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Supersymmetry: Example Sheet 3

Due by lecture on March 7th

1. Consider the Wess-Zumino model consisting of one chiral superfield Φ with standard Kähler potential $K = \Phi^\dagger \Phi$ and tree-level superpotential:

$$W_{\text{tree}} = \frac{1}{2} m \Phi^2 + \frac{1}{3} g \Phi^3$$

Consider the couplings g and m as spurion fields, and define two $U(1)$ symmetries of the tree-level superpotential, where both $U(1)$ factors act on Φ, m and g. One of the $U(1)$ symmetries should be an R-symmetry (i.e. also act on θ).

Using these symmetries, infer the most general form that any loop corrected superpotential can take in terms of an arbitrary function $F(\Phi, m, g)$. Consider the weak coupling limit $g \to 0$ and $m \to 0$ to fix the form of this function and thereby show that the superpotential is not renormalised.

2. Consider a renormalisable $\mathcal{N} = 1$ SUSY Lagrangian for chiral superfields with F-term supersymmetry breaking. By analysing the scalar and fermion mass matrix, show that

$$\mathrm{STr}(M^2) = \sum_j (-1)^{2j+1}(2j+1) m_j^2 = 0$$

where j is the particle spin. Verify that this relation holds for the O'Raifertaigh model, and explain, why this forbids single-sector susy breaking models where the MSSM superfields are the dominant source of supersymmetry breaking.

3. Consider a chiral superfield Φ of charge q coupled to an Abelian vector superfield V. Show that a non-vanishing vev for D, the auxiliary field of V, can break supersymmetry, and determine the Goldstino in this case. Write down the D-term scalar potential, and find the condition that the FI term and the charge q have to satisfy for supersymmetry to be broken. Find the spectrum of the model once supersymmetry is broken, and find the mass splitting of the multiplet.

4. Consider a renormalisable $\mathcal{N} = 1$ supersymmetric theory with chiral superfields Φ_i and vector superfields V_a with both D- and F-term supersymmetry breaking. Show that in the vacuum

$$\frac{\partial V}{\partial \varphi^i} = F^j \frac{\partial^2 W}{\partial \varphi^i \partial \varphi^j} - g^a D^a \varphi_j^\dagger (T^a)_i{}^j = 0$$

where g^a and T^a are the different gauge couplings and generators. Also note that as the superpotential is gauge invariant, the gauge variation of W is

$$\delta^a_{\text{gauge}} = \frac{\partial W}{\partial \varphi^i} \delta^a_{\text{gauge}} \varphi^i = -F_i^\dagger (T^a)_{ij} \varphi_j \ .$$

Write these two conditions in the form of a matrix M acting on a 'two-vector' with components $\langle F^j \rangle$ and $\langle D^a \rangle$. Identify this matrix and show that it is the same as the fermion mass matrix. Argue that it always has one zero eigenvalue, which can be identified as the goldstino.

5. This problem studies the vacuum structure of $\mathcal{N} = 4$ SYM written in $\mathcal{N} = 1$ language.

Consider a $SU(2)$ $\mathcal{N}=1$ supersymmetric theory with three chiral superfields in the adjoint representation $\Phi_1,\ \Phi_2,\ \Phi_3$ with superpotential

$$W = \epsilon_{ijk}\ \mathrm{Tr}\left(\Phi_i\left[\Phi_j, \Phi_k\right]\right).$$

Assuming a canonical Kähler potential, calculate the scalar potential and find the form of solutions that have vanishing F- and D-terms. Find the flat scalar field directions which preserve supersymmetry and the parts of the gauge group that are broken along these directions. The above is the $\mathcal{N}=4$ theory in $\mathcal{N}=1$ language. We can explicitly break the $\mathcal{N}=4$ theory down to either $\mathcal{N}=2$ or $\mathcal{N}=1$ supersymmetry by adding terms to the superpotential:

$$W = \epsilon_{ijk}\ \mathrm{Tr}\left(\phi_i\left[\Phi_j, \Phi_k\right]\right) + m_1\mathrm{Tr}(\Phi_1^2) + m_2\mathrm{Tr}(\Phi_2^2) + m_3\mathrm{Tr}(\Phi_3^2)$$

If all masses are non-zero, the resulting theory is $\mathcal{N}=1$ supersymmetric. Show that the field equations now become

$$[\Phi_i, \Phi_j] = -\epsilon_{ijk} m_k \Phi_k$$

Which matrices Φ_i satisfy this equation?

6. Experimentally the proton lifetime is bounded

$$\tau_{\text{proton}} > 6 \times 10^{32} \text{ years} = 2.4 \times 10^{64}\ \text{GeV}^{-1}\ .$$

Use this to bound the two 'Yukawa' couplings leading to proton decay

$$p \to e^+ + \pi^0$$

from the baryon and lepton number violating operators in the MSSM. Show explicitly that all gauge invariant baryon and lepton number violating operators in the MSSM can be forbidden using R-parity.

7. This problem examines no-scale supergravity. Consider $\mathcal{N}=1$ supergravity with three chiral superfields S, T and C. The Kähler potential is (M_{Planck}):

$$K = -\log\left(S+\bar{S}\right) - 3\log\left(T+\bar{T}-C\bar{C}\right)$$

The superpotential is

$$W = C^3 + ae^{-\alpha S} + b$$

where $a,\ b$ are arbitrary complex numbers and $\alpha > 0$. Compute the F-term scalar potential. What is the vacuum energy in the minimum and are there flat directions in the potential? Find the F-terms for S, T, and C. Verify that SUSY is broken and calculate the gravitino mass. If the gauge kinetic function is $f = T$, what is the gaugino mass for this gauge group?

8. Show that the MSSM Higgs potential is given by

$$\begin{aligned} V_H &= (|\mu|^2 + m_{H_u}^2)|H_u|^2 + (|\mu|^2 + m_{H_d}^2)|H_d|^2 - B\mu(H_u^+ H_d^- - H_d^0 H_u^0 + \text{h.c.}) \\ &\quad + \frac{g^2+g'^2}{8}\left(|H_u|^2 - |H_d|^2\right)^2 + \frac{g^2}{2}|H_u^\dagger H_d|^2 \end{aligned}$$

where g and g' denote the $SU(2)$ gauge coupling and $U(1)_Y$ coupling respectively. The following identity for Pauli-matrices is useful in the derivation:

$$\sum_a \sigma^a_{mn}\sigma^a_{rs} = 2\delta_{ms}\delta_{nr} - \delta_{mn}\delta_{rs}.$$