Optical Properties of Solids: Problem Set
Trinity Term 2016

1. **Reflection and absorption of EM radiation.** Light with angular frequency $\omega = 7.2 \times 10^{12}\,\mathrm{rad\,s^{-1}}$ is incident on a sample of thickness $20\,\mu\mathrm{m}$ with index of refraction $n = 11.7$ and extinction coefficient $\kappa = 8.5$.

   (a) What is the speed of the wave in the sample?

   (b) What is the wavelength of the wave before and after entering the sample?

   (c) What fraction of light entering the sample is present in the beam just before exiting?

   (d) What are the real and imaginary parts of the dielectric constant ϵ' and ϵ'' at this frequency?

2. **Absorption coefficient of AlAs.** Explain the difference between direct and indirect optical transitions in a semiconductor. Why is it difficult to make light-emitting devices out of indirect gap material?

   Figure 1 shows data for the absorption coefficient α of of the III-V semiconductor AlAs at 300 K. What can you deduce from the data about the band structure of AlAs?

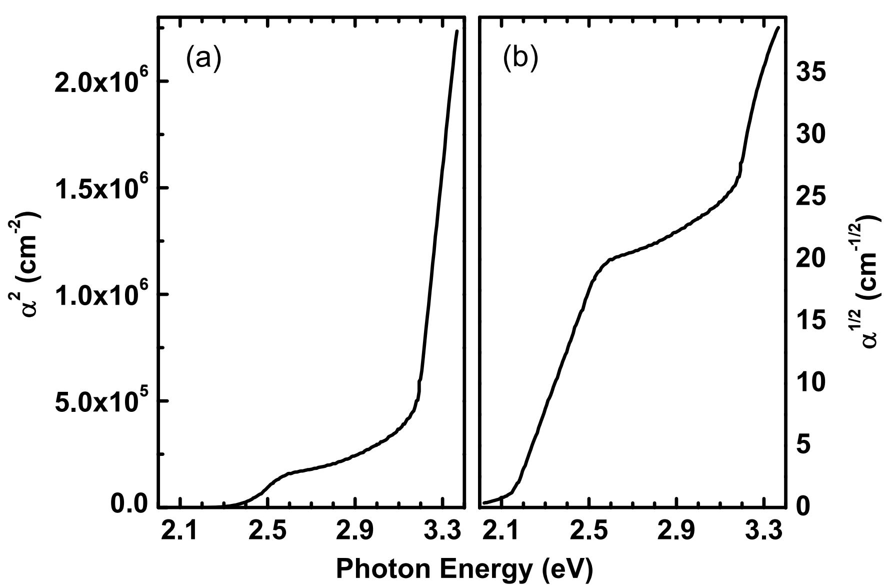

Figure 1: Absorption of AlAs at 300 K. For ease of analysis α is given both in terms of (a) α^2 and (b) $\sqrt{\alpha}$.

3. **Excitons in cuprous oxide.** Estimate the exciton binding energy of Cu_2O from the absorption spectrum shown in Figure 2.

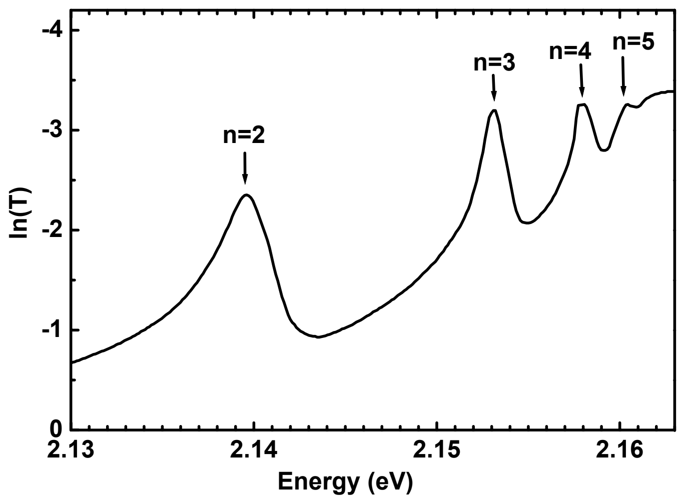

Figure 2: Absorption spectrum of CuO_2 at 77 K. (T is the fraction of light transmitted.)

Given that $\epsilon_\infty \approx 10$ obtain an estimate of the electron-hole reduced mass. The n=1 exciton transition is "forbidden" by symmetry in this material; suggest an alternative optical probe.

4. **Absorption in a GaAs/AlAs heterostructure.** Figure 3 shows the absorption spectrum of a sample containing a number of identical GaAs/AlAs quantum wells, measured at 4 K using the photoluminescence excitation (PLE) technique.

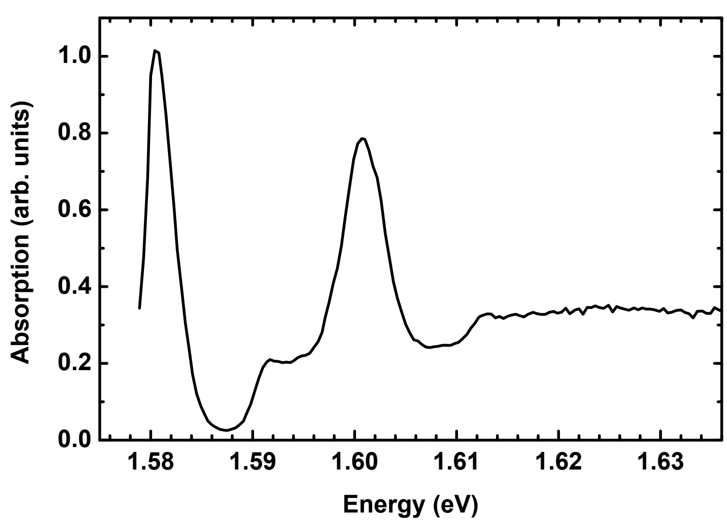

Figure 3: Absorption spectrum (PLE Intensity) of a GaAs/AlAs quantum well at 4K.

(a) Account for the shape of the absorption spectrum.

(b) Estimate the width of the quantum wells by assuming that the wells behave like perfectly confined two-dimensional systems. Would you expect the true well width to be larger or smaller than the answer you have calculated?

(c) Deduce the binding energies of the heavy and light hole excitons and comment on the values you obtain.

(d) What values would you expect for the binding energies in the limit of very wide and very narrow quantum wells?

GaAs data: $E_g(4\,K) = 1.519\,\mathrm{eV}$; effective masses: electrons: $m_e^* = 0.07$; heavy holes: $m_{hh}^* = 0.45$ $m_{lh}^* = 0.085$.

5. **Free electon absorption in InSb.** Figure 4 shows data for the reflectivity of n-type indium antimonide as a function of photon energy.

(a) Explain the general shape of the curves and comment in particular on how and why the sharp drop in relectivity correlates with the free carrier density.

(b) Using the given data, estimate the value of the dielectic constant in the high-frequency limit, ϵ_∞. For each free-carrier density, determine the effective electron mass of InSb. Comment on the result you obtain.

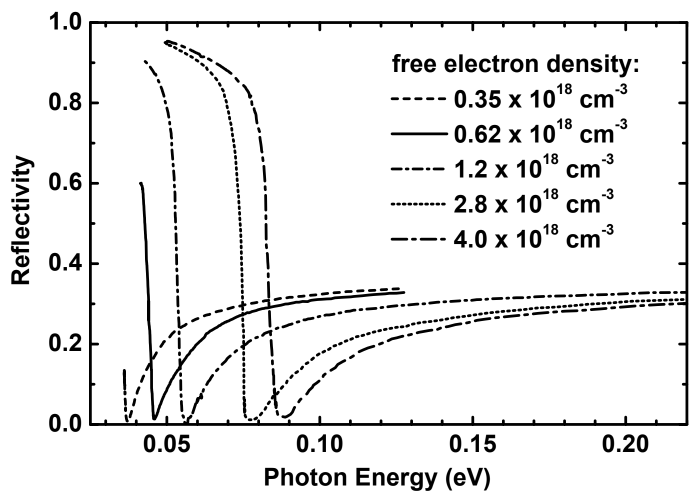

Figure 4: Reflectivity of n-type InSb for a range of free-electron densities.

6. **IR reflectivity of an ionic solid.** A linear chain of ions is often used to model the dielectric properties of an ionic solid. Here, the ions have masses m_1 and m_2 and charges $+q$ and $-q$, respectively, and they are subject to an interionic restoring force $-kx$, where k is a constant and x is the separation of an ion from its neighbour. Using such a model, show that the relative permittivity ϵ_r of the solid in the near infrared is of the form:

$$\epsilon_r(\omega) = \epsilon_\infty + (\epsilon_s - \epsilon_\infty)\,\frac{\bar{\omega}^2}{\bar{\omega}^2 - \omega^2} \tag{1}$$

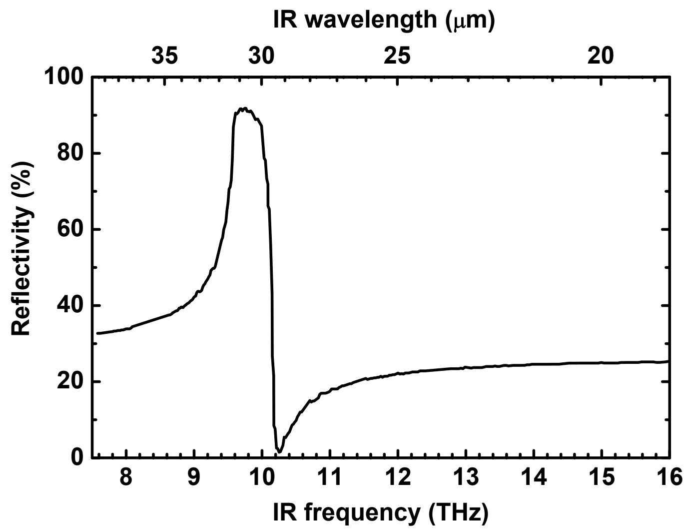

Figure 5: Infrared reflectivity of AlSb.

where ϵ_s and ϵ_∞ are relative permittivities at zero and very high (optical) frequencies, respectively. Derive an expression relating ϵ_s, ϵ_∞, $\bar{\omega}$ and ω_0, where ω_0 is the angular frequency at which $\epsilon_r = 0$. Show that $\bar{\omega}$ and ω_0 are equal to the angular frequencies of the transverse and longitudinal optic phonon modes, respectively, close to the Brillouin zone centre.

Figure 5 shows the reflectivity at normal incidence of a crystal of AlSb in the infrared. Account briefly for the descrepancies between the reflectivity predicted by your model and the experimental data shown in the figure. Using your model estimate the frequencies of the transverse and longitudinal optic phonons close to the Brillouin zone centre, and the permittivities ϵ_s and ϵ_∞. Hence estimate the value of the constant k for AlSb. Describe how you would estimate the lifetime of transverse optical phonons in AlSb using the data in the figure.

[The molar masses of Al and Sb are 27 g and 122 g, respectively.]

7. **Nonlinear Optics.**

(a) A nonlinear substance is subjected to an applied optical-frequency field $\mathbf{E}$, whose cartesian components are E_i (i=1, 2, 3). The polarization of the substance is given by

$$P_i = \epsilon_0 \sum_j \chi_{ij} E_j + \epsilon_0 \sum_{jk} \chi^{(2)}_{ijk} E_j E_k + \epsilon_0 \sum_{jkl} \chi^{(3)}_{jkl} E_j E_k E_l + \cdots \quad (2)$$

Explain qualitatively the origin of the given terms that contribute to $\mathbf{P}$. How does the crystal symmetry affect the nonlinear properties of solids? Explain why $\chi^{(2)}$ is found to be zero in centrosymmetric crystals.

(b) Describe how the phase matching condition for second-harmonic generation may be fulfilled in a uniaxial crystal. Show that for a positive uniaxial crystal (i.e. $n_e > n_o$) with $\chi^{(2)} \neq 0$ second-harmonic generation may occur if the fundamental travels at an angle θ with respect to the optic axis, where

$$\sin^2\theta = \frac{(n_o^{\omega})^{-2} - (n_o^{2\omega})^{-2}}{(n_o^{\omega})^{-2} - (n_e^{\omega})^{-2}} \tag{3}$$

(c) The nonlinear optical coefficient tensor of a crystal from the orthorhombic crystal class is given by

$$\mathbf{d} = \begin{pmatrix} 0 & 0 & 0 & d_{14} & 0 & 0 \\ 0 & 0 & 0 & 0 & d_{25} & 0 \\ 0 & 0 & 0 & 0 & 0 & d_{36} \end{pmatrix} \tag{4}$$

A laser beam is incident on the crystal. The beam travels in the xy plane, and is polarized with its electric field in the xy plane. Show that a second harmonic beam is produced which is polarized along the z axis. Show also that the magnitude of the second harmonic beam is a maximum if the incoming beam travels at 45° to the x axis.

(d) Explain how the $\chi^{(3)}$ terms of the nonlinear polarization give rise to an intensity-dependent refractive index in isotropic media. Describe why this effect may lead to self-focussing of intense laser beams in the medium.