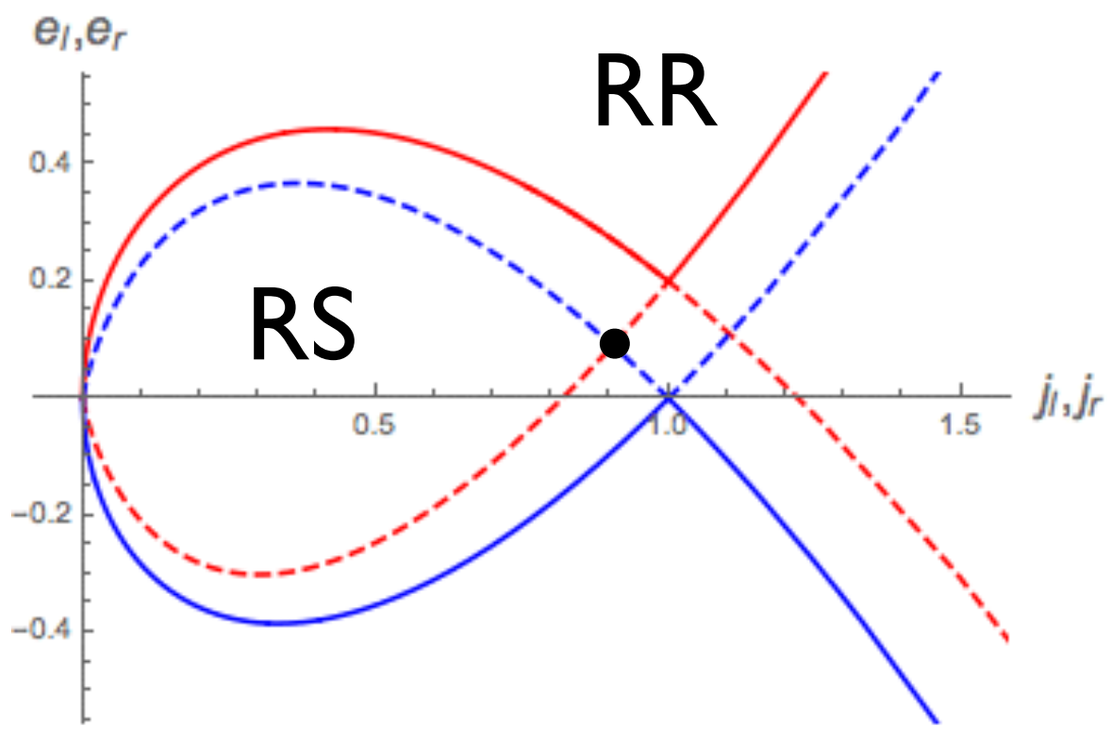

Infinite d

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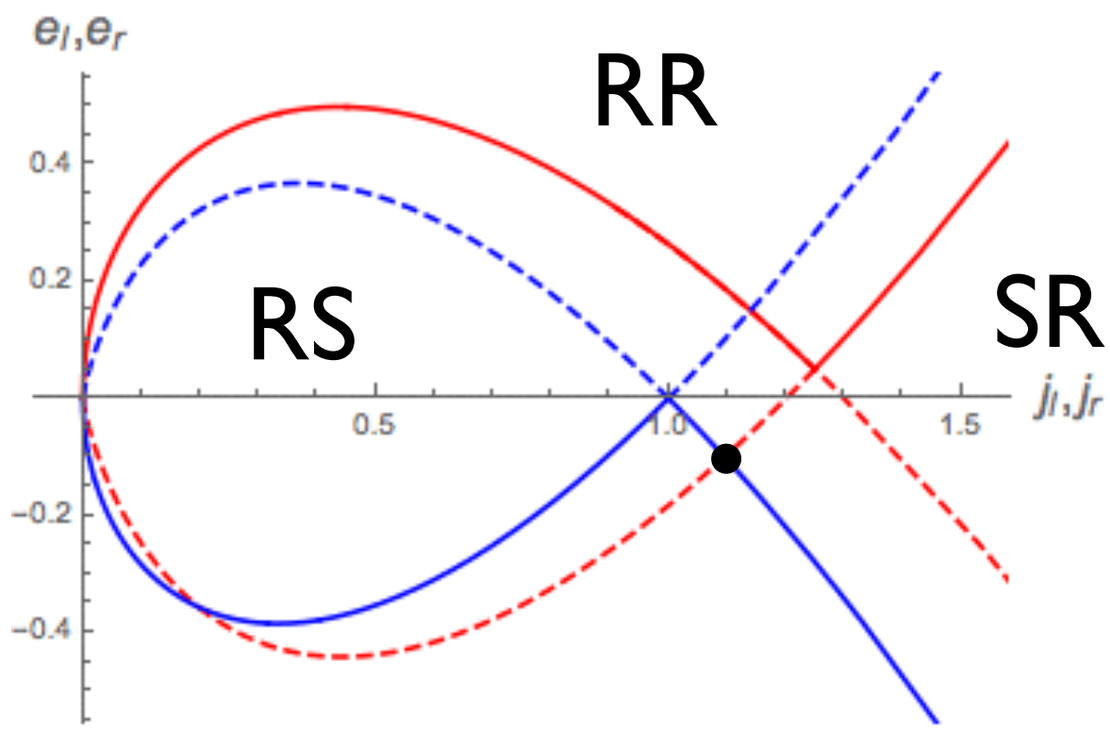
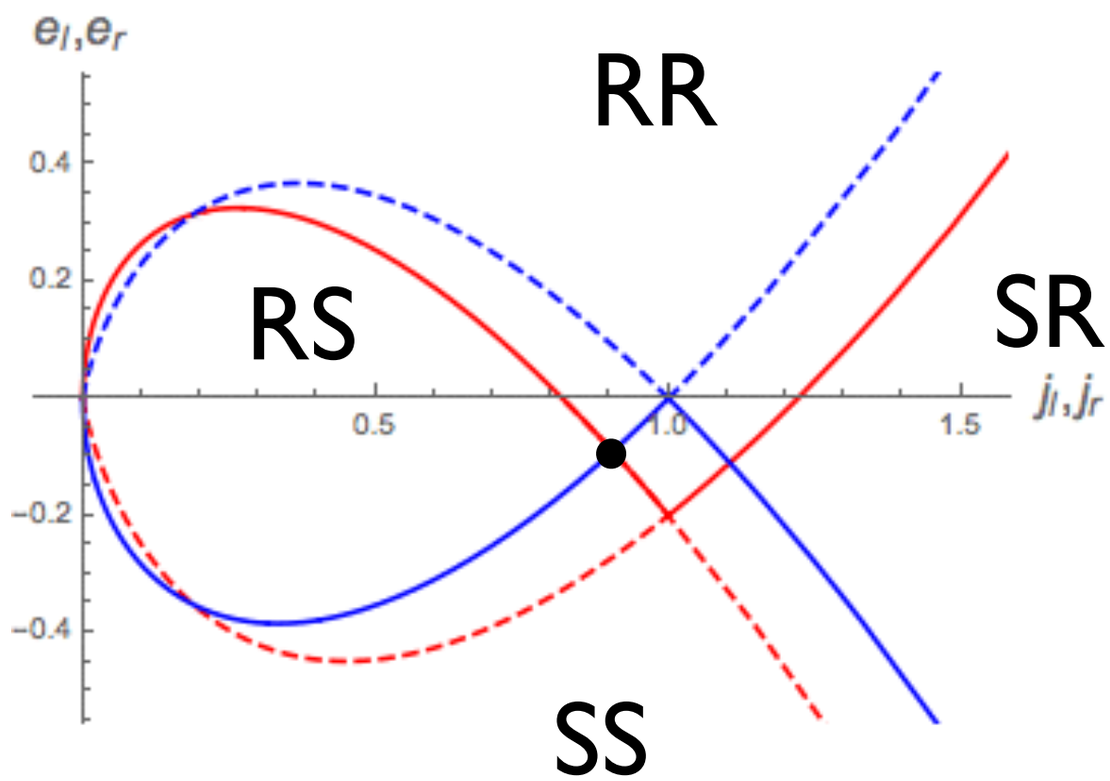


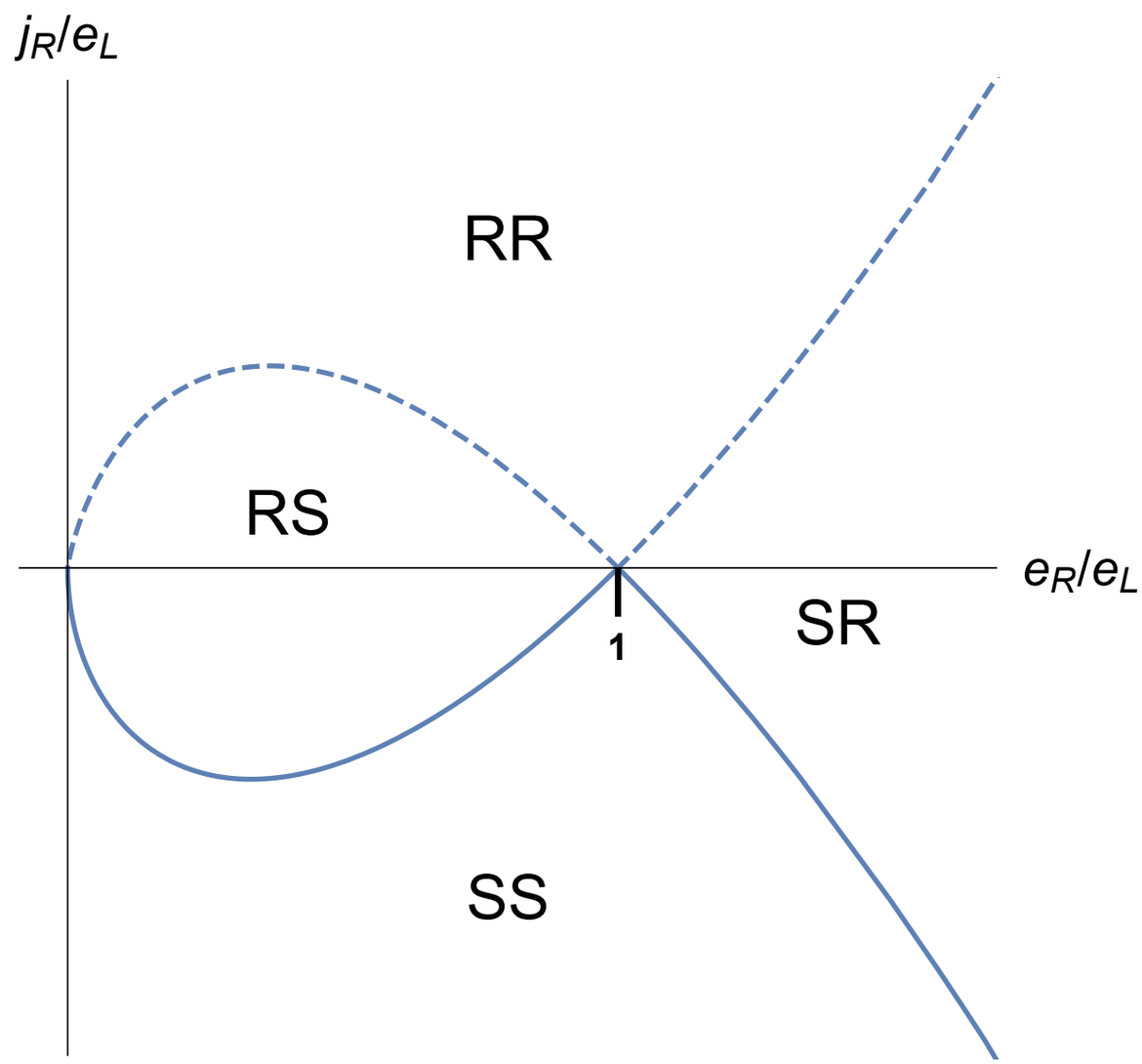
Infinite d

We can now glue solutions like we did before:



Infinite d

We obtain a "phase" diagram,



Holography

Holography

Out of equilibrium we would like to solve the equations of motion for:

$$ds^2 = dt(2dr - g_{tt}dt - 2g_{t\chi}d\chi) + g_{\chi\chi}d\chi^2 + g_{\perp\perp}d\chi_\perp^2$$

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with

$$g_{tt} = \mathcal{O}(d^0) \qquad g_{\chi\chi} = \mathcal{O}(d^{-1})$$

$$g_{t\chi} = \mathcal{O}(d^{-1}) \qquad g_{\perp\perp} = \mathcal{O}(d^{-1})$$

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and

$$R = r^d$$

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(Emparan, Suzuki, Tanabe, 2015 (see also Bhattacharyya et. al. 2015))

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one finds

$$\frac{g_{tt}}{r^2} = 1 - \frac{e}{R} + \mathcal{O}(d^{-1}) \qquad g_{t\chi} = \frac{j}{dR} + \mathcal{O}(d^{-2})$$

$$\frac{g_{\chi\chi}}{r^2} = \frac{1}{d} + \mathcal{O}(d^{-2}) \qquad \frac{g_{\perp\perp}}{r^2} = \frac{1}{d} + \mathcal{O}(d^{-3})$$

where:

$$\partial_t e - \partial_\chi^2 e = -\partial_\chi j \qquad \partial_t j - \partial_\chi^2 j = -\partial_\chi \left(\frac{j^2}{e} + e \right)$$

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which come from a conservation of

$$T^{\mu\nu} = \begin{pmatrix} e & j - \partial_\chi e \\ j - \partial_\chi e & e + \frac{j^2}{e} - 2\partial_\chi j + \partial_\chi^2 e \end{pmatrix} + \begin{pmatrix} \partial_\chi^2 g & -\partial_\chi \partial_t g \\ -\partial_\chi \partial_t g & \partial_t^2 g \end{pmatrix}$$

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This is compatible with

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Holography

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$$\frac{\eta}{s} = \frac{1}{4\pi} \qquad \tau_0 = \frac{1}{2}$$

once ‘g’ is chosen appropriately

Holography

one finds

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Holography

EOM's are exactly first order in derivatives

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$$T^{\mu\nu} = \begin{pmatrix} e & j - \partial_\chi e \\ j - \partial_\chi e & e + \frac{j^2}{e} - 2\partial_\chi j + \partial_\chi^2 e \end{pmatrix} + \begin{pmatrix} \partial_\chi^2 g & -\partial_\chi \partial_t g \\ -\partial_\chi \partial_t g & \partial_t^2 g \end{pmatrix}$$

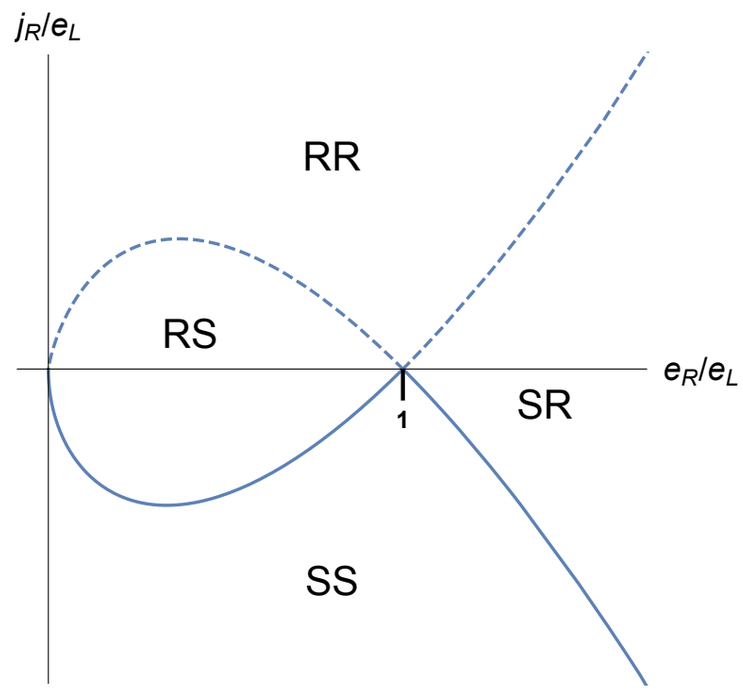
This is compatible with

$$\frac{\eta}{s} = \frac{1}{4\pi} \qquad \tau_0 = \frac{1}{2}$$

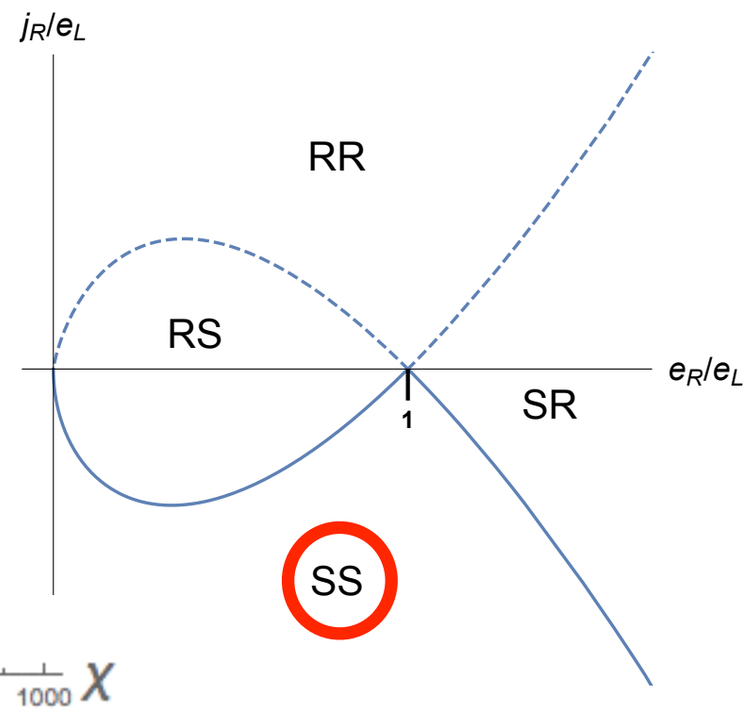
Stress tensor is second order

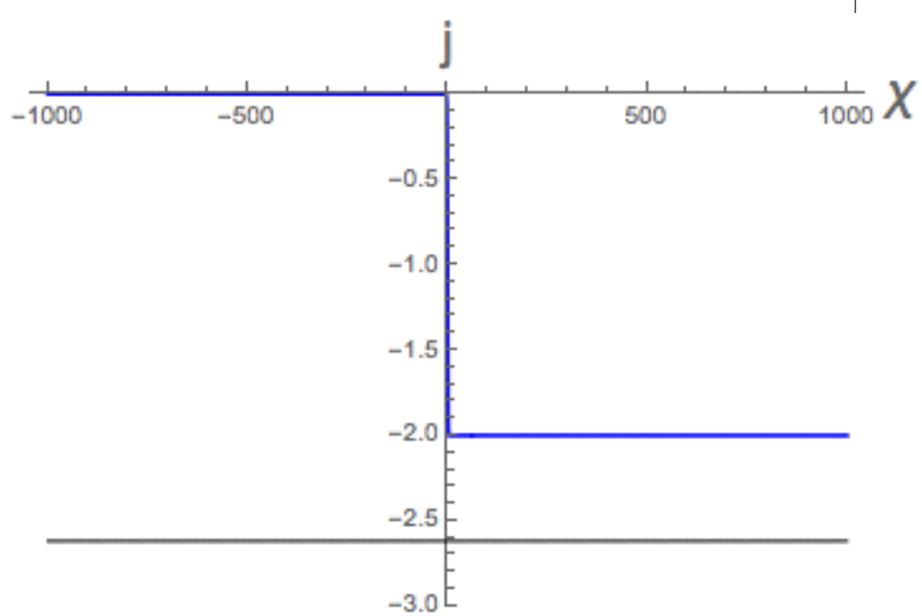
once 'g' is chosen appropriately

Holography



Holography





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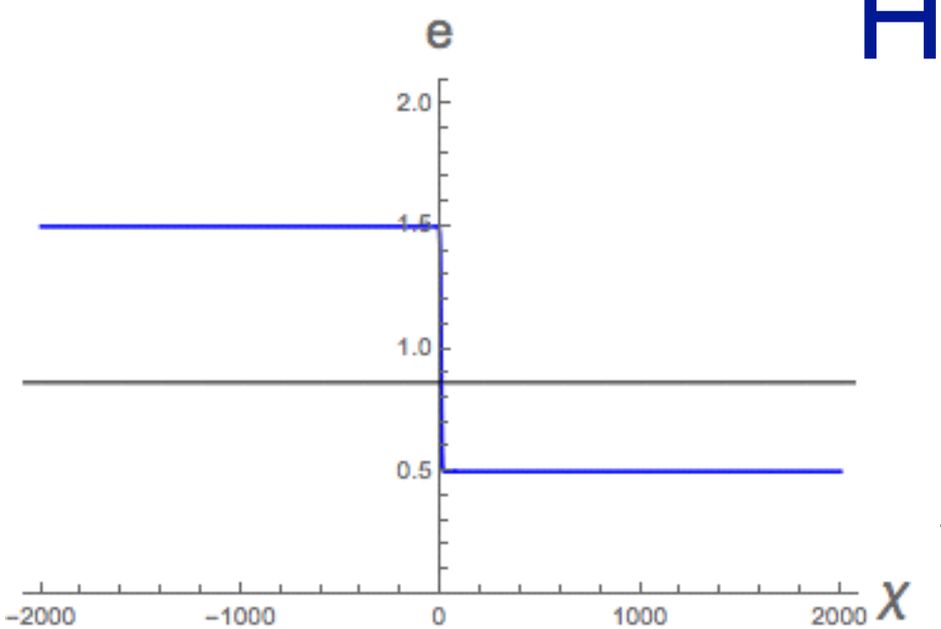
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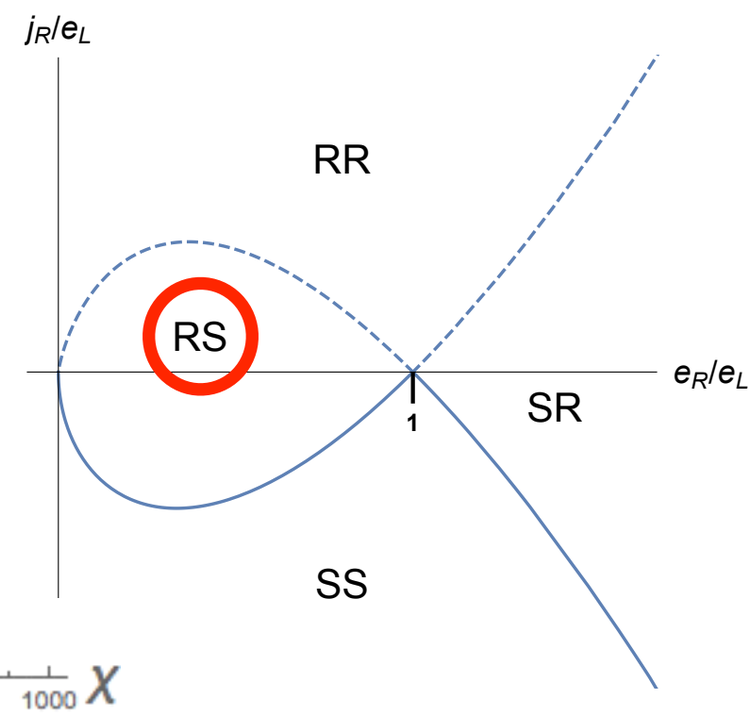
0
1000
2000
X


j_R/e_L
RR
RS
e_R/e_L
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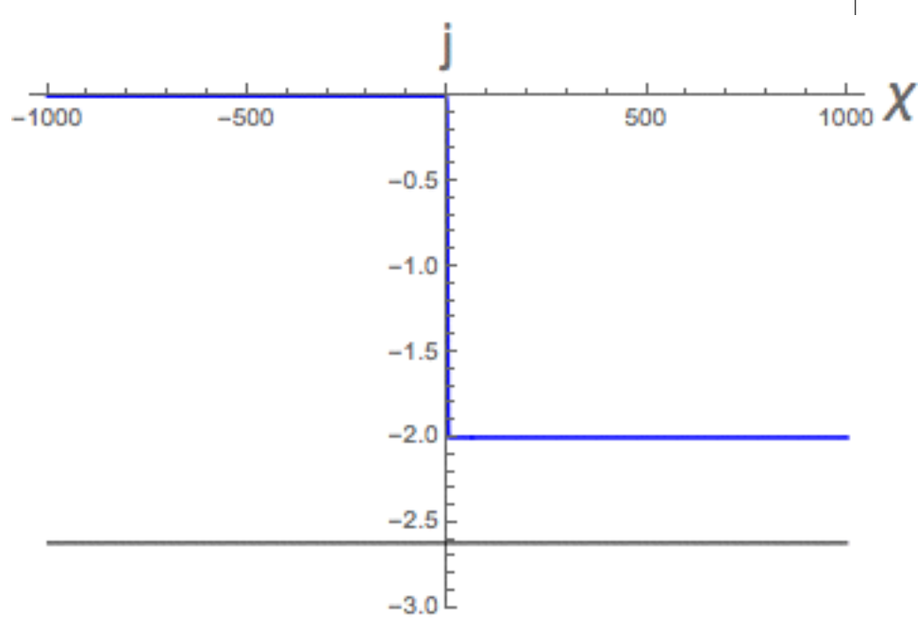
SR
SS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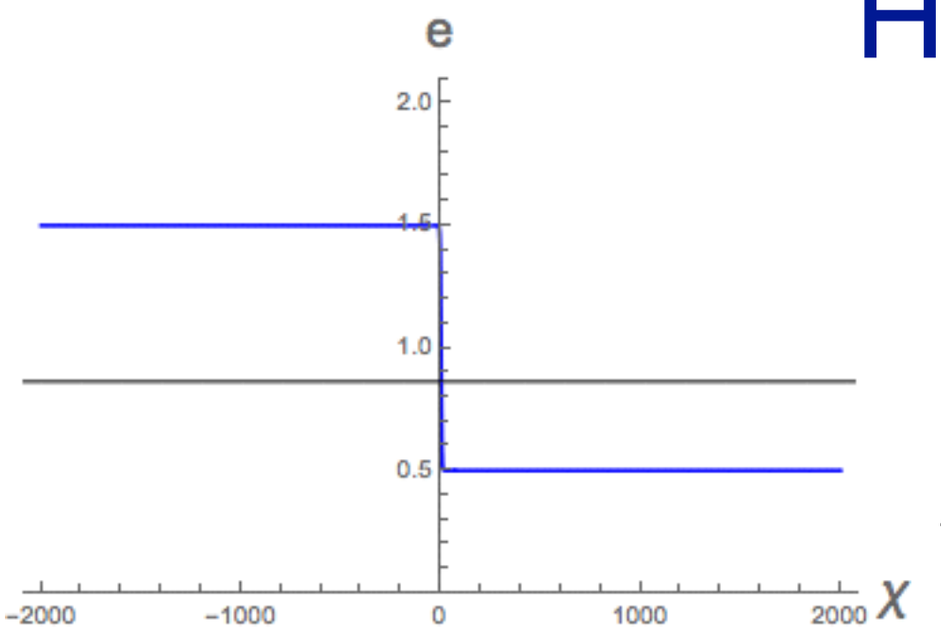
−3.0

Holography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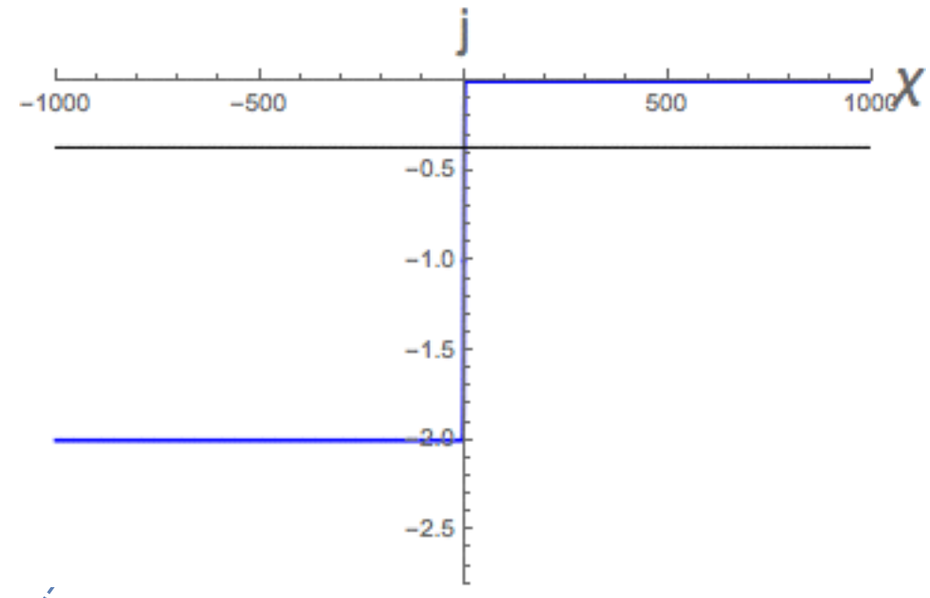
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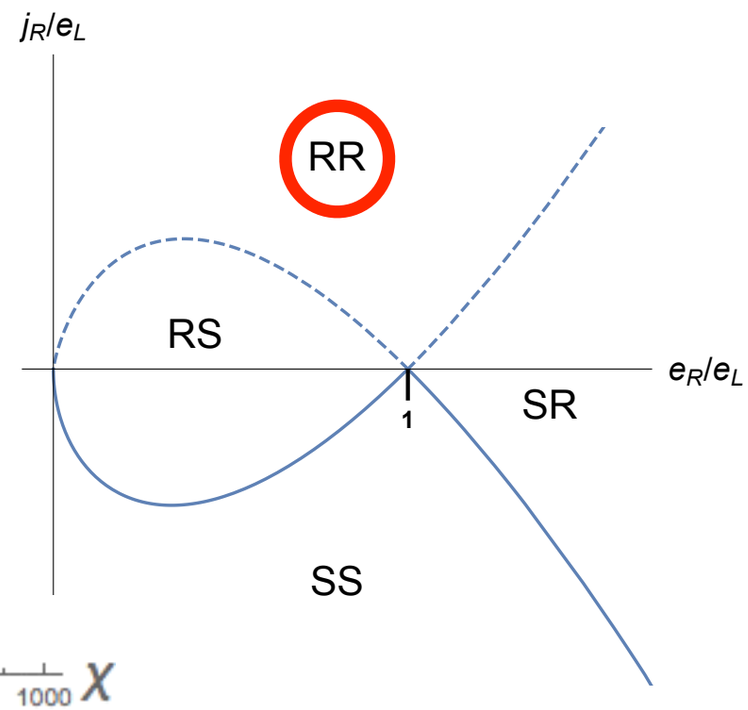
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RS
e_R/e_L
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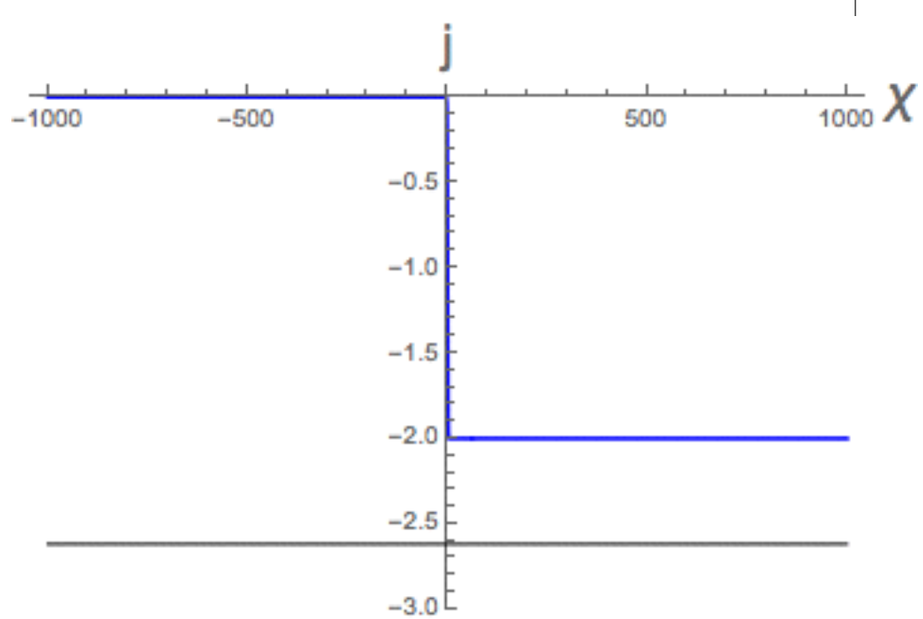
SR
SS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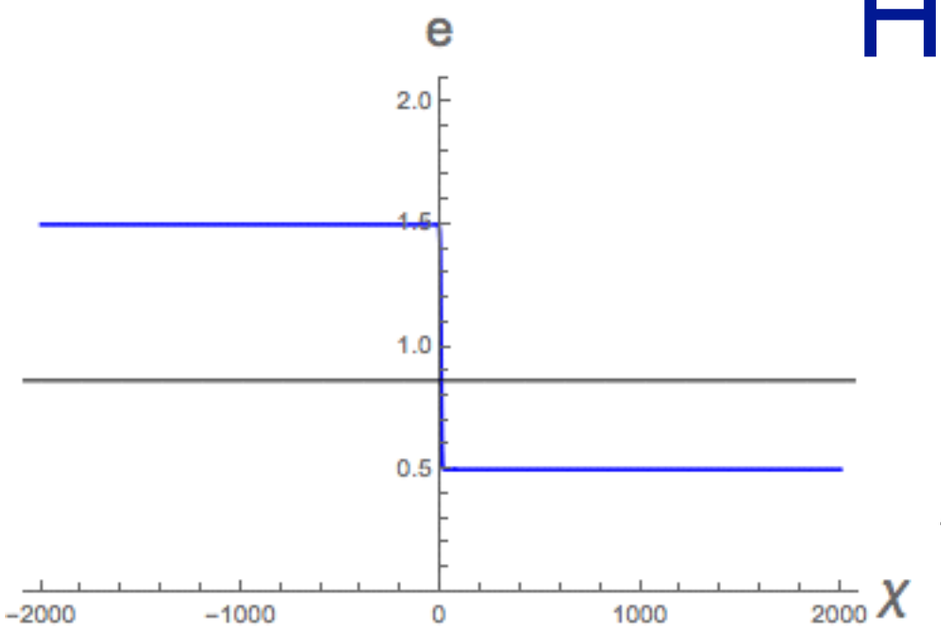
Holography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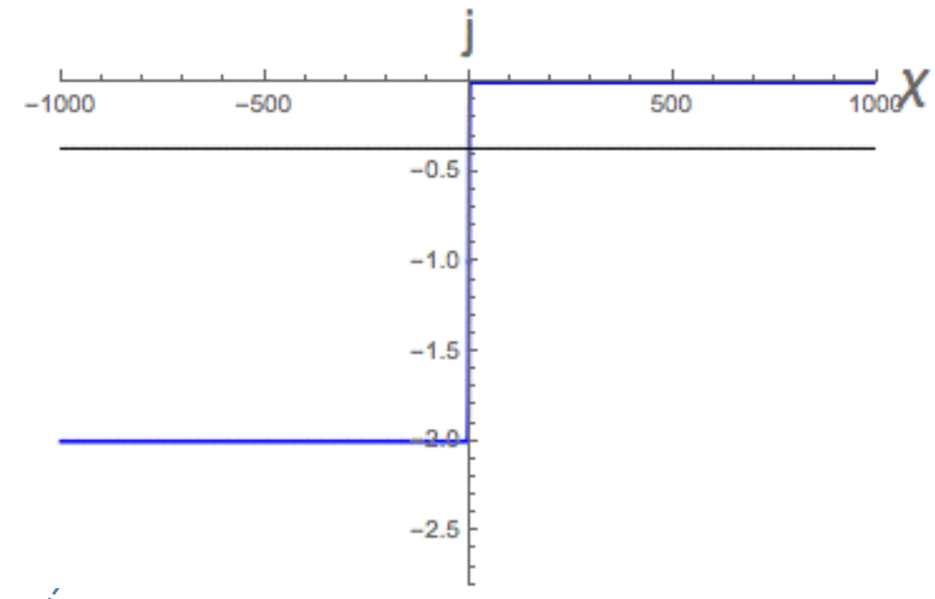
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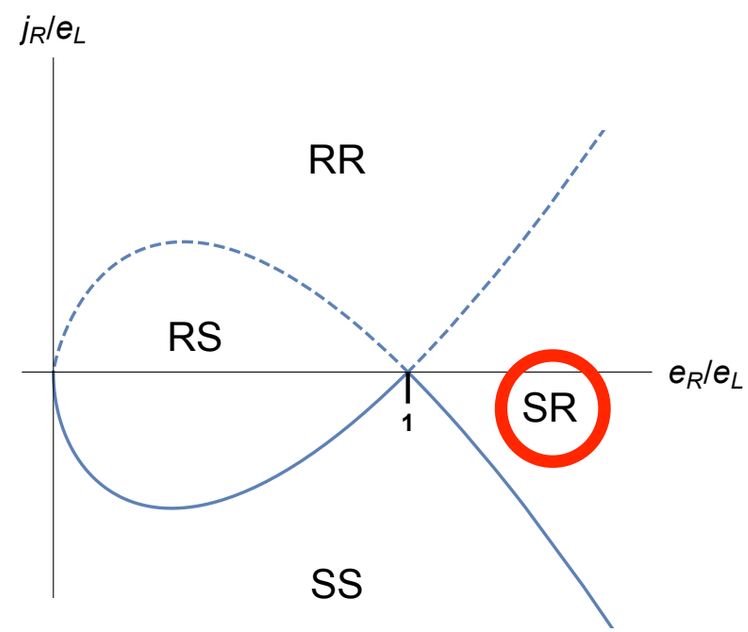
−2.5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RS
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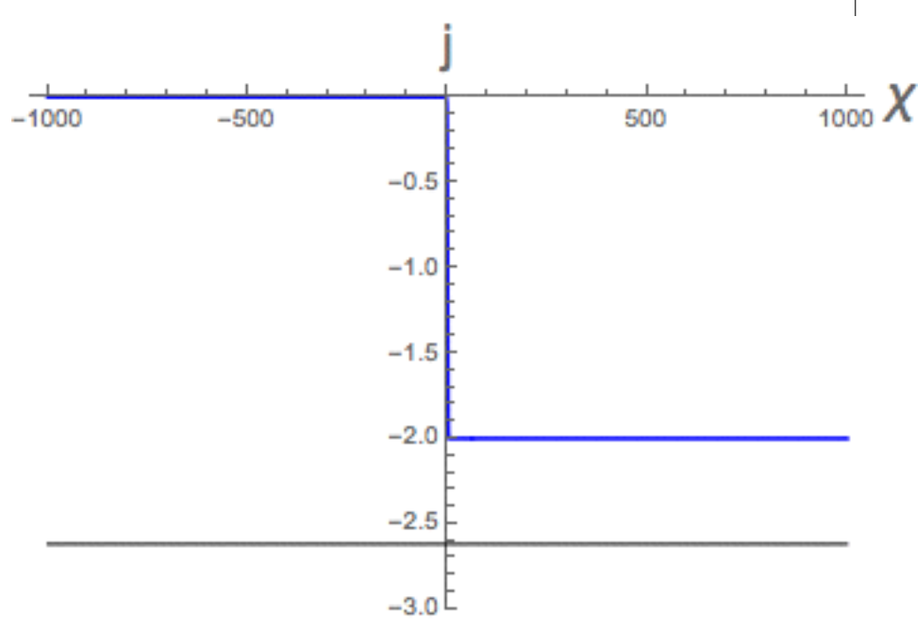
SR
SS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−2.5
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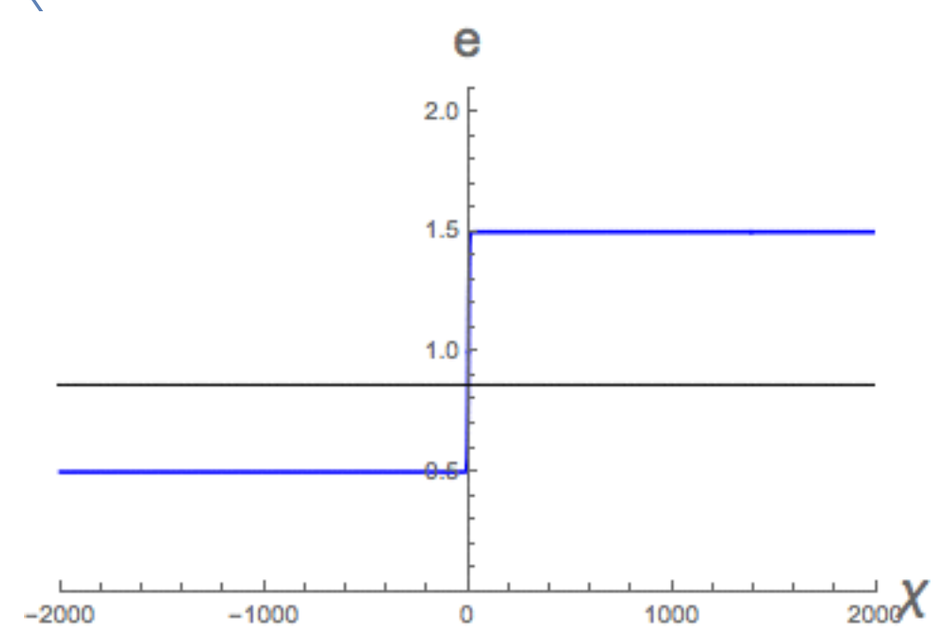
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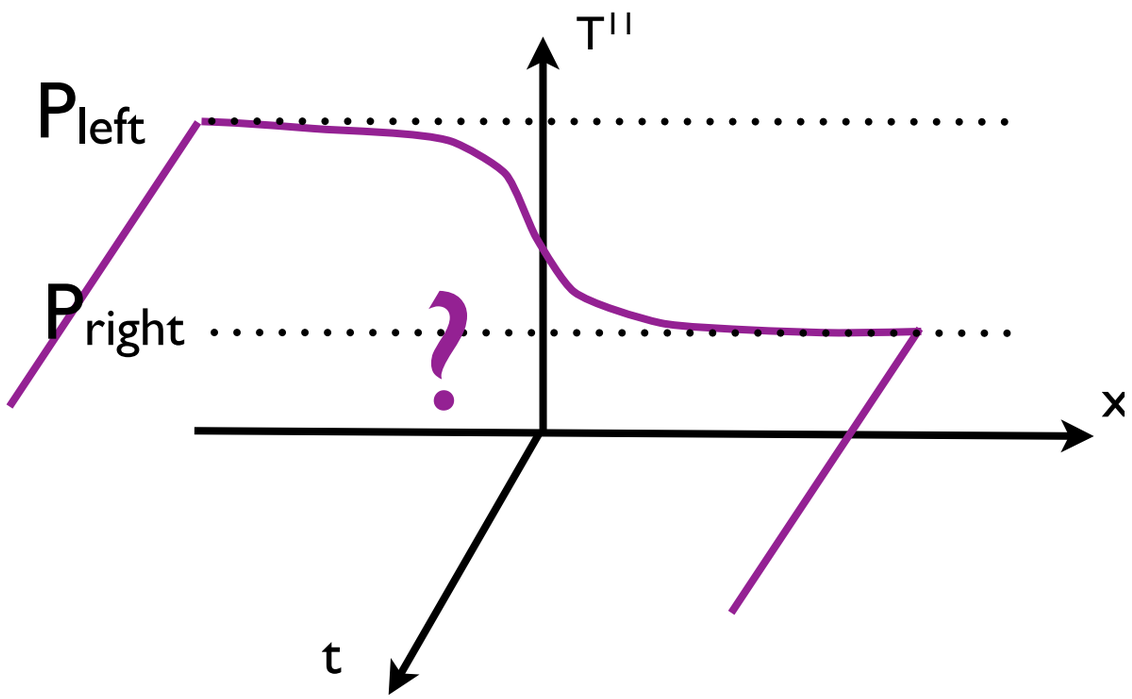
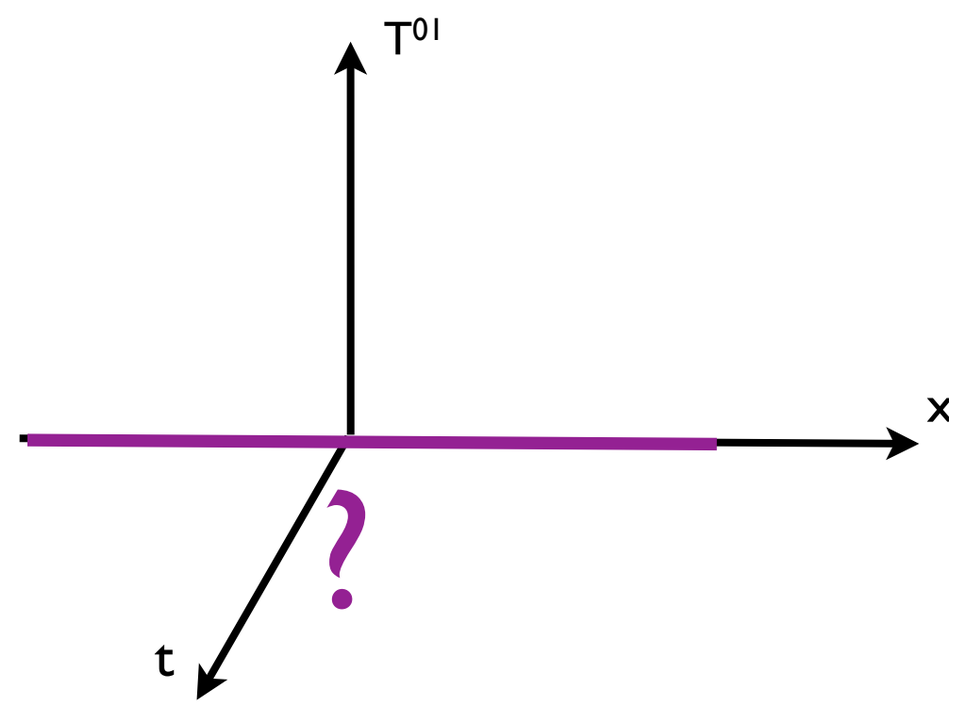
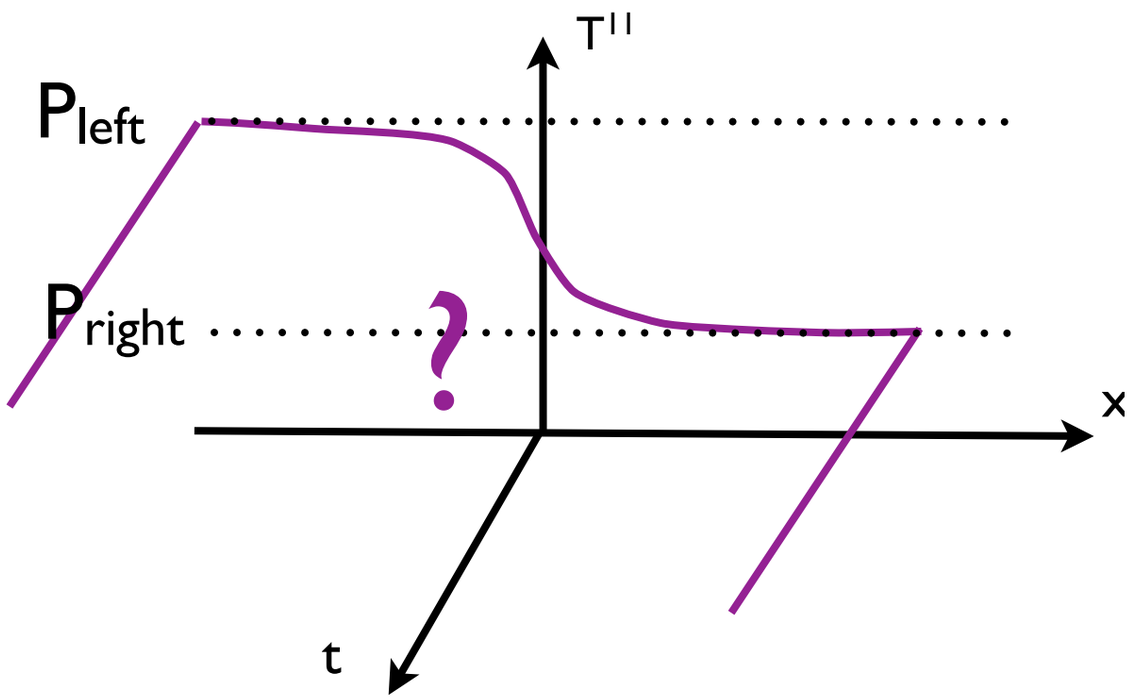
1000
2000
X

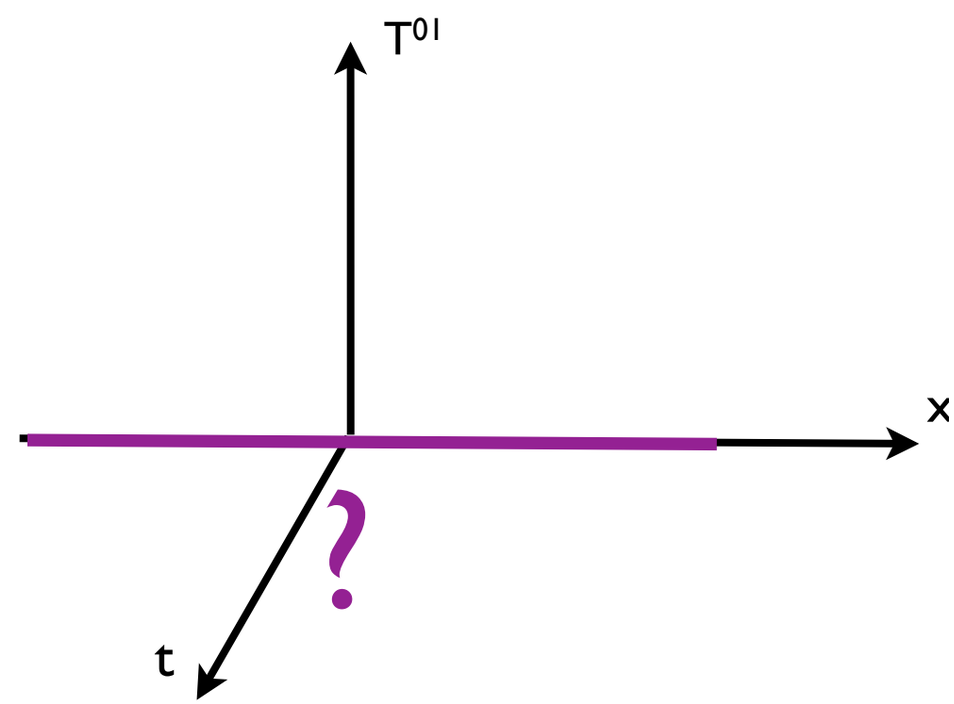
Summary





Summary





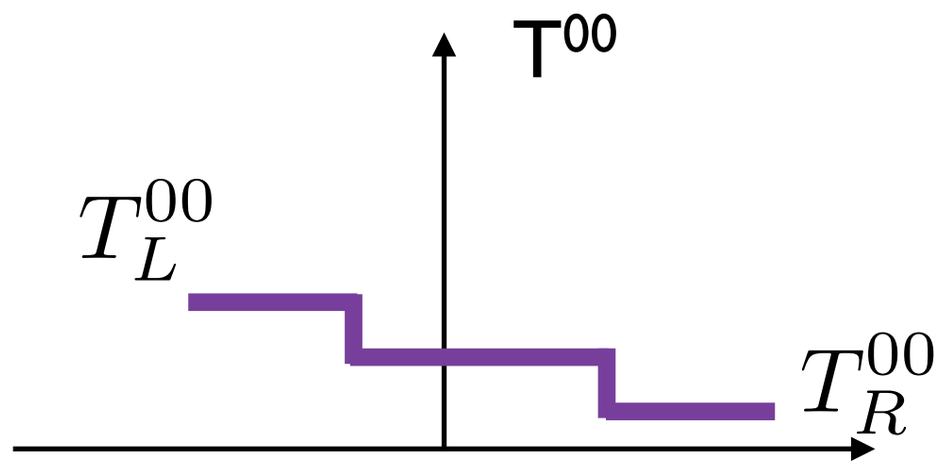
In a 2d CFT we find

$$T^{00} = T_+(\infty) + T_-(-\infty) = \frac{1}{2}\left(P_{\text{left}} + P_{\text{right}}\right) \ ,$$

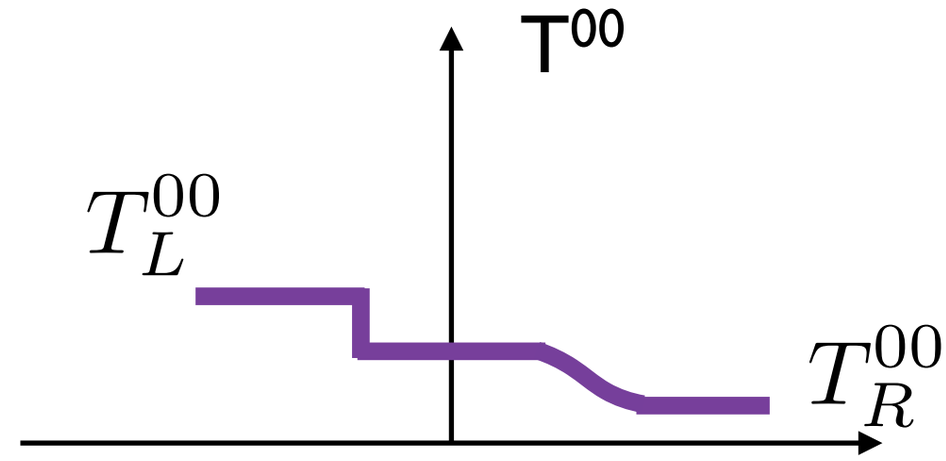
$$T^{01} = T_-(-\infty) - T_+(\infty) = \frac{1}{2}\left(P_{\text{left}} - P_{\text{right}}\right)$$

d>2

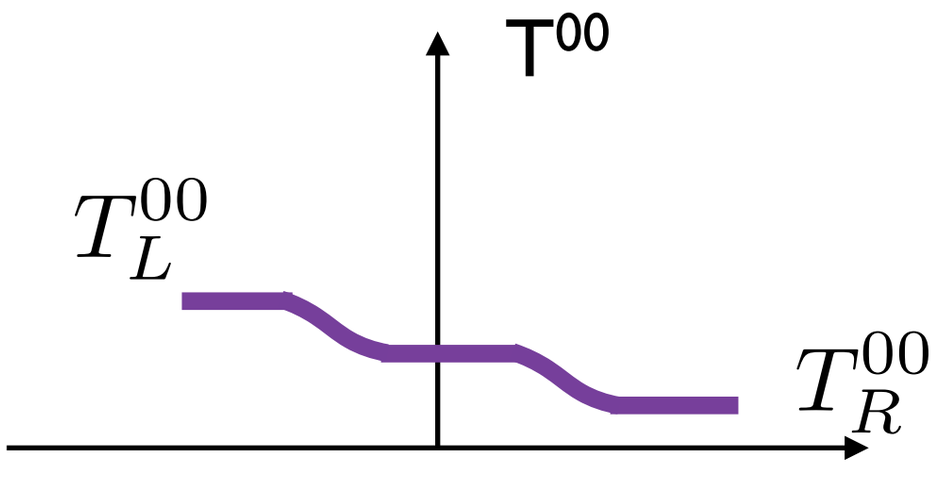
We have 4 possibilities:



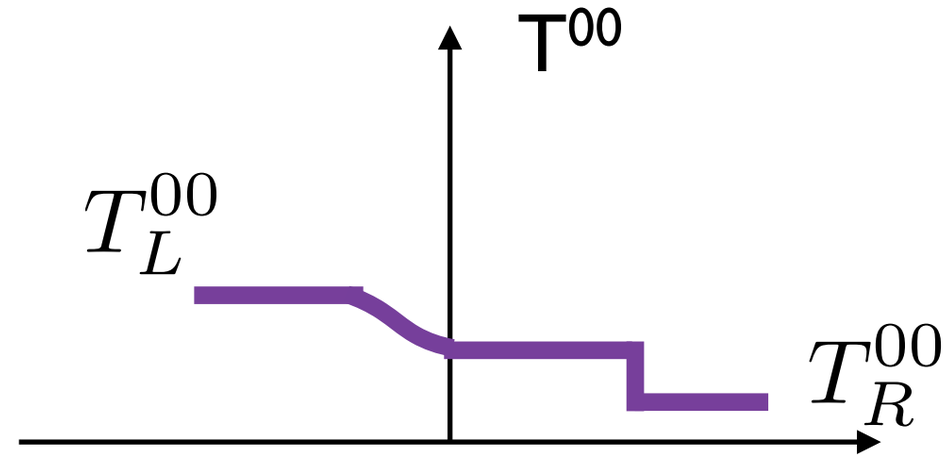
(SS)



(SR)

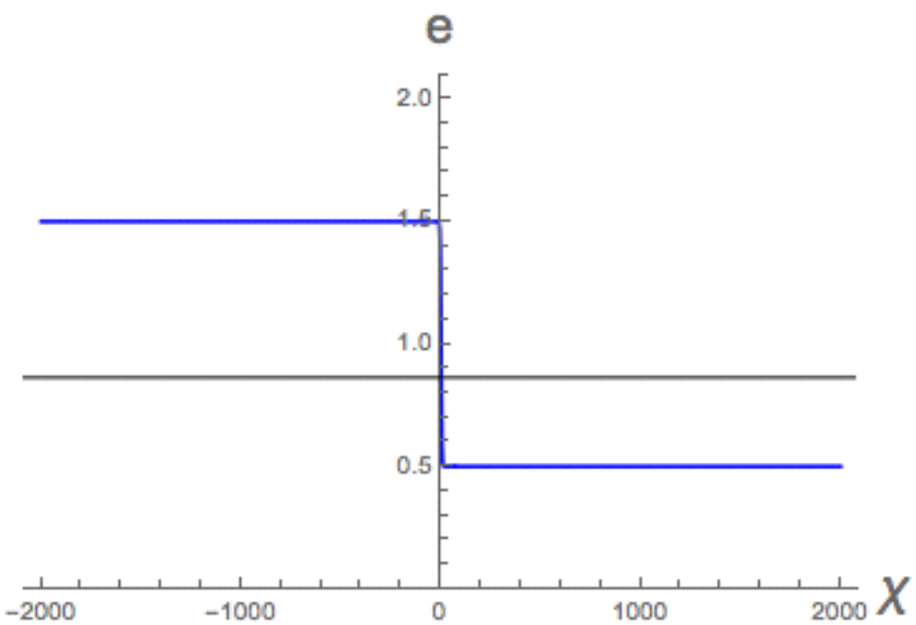


(RR)



(RS)

Large d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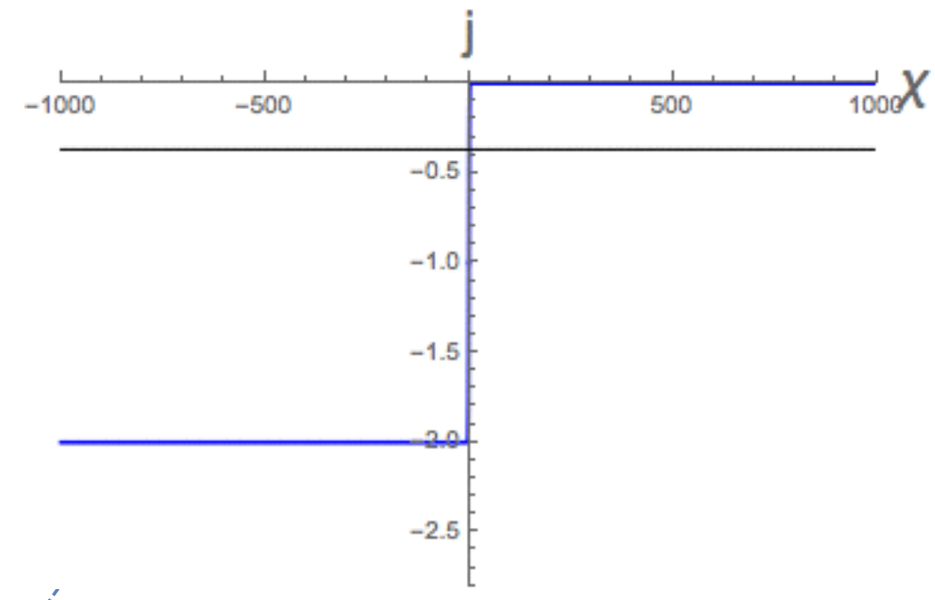
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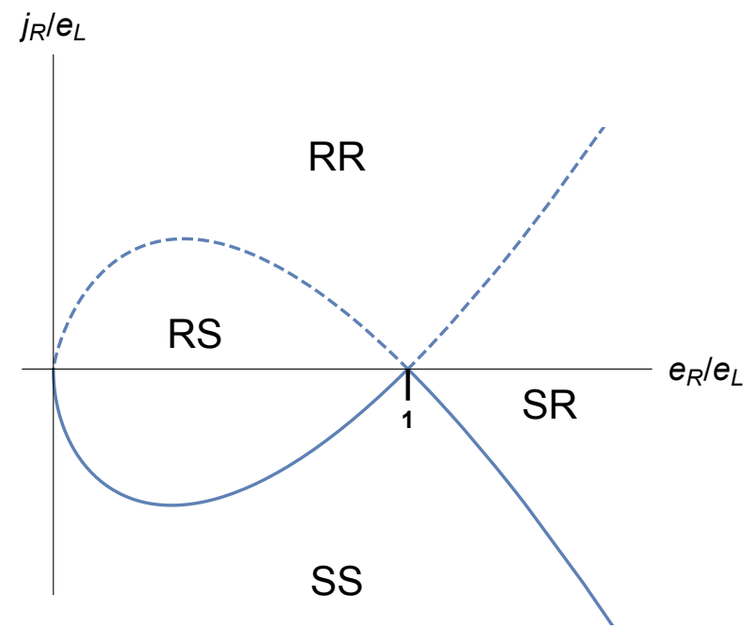
j_R/e_L
RR
RS
e_R/e_L
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SR
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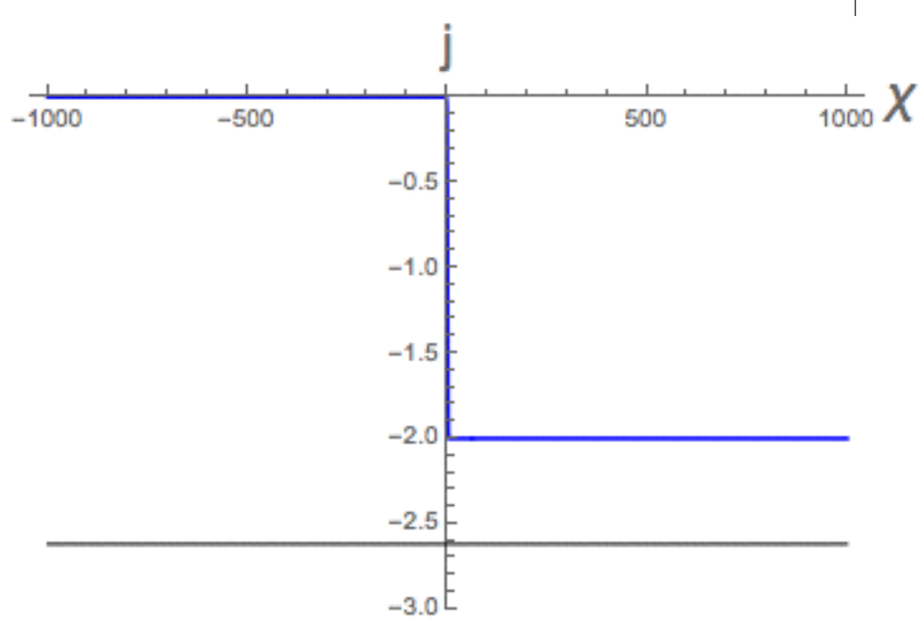
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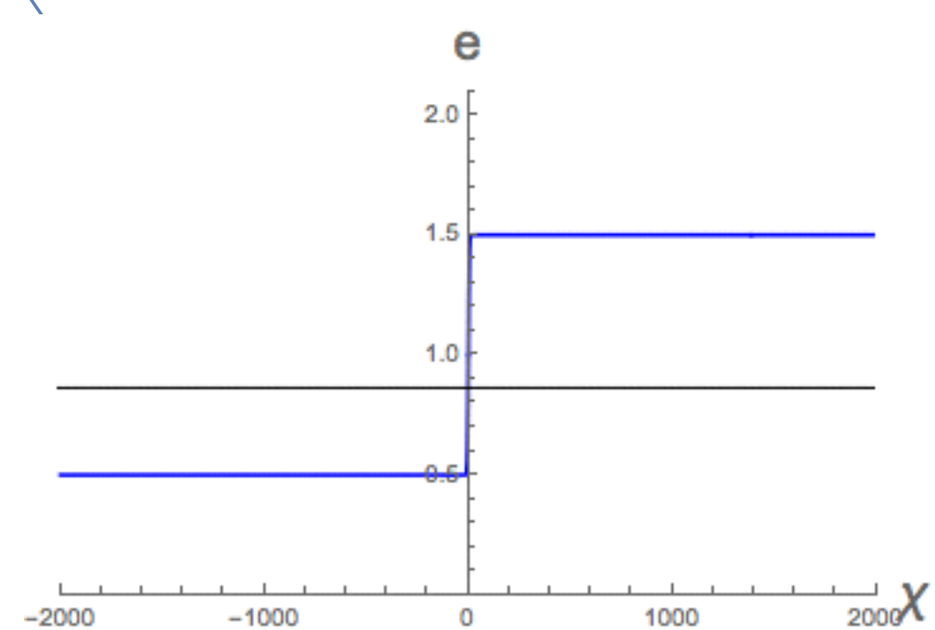
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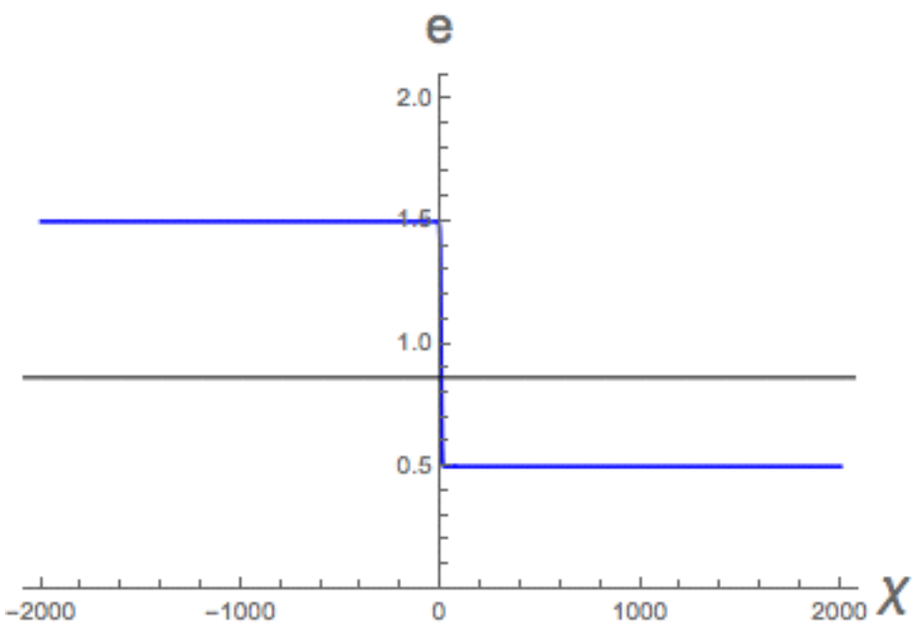
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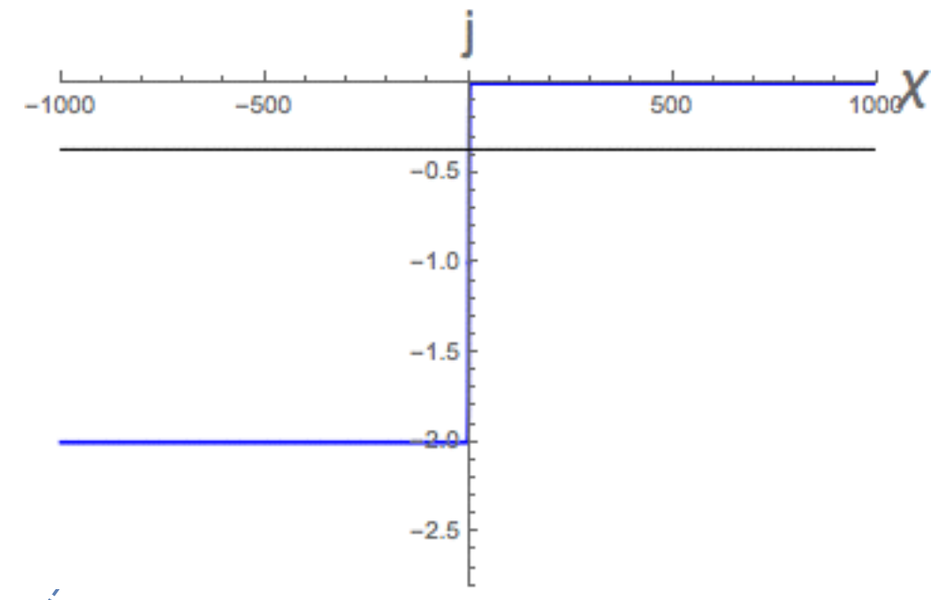
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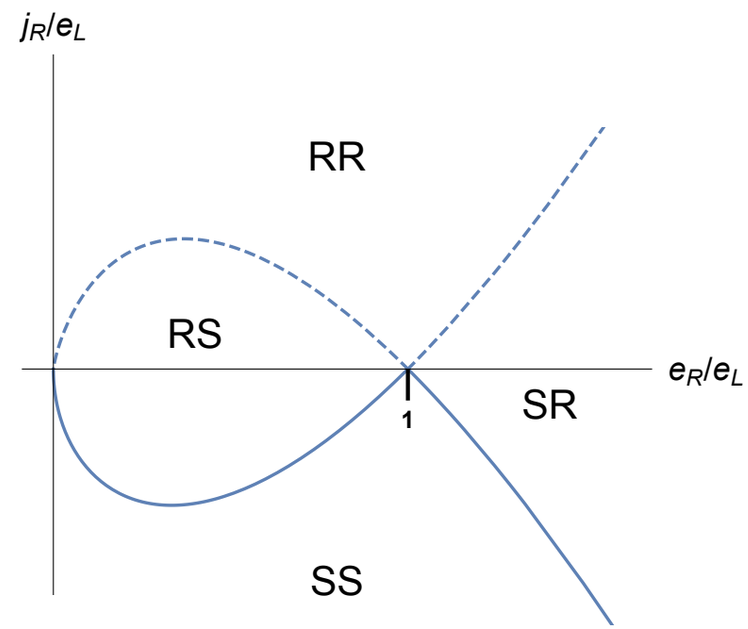
−2.5


j_R/e_L
RR
RS
e_R/e_L
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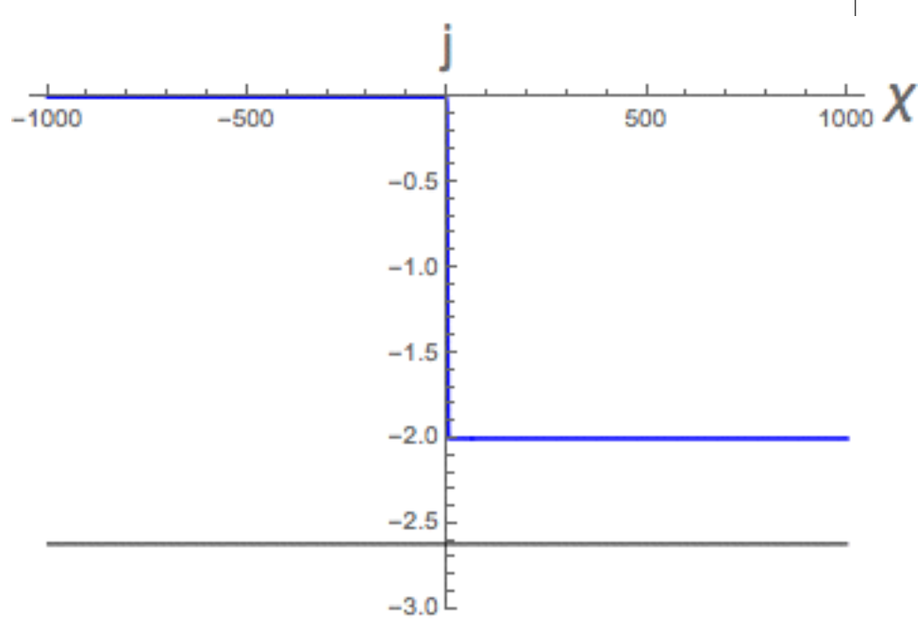
SR
SS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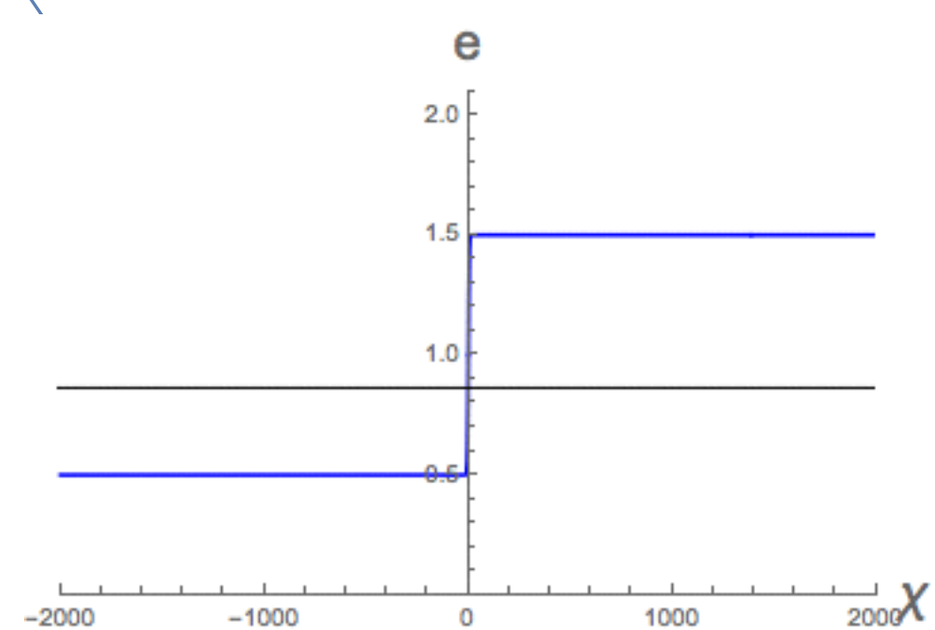
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Thank you