# Scale Invariance and Quantum Hydrodynamics in Expanding Strongly Interacting Fermi Gases

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# JETLab Group

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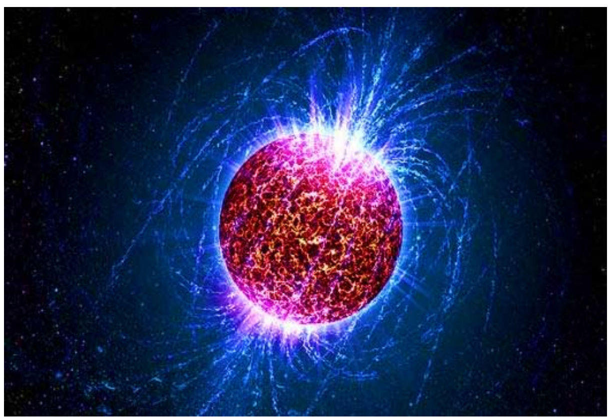
# Outline

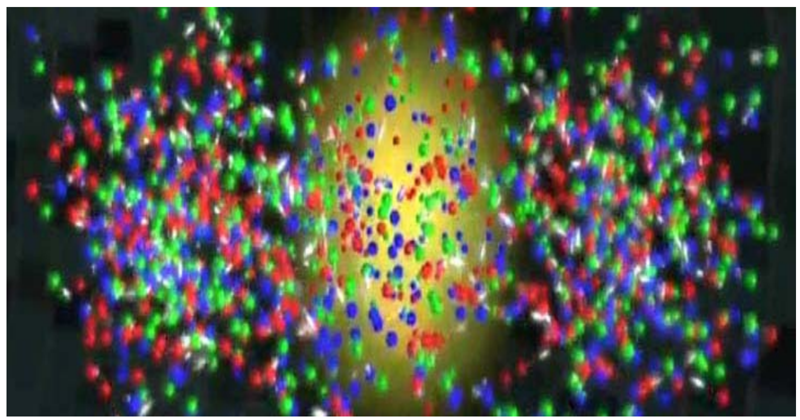
- *Introduction: Optically trapped Fermi gases:*
  - Creating a strongly interacting Fermi gas
  - Universal energy and entropy, *Quantum viscosity, KSS* conjecture
- *Scale Invariance in Expanding Fermi gases:*
  - Defining and observing scale invariant expansion: *"Ballistic"* flow of a *Hydrodynamic* gas
  - Observation of conformal symmetry breaking
  - Vanishing *Bulk* viscosity
- *Searching for Perfect fluids*
  - Measuring *Shear* viscosity on and off resonance
  - Comparison with the KSS bound
- *Future Prospects*

# Why Study Strongly Interacting Fermi Gases?

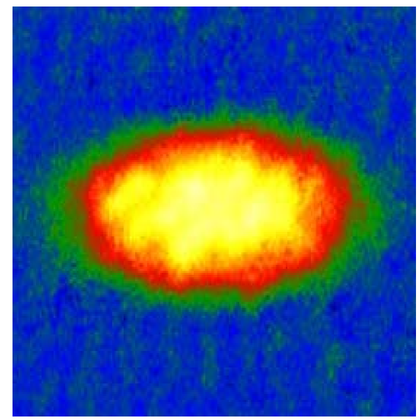
## Strongly Interacting Fermionic Systems



Neutron Star



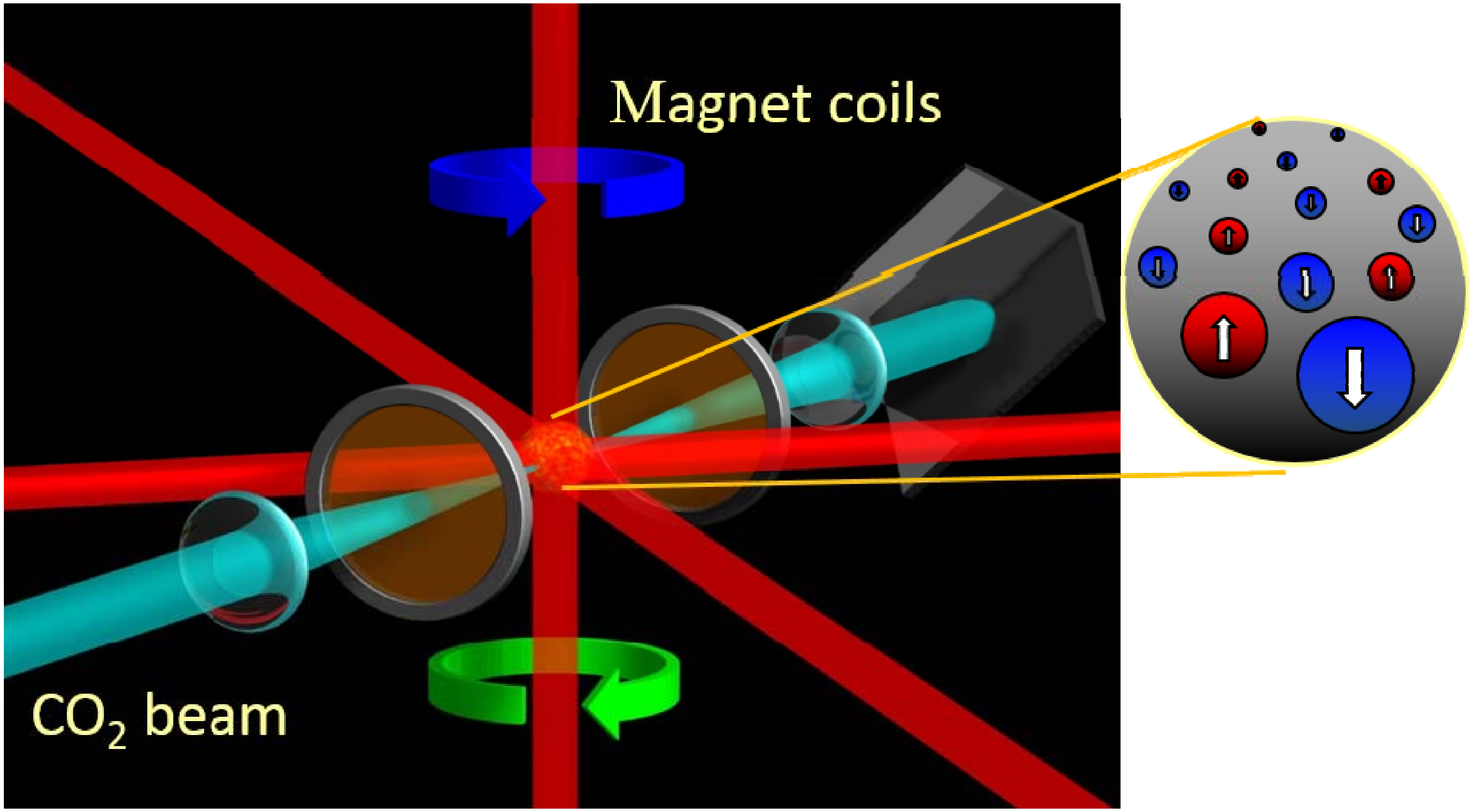
Quark Gluon Plasma



Ultra-Cold Fermi Gas

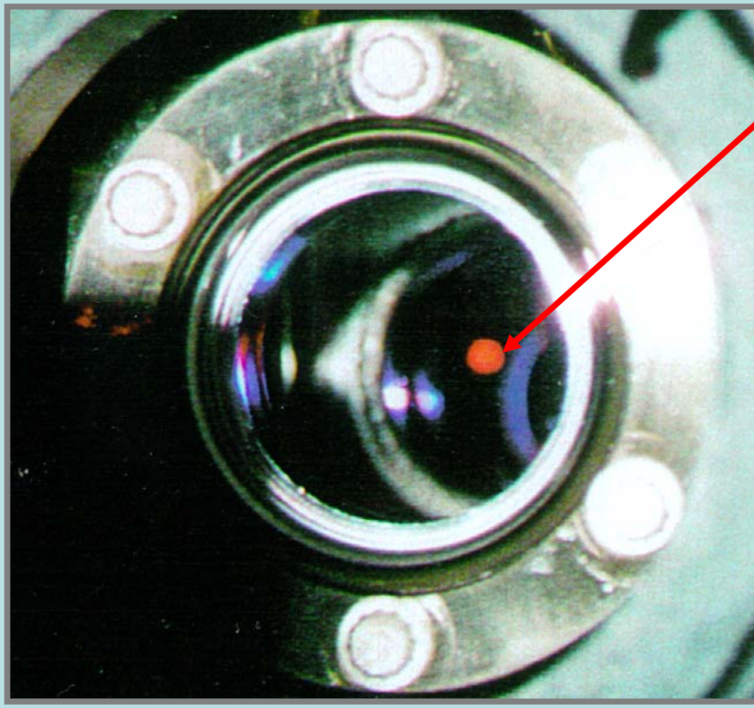
**High Temperature Superconductors**

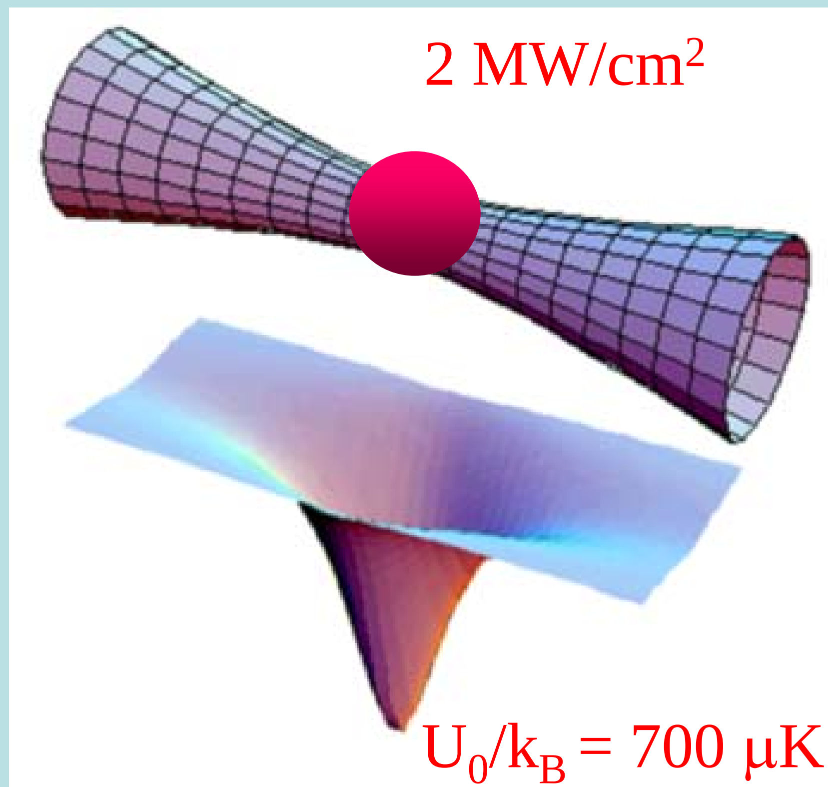
# Creating a *Scale-Invariant* Strongly-Interacting Fermi gas



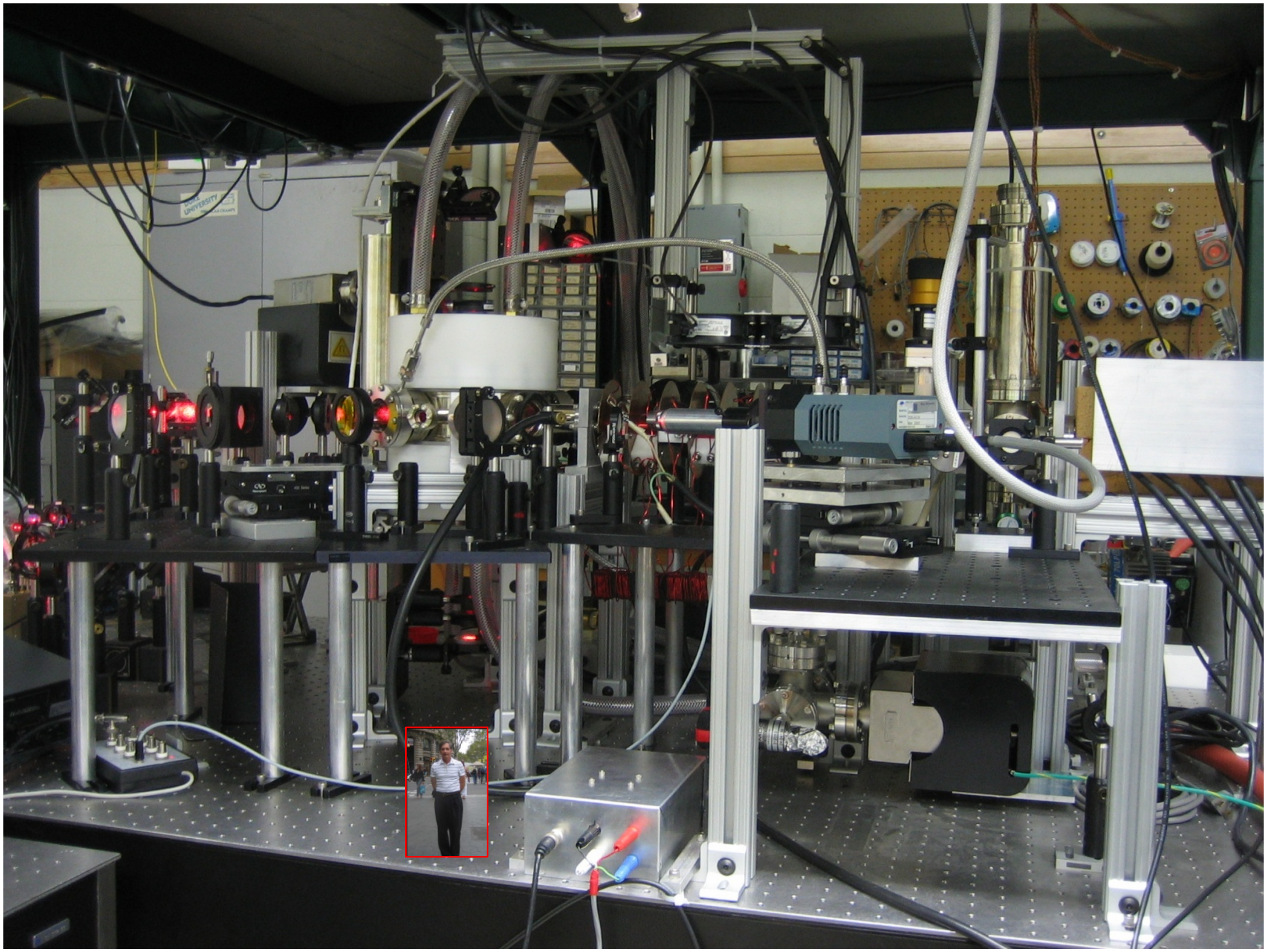
# Creating a *Scale-Invariant* Strongly-Interacting $^6$Li Fermi gas

Atoms precooled
in a magneto-optical trap
to 150 μK

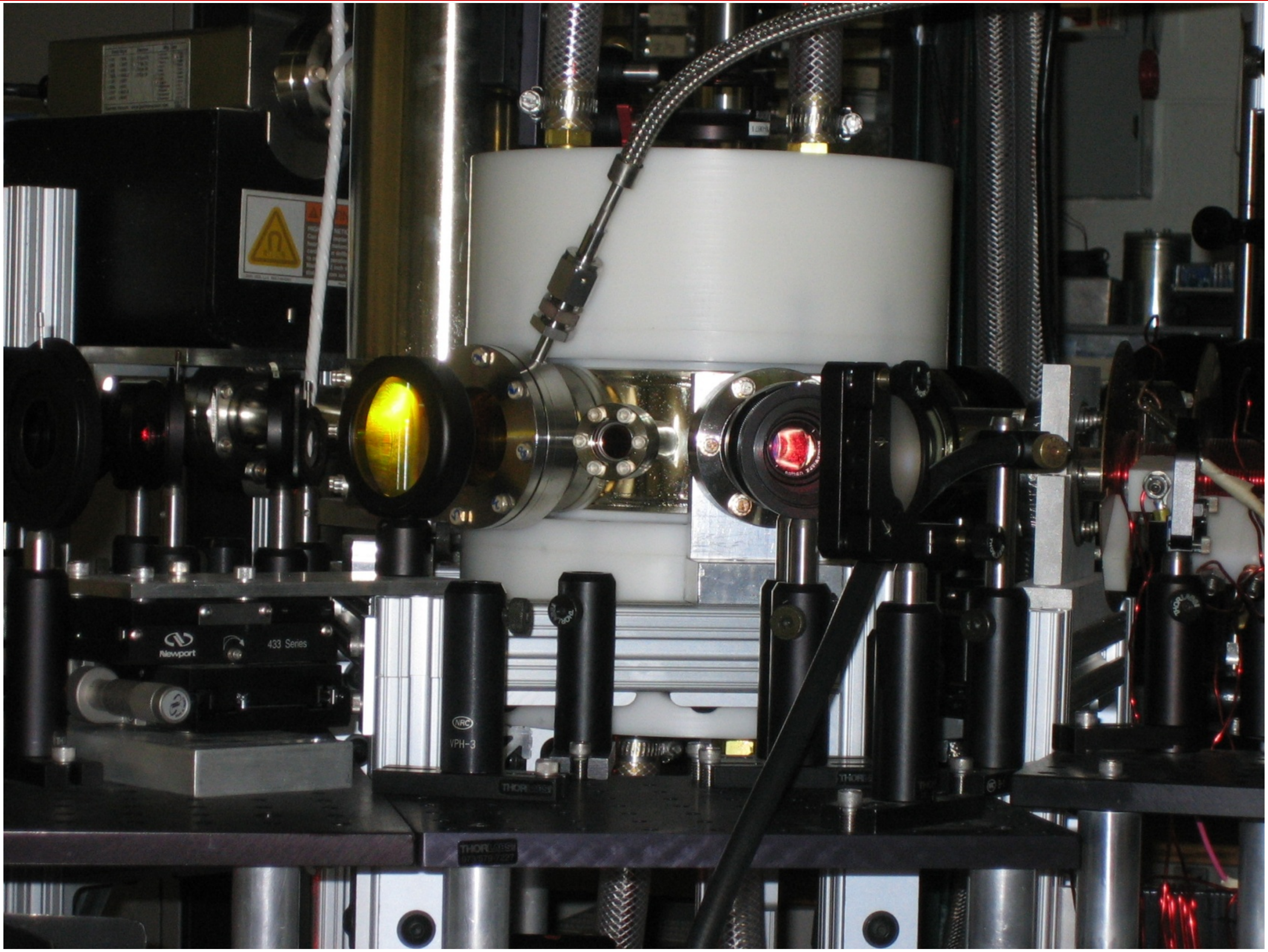




# Experimental Apparatus



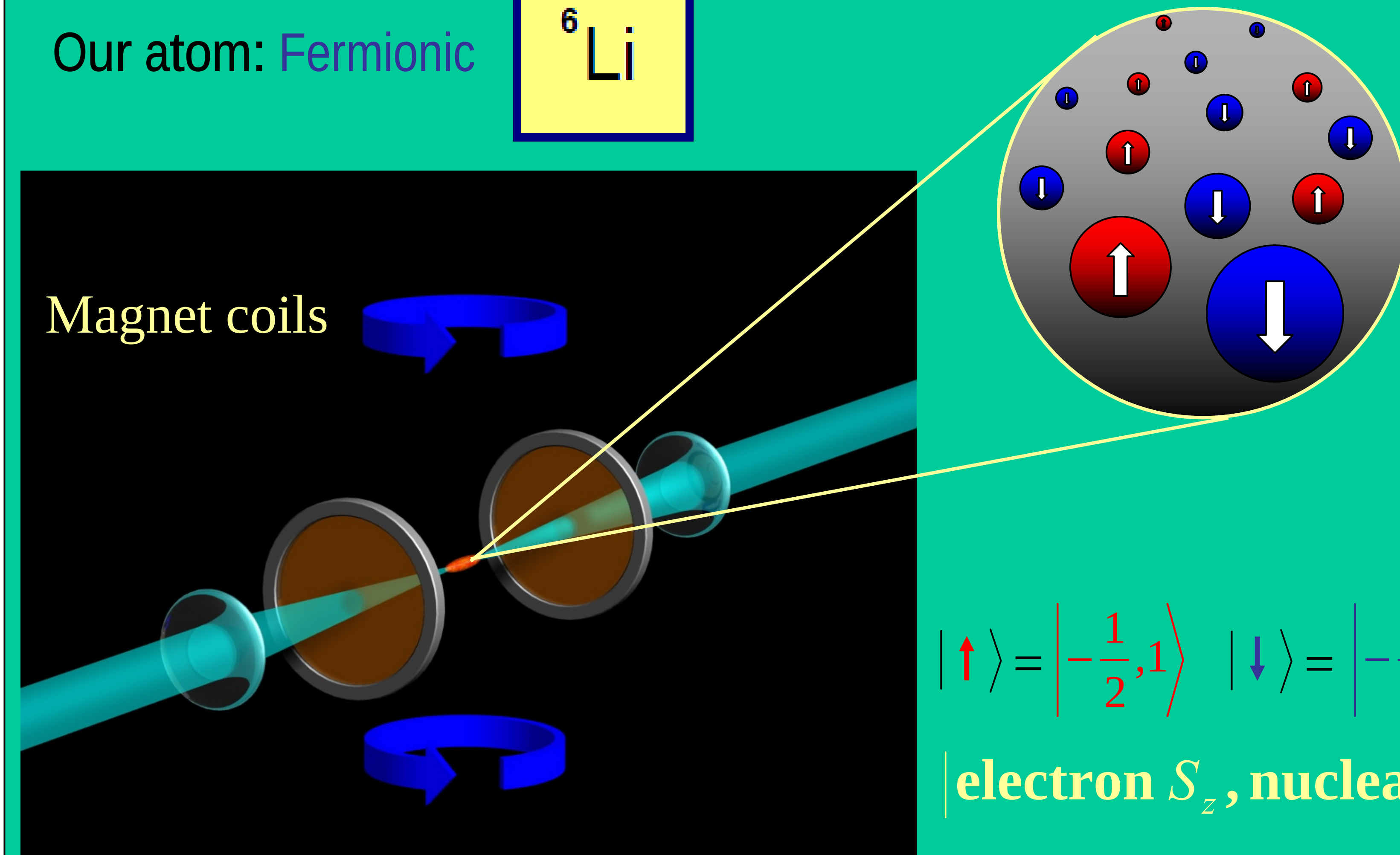
# Experimental Apparatus



# Optically Trapped Fermi Gas

Our atom: Fermionic

$^{6}\text{Li}$



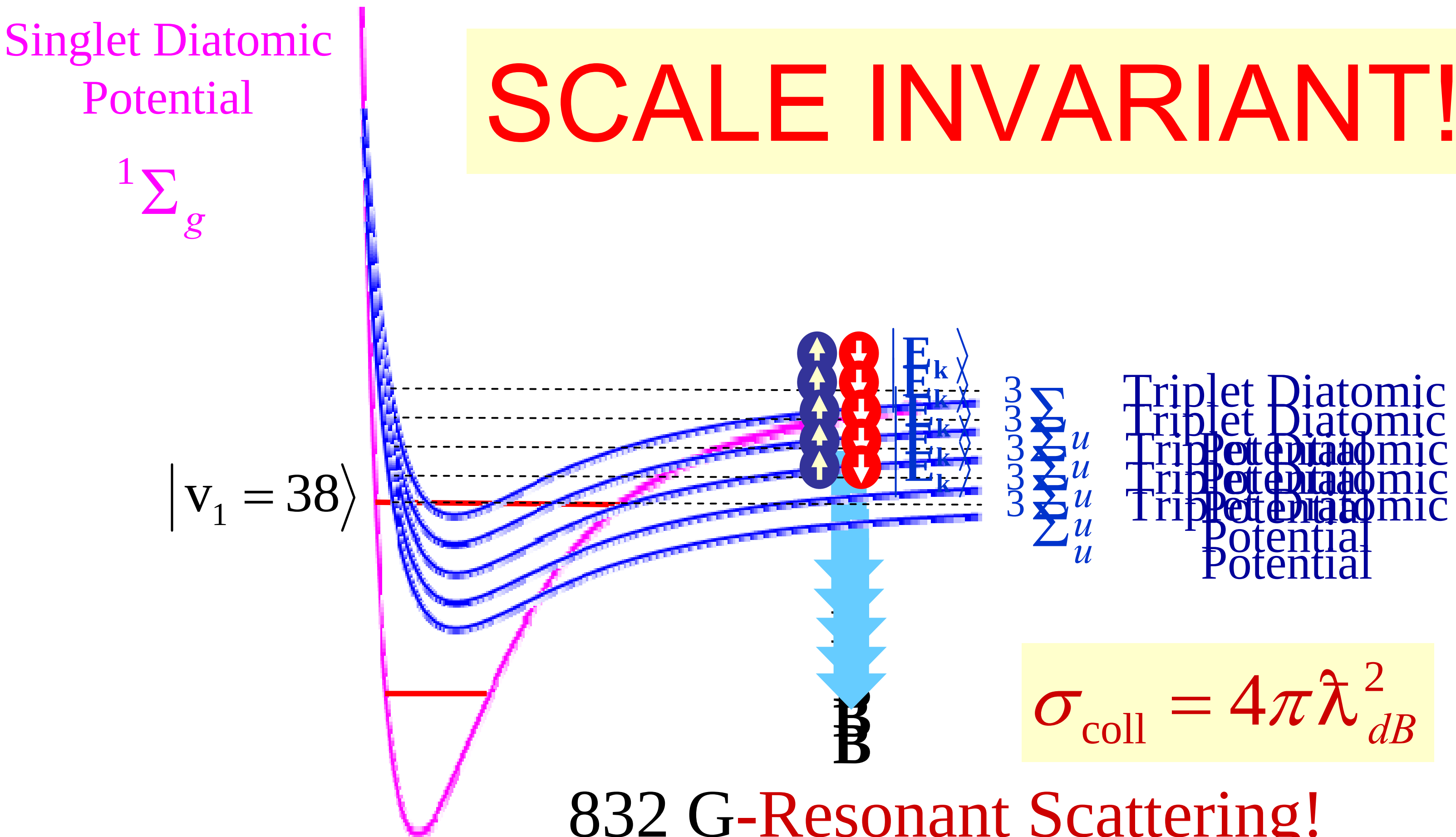

$$|\uparrow\rangle = \left|-\frac{1}{2},1\right\rangle \quad |\downarrow\rangle = \left|-\frac{1}{2},0\right\rangle$$

$$|\textbf{electron } S_z\textbf{, nuclear } I_z\rangle$$

# Feshbach Resonance

Resonant Coupling between Colliding Atom Pair – Bound Molecular State

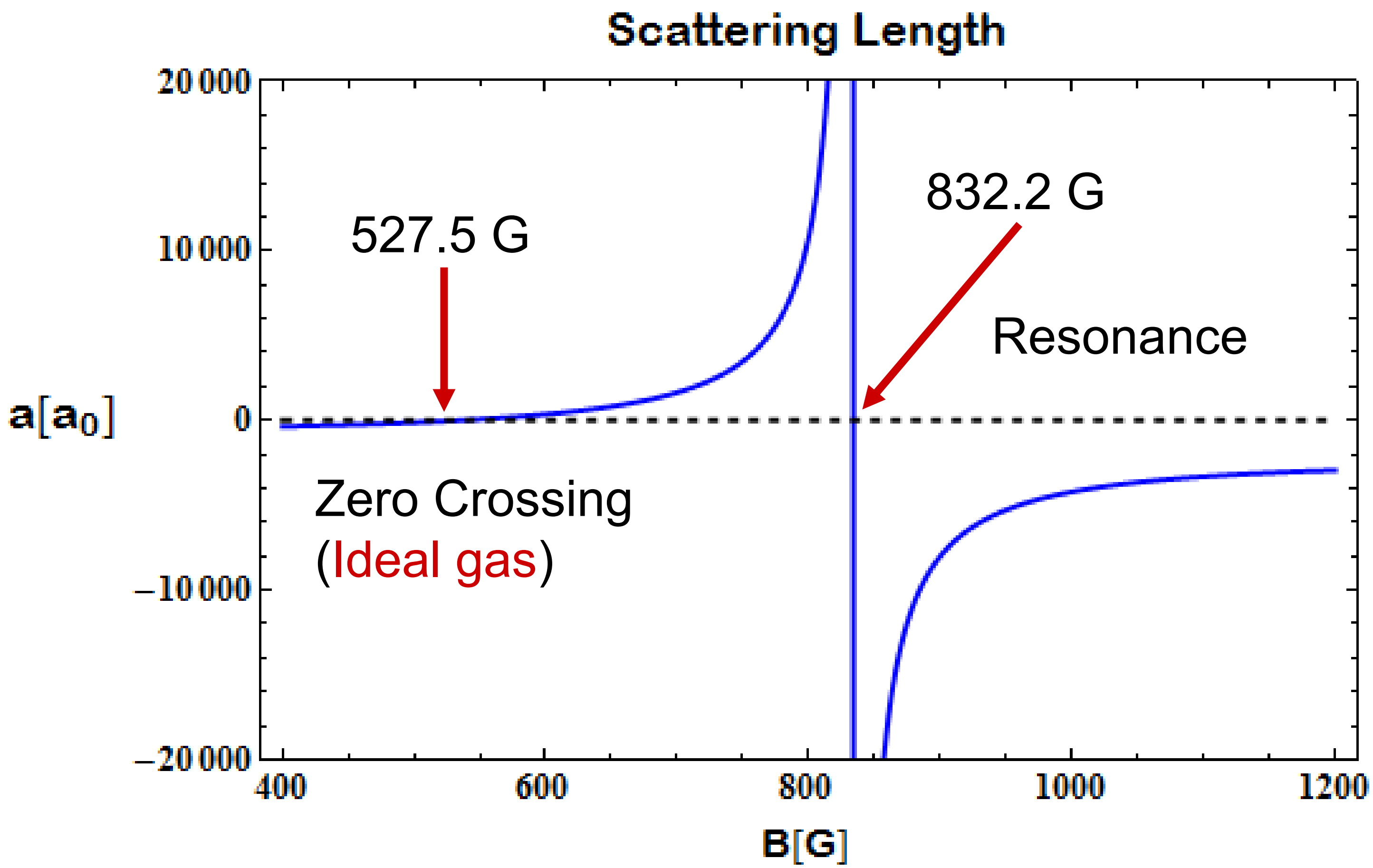
SCALE INVARIANT!




$$\sigma_{\text{coll}} = 4\pi ƛ_{dB}^2$$

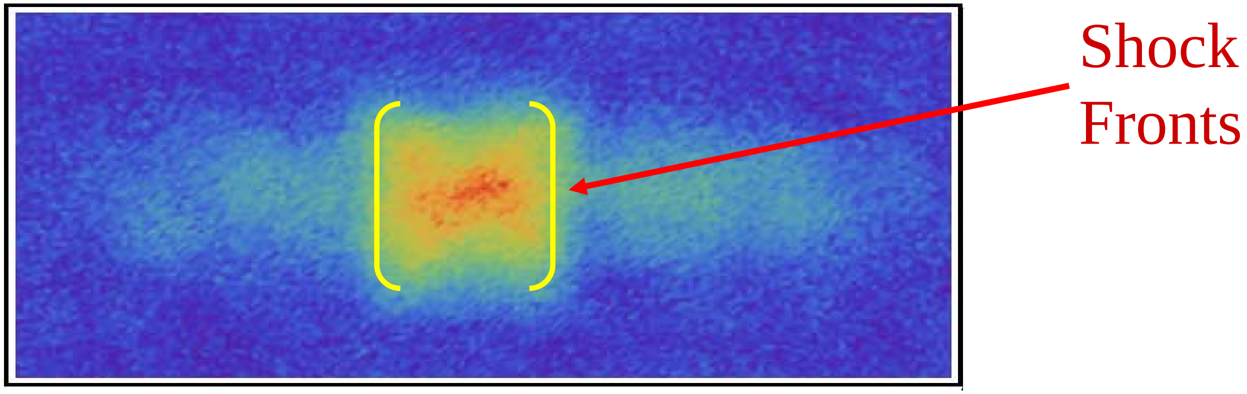
832 G-Resonant Scattering!

# Tunable Strong Interactions



# *Strong Interactions:*
# *Shock waves* in Fermi gases

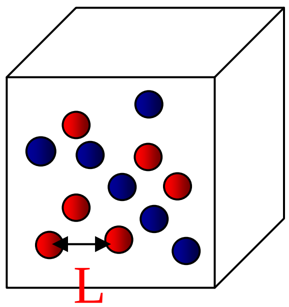
- Trapped gas is divided into two clouds with a repulsive optical potential.
- The repulsive potential is extinguished, the two clouds accelerate towards each other and collide.




Really strong interactions!

# *Universal Regime*: Natural Units

*Universal Regime:* *For resonant* scattering,
the scattering cross section is the square of the de Broglie wavelength, which is independent of the details of the collisional interactions!




Atom spacing L
becomes the *only* length scale.

*Heisenberg Uncertainty Principle:* $\Delta x \Delta p \approx \hbar$ $\quad\quad$ $\Delta p \approx p \approx \dfrac{\hbar}{L}$

Physical Properties, like Energy and Temperature have *Natural Units* determined by L.

Fermi Energy: $\dfrac{\hbar^2}{2mL^2}$

# Universal Thermodynamics

When the interparticle spacing sets the scale of energy and temperature, the pressure p is a function only of density n and temperature T:

$$p(n,T)$$

Using elementary thermodynamics, one then can show that

$$\boxed{p = \frac{2}{3}\varepsilon}$$

$\varepsilon =$ energy density

(Ho, 2004)

This elementary result has several amazing consequences.

# Global energy E measurement

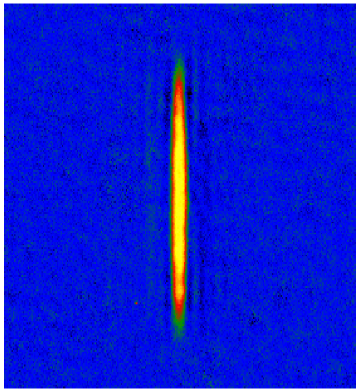
***Universal* Gas obeys the Virial Theorem**

$$\boxed{E = \langle U \rangle + \tfrac{1}{2}\langle \mathbf{r}\cdot\nabla U \rangle}$$

Thomas (2005)
Castin (2004)
Werner and Castin (2006)
Son (2007)

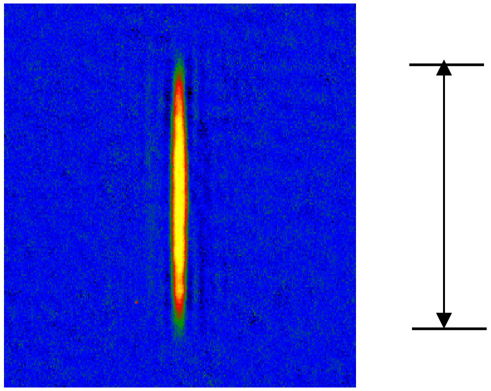
**In a HO potential:** $E = 2\langle U \rangle$

**Energy per particle** $\boxed{\mathrm{E} = 3m\omega_z^2\langle z^2 \rangle}$



For a *universal* quantum gas, the energy E is determined by the *cloud size*

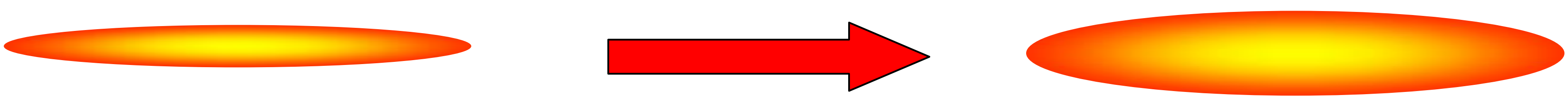
# Measuring the Energy E and Entropy S



For a *universal* quantum gas,
the energy E is determined
by the *cloud size*

For a *weakly interacting* quantum gas
the entropy S can always be determined
from the *cloud size*  (textbook problem)

Experiment

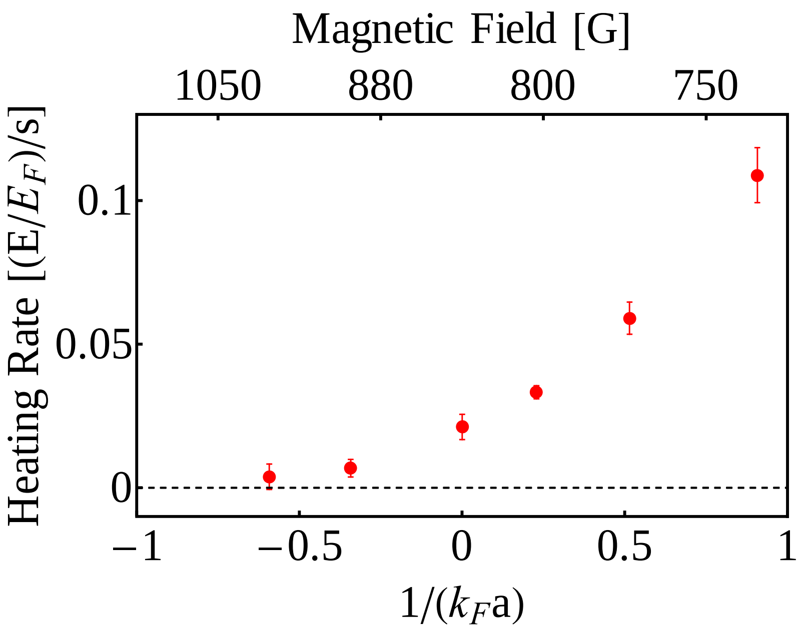


**Start: 832 G**
**Universal**
**Strongly interacting**

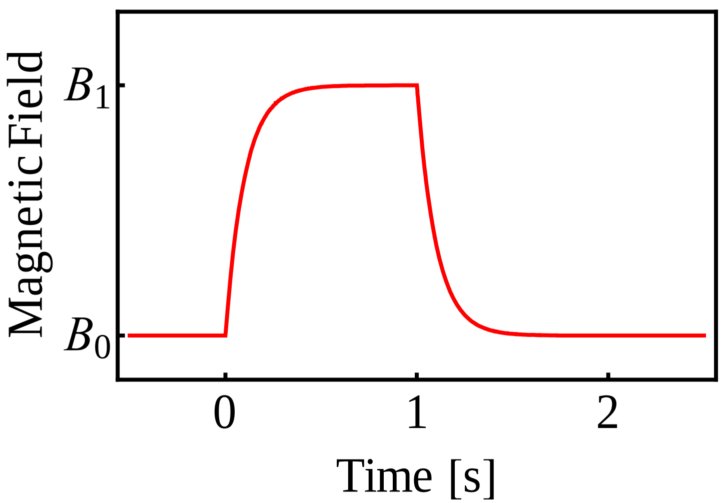
*Sweep* magnetic field

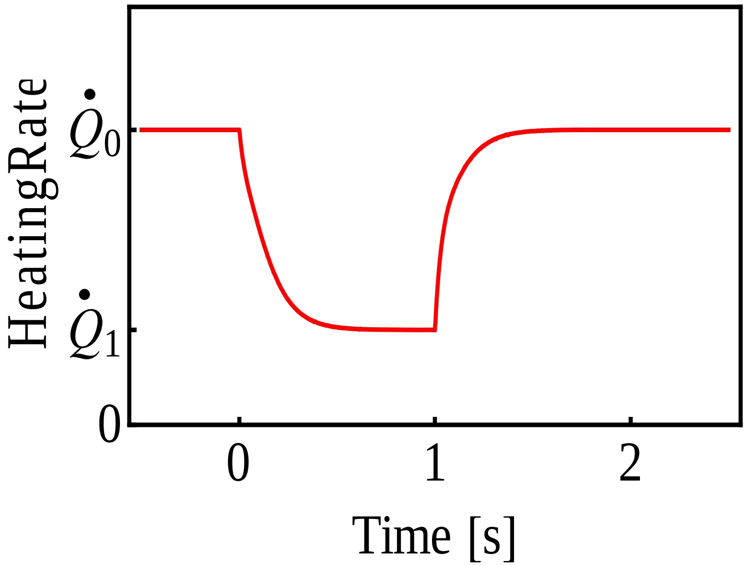
**End: 1200 G**
**Weakly interacting**

# Is the B-Field Sweep Adiabatic?

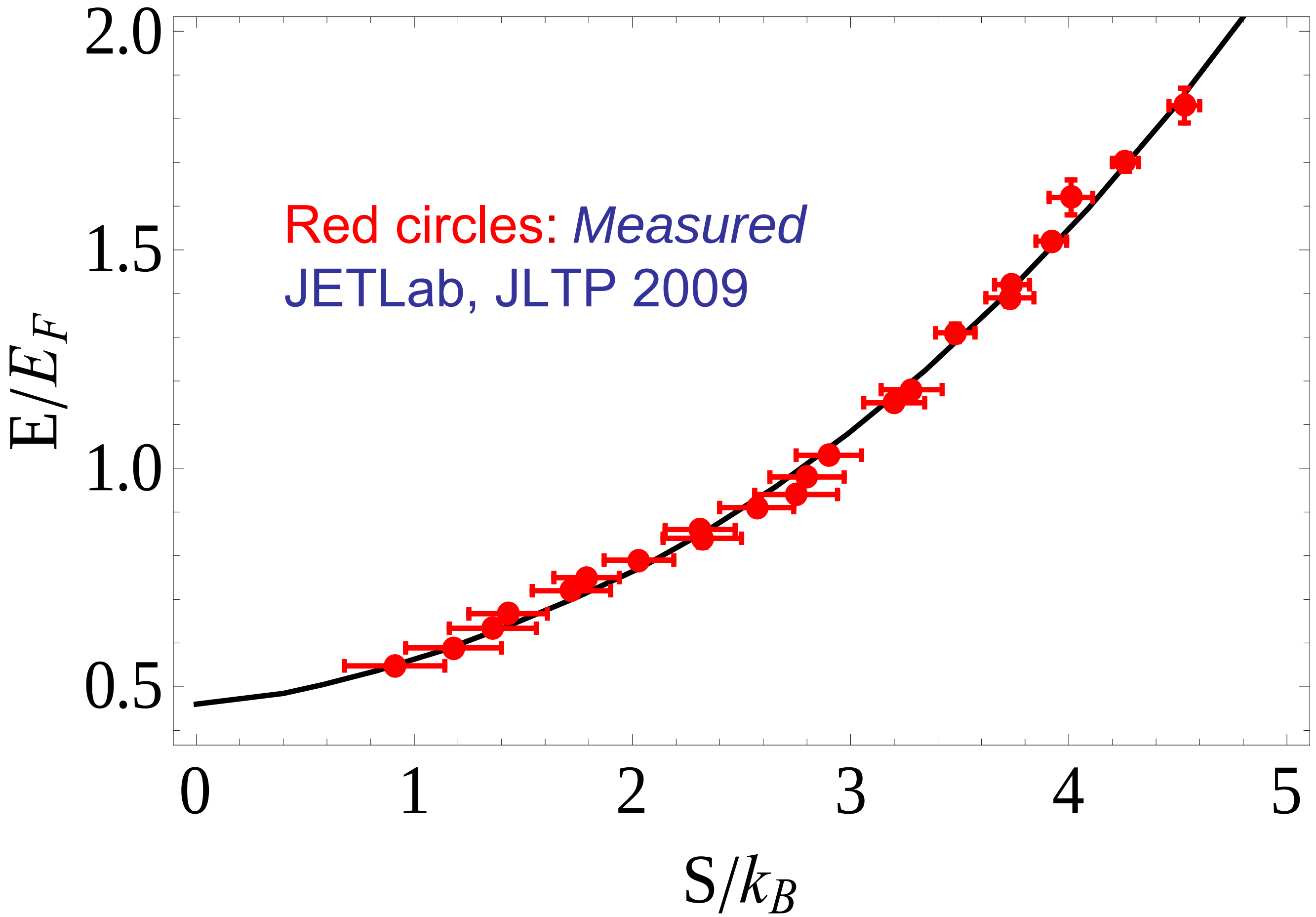



| $B_0$ | $B_1$ | ΔE measured | $\int Q\,dt$ |
|---|---|---|---|
| 832 G | 770 G | 0.081(8) | .077 |
| 832 G | 800 G | 0.053(6) | .054 |
| 832 G | 900 G | 0.028(3) | .029 |



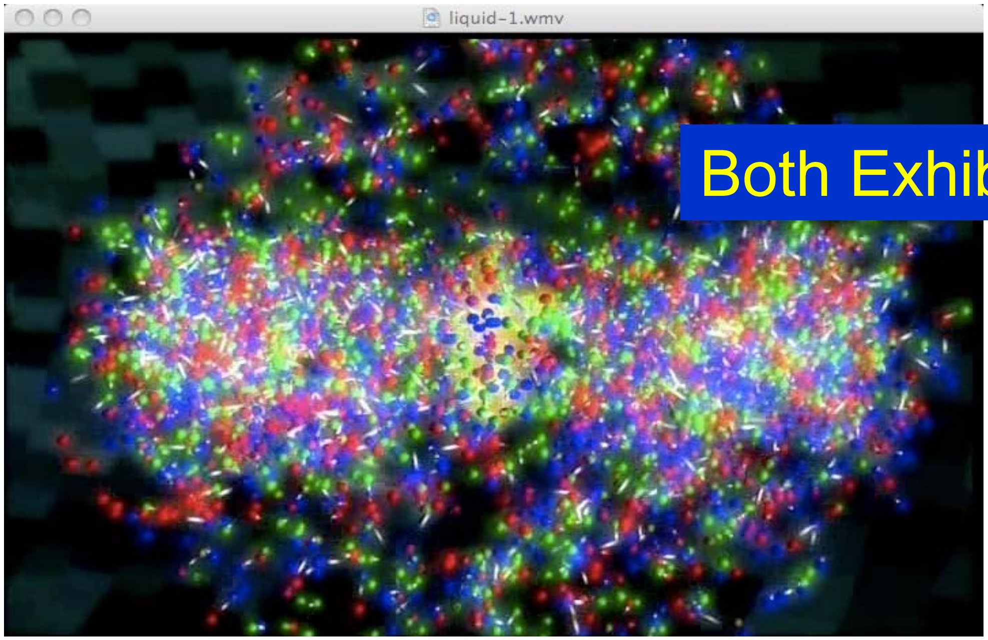
# Energy per particle versus Entropy per Particle



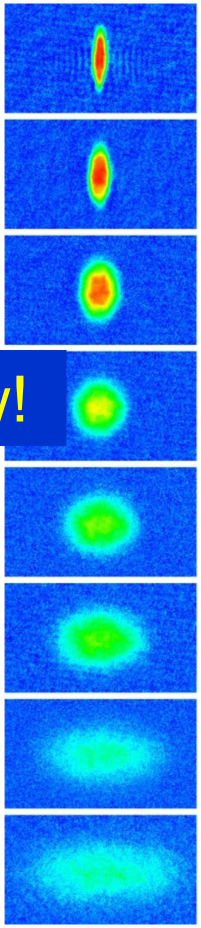

Solid line—from measured equation of state: Ku et al., *Science*, 2012

# Perfect Fluidity—Viscosity

Quark-Gluon Plasma: $T = 10^{12}$ K




Both Exhibit Elliptic Flow!



JETLab 2002

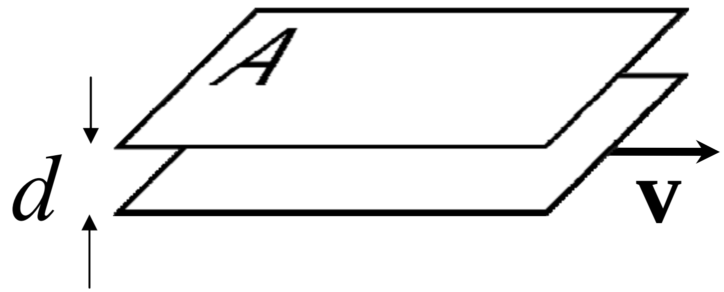
**Computer simulation of RHIC collision**

**BIG BANG**

Ultra-cold Atomic Fermi gas: $T = 10^{-7}$ K

# *Universal Regime:* Viscosity Scale

Shear forces:




$$\mathbf{F} = \eta \frac{A}{d} \mathbf{v}$$

Viscosity scale:

$$\eta \approx \frac{p}{A}$$

$$p \approx \frac{\hbar}{L} \qquad A \approx L^2$$

$$\boxed{\eta \approx \frac{\hbar}{L^3} = \hbar n}$$

n = density

$$\eta \approx \frac{\hbar}{\lambda_T^3} \propto T^{3/2}$$

High Temperature Limit

*Quantum* scale—requires Planck's constant!

# *Quantum* Viscosity

PHYSICS

Viscosity: $\eta = \alpha_s \hbar n$  n = density ( particles/cc)

dimensionless shear viscosity coefficient

| | | |
|---|---|---|
| Water: | n = 3.3 x $10^{22}$ | $\eta = 300\ \hbar n$ |
| Air: | n = 2.7 x $10^{19}$ | $\eta = 6000\ \hbar n$ |
| Fermi gas: | n = 3.0 x $10^{13}$ | $\eta = 0.5\ \hbar n$ |
| Nuclear Matter: | n = 3.0 x $10^{38}$ | $\eta = ?\ \hbar n$ |

# The Minimum Viscosity Conjecture

Resistance to flow—hydrodynamic properties

$$\frac{\text{viscosity}}{\text{entropy}} = \frac{\propto \text{surface area black hole}}{\propto \text{surface area black hole}} \geq \frac{1}{4\pi}\frac{\hbar}{k_B}$$

Kovtun, Son, Starinets, PRL 2005

Disorder—thermodynamic properties

Minimum defines a *Perfect* Fluid

# Minimum Viscosity Conjecture
# *Experimentalist's* Approach!

Viscosity $\hbar\, n$ —Hydrodynamics

$$\boxed{\frac{\eta}{s} \approx \frac{\hbar}{k_{\mathrm{B}}}}$$

Entropy density $k_B\, n$ —Thermodynamics

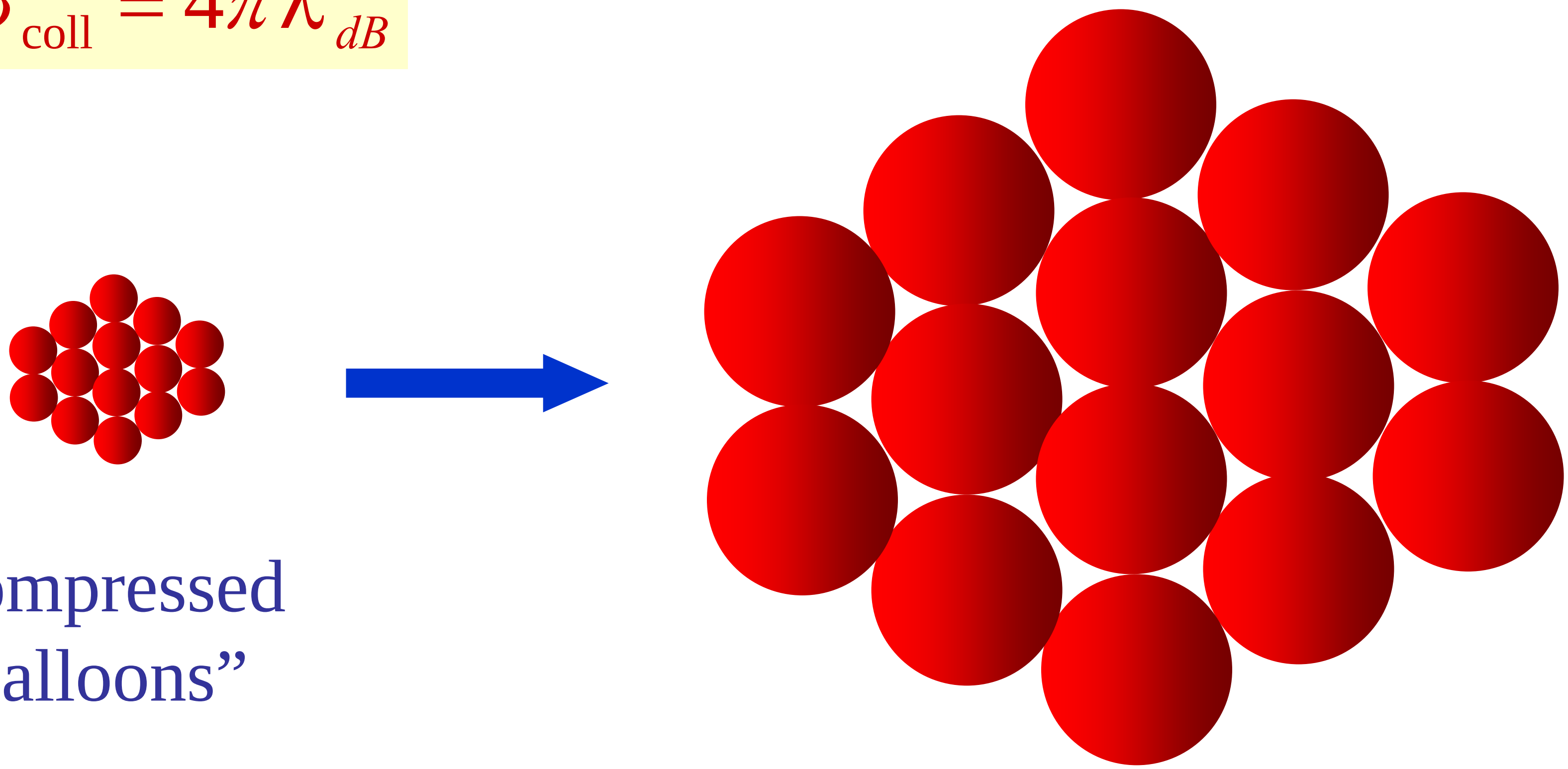
Density cancels!

In a $^6$Li gas we can *measure* η and s.

Is the Expansion Scale-Invariant?

# *Scale Invariance*
## in Expanding *Resonant* Fermi Gases

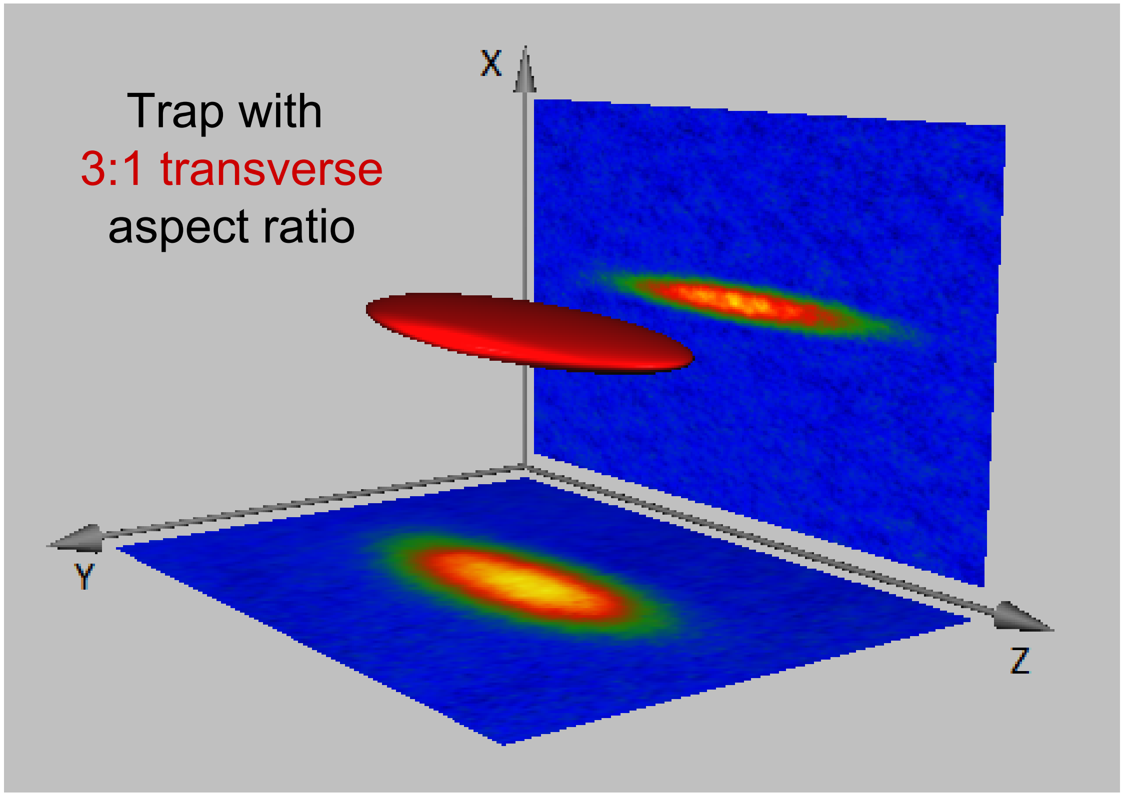
$$\sigma_{\text{coll}} = 4\pi ƛ_{dB}^{2}$$



Compressed
"Balloons"

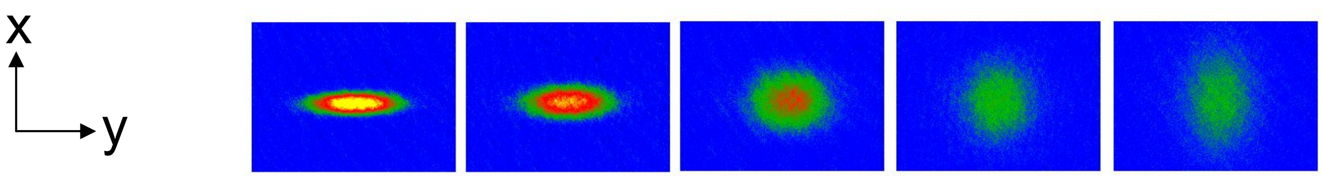
Expanded "Balloons"

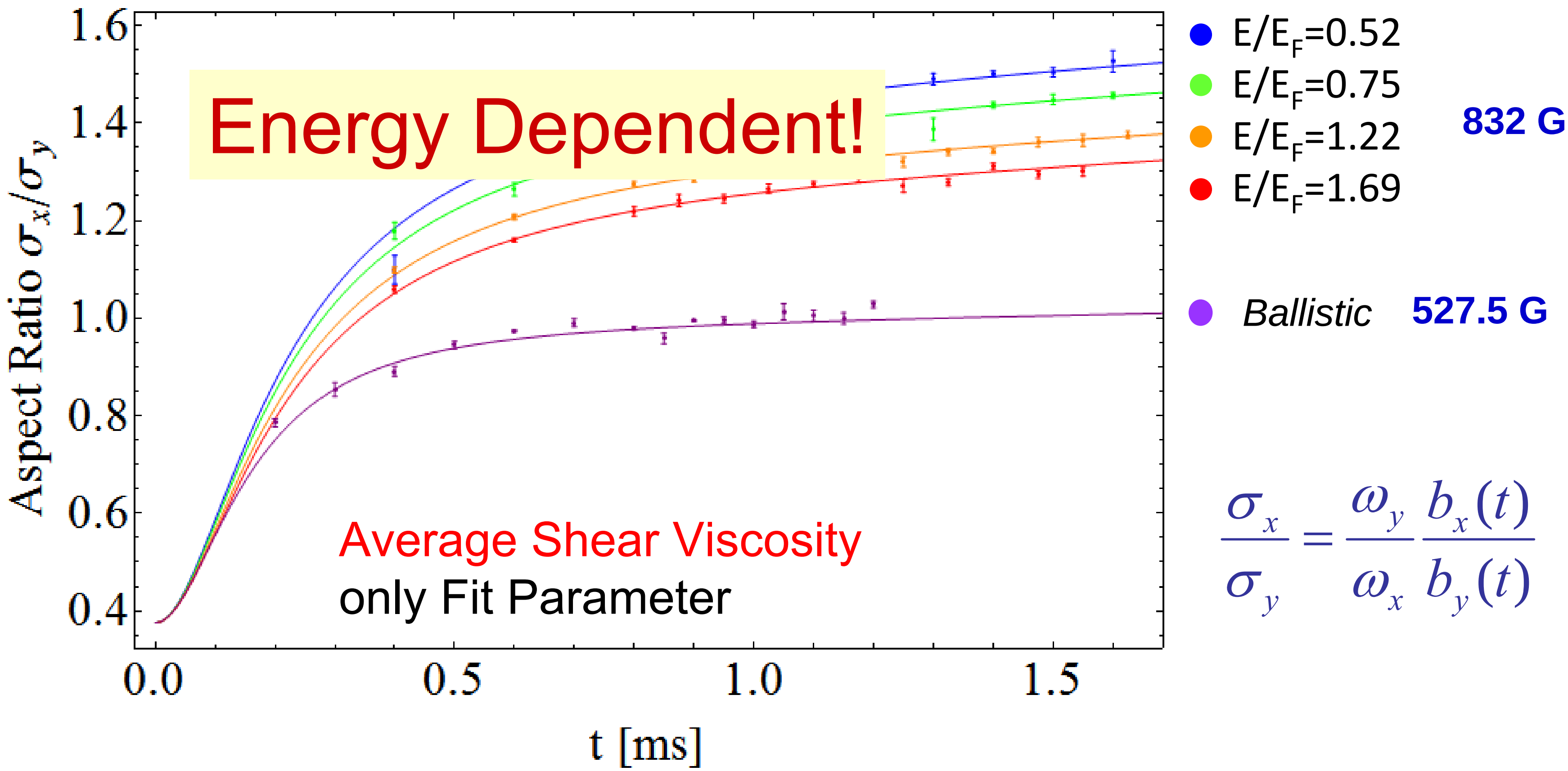
# Measuring the cloud in 3D




- Measure *all three* cloud radii using two cameras.

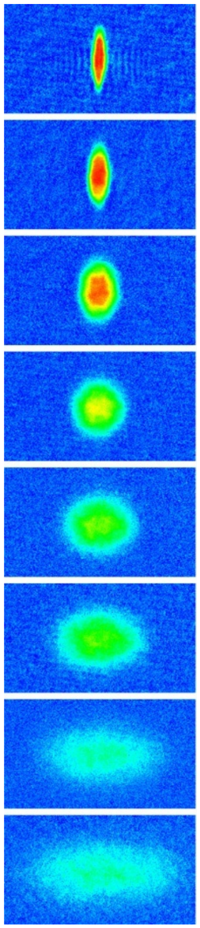
# Aspect Ratio versus Expansion Time




$$\frac{\sigma_x}{\sigma_y} = \frac{\omega_y}{\omega_x}\frac{b_x(t)}{b_y(t)}$$

# Scale-Invariance: Connecting *Strongly* to *Weakly* Interacting



- Anti-de Sitter-Conformal Field Theory Correspondence: Connects strongly interacting fields in 4-dimensions to weakly interacting gravity in 5-dimensions: Perfect fluid conjecture

- Can we connect elliptic flow of a resonant gas to the ballistic flow of an *ideal* gas in 3D?

For both, the pressure is 2/3 of the energy density:

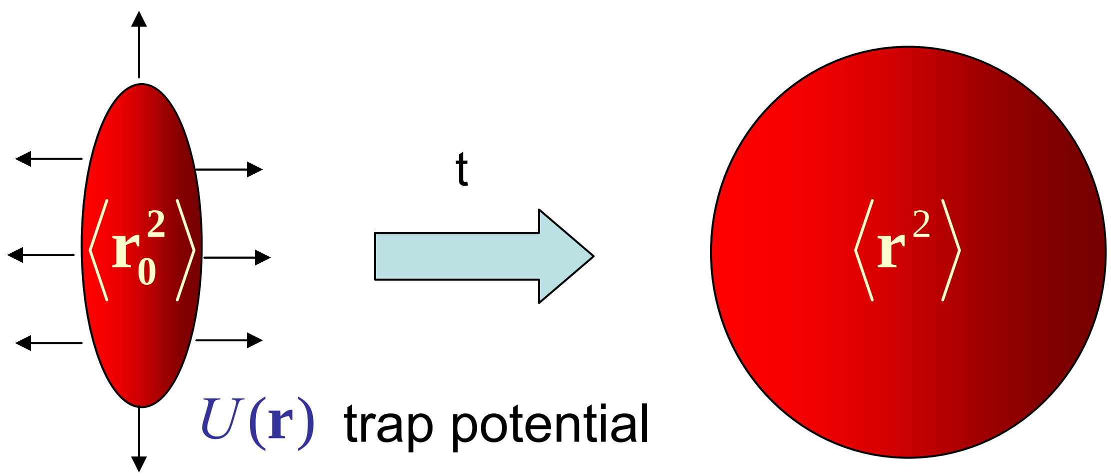
$$\boxed{\Delta p \equiv p - \frac{2}{3}\varepsilon = 0}$$

Scale Invariant?

<u>Elliptic Flow</u>: Observe 2 dimensions + time

# Scale Invariance: Ideal Gas

<u>Ideal gas:</u> $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$ Ballistic flow




$$\langle \mathbf{r}^2 \rangle = \langle \mathbf{r}^2 \rangle_0 + t^2 \langle \mathbf{v}^2 \rangle_0$$

Cloud average

How does the *mean square radius* evolve in time? $\langle \mathbf{r}^2 \rangle = \langle x^2 + y^2 + z^2 \rangle$

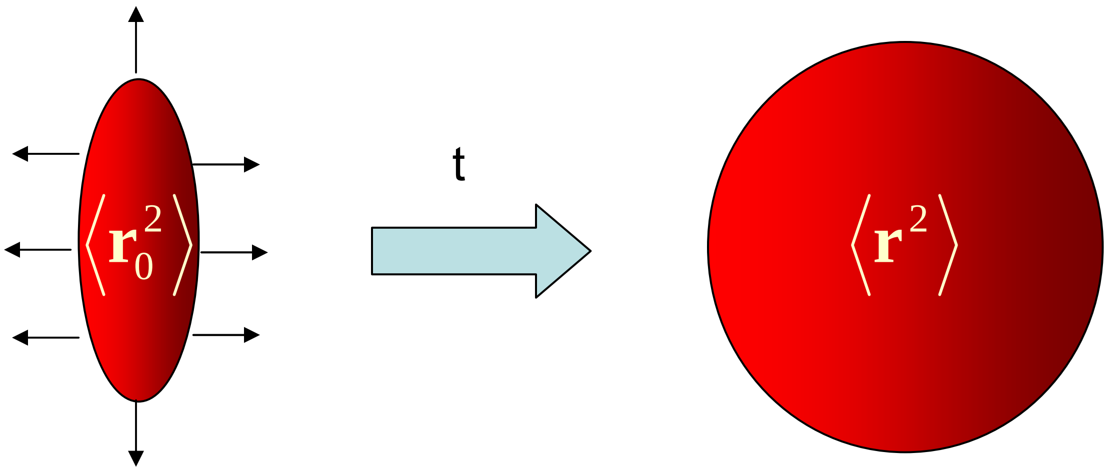
Virial Theorem:

$$m\langle \mathbf{v}^2 \rangle_0 = \langle \mathbf{r} \cdot \nabla \mathbf{U} \rangle_0$$

$$\boxed{\langle \mathbf{r}^2 \rangle = \langle \mathbf{r}^2 \rangle_0 + \frac{t^2}{m} \langle \mathbf{r} \cdot \nabla \mathbf{U} \rangle_0}$$

Ballistic Flow

# Defining Scale Invariant Flow




$$\langle \mathbf{r}^2 \rangle = \langle x^2 + y^2 + z^2 \rangle$$

Cloud average

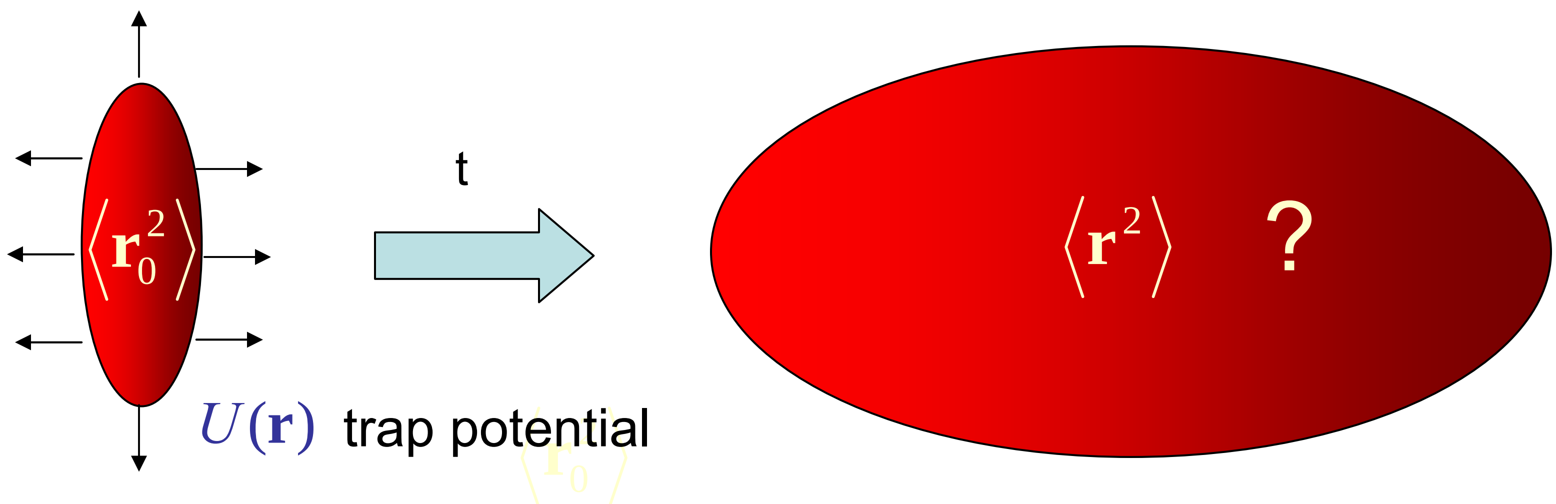
$$\frac{m\left(\langle \mathbf{r}^2 \rangle - \langle \mathbf{r}^2 \rangle_0\right)}{\langle \mathbf{r} \cdot \nabla \mathbf{U} \rangle_0} = t^2$$

t = expansion time

Defines Scale Invariant Flow!

# Scale Invariance: Resonant Gas

<u>Hydrodynamic gas:</u> Elliptic flow




How does the *mean square radius* evolve in time? $\langle \mathbf{r}^2 \rangle = \langle x^2 + y^2 + z^2 \rangle$

# Scale Invariant Expansion

Using the hydrodynamic equations and energy conservation it is easy to show that

$$\frac{d^2}{dt^2}\frac{m\langle \mathbf{r}^2\rangle}{2} = \langle \mathbf{r}\cdot\nabla \mathbf{U}\rangle_0 + \cancel{\frac{3}{N}\int d^3\mathbf{r}\,\left(\Delta\mathbf{p}-\Delta\mathbf{p}_0\right)} - \cancel{\frac{3}{N}\int d^3\mathbf{r}\,\zeta_B \nabla\cdot\mathbf{v}}$$

Initial trap potential

*Conformal symmetry breaking* $\Delta p$

Bulk viscosity

$$\boxed{\Delta p \equiv p - \frac{2}{3}\varepsilon}$$

# Scale Invariance!

Resonant gas $\Delta p \equiv p - \frac{2}{3}\varepsilon = 0$

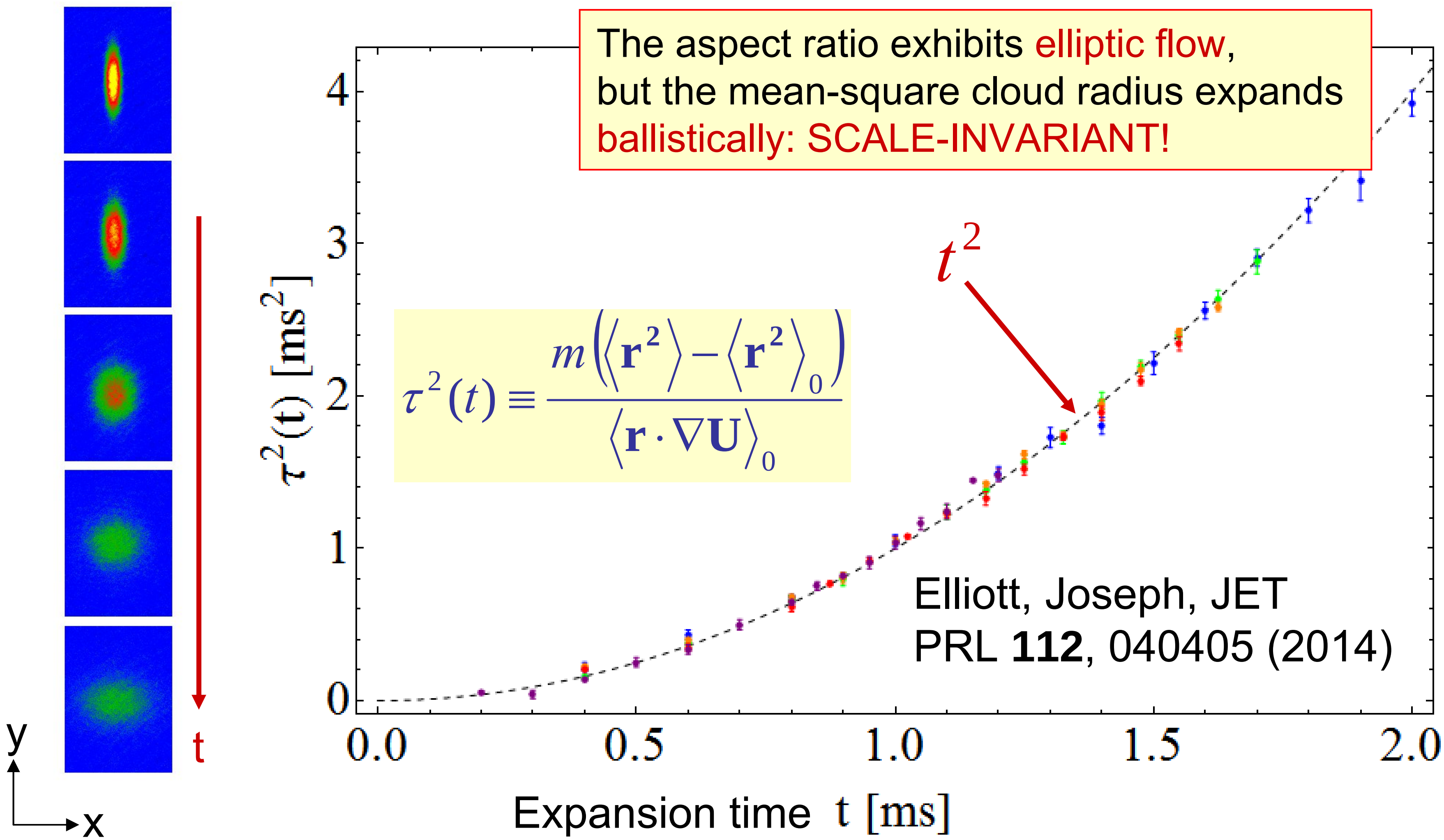
The bulk viscosity is predicted to vanish so

$$\boxed{\left\langle \mathbf{r}^2 \right\rangle = \left\langle \mathbf{r}^2 \right\rangle_0 + \frac{t^2}{m}\left\langle \mathbf{r}\cdot\nabla\mathbf{U}\right\rangle_0}$$

Ballistic Flow!

Can we observe *ballistic* flow of an *elliptically* expanding gas?

# Scale-invariant "Ballistic" Expansion of a Resonant Fermi gas



# Summary:
# Scale-Invariant Expansion

String theory has sometimes been characterized as an elegant scale-invariant theory of everything with one minor defect: It Predicts Nothing!

JET:

"Now we have an experiment that Measures Nothing to compare to it!"

# Conformal Symmetry Breaking

Observing the conformal symmetry breaking pressure:

$$\frac{d^2}{dt^2}\frac{m\left\langle \mathbf{r}^2\right\rangle}{2} = \left\langle \mathbf{r}\cdot\nabla\mathbf{U}\right\rangle_0 + \frac{3}{N}\int d^3\mathbf{r}\left(\Delta\mathbf{p}-\Delta\mathbf{p}_0\right) - \cancel{\frac{3}{N}\int d^3\mathbf{r}\,\zeta_B\nabla\cdot\mathbf{v}}$$

Initial trap potential

*Conformal symmetry breaking* $\Delta p$

Bulk viscosity

$$\boxed{\Delta p \equiv p - \tfrac{2}{3}\varepsilon}$$

# *Pressure Change $\Delta p$*

Dimensional analysis, to leading order in $1/k_F a$, requires

$$p - \tfrac{2}{3}\varepsilon \equiv \Delta p = n\varepsilon_F(n)\frac{f_p(\theta)}{k_F a}$$

Vanishes for infinite scattering length.

Time dependence: $k_F = (3\pi^2 n)^{1/3} \propto \Gamma^{-1/3}$ $\Gamma(t)$ = volume scale factor

Assuming the temperature drops adiabatically, the reduced temperature $\theta$ is time-independent:

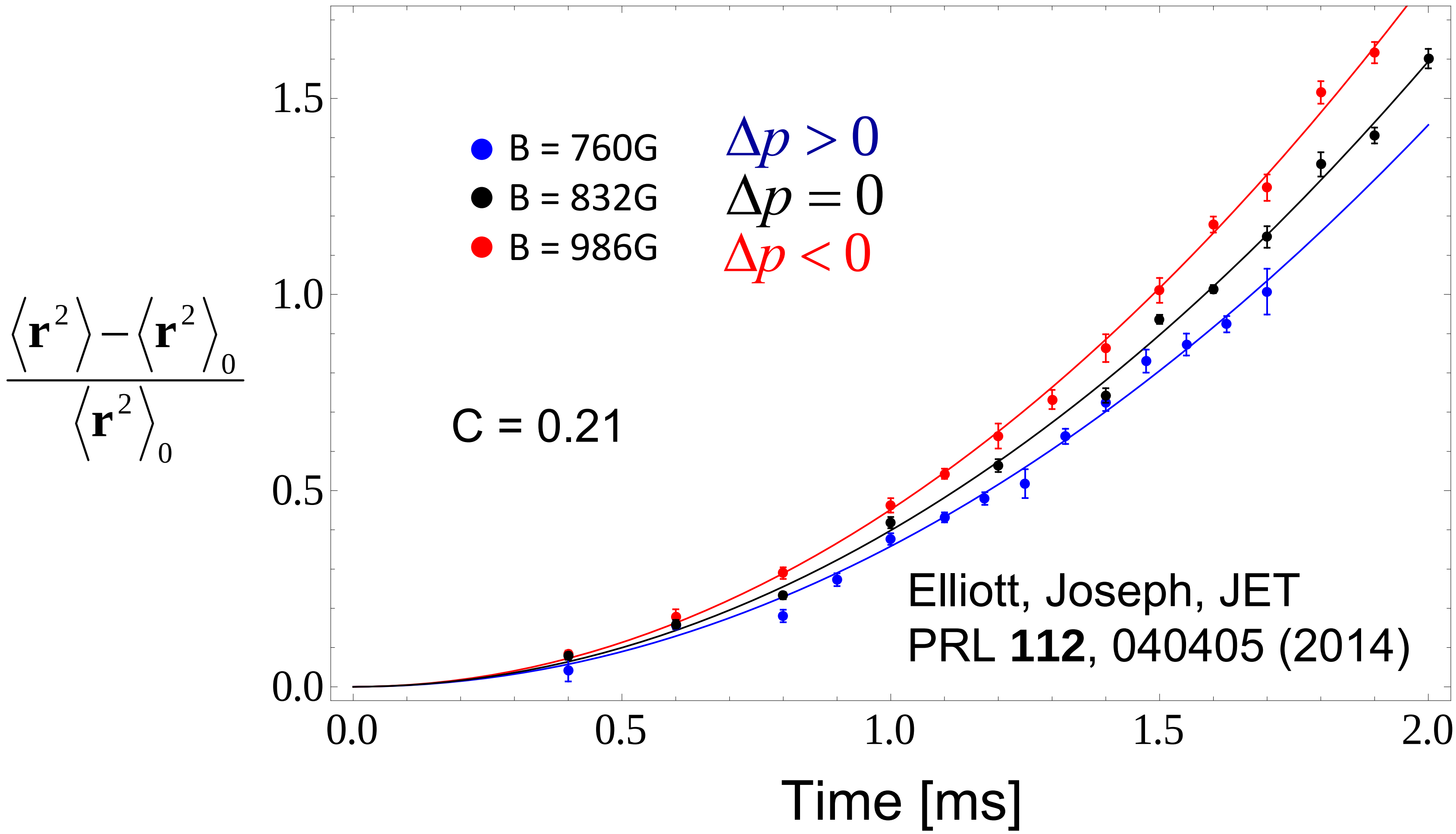
$$\theta \equiv \frac{k_B T}{\varepsilon_F} \propto \frac{T}{k_F^2}$$

$$\boxed{\frac{1}{N}\int d^3\mathbf{r}\,\Delta p = \frac{C}{3}\frac{\langle \mathbf{r}\cdot\nabla U\rangle_0}{k_{FI}a}\Gamma^{-1/3}(t)}$$

$$\frac{1}{k_{FI}a} = \pm 0.6$$

Changes sign with the scattering length

# Breaking Scale Invariance



# Bulk Viscosity at Resonance

Measuring the bulk viscosity:

$$\frac{d^2}{dt^2}\frac{m\langle \mathbf{r}^2\rangle}{2} = \langle \mathbf{r}\cdot\nabla\mathbf{U}\rangle_0 + \cancel{\frac{3}{N}\int d^3\mathbf{r}\left(\Delta\mathbf{p}-\Delta\mathbf{p}_0\right)} - \frac{3}{N}\int d^3\mathbf{r}\,\zeta_B\nabla\cdot\mathbf{v}$$

Initial trap potential

*Conformal symmetry breaking* $\Delta p = 0$

Bulk viscosity

Evaluate last term in scaling approximation:

$\Gamma$(t) = volume scale factor

$$\nabla\cdot\mathbf{v} = \dot{\Gamma}/\Gamma$$

# *Bulk Viscosity*

Dimensional analysis, to leading order in $1/k_Fa$, with $\zeta_B \geq 0$ requires

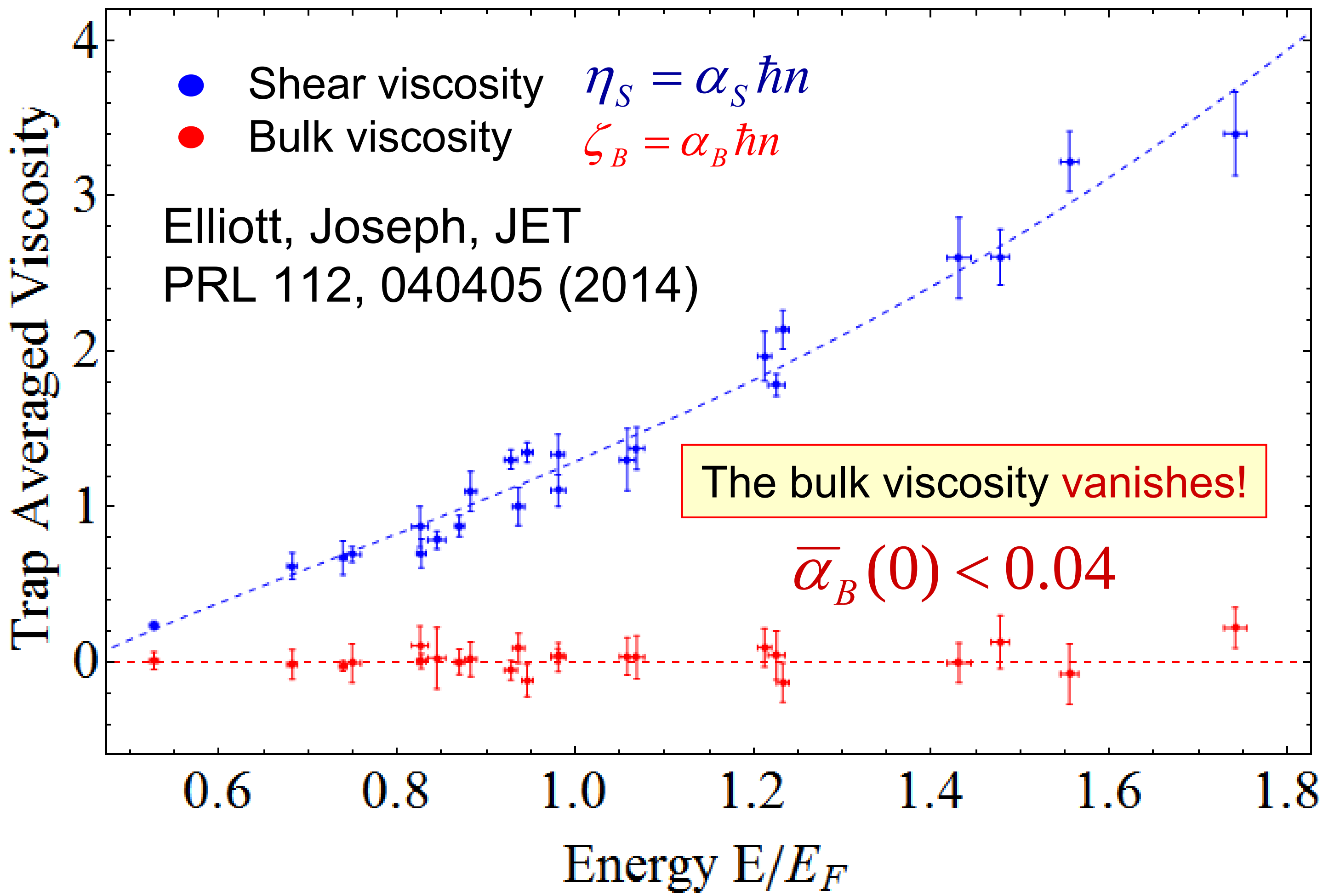
$$\zeta_B \equiv \hbar n \frac{f_B(\theta)}{(k_F a)^2} \equiv \alpha_B \hbar n$$

Time dependence: $k_F = (3\pi^2 n)^{1/3} \propto \Gamma^{-1/3}$ $\quad\Gamma(t)$ = volume scale factor

Assuming the temperature drops adiabatically, the reduced temperature $\theta$ is time-independent: $\theta \equiv \frac{k_B T}{\varepsilon_F} \propto \frac{T}{k_F^2}$

$$\boxed{\frac{1}{N}\int d^3\mathbf{r}\,\varsigma_B \equiv \hbar\overline{\alpha}_B(0)\,\Gamma^{2/3}(t)}$$

# Bulk Viscosity at Resonance



# Measuring the Shear Viscosity

From the Navier-Stokes and continuity equations, it is easy to show that a single component fluid obeys:

Pressure Trap potential

$$\frac{d^2}{dt^2}\frac{m\langle x_i^2\rangle}{2} = m\langle \mathbf{v}_i^2\rangle + \frac{1}{N}\int d^3\mathbf{r}\, p - \langle x_i \partial_i U\rangle - \hbar\langle \alpha_S \sigma_{ii} + \alpha_B \nabla\cdot\mathbf{v}\rangle$$

Stream KE Shear and Bulk Viscosity

<u>Equilibrium:</u>

$$\frac{3}{N}\int d^3\mathbf{r}\, p_0 = \langle \mathbf{r}\cdot\nabla U\rangle_0 \equiv \tilde{E}$$

Measured from the cloud profile and trap parameters

Need to find the time-dependent volume integral of the pressure:

# Energy Conservation

For a temporally constant potential energy U,
the internal energy change during expansion is: $dE_{\text{int}} = dQ - p\,dV$

$$\Rightarrow \quad \dot{E}_{\text{int}} = \dot{Q} - p\dot{V}$$

The local volume dilates at a rate: $\dot{V} = d^3\mathbf{r}\,\nabla\cdot\mathbf{v}$

$$E_{\text{int}} = \int d^3\mathbf{r}\,\varepsilon$$

energy density

$$\boxed{\frac{d}{dt}\int d^3\mathbf{r}\,\varepsilon = \dot{Q} - \int d^3\mathbf{r}\,(\nabla\cdot\mathbf{v})p}$$

$$\boxed{\frac{2}{3}\varepsilon = p - \Delta p}$$

*$\Delta p = 0$ for resonantly interacting gas*

Easy to solve in scaling approximation:

$\Gamma(t)$ = volume scale factor

$$\boxed{\nabla\cdot\mathbf{v} = \dot{\Gamma}/\Gamma}$$

# Scaling Approximation

$$n(x,y,x,t) = \frac{n_0(x/b_x, y/b_y, z/b_z)}{\Gamma}$$

$$\Gamma = b_x b_y b_z$$

Volume scale factor

$$\mathrm{v}_i = x_i \dot{b}_i / b_i$$

Velocity field is linear in the spatial coordinates

$$\langle x_i^2 \rangle = \langle x_i^2 \rangle_0 b_i^2(t)$$

$$\langle \mathrm{v}_i^2 \rangle = \langle x_i^2 \rangle_0 \dot{b}_i^2(t)$$

$$\overline{\omega_i^2} \equiv \frac{\langle \mathbf{r} \cdot \nabla U \rangle_0}{3m\langle x_i^2 \rangle_0}$$

$$\sigma_{ii} = 2\frac{\dot{b}_i}{b_i} - \frac{2}{3}\frac{\dot{\Gamma}}{\Gamma}$$

Cloud-averaged shear viscosity coefficient

Pressure correction factors

$$\boxed{\ddot{b}_i = \frac{\overline{\omega_i^2}}{\Gamma^{2/3} b_i}\left[1 + C_Q(t) + C_{\Delta p}(t)\right] - \frac{\hbar \overline{\alpha}_S \sigma_{ii}}{m\langle x_i^2 \rangle_0 b_i} - \omega_{i\,mag}^2 b_i}$$

# Pressure Correction Factors

Velocity field is linear in the spatial coordinates $\mathrm{v}_i = x_i \frac{\dot{b}_i}{b_i}$ $\qquad \nabla \cdot \mathbf{v} = \frac{\dot{\Gamma}}{\Gamma}$

Heating rate per particle:

$$\frac{\dot{Q}}{N} = \frac{\hbar}{2}\left( \overline{\alpha}_S \sum_i \sigma_{ii}^2 + 2\overline{\alpha}_B (\nabla \cdot \mathbf{v})^2 \right)$$

Heating correction factor:

$$\dot{C}_Q(t) = \frac{2\dot{Q}(t)}{N} \frac{\Gamma^{2/3}(t)}{\langle \mathbf{r} \cdot \nabla U \rangle_0}$$

Conformal symmetry breaking pressure:

$$\frac{1}{N}\int d^3\mathbf{r}\,\Delta p = \frac{C}{3} \frac{\langle \mathbf{r} \cdot \nabla U \rangle_0}{k_{FI}a} \Gamma^{-1/3}(t)$$

Pressure correction factor:

$$C_{\Delta p}(t) = -\frac{C}{k_{FI}a}\left(\Gamma^{1/3}(t) - 1\right)$$

# Cloud-Averaged Viscosity

Shear Viscosity: $\eta_S \equiv \alpha_S \hbar n$

Cloud-averaged shear viscosity coefficient:

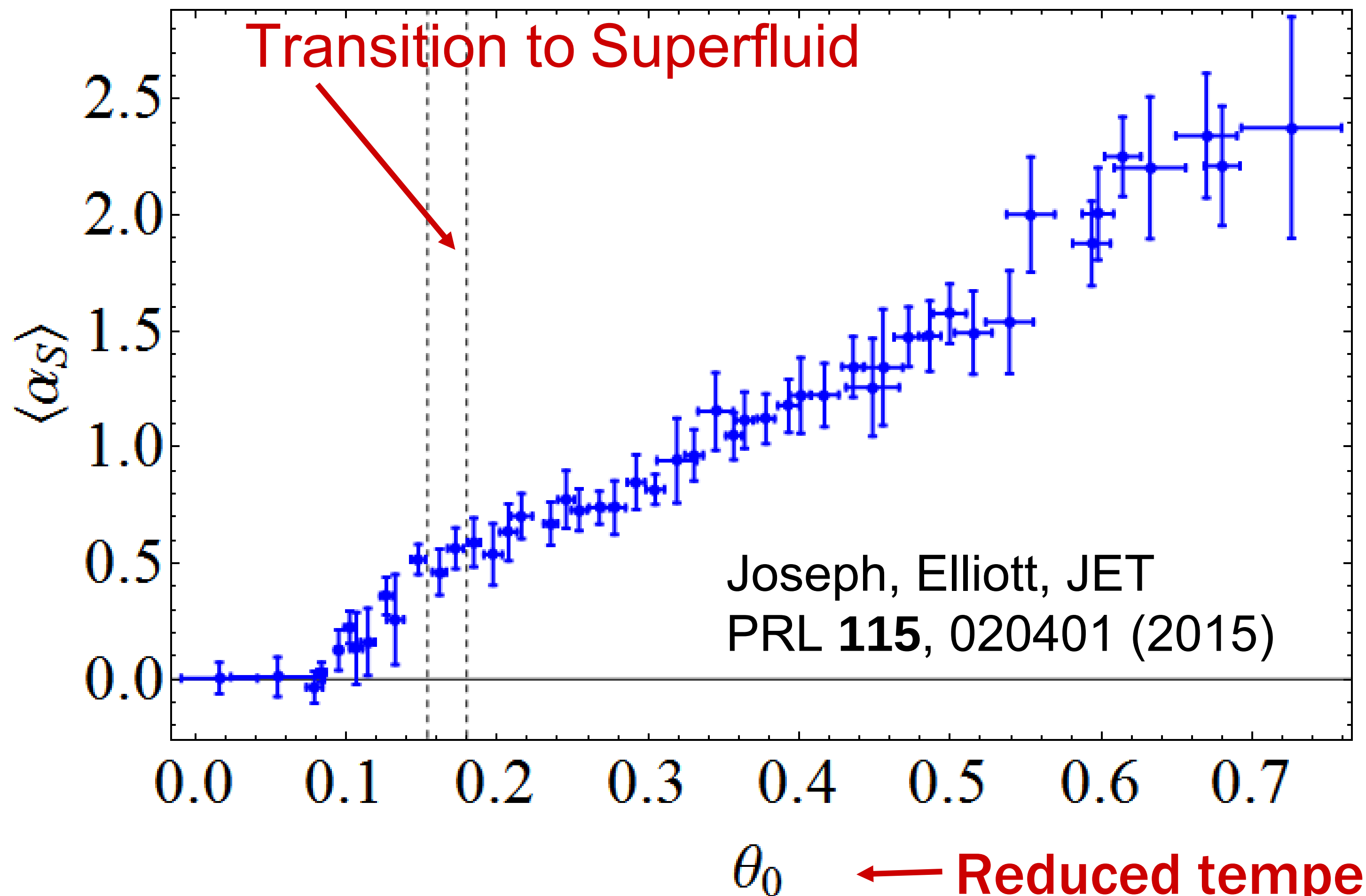
$$\boxed{\overline{\alpha}_S \equiv \frac{1}{N\hbar}\int d^3\mathbf{r}\,\eta_S(\mathbf{r}) = \int d^3\mathbf{r}\,\frac{n(\mathbf{r})}{N}\alpha_S[\theta(\mathbf{r})] = \langle \alpha_S \rangle_0}$$

*Temporally constant in the adiabatic approximation = Trap average.

Volume integrated KSS bound:

$$\boxed{\overline{\alpha}_S \equiv \frac{1}{N\hbar}\int d^3\mathbf{r}\,\eta_S(\mathbf{r}) \geq \frac{1}{N}\int d^3\mathbf{r}\,\frac{s(\mathbf{r})}{4\pi k_B} = \frac{1}{4\pi}\frac{S/N}{k_B}}$$

# Shear Viscosity at Resonance versus Reduced Temperature

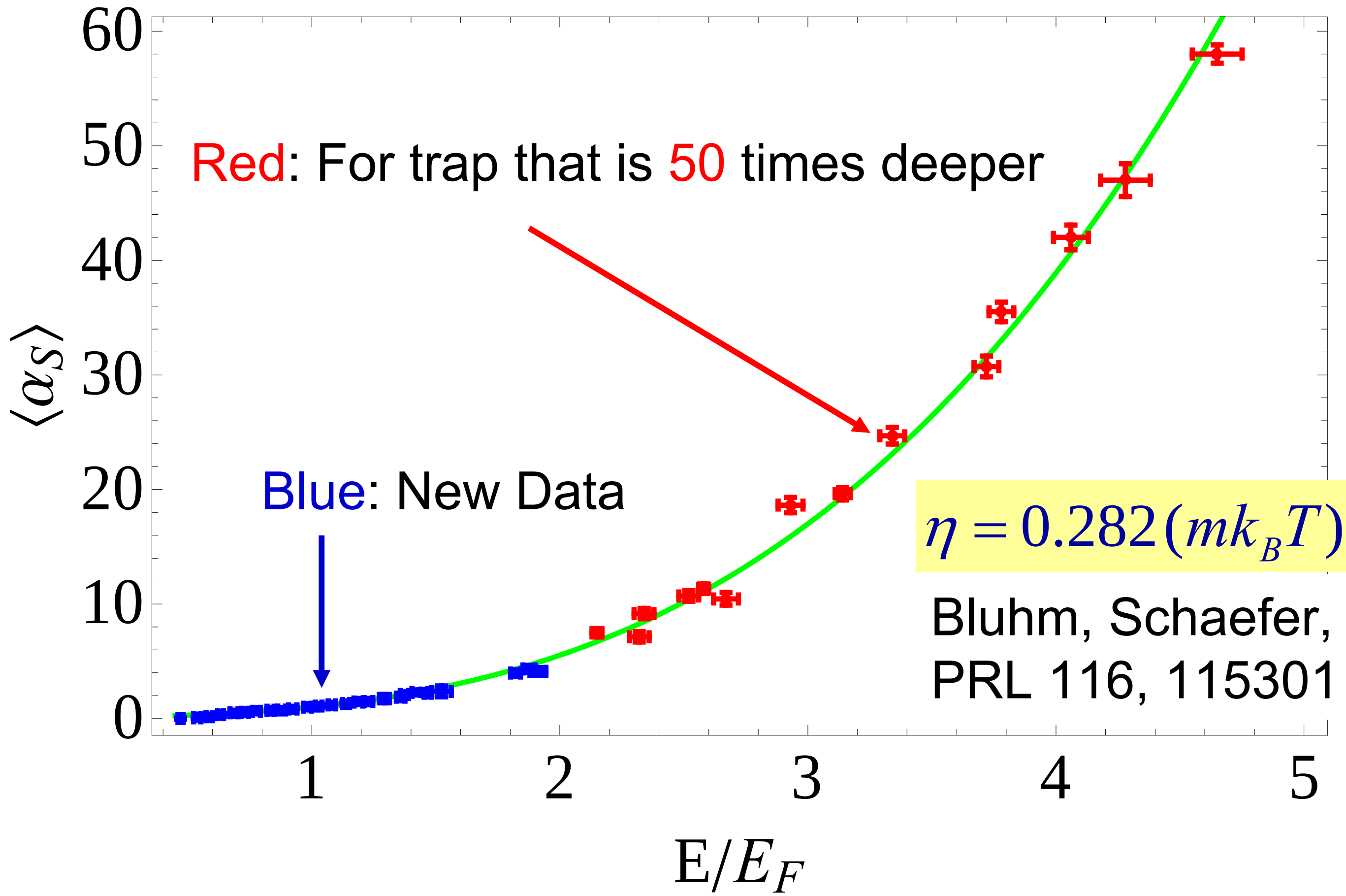



**Reduced temperature at the trap center**

*EoS from Ku et al., *Science*, 2012

# Shear Viscosity: Universal Scaling

$$\boxed{\eta = \alpha_S \hbar n}$$

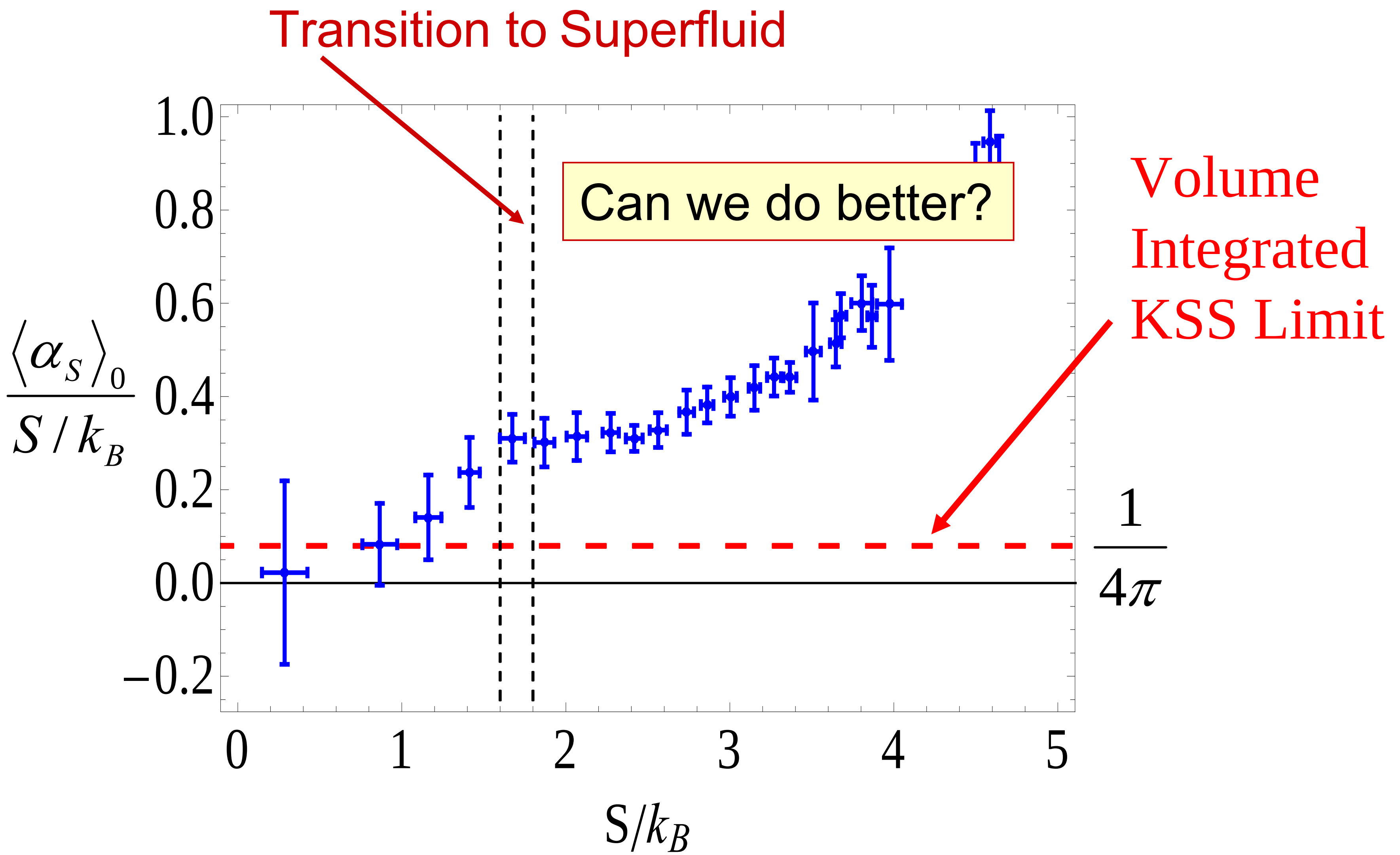



JETLab, *Science* **331**, 58 (2011)

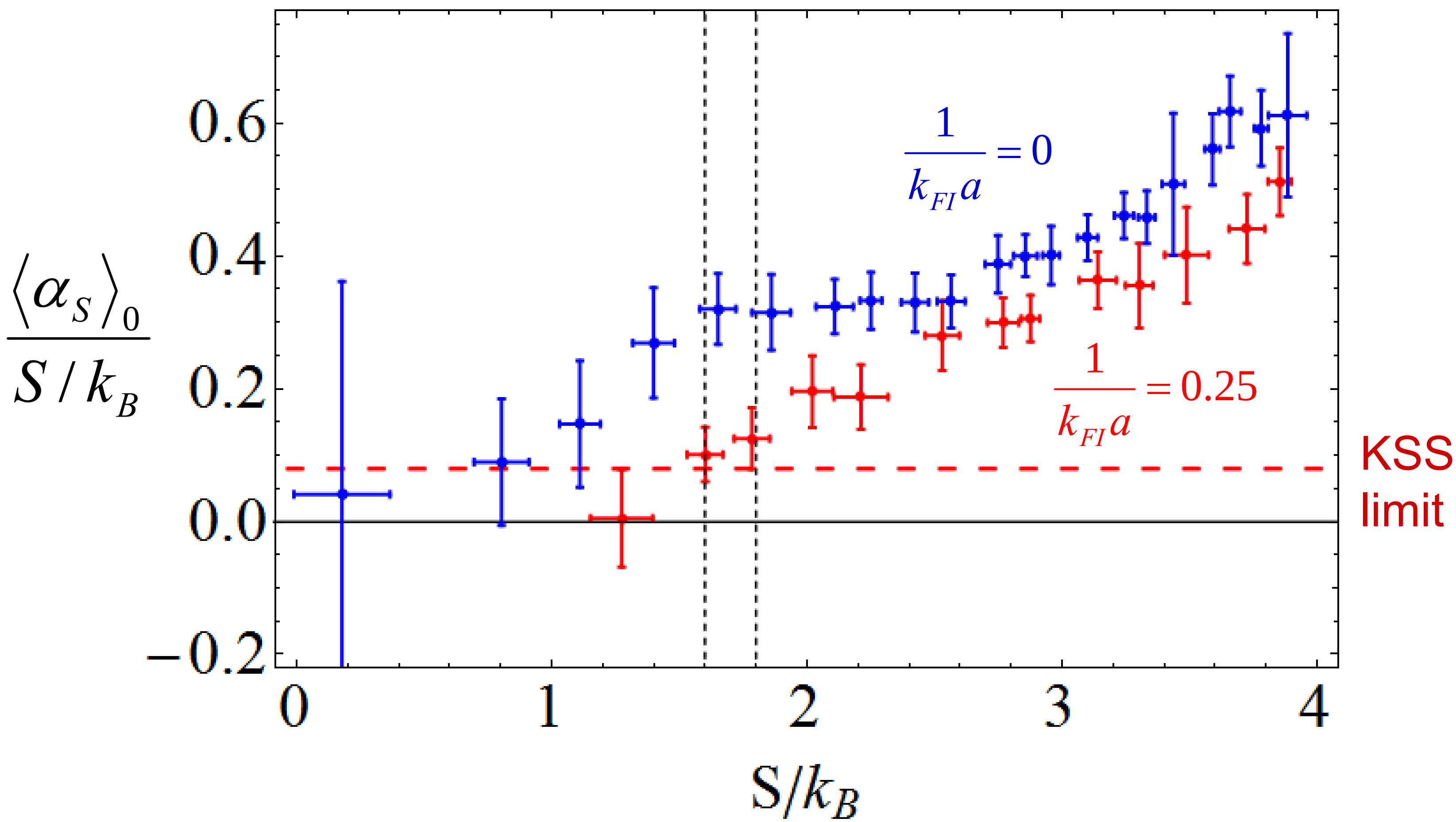
$$\eta = 0.282 (m k_B T)^{3/2} / \hbar^2$$

Bluhm, Schaefer, PRL 116, 115301 (2016)

# Shear Viscosity/Entropy versus Entropy: Resonance



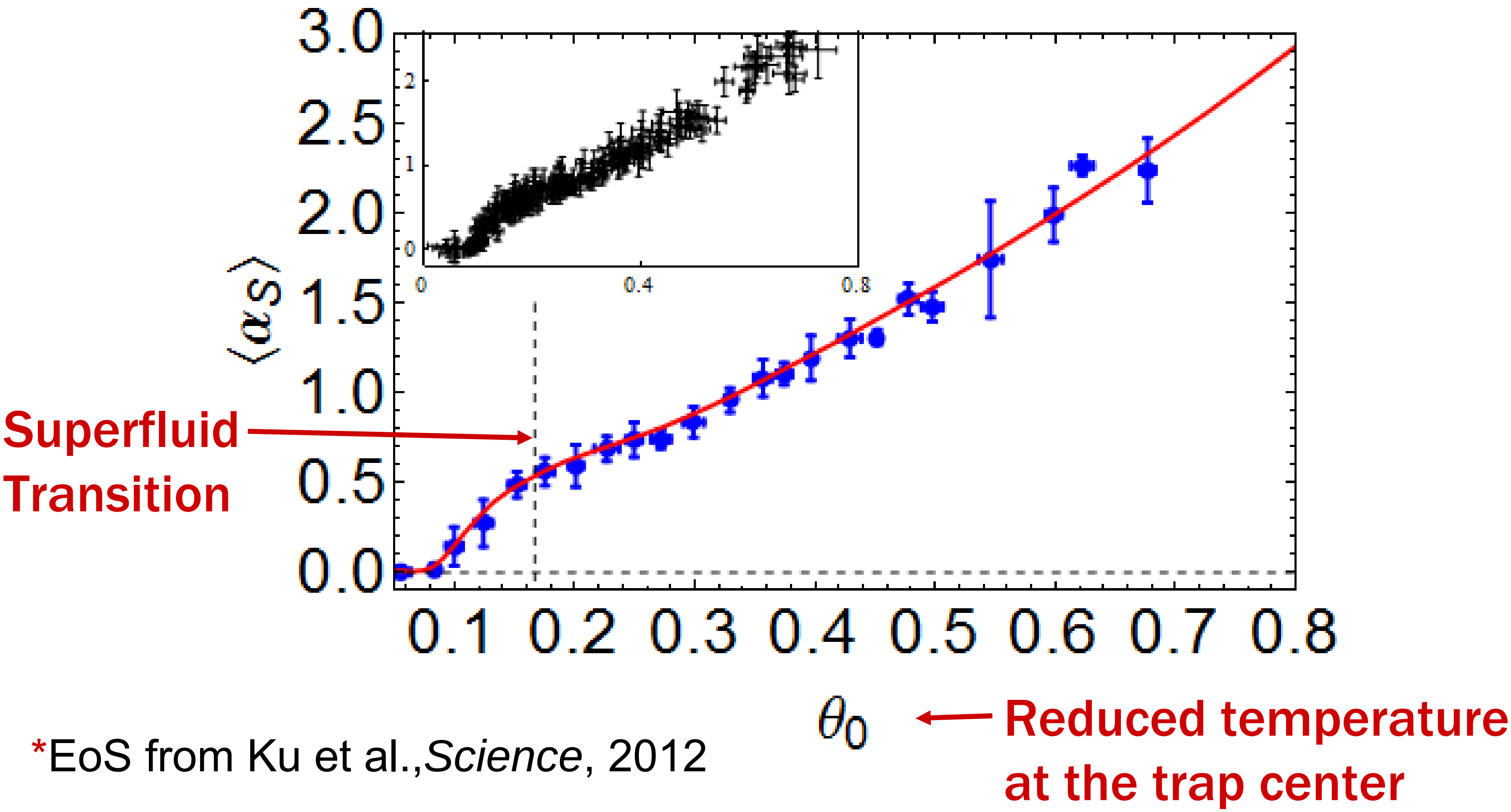
# Shear Viscosity/Entropy versus Entropy: Perfect Fluid?



# Problems with Viscosity Measurement

- We have measured the "Cloud-Averaged" shear viscosity.
- Integration "Volumes" for entropy and viscosity may not be the same.
- What can we say about the "Local" shear viscosity?
  - Inverting cloud-averaged data
  - Local ratio of shear viscosity to entropy density
  - Comparison with predictions

# Cloud-Averaged Shear Viscosity versus Reduced Temperature




*EoS from Ku et al., *Science*, 2012

# Obtaining Local Viscosity from Cloud-Averaged Viscosity Data

$$\langle \alpha_S \rangle_0 = \frac{1}{N}\int_0^\infty d^3r\, \alpha_S \underbrace{[\theta(r)]\, n(r)}_{\propto T^{3/2} \text{ as } r \to \infty} \to \infty$$

A problem!

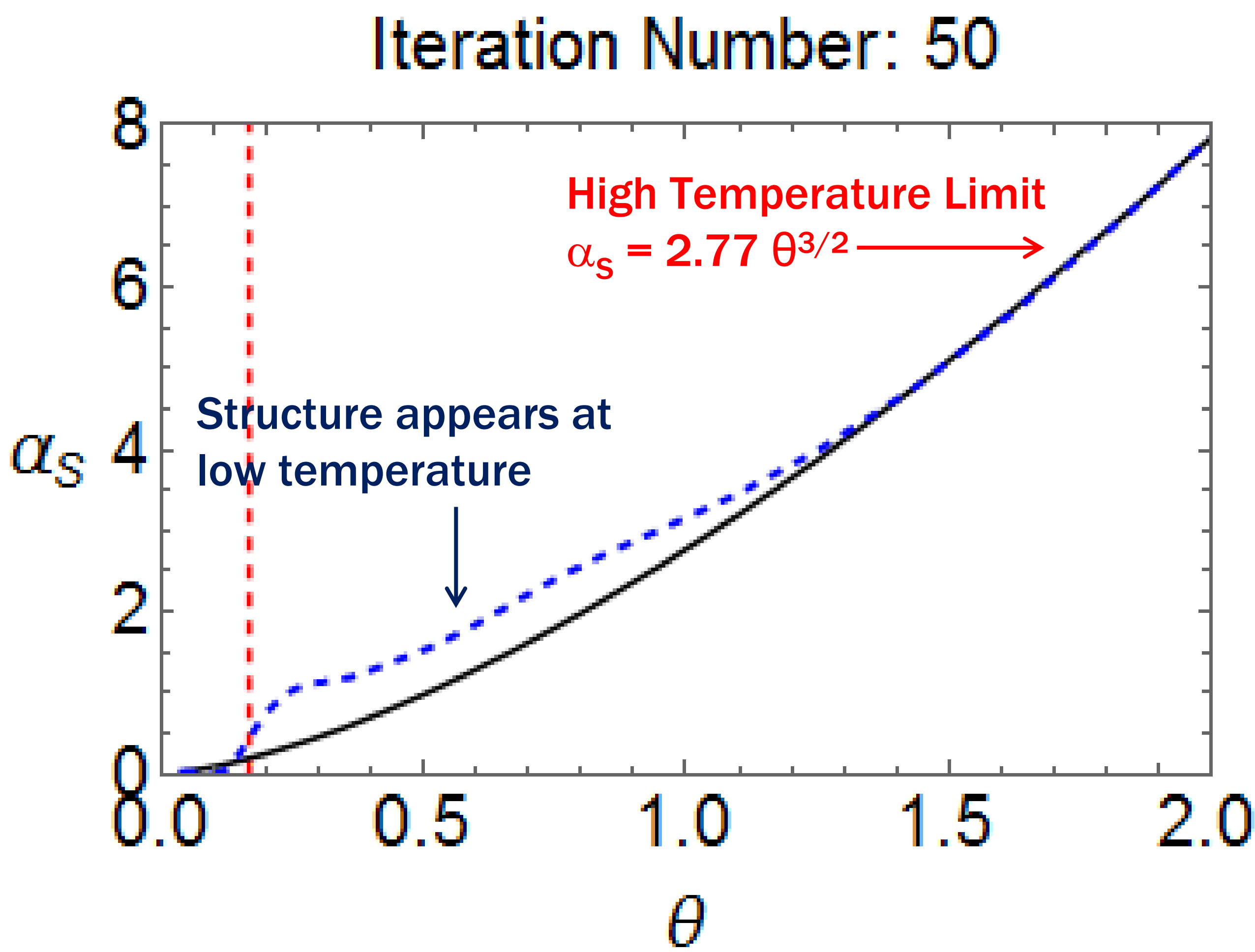
Cutoff radius: $$\langle \alpha_S \rangle_0 = \frac{1}{N}\int_0^{R_C} d^3r\, \alpha_S [\theta(r)]\, n(r)$$

Choose $R_C$ to agree with high temperature data:

$$\boxed{R_C = 0.98 \langle r^2 \rangle^{1/2}}$$

Now we can estimate $\alpha_S(\theta)$ by image processing methods!

# *Local* Shear Viscosity versus Reduced Temperature

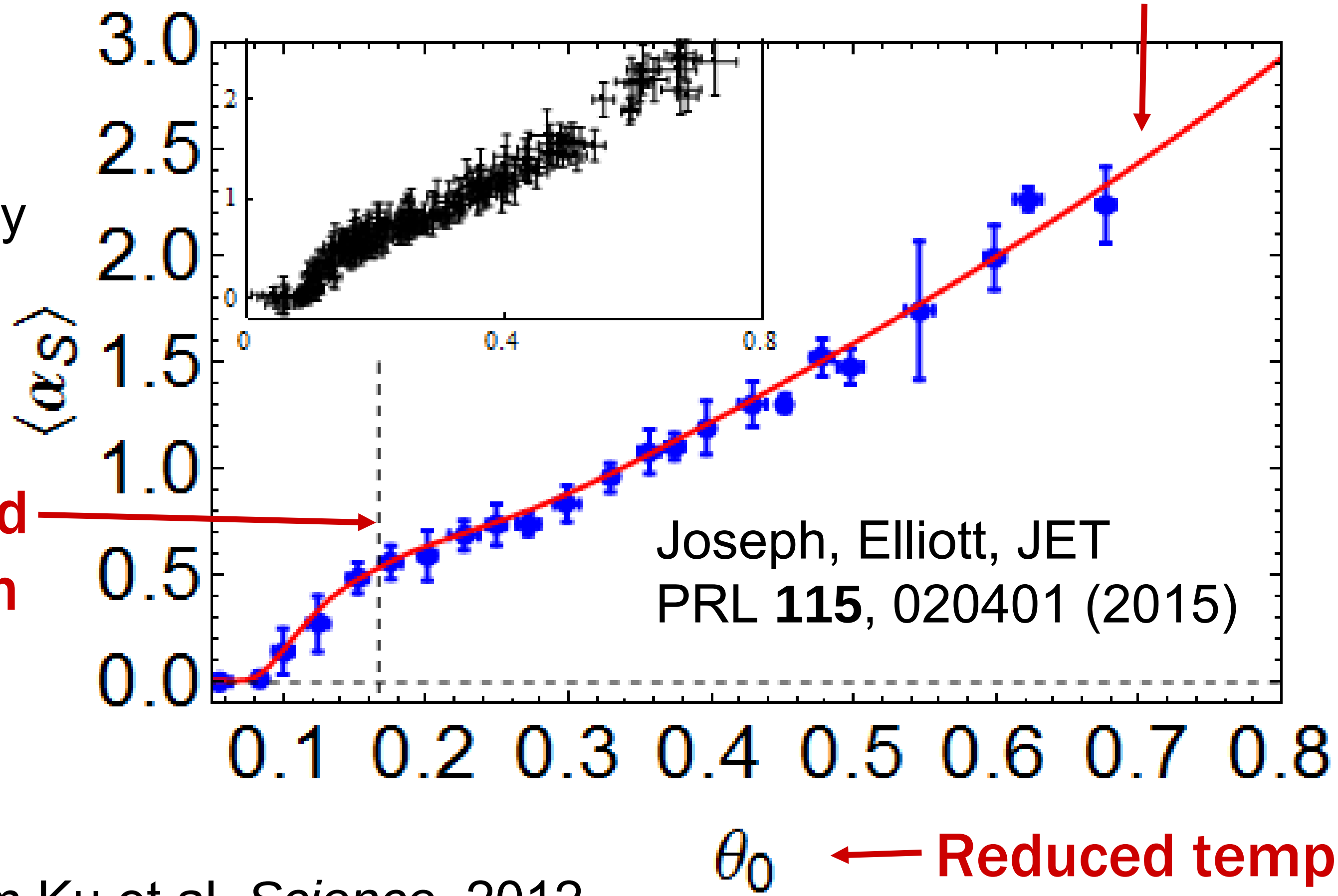



Bruun, Smith
Phys. Rev. A **75**
043612 (2007)

# Cloud-Averaged Shear Viscosity versus Reduced Temperature

Integrated local viscosity

Shear viscosity ($\hbar n$)

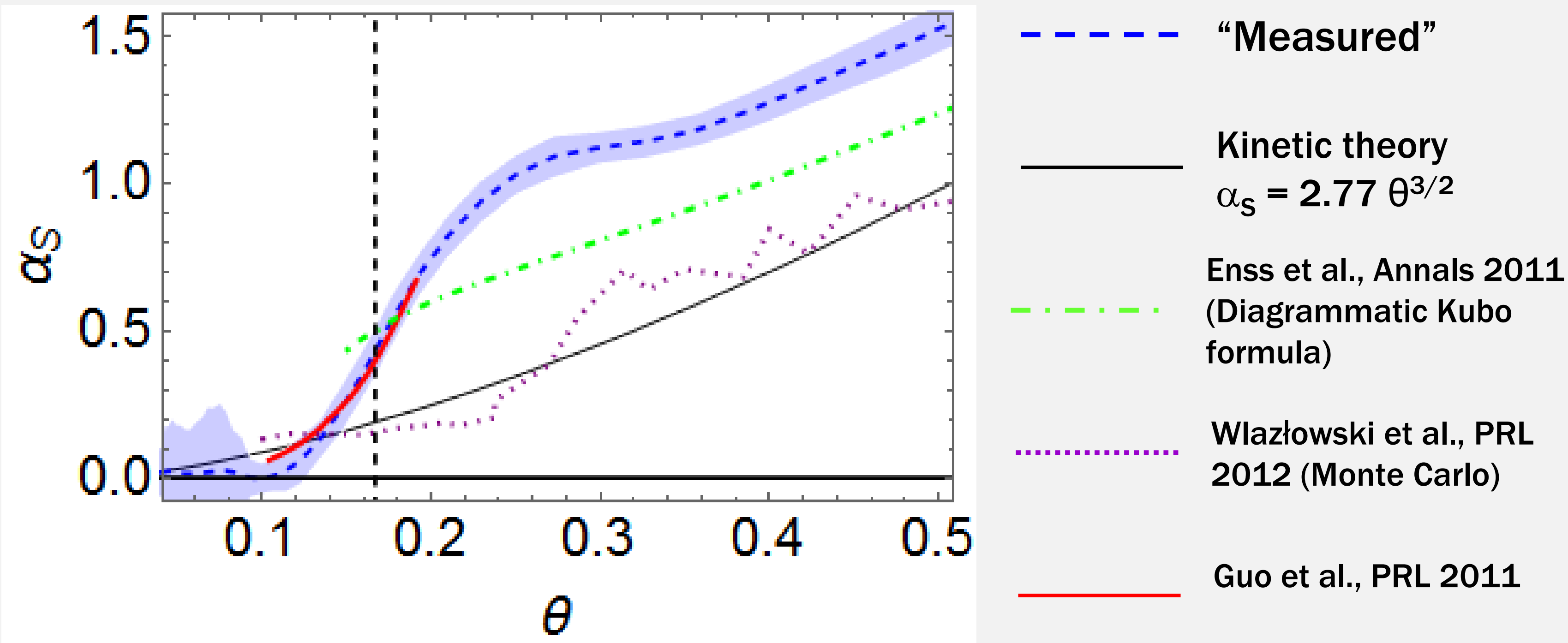



Superfluid Transition

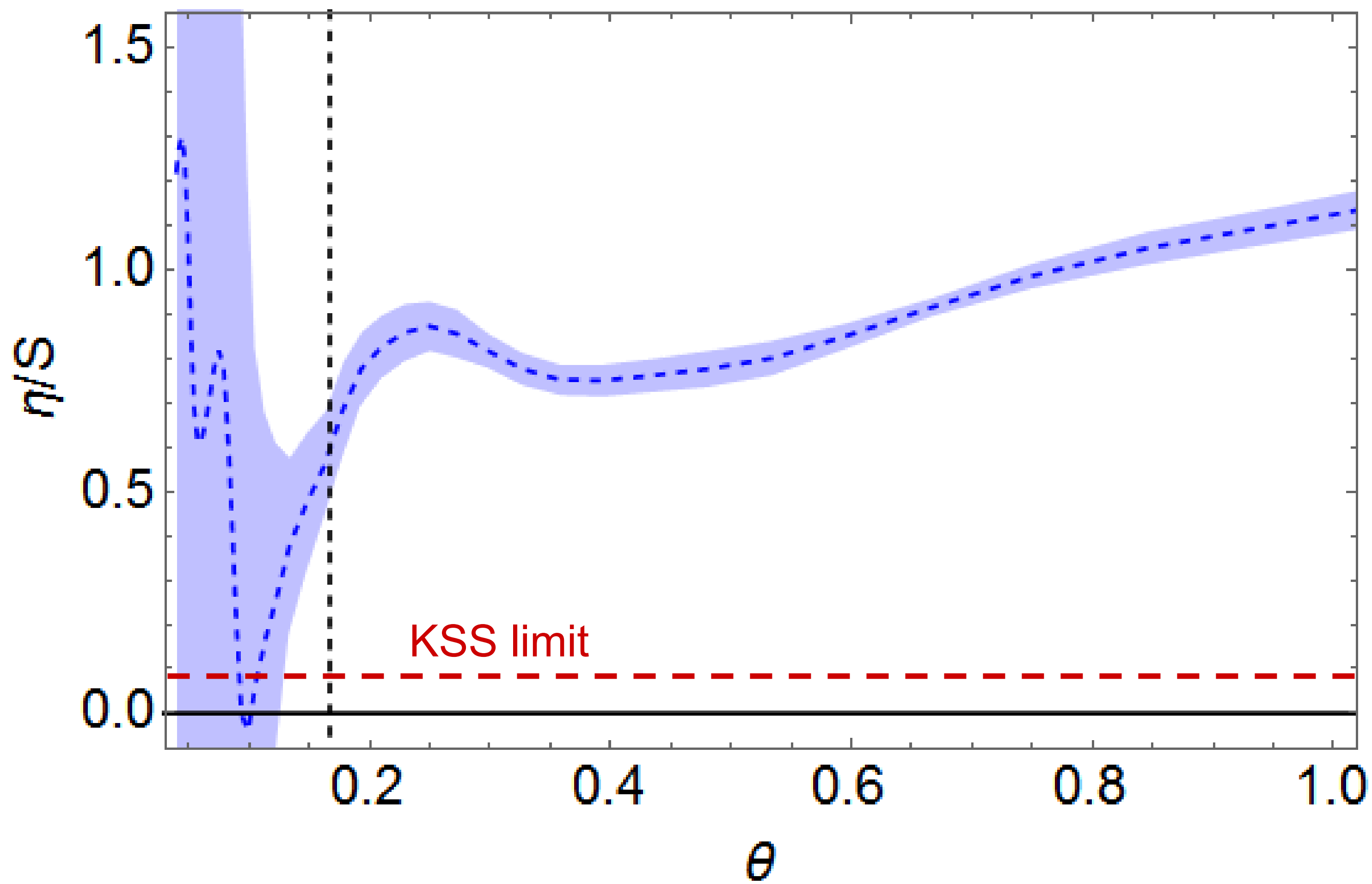
Reduced temperature at the trap center

*EoS from Ku et al., *Science*, 2012

# Local Shear Viscosity (Comparison to Theory)



# Ratio of the Local Shear Viscosity to the Entropy Density*




*EoS from Ku et al., *Science*, 2012

# Summary: Image Processing

Quote on extrapolating QMC data
in a recent viscosity theory paper:

"As a result the integral equation [3] belongs to a class of numerically ill-posed problems. Therefore, the use of special techniques is warranted in order to extract numerically stable results."

JET:

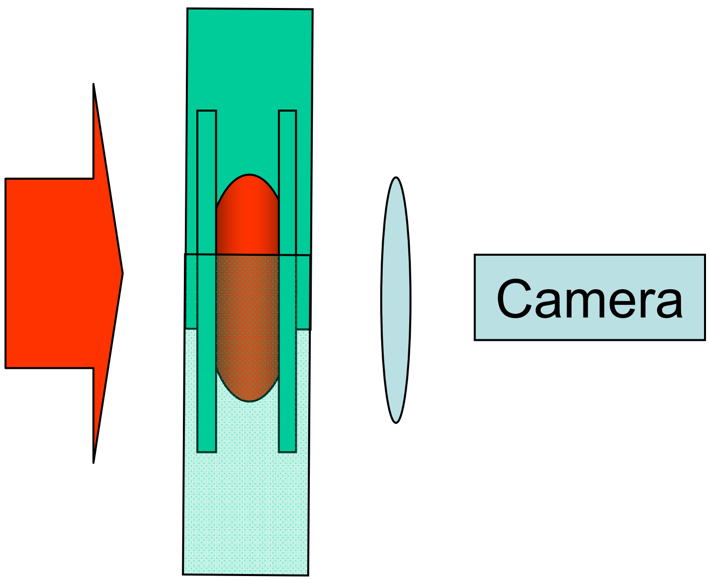
"Now we have numerically ill-posed measurements to compare to numerically ill-posed predictions!"

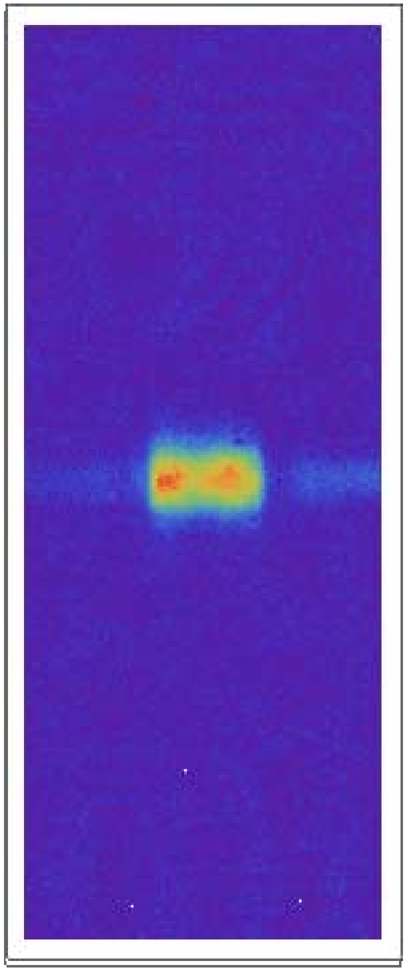
# Summary

- Testing "string" theory
  - Scale invariant hydrodynamics and thermodynamics
- Scale invariance in expanding Fermi gases:
  - "Ballistic" flow of resonant, hydrodynamic gas
  - Bulk viscosity very small compared to shear viscosity
- Perfect fluidity and shear viscosity:
  - Need for direct measurement of local shear viscosity
  - Need for non-relativistic conformal field theory or a trapped "relativistic" gas

# 1D Flow in a Rectangular Pipe

Use a micro-mirror array to create a four sheet repulsive optical potential:



# December 2010 Celebration!

Elliptic Flow Birthday Cake!

# Birthday Party December 2015

# *Bulk Viscosity* $\zeta_B$

*Near* unitarity, the bulk viscosity generally takes the form:

$$\zeta_B = \frac{f_B(\theta)}{(k_F a)^2} \hbar n \equiv \alpha_B \hbar n$$

$\theta$ = reduced temperature,
a = s-wave scattering length,
$k_F$ = local Fermi wave vector

The trap-average gives:

$$\boxed{\overline{\alpha}_B(t) = \overline{\alpha}_B(0)\,\Gamma^{2/3}(t)}$$

High T: Dusling and Schaefer point out that the bulk viscosity must be second order in the fugacity $z \simeq n\lambda_T^3/2$

$$\zeta_B = \frac{1}{24\pi\sqrt{2}} \frac{\lambda_T^2}{a^2} \frac{\hbar}{\lambda_T^3} z^2$$

$$\boxed{\overline{\alpha}_B(0) = \frac{9}{32} \frac{1}{(k_{FI} a)^2} \left(\frac{E_F}{E}\right)^4 \equiv c_B \left(\frac{E_F}{E}\right)^4}$$