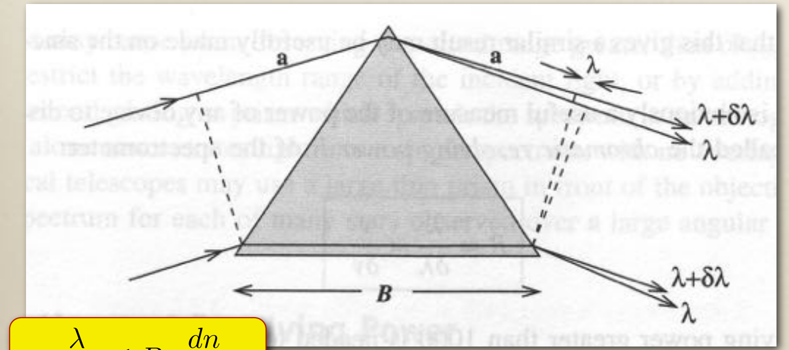
Chromatic Resolving Power

Prism ... diffraction limited by entrance aperture



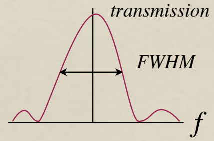
$$\frac{\lambda}{\Delta\lambda} \leq B \cdot \frac{dn}{d\lambda}$$

Resolving Power

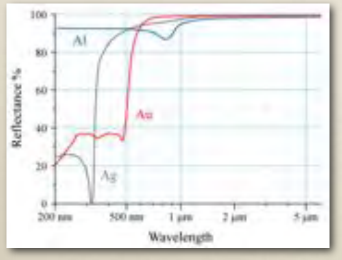
Spectroscopic Resolution *– some alternatives*

- *chromatic resolving power* $\lambda/\Delta\lambda$
- *Finesse* $\mathcal{F} = FSR/FWHM$
- ***instrumental width*** $\Delta\lambda$ or $\Delta\bar{\nu} = \Delta\lambda/\lambda^2$
- *instrumental line profile ...*

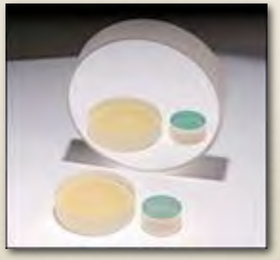
apparent spectral energy distribution for monochromatic light entering the device



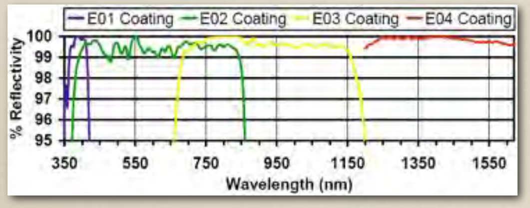
Multi-Layer Coatings



metal reflectance

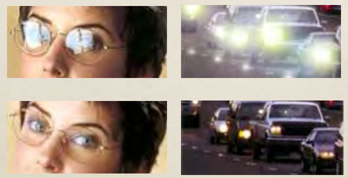


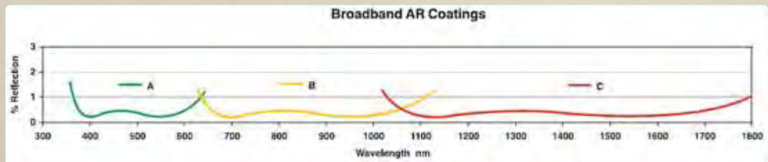
reflectance of commercial multi-layer coatings



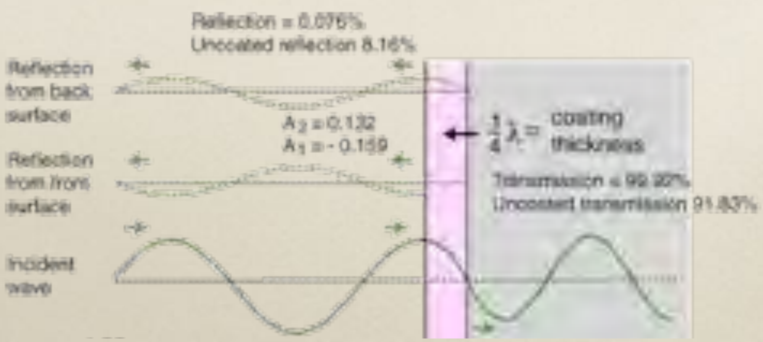
Multi-Layer Coatings

anti-reflection coating





Anti-Reflection Coating

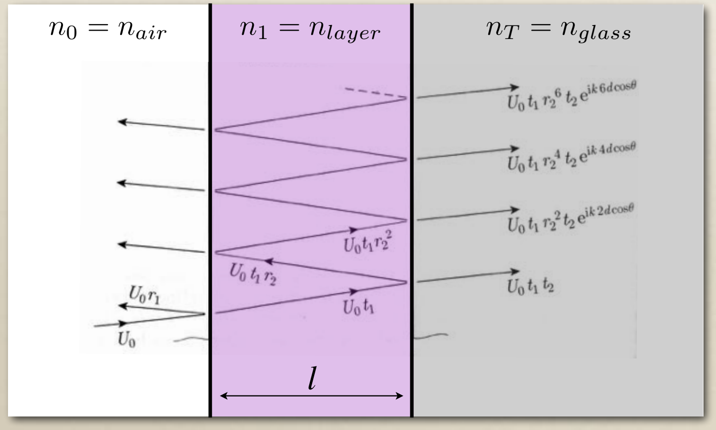


- *reflected waves out of step*
- *equal amplitudes*

} *destructive interference*

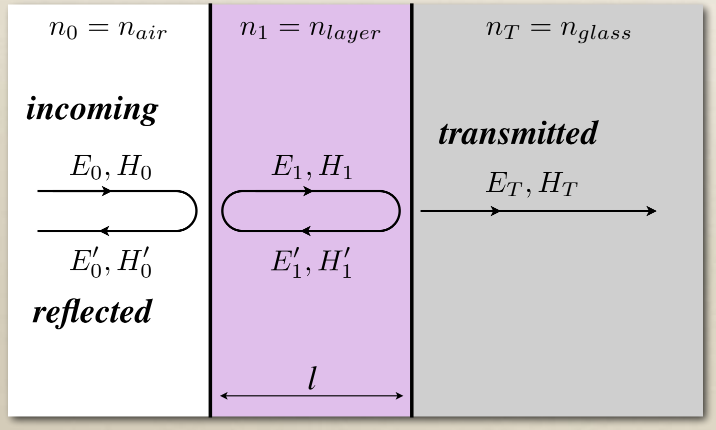
Multi-Layer Coatings

single layer ➙ FPI



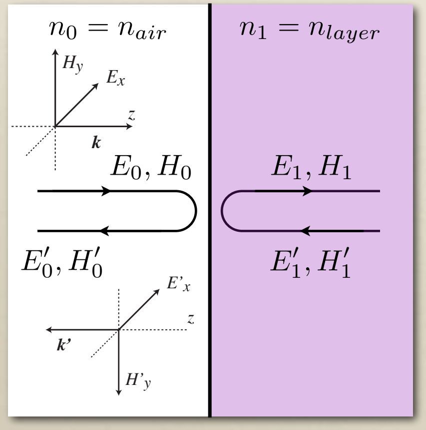
Multi-Layer Coatings

single layer ➙ circulating wave



Multi-Layer Coatings

single boundary



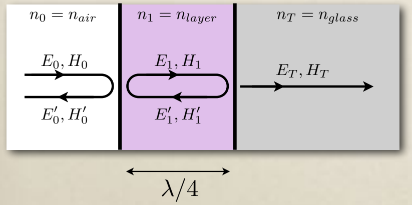
from Maxwell's equations

$$Z = \frac{E_x}{H_y} = \sqrt{\frac{\mu\mu_0}{\epsilon\epsilon_0}} = \frac{Z_0}{n}$$

continuity of E and H

$$E_0 + E'_0 = E_1 + E'_1$$
$$H_0 + H'_0 = H_1 + H'_1$$

Anti-Reflection Coating

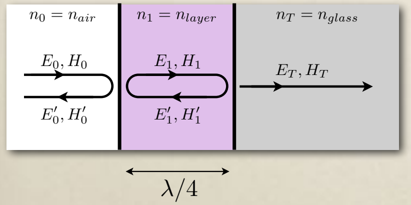


$$\begin{pmatrix} E_0 \\ E_0' \end{pmatrix} = \mathcal{M}_{01} \cdot \mathcal{M}_{\lambda/4} \cdot \mathcal{M}_{1T} \cdot \begin{pmatrix} E_T \\ 0 \end{pmatrix} = -\frac{i}{2n_1} E_T \begin{pmatrix} n_T + n_1^2 \\ n_T - n_1^2 \end{pmatrix}$$

Reflection

$$R = \left|\frac{E_0'}{E_0}\right|^2 = \left|\frac{n_T - n_1^2}{n_T + n_1^2}\right|^2 \longrightarrow 0 \qquad \textit{for} \qquad n_1 \approx \sqrt{n_T}$$

Anti-Reflection Coating



$$R = \left|\frac{E_0'}{E_0}\right|^2 = \left|\frac{n_T - n_1^2}{n_T + n_1^2}\right|^2 \longrightarrow 0$$

for $n_1 \approx \sqrt{n_T}$

$l = \lambda/4$

AR-layer

Compare to Fabry-Perot Interferometer

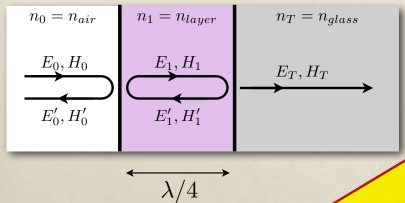
$$I_{out} = I_{max}\left[1 + \left(\frac{2\mathcal{F}}{\pi}\sin\frac{\delta}{2}\right)^2\right]^{-1} \quad \text{with} \quad \delta = 2kl\cos\theta$$

for $\theta = 0 \implies I_{out} = I_{max}$ if $p\lambda = 2l$

$l = \lambda/2$

FPI

Anti-Reflection Coating



$R = \left|\frac{E_0'}{E_0}\right|^2 = \ldots \longrightarrow 0$

fo... $\sqrt{n_T}$

$l = \lambda/4$

AR-layer

Compare to Fab... Interferometer

$I_{out} = I$ ... $\sin\frac{\delta}{2}\Big)^2\Big]^{-1}$ with $\delta = 2kl\cos\theta$

... $= 0 \implies I_{out} = I_{max}$ if $p\lambda = 2l$

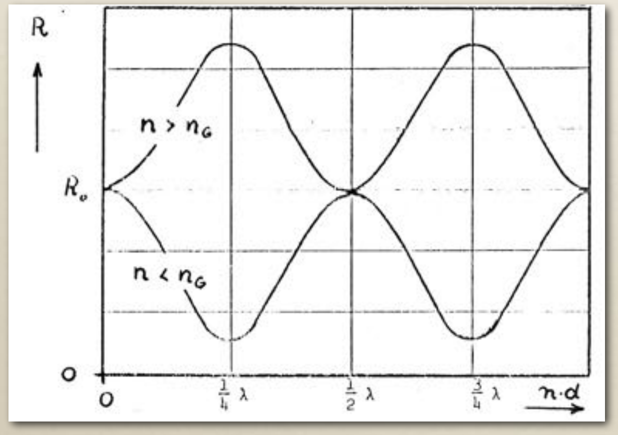
$l = \lambda/2$

FPI

phase jumps by π when reflecting of the optically thicker medium

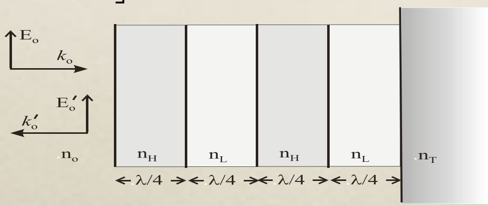
Anti-Reflection Coating

reflection vs. layer thickness



Multi-layer Stack

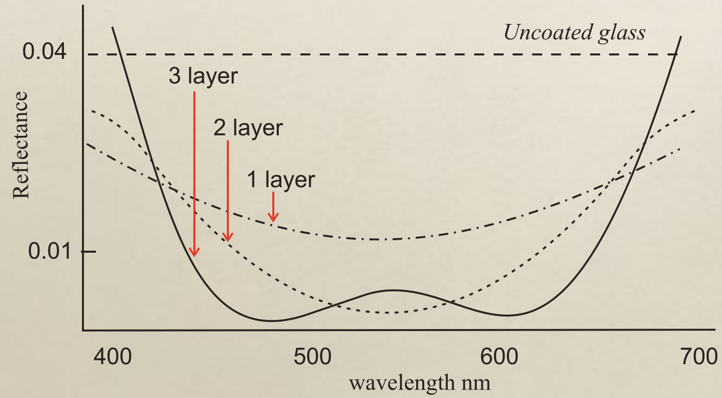
$$\begin{pmatrix} E_0 \\ E_0' \end{pmatrix} = \left[\prod_{l=1}^{k} \mathcal{M}_{l-1,l} \cdot \mathcal{M}_{\lambda/4}\right] \cdot \mathcal{M}_{nT} \cdot \begin{pmatrix} E_T \\ 0 \end{pmatrix}$$



$$R = \left| \frac{(n_1 n_3 n_5 ... n_{k-1})^2 n_T - n_0 (n_2 n_4 ... n_k)^2}{(n_1 n_3 n_5 ... n_{k-1})^2 n_T + n_0 (n_2 n_4 ... n_k)^2} \right|^2 \qquad k \text{ even}$$

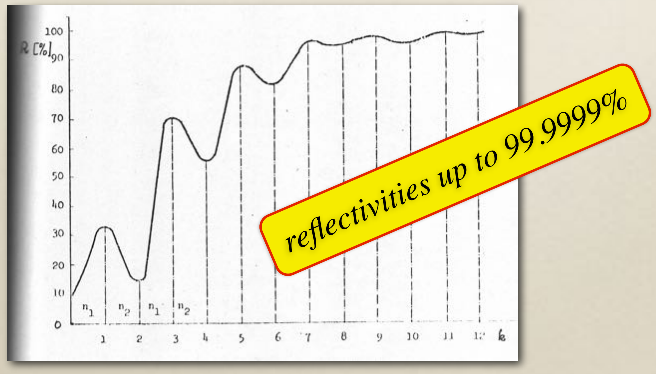
$$R = \left| \frac{(n_1 n_3 n_5 ... n_k)^2 - n_0 n_T (n_2 n_4 ... n_{k-1})^2}{(n_1 n_3 n_5 ... n_k)^2 + n_0 n_T (n_2 n_4 ... n_{k-1})^2} \right|^2 \qquad k \text{ odd}$$

Anti-Reflection Coating



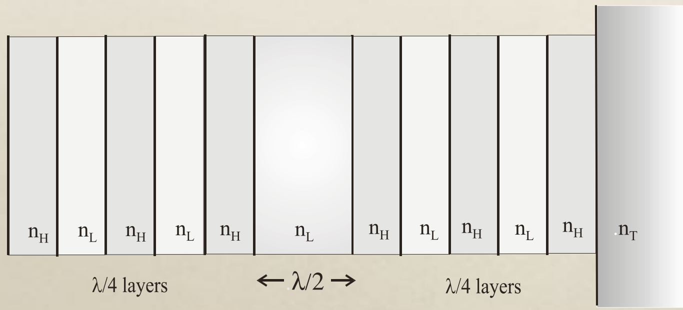
Dielectric Mirror

reflection vs. number of layers



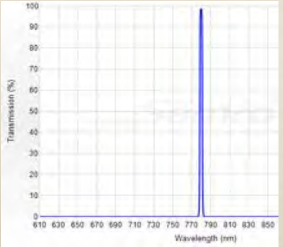
$n_1 = 2.3$ and $n_2 = 1.35$

Multi-Layer Coatings

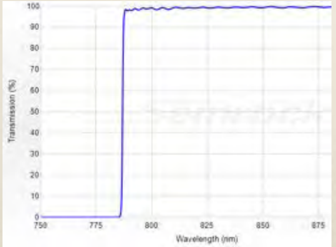


Interference filter; composed of multi-layer stacks

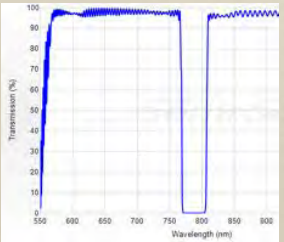
Interference Filter



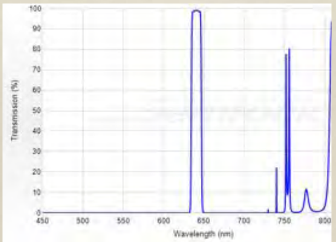
single line



edge



notch



band pass

Impedance ... a sleeker way

Maxwell's equations

$$\vec{\nabla} \cdot \vec{B} = 0 \qquad \vec{\nabla} \cdot \vec{D} = \rho$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial}{\partial t}\vec{B} \qquad \vec{\nabla} \times \vec{H} = \vec{j} + \frac{\partial}{\partial t}\vec{D}$$

flux densities $\quad \vec{D} = \epsilon_r \epsilon_0 \vec{E} \qquad \vec{B} = \mu_r \mu_0 \vec{H}$

speed $\quad c = 1/\sqrt{\mu_0 \epsilon_0} \qquad n = \sqrt{\mu_r \epsilon_r} \approx \sqrt{\epsilon_r}$

wave equations for E and H

$$\nabla^2 \vec{E} = \frac{1}{v^2}\frac{\partial^2 \vec{E}}{\partial t^2}; \qquad \nabla^2 \vec{H} = \frac{1}{v^2}\frac{\partial^2 \vec{H}}{\partial t^2}; \qquad v = \frac{c}{n} = \frac{1}{\sqrt{\mu_r \mu_0 \epsilon_r \epsilon_0}}$$

Impedance ... a sleeker way

$$\nabla^2 \vec{E} = \frac{1}{v^2}\frac{\partial^2 \vec{E}}{\partial t^2}; \qquad \nabla^2 \vec{H} = \frac{1}{v^2}\frac{\partial^2 \vec{H}}{\partial t^2}; \qquad v = \frac{c}{n} = \frac{1}{\sqrt{\mu_r \mu_0 \epsilon_r \epsilon_0}}$$

plane wave (same for H)

$$\vec{E}(\vec{r}, 1) = \vec{E}_0 \; e^{i(\vec{k}\cdot\vec{r} - \omega t)}; \qquad |\vec{k}| = k = \frac{2\pi}{\lambda}; \qquad v = \frac{\omega}{k} = \frac{c}{n}$$

from Maxwell's equations: $\quad \omega \vec{B} = \vec{k} \times \vec{E}$

$$E_x(z,t) = E_0 \; e^{i(kz - \omega t)} \quad \Longrightarrow$$

$$\frac{E_x}{H_y} = Z \times \frac{k}{|k|} \quad \text{with} \quad Z = \sqrt{\frac{\mu_r \mu_0}{\epsilon_r \epsilon_0}} \approx \frac{Z_0}{n}$$

Impedance ... a sleeker way

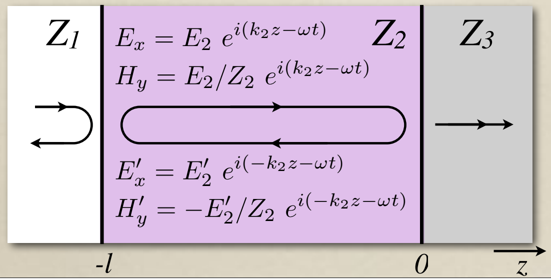
$$\frac{E_x}{H_y} = Z \times \frac{k}{|k|} \quad \text{with} \quad Z = \sqrt{\frac{\mu_r \mu_0}{\epsilon_r \epsilon_0}} \approx \frac{Z_0}{n}$$

impedance Z; vacuum impedance Z_0 = 377 Ω

continuity

$$E_2 + E_2' = E_3 + E_3'$$
$$H_2 + H_2' = H_3 + H_3'$$

$$\frac{E_2}{E_2'} = \frac{Z_3 - Z_2}{Z_3 + Z_2}$$



Impedance ... a sleeker way

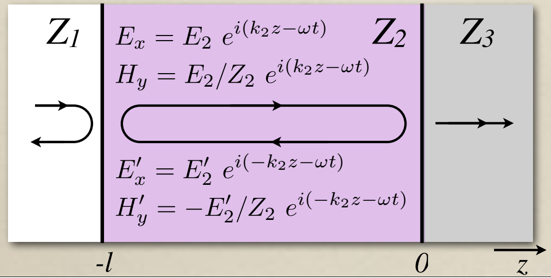
load impedance at z=0 $$Z_L = \left(\frac{E_x + E'_x}{H_y + H'_y}\right)_0 = \frac{E_2 + E'_2}{E_2 - E'_2} Z_2$$

input impedance at z=-l (for a λ/4 layer) $$Z_{in} = \left(\frac{E_x + E'_x}{H_y + H'_y}\right)_{-l} \longrightarrow \frac{Z_2^2}{Z_L}$$

at boundaries

$$Z_{L,m} = Z_{in,m+1}$$

$$\frac{E_2}{E'_2} = \frac{Z_3 - Z_2}{Z_3 + Z_2}$$



Impedance ... a sleeker way

load impedance at z=0 $$Z_L = \left(\frac{E_x + E'_x}{H_y + H'_y}\right)_0 = \frac{E_2 + E'_2}{E_2 - E'_2} Z_2$$

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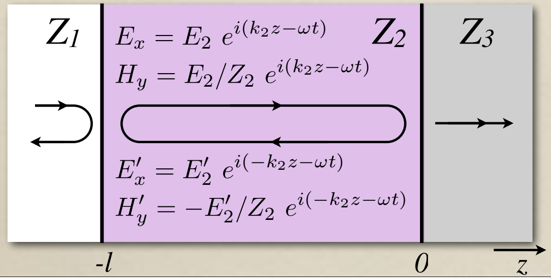
at boundaries

$$Z_{L,m} = Z_{in,m+1}$$

$$\frac{E_2}{E'_2} = \frac{Z_3 - Z_2}{Z_3 + Z_2}$$



Impedance ... a sleeker way

load impedance at z=0 $$Z_L = \left(\frac{E_x + E'_x}{H_y + H'_y}\right)_0 = \frac{E_2 + E'_2}{E_2 - E'_2} Z_2$$

input impedance at z=-l (for a λ/4 layer) $$Z_{in} = \left(\frac{E_x + E'_x}{H_y + H'_y}\right)_{-l} \longrightarrow \frac{Z_2^2}{Z_L}$$

input impedance at z=-l (for a λ/2 layer) $$Z_{in} = Z_L$$

at boundaries

$$Z_{L,m} = Z_{in,m+1}$$

Stack of λ/4 Layers

load impedance at z=0 $$Z_L = \left(\frac{E_x + E'_x}{H_y + H'_y}\right)_0 = \frac{E_2 + E'_2}{E_2 - E'_2} Z_2$$

input impedance at z=-l (for a λ/4 layer) $$Z_{in} = \left(\frac{E_x + E'_x}{H_y + H'_y}\right)_{-l} \longrightarrow \frac{Z_2^2}{Z_L}$$

from the far side ....

$$Z_{in,k} = Z_k^2/Z_{L,k} \longleftarrow Z_{L,k} = Z_T$$

$$\vdots$$

$$Z_{in,n} = Z_n^2/Z_{L,n} \longleftarrow Z_{L,n} = Z_{in,n+1}$$

$$\vdots$$

$$\frac{E_0 + E'_0}{E_0 - E'_0} Z_0 = Z_{in} \longleftarrow Z_L = Z_{in} = Z_{in,1}$$

Stack of λ/4 Layers

from the far side ....

$$Z_{in,k} = Z_k^2/Z_{L,k} \longleftarrow Z_{L,k} = Z_T$$

$$\vdots$$

$$Z_{in,n} = Z_n^2/Z_{L,n} \longleftarrow Z_{L,n} = Z_{in,n+1}$$

$$\vdots$$

$$\frac{E_0 + E_0'}{E_0 - E_0'} Z_0 = Z_{in} \longleftarrow Z_L = Z_{in} = Z_{in,1}$$

Reflectivity

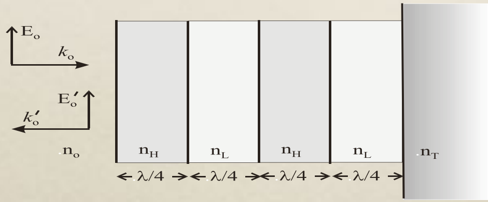
$$R = \left|\frac{E_0'}{E_0}\right|^2 = \left|\frac{Z_{in} - Z_0}{Z_{in} + Z_0}\right|^2$$

AR-Coating ⇔ impedance matching $Z_0 = Z_{in}$

Mirror ⇔ impedance mismatch: $Z_{in} = 0$ or $Z_{in} >> Z_0$

Multi-layer Stack

$$\begin{pmatrix} E_0 \\ E_0' \end{pmatrix} = \left[\prod_{l=1}^{k} \mathcal{M}_{l-1,l} \cdot \mathcal{M}_{\lambda/4}\right] \cdot \mathcal{M}_{nT} \cdot \begin{pmatrix} E_T \\ 0 \end{pmatrix}$$

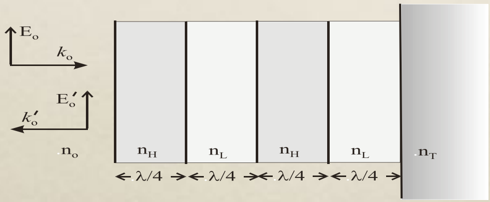


$$R = \left|\frac{(n_1 n_3 n_5 ... n_{k-1})^2 n_T - n_0 (n_2 n_4 ... n_k)^2}{(n_1 n_3 n_5 ... n_{k-1})^2 n_T + n_0 (n_2 n_4 ... n_k)^2}\right|^2 \qquad k \text{ even}$$

$$R = \left|\frac{(n_1 n_3 n_5 ... n_k)^2 - n_0 n_T (n_2 n_4 ... n_{k-1})^2}{(n_1 n_3 n_5 ... n_k)^2 + n_0 n_T (n_2 n_4 ... n_{k-1})^2}\right|^2 \qquad k \text{ odd}$$

Multi-layer Stack

$$R = \left|\frac{E_0'}{E_0}\right|^2 = \left|\frac{Z_{in} - Z_0}{Z_{in} + Z_0}\right|^2$$

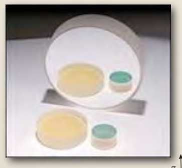


$Z_{in} = (Z_2/Z_3)^{2p} Z_T$ ***k=2p (even)***

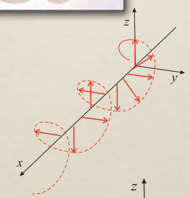
$Z_{in} = (Z_2/Z_3)^{2p} Z_2^2/Z_T$ ***k=2p+1 (odd)***

Summary & Outlook

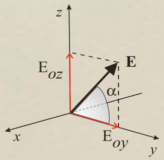
- ☑ ***Multi-Layer Coatings***
  - ☑ *AR Coatings / FPI Comparison*
  - ☑ *Impedance & multiple layers*



- ☐ ***Polarisation***
  - ☐ *linear, circular, elliptical*



- ☐ ***Polarisation optics***
  - ☐ *uni-axial crystals*
  - ☐ *polarising prisms*
  - ☐ *wave plates (λ/2 and λ/4)*



- ☐ ***Interference with polarised light***